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Mathematical Knowledge for Teaching in Elementary Pre-Service Teacher Training

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This is to certify that the doctoral study by

Jason S. Proctor

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Walden University
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Abstract

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MAT, Marygrove College, 2005

BS, Ferris State University, 2001

Proposal Submitted in Partial Fulfillment

of the Requirements for the Degree of

Doctor of Education

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Abstract

It was unclear how the teacher education curriculum at a regional university in the south central region of the United States developed mathematical knowledge for teaching (MKT) in prospective elementary teachers. Understanding how MKT develops during teacher training is important because MKT has been linked to student achievement. The purpose of this sequential explanatory mixed methods study was to examine how prospective elementary teachers' MKT developed while enrolled in a math and science strategies course. Guided by Ball et al.'s MKT framework and Silverman and Thompson's development of this framework, this study investigated changes in prospective teachers' MKT levels and teacher candidates' perceptions of instructional tasks that assisted in the development of MKT during the course. During the quantitative phase, teacher candidates ($N = 30$) completed the Number Concepts and Operations assessment as a pre- and posttest. Paired t test results showed no significant changes in candidates' MKT levels. During the qualitative phase, volunteers were interviewed about their perceptions of how the course influenced their development of MKT. Thematic analyses revealed that teacher candidates recognized instruction that developed MKT, perceived the strategies course to have little to no influence on MKT, and felt unprepared to teach math. Findings were used to develop a revised curriculum plan for developing prospective teachers' MKT. The findings may lead to positive social change in the form of curriculum revisions aimed at developing teacher candidates' MKT to improve future instruction. The project may be shared with other colleges to improve curriculum with the goal of improving the quality of math instruction statewide.

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Dedication

I dedicate this doctoral study to my family. Throughout this journey you have showered me with support and provided me with countless memories. To my wife, thanks for choosing me as your partner and believing in me. To my children, thank you for the constant hugs and regular play breaks. Completing this doctoral study has been a major accomplishment, but it pales in comparison to the pride I take in being your Daddy. I love you all, whole big bunches!

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Lastly, I would like to acknowledge the support of my doctoral committee, education peers, and family members. Dr. Sarah Hough, Dr. Barbara Calabro, Dr. Dannett Babb, and Dr. Joel Goodin, I thank you for your timeliness in providing quality feedback. Tonya Garrett, thanks for being my sounding board and helping me clarify my ideas about math education. To my wife and children, thanks for your understanding when I stole time away to write and for reminding me that it is good to regularly step away from the project.

Table of Contents

List of Tables

List of Figures

Section 1: The Problem

The Local Problem

It was unclear how the teacher education curriculum at a regional university in the west south central region of the United States developed mathematical knowledge for teaching (MKT) in prospective elementary teachers. Teacher educators throughout the United States and Canada have been investigating the influence of teacher training programs on teacher candidates' MKT (Cardetti & Truxaw, 2014; Goodson-Epsy et al., 2014; Kajander & Holm, 2016; Tyminski, Zambak, Drake, & Land, 2014). MKT details a teacher's understanding of mathematics, content knowledge (CK), and the teaching and learning of mathematics or pedagogical content knowledge (PCK; Ball, Thames, & Phelps, 2008). Specific to teaching practices, Ball et al. (2008) identified domains of MKT, such as specialized content knowledge and knowledge of content and students, that describe a teacher's ability to select appropriate tasks, anticipate errors, and design instruction to advance learning. Recent curriculum changes have reduced teacher candidates' exposure to math pedagogy coursework while math content scores have been declining at the study site (T. Garrett, College of Education Director of Assessment, personal communication, December 1, 2017). Teacher candidates express a desire for additional training in math strategies that is supported by local superintendents suggesting new teachers need more training in instructional design (S. Farmer, personal communication, February 23, 2018). A failure to develop MKT during teacher preparation may negatively influence teacher candidates' future instructional practices and students' achievement.

The depth of understanding of content and instructional practices of a teacher impacts the potential for student learning. CK alone is insufficient to support the teaching of mathematics and a lack of PCK negatively affects a teacher's instructional practice (Baki & Arslan, 2016; Bartell, Webel, Bowen, & Dyson, 2013; Maher & Muir, 2013). Teacher's MKT is linked to student achievement (Baki & Arslan, 2016; Hill, Umland, Litke, & Kapitula, 2012; Leong, Meng, & Abdul Rahim, 2015). Developing the CK and PCK of prospective teachers is essential to promote the successful teaching and learning of mathematics. Therefore, it is critical that teacher education programs ensure that curriculum requirements intentionally address MKT development.

Teacher preparation programs have used a mixture of content and methods coursework. Presenting content and instructional practices in a blended format affect the development of MKT more than addressing the concepts individually (Auslander, Smith, Smith, Hart, & Carothers, 2016; Hoover, Mosvold, Ball, Lai, 2016; Son & Lee, 2016). Recommendations include the use of special content courses designed for teachers (Holm, Kajander, & Avosh, 2016) and additional methods coursework versus additional content coursework (Smith, Swars, Smith, Hart, & Haardorfer, 2012). Math methods coursework has shown to improve the CK and PCK of teacher candidates and the ability to lead discussions (Cardetti & Truxaw, 2014; Goodson-Epsy et al., 2014; Tyminski et al., 2014). However, the current curriculum requirements for elementary education majors at the study site do not reflect research-based recommendations to blend content and instructional practices, focusing instead on content knowledge. Therefore, investigating how teacher candidates' MKT changes while enrolled in a math and science strategies course may inform the future revision of the teacher education curriculum.

Rationale

Recent curriculum changes at the study site may have a negative impact on the mathematical preparation of elementary education majors. Before the curriculum revision, elementary teacher candidates were required to take three math methods courses for teachers. The current requirements include 12 credit hours of college mathematics with no direct relationship to the content prospective teachers will be expected to teach. The only remaining course that addresses math pedagogy and the learning of elementary age students is a math and science strategies course taught within the college of education. Since the curriculum change, elementary teacher candidates' scores have declined regarding subject matter certification tests in math (T. Garrett, personal communication, December 1, 2017). Also graduates from the local site express a desire for additional training in the teaching of mathematics. Local P-12 superintendents expressed a need for new teachers to have a better understanding of instructional design and strategies (S. Farmer & D. Glover, personal communication, February 23, 2018). Without the opportunity to explore mathematical content in the context of teaching, teacher candidates at the study site may be missing out on opportunities to develop a deep conceptual understanding of content and instructional skills. Therefore, it was a goal of this study to further the understanding of how to develop MKT in prospective teachers.

The purpose of this sequential explanatory mixed methods study was to examine how prospective elementary teachers' MKT develops while enrolled in a math and science strategies course at a regional university. Quantitative data were collected using the Number and Operations Content Knowledge assessment and the Perception of Course survey, both developed by the Learning Mathematics for Teaching project (2008b) before and after taking the required math and science strategies course for pre-service teachers (PSTs) offered by the teacher education department. The assessment measures a teacher's knowledge to teach number and operations content rather than assessing only their knowledge of the content. The MKT measure results highlighted changes in the prospective teachers' MKT, which was used to inform the selection of participants for the qualitative phase of the study. In the qualitative component of the study, I used items from the MKT assessment to develop questions to gain PSTs' perspectives on how the coursework influenced their development in the subdomains of the MKT framework that focus on knowledge of students and teaching. These subdomains specifically address the pedagogical actions and knowledge of teachers necessary to successfully teach mathematics.

Definition of Terms

Conceptual understanding: Conceptual understanding is "the comprehension and connection of concepts, operations, and relations" (NCTM, 2014, p. 7).

Content knowledge: CK of mathematics taught in schools includes a combination of understanding how to do mathematics, recognizing errors in student thinking, and knowing how topics develop and relate across a curriculum (Ball et al., 2008; Shulman, 1987).

Pedagogical content knowledge: PCK extends the ability to do the math to the ability to teach math. PCK includes an understanding of how to represent content and adapt instruction based on the needs of the learner (Ball et al., 2008; Shulman, 1987).

Mathematical knowledge for teaching: Mathematical knowledge for teaching is a framework that contextualizes the tasks of teaching mathematics such as understanding content, recognizing errors, anticipating misconceptions, selecting examples, and identifying instructional strategies to advance student learning (Ball et al., 2008).

MKT measures: The Learning Mathematics for Teaching Project supervises the development of instruments to test the effectiveness of mathematics-focused training. The assessment items are referred to as MKT measures. The assessment items represent tasks common to teaching mathematics including evaluation of student work, using multiple representations, and anticipating student errors (Hill, Schilling, & Ball, 2004).

Significance of the Study

This study addressed a local problem by examining how prospective elementary pre-service teachers' MKT developed while enrolled in a math and science strategies course. The results of the study may provide faculty members at the study site with various forms of data about MKT development. First, the quantitative data might provide faculty members with information about MKT changes when PSTs are exposed to the math and science strategies course content. Likewise, qualitative data may provide the study site with information on what learning experiences prospective teachers found most beneficial and identify areas where PSTs feel instruction or content was lacking in the ability to meet their perceived needs. The qualitative findings may also provide an understanding of how specific subdomains of MKT change while PSTs are enrolled in the strategies course. The combined results may provide the study site with insight into whether or not course objectives are being met and inform any future course revisions. The methods and results may be shared with colleges of education and mathematics departments at other universities to guide similar inquiries into the development of elementary teachers. The methods and results may also inform the evaluation of professional development sessions for in-service teachers.

Research Questions and Hypotheses

The knowledge necessary to successfully teach mathematics is unique, but it is unclear how to develop that knowledge in prospective teachers. Math content courses focused on elementary school-related content presented within a teaching context support CK development and PCK development in later methods coursework (Cardetti & Truxaw, 2014; Kajander & Holm, 2016). However, there is no clear direction on how PSTs develop MKT or best practices to develop MKT to guide teacher educators. Therefore, the following research questions were selected to understand how MKT develops in elementary teacher candidates while enrolled in a strategies course and to examine candidates' perceptions of the role of course content and instruction in the development in each subdomain of MKT.

RQ1: What is the difference between pre and posttest scores of elementary PSTs' MKT after taking a math strategies course?

$H_0 1$: There is no significant difference in terms of MKT between pre- and posttest scores of PSTs after taking a math strategies course.

H_a1: There is a significant difference in terms of MKT between pre- and posttest scores of PSTs after taking a math strategies course.

RQ2a: To what extent did elementary PSTs perceive their math strategies course changed their MKT?

RQ2b: In what ways do elementary PSTs perceive the content and instruction in their math strategies course influenced their development of particular MKT components?

Review of the Literature

This section provides a synthesis of the research literature on the development of MKT and how it relates to the development of prospective teachers. It includes an overview of the components of MKT and what types of interventions are necessary to develop strong MKT levels in elementary PSTs. The impact of mathematics content and methods coursework on the MKT of PSTs is also discussed, as well as the need for further research to understand how to develop future elementary teachers best to teach mathematics.

Peer-reviewed journals provided the resources to reach the desired level of saturation necessary for this literature review. Several databases available through the Walden University Library provided access to current literature, including Education Source, ERIC, SAGE Journals, and Thoreau Multi-Database Search. Search terms included: *mathematical knowledge for teaching*, *pre-service teachers*, *math content coursework*, *math methods coursework*, *content knowledge*, *pedagogical knowledge*, and *elementary*.

Conceptual Framework

The conceptual framework for this study focused on the development of MKT in teachers. MKT is commonly used to describe the different types and development of knowledge for in-service and PSTs of mathematics (Jenlink, 2016; Kang, 2016; Wilson, Sztajn, Edgington, & Confrey, 2014). Shulman (1987) first proposed different categories of teacher knowledge that included knowledge of content, pedagogy, curriculum, learners, and educational contexts. Ball et al. (2008) refined Shulman's model and proposed the MKT framework as a construct to conceptualize mathematical knowledge specific to the discipline of teaching. Figure 1 shows the domains of MKT and their relationship to Shulman's categories of subject matter knowledge and pedagogical content knowledge.

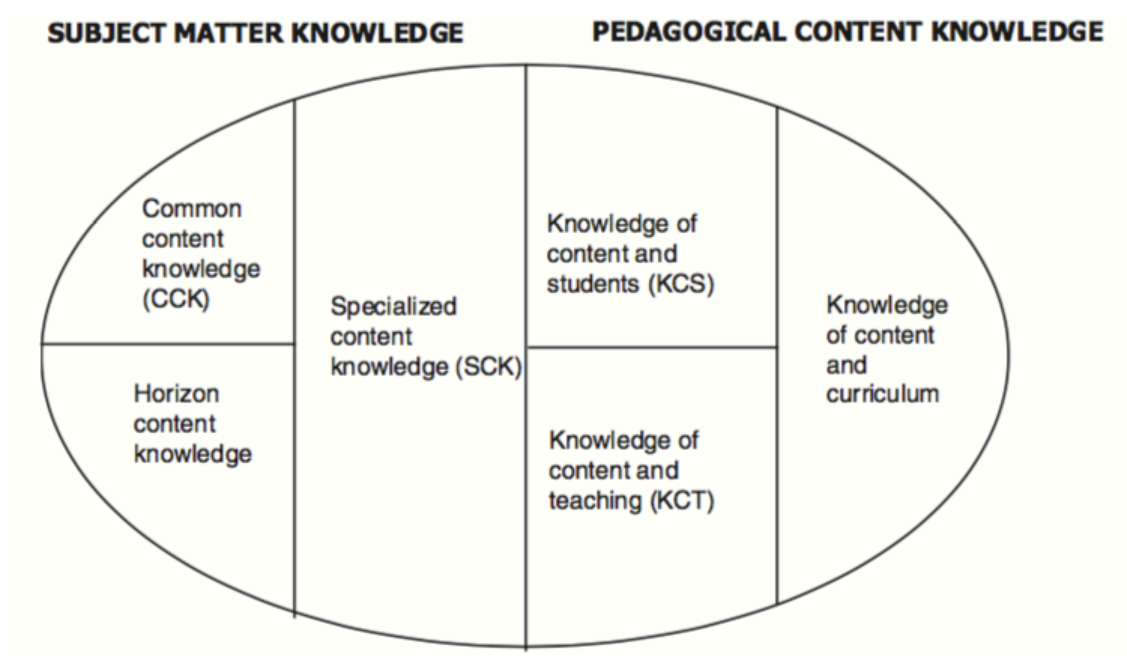

Figure 1. Domains of mathematical knowledge for teaching (Ball et al., 2008).

The subject matter domains represent the complexity of CK necessary to teach mathematics. Ball et al. (2008) defined common content knowledge (CCK) as “the mathematical knowledge and skill used in settings other than teaching” (p. 399). CCK is a measure of an individual’s ability to obtain or recognize correct answers to math problems. Ball et al. identified knowledge that extended beyond obtaining correct solutions as specialized content knowledge (SCK). SCK is defined as “the mathematical knowledge and skill unique to teaching” (p. 400). SCK highlights the work teachers do when identifying student errors or evaluating the merit of a student’s approach to a problem. Lastly, Ball et al. recognized horizon content knowledge as “an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (p. 403). Horizon knowledge is useful in helping teachers understand the mathematical foundation they are setting with their students and what pedagogical approaches may assist in allowing a student to build upon their knowledge in future learning experiences.

The pedagogical domains represent a teacher’s ability to blend their knowledge of mathematics and instruction to advance students’ understanding of mathematics. Ball et al. (2008) defined knowledge of content and students (KCS) as “knowledge that combines knowing about students and knowing about mathematics” (p. 401). KCS is represented in a teacher’s ability to identify mathematical tasks that students will find interesting along with anticipating common errors students are most likely to make. Ball et al. described the knowledge of content and teaching (KCT) as the combination of “knowing about teaching and knowing about mathematics” (p. 401). KCT is the knowledge teachers use to design instruction with a focus on the impact of student learning. Lastly, Ball et al. included Shulman's (1987) category of curricula knowledge but expressed that the domain was currently unclear and may comprise a portion of the KCT domain. Investigating MKT changes across the subdomains may be useful in understanding how MKT develops for prospective teachers enrolled in the math and science strategies course.

Ball et al.'s (2008) theory applies to this study by addressing the first research question that describes changes to prospective teachers' MKT over the duration of the strategies course. The first research question paid specific attention to changes in teacher candidates' CK as measured by the MKT measures designed by Hill et al. (2004). Understanding changes in CK informed the qualitative phase of the study through the development of interview questions and identification of potential participants. While investigating CK is helpful to understand changes in MKT, another perspective is necessary to understand how content and pedagogical knowledge develop in elementary PSTs. Therefore, this study used a second framework to understand how MKT develops.

Silverman and Thompson (2008) proposed a framework that describes the transformative process teachers must go through in relation to mathematical content to develop MKT. Silverman and Thompson asserted that MKT develops when teachers connect content and pedagogical knowledge to create a new understanding of how to support student learning. To assist the development of MKT, teacher educators must be intentional in designing learning experiences that engage PSTs in the process of exploring content while considering how their future students may approach similar tasks. The key to developing MKT is helping PSTs consider their future students' point of view when encountering new content.

Developing MKT is a process that blends a teacher's understanding of content, teaching, and students. According to Silverman and Thompson (2008, p. 508), MKT is developed when a teacher:

1. Has developed a KDU (key developmental understanding) within which that topic exists.
2. Has constructed models of the variety of ways students may understand the content (decentering).
3. Has an image of how someone else might come to think of the mathematical idea in a similar way.
4. Has an image of the kinds of activities and conversations about those activities that might support another person's development of a similar understanding of the mathematical idea.
5. Has an image of how students who have come to think about the mathematical idea in the specified way are empowered to learn other and related mathematical ideas.

When teachers consider potential student thinking about content and design instruction to help advance student thinking based on their current level of understanding, then they have developed a new knowledge set for teaching mathematics. Silverman and Thompson's transformative model is useful in assessing how MKT develops in prospective teachers.

Silverman and Thompson (2008) acknowledged the skills needed to teach mathematics as identified by Ball et al. (2008) while also considering the process teachers must go through to develop such understanding. Silverman and Thompson's theory applies to the study by addressing RQ2 explaining elementary PSTs' perceptions of how the instruction and content presented in the strategies course influenced their development of MKT components. The steps of the transformative model were used to develop interview questions. For example, the second and third stages of Silverman and Thompson's model relate to Ball et al.'s subdomains of SCK and KCS. Based on sample MKT items released by the Learning Mathematics for Teaching project open-ended questions were developed to probe teacher candidates' KCS and KCT. Candidate interviews helped to explain PSTs' perceptions of how the course provided opportunities to consider how students may develop different mathematical approaches. Therefore, the combination of both theories related well to the use of a mixed methods approach blending a quantitative assessment of changes in CK with PSTs' perceptions of specific learning experiences that supported their development across all MKT domains.

Review of the Broader Problem

Successful teaching necessitates more than mastery of the subject area. CK and PCK are distinct yet essential components of a teacher's MKT (Depaepe et al., 2015; Kleickmann et al., 2015). Depaepe et al. (2015) asserted that CK is independent of PCK, but PCK is dependent on CK. The argument that PCK is dependent on CK is reflected in the MKT model with a joint partnership between CK and PCK. CK without PCK fails to support quality teaching (Bartell et al., 2013; Boerst, Sleep, Ball, & Bass, 2011; Baki &

Arslan, 2016). Likewise, extensive evidence exists of the positive influence teachers' MKT has on student achievement (Baki & Arslan, 2016; Leong et al., 2015; Shirvani, 2015). Educational leaders are aware of the need to develop teachers' CK and PCK, which is reflected in current calls to action in the mathematics education field.

The current positions of the National Council of Teachers of Mathematics (NCTM, 2014) and Common Core State Standards (CCSS; NGO & CCSSO, 2010) require students to develop conceptual understanding through investigations and discussions. However, teacher training has failed to develop skills for the teaching and learning of mathematics in this manner (Kaya & Aydin, 2016; Kosko, 2016; Teuscher, Moore, & Carlson, 2015). The Conference Board of the Mathematical Sciences (2012) maintained that math coursework for prospective elementary teachers should focus on the math they will teach from a teacher's perspective. Additional recommendations included math content courses that engage elementary PSTs in demanding math tasks, collaboration, and discourse focused on reasoning and reflection (McGalliard & Wilson, 2017). The most recent standards reform movement along with the research on MKT has informed a push to modify math instruction for elementary PSTs.

Complexity of teacher knowledge. Quantifying the knowledge a teacher must possess to successfully teach math is difficult. Fauskanger (2015) stated it was difficult to measure teachers' MKT by showing that teachers' responses on a multiple choice MKT assessment often conflicted with their constructed responses. Fauskanger's results confirmed the findings of Hill et al. (2012) that highlight the difficulty of differentiating between different types of math knowledge using a single assessment. Carillo-Yanez et al. (2018) criticized the MKT model for the lack of clearly defined actions within each domain and even proposed a revised model with restructured domains. However, one common theme remains that measuring teacher knowledge is difficult and there appears to be no single distinguishing factor of MKT that ensures student achievement. Hill, Charalambos, and Chin (2018) investigated multiple teacher characteristics such as preparation, experience, knowledge, and disposition and their influence on student learning, and concluded that even though some of the characteristics they investigated showed a positive relationship to student outcomes no single component stood out as a definitive characteristic to be effective at teaching mathematics.

MKT related to teaching practice and student achievement. A teacher's overall MKT level may influence student achievement. Students of teachers with higher MKT levels and stronger content and pedagogical knowledge outperform students in classrooms with teachers with lower MKT (Baki & Arslan, 2016; Behlol, Akbar, & Sehrish, 2018; Leong et al., 2015; Ojose, 2014; Shirvani, 2015; Strand & Mills, 2014). Baki and Arslan (2016) revealed that a lack of PCK negatively impacts the classroom practices of a teacher. Ojose (2014) and Tajudin (2014) showed that if a teacher has a deficit of knowledge in either content or pedagogy, then they are more likely to rely on teaching mathematics through procedures and fail to develop conceptual understanding within their students. Behlol et al. (2018) provided an example of how a teacher's lack of pedagogical knowledge in the area of using problem-solving approaches to teach mathematics may hinder student achievement.

Behlol et al. (2018) showed that teachers who used problem-solving approaches versus traditional lecture and procedure heavy instruction had greater student success. Using a problem-solving approach connects to the KCT domain in the MKT model as an example of using different instructional strategies. Without specific training, teachers may not develop the ability to engage students in such rigorous activities. Teachers who lack high MKT, specifically in the KCT domain, may struggle to plan effective lessons (Linder & Simpson, 2017) and may not have the ability to adjust curriculum or instruction to best meet the needs of their students (Lui & Bonner, 2016). Hill and Chin (2018) argued that a teacher's understanding of student thinking is positively related to student mathematical achievement and teachers who have a better understanding of student thinking are more likely to adjust instruction in a manner to support student achievement. The ability to understand student thinking and make appropriate instructional changes relates directly to the subdomains of KCS and KCT.

MKT, facilitating discussions, and eliciting student thinking. Teachers must have the ability to develop conceptual understanding of mathematical topics through the use of small and whole group discussions. Ball et al.'s (2008) KCT domain considered with Silverman and Thompson's (2008) fourth stage of MKT development connects a teacher's ability to select activities and facilitate discussions that advance student thinking. Also, the CCSS for mathematics and the NCTM encourage teachers to use discussion to develop students' conceptual understanding of mathematical ideas. There are benefits in terms of peer discussion and the use of probing questions by a teacher on developing the conceptual understanding of mathematical ideas in students (Brodie, 2011; Kaya & Aydin, 2016; Kosko & Gao, 2014; Kosko, 2016; Sahin, 2015). However, the use of questioning methods by teachers often falls short of the types of questions suggested by research that best push student thinking or elicit ideas regarding how students approach solving math problems. Instead, teachers often rely on questions that lead student thinking or require lower level thinking such as requesting specific information (Brodie, 2011; Kaya & Aydin, 2016; Yang, 2013). Job restrictions, misunderstanding reform initiatives, and lack of teacher training have been linked to reasons why teachers fail to effectively use questions to lead discussions (Kosko, 2016; Kosko & Gao, 2014). If teachers have low MKT levels, their students may not benefit from opportunities to develop conceptual understanding of mathematical ideas through discussion or exploration of their peers' ideas.

MKT and student engagement. Observing teaching practices are a standard method used by researchers to examine the impact of teachers on student learning. Regular use of active learning techniques by teachers increases student engagement and achievement (Ing et al., 2015; Webb et al., 2014). To identify, design, and sequence learning tasks that provide students opportunities to engage with mathematical ideas actively, teachers must have unique knowledge, specifically KCS and KCT (Ball et al., 2008). Chapman (2013) pointed out that teachers approach the selection of tasks differently, have differing views on the value of active learning tasks, and may benefit from additional training. Likewise, Olson (2013) maintains that teachers' perceptions of the teaching process influence how they engage students in learning. After additional training in how to use student-centered tasks, teachers' beliefs change toward active learning, however lasting impact on teacher practices is unclear (Gningue, Peach, & Schroder, 2013; Polly, Neale, Pugalee, 2016). Franke et al. (2015) identified challenges students struggle with when engaging with the mathematical ideas of their peers and the complexity of a teachers' decision-making process necessary to support student engagement. It is clear that student engagement is critical for learning and that a teacher's depth of MKT may limit their ability to engage students actively.

Call for a blended approach to MKT development. There is a common recommendation to address content and pedagogy during the mathematical preparation of teachers (Aslan-Tutak & Adams, 2015; Depaepe et al., 2015; McGalliard & Wilson, 2017; Zambak & Tyminski, 2017). The recommendation to address content and pedagogy is supported by evidence that blending the exploration of content and instructional practices improves MKT (Auslander et al., 2016; Hoover et al., 2016; Qian & Youngs, 2016; Son & Lee, 2016). Rhine (2016) also pointed out that KCS is the domain that most distinguishes master teachers from novice teachers. Similarly, Edelman (2017) stressed that prospective teachers need more instruction to develop knowledge of students, teaching, and curriculum. It is clear that CK is necessary (Lachner & Nuckles, 2016) to support teaching, but Fernandez (2014) pointed out that teacher preparation programs must have a clear plan on how to build PCK. The most common method teacher preparation programs address mathematics CK and PCK is through a combination of content, special content, and methods coursework. The math and science strategies course in this study would be considered a methods course.

The impact of coursework on MKT development. The impact of traditional university coursework on the development of MKT is unclear. University mathematics does not affect MKT development (Qian & Youngs, 2016). In contrast, traditional content courses do have a positive impact on the CCK of elementary PSTs (Copur-Gencturk & Lubienski, 2013), but fail to impact PCK (Kajander & Holm, 2016). A potential reason for the minimal impact traditional content courses have on the development of MKT may be the relevancy of the content. Elementary PSTs often find traditional mathematics coursework as disconnected from their future classrooms (Koponen, Asikainen, Viholainen, & Hirvonen, 2016). However, math content courses designed specifically for teachers provide an avenue to explore mathematical content and address multiple MKT domains.

Special content courses often engage PSTs in the exploration of mathematical concepts they are expected to teach from the teacher and student perspectives. Special content courses have been shown to raise CK (Holm & Kajander, 2012; Matthews, Rech, & Grandgenett, 2010), SCK (Copur-Gencturk & Lubienski, 2013), and KCS (Warshauer, Strickland, Namakshi, Hickman, & Bhattacharyya, 2015). Special content courses prepare PSTs for furthering their development of PCK in later methods coursework, especially when compared to traditional math courses (Cardetti & Truxaw, 2014; Kajander & Holm, 2016). Thus, Holm et al. (2016) strongly advocate for the need of special content courses to support SCK development to best maximize learning in methods coursework.

Math methods courses are essential to the development of MKT for prospective teachers. Smith et al. (2012) provided evidence that the MKT levels of elementary PSTs were not influenced by additional content coursework but by additional methods coursework. Smith et al.'s findings are supported when reviewing international trends where the development of MKT is unrelated to the number of content courses taken but to what content is covered; where elementary PSTs concentrate on studying school mathematics and pedagogy (Koponen et al., 2016; Qian & Youngs, 2016; Schmidt, Burroughs, Cogan, & Houang, 2017). Therefore, methods courses seem to have a significant impact on the development of MKT in PSTs.

Methods courses impact the development of MKT across domains. Similar to special content courses, math methods courses improve CK and PCK (Auslander et al., 2016; Cardetti & Truxaw, 2014; Goodson-Epsy et al., 2014; Qian & Youngs, 2016). Auslander et al. (2016) stressed that methods coursework focused on connecting teaching, learning, and student thinking resulted in stronger SCK when compared to traditional content courses. Tyminksi et al. (2014) reported how methods coursework improved PSTs ability to lead quality discussions, which could be classified as KCT. Jansen, Berk, and Meikle (2017) reported on six first-year teachers that graduated from the same teacher preparation program. Jansen et al. found that the each of the first-year teachers demonstrated stronger abilities to teach for conceptual understanding when presenting topics covered in the math methods coursework of the elementary teacher program. Jansen et al. suggest that the methods coursework help support stronger CK and PCK for the teachers once in the field. Thus, methods coursework appears to provide

PSTs with the opportunity to deepen their conceptual understanding of mathematics while exploring how to teach content in a manner conducive to student learning. The research results from methods coursework, though scarce, aligns with Silverman and Thompson's (2008) transformative model where teachers must consider content from a learner's perspective to transform their understanding into new mathematical knowledge for teaching. This review of the literature has identified how specific coursework may impact the development of MKT in pre-service teachers. However, more research is necessary to identify the best practices to develop MKT within those courses.

Need for additional research on MKT development. Recently, a working group conducted a thorough review of research from 1978 to 2012 with a focus on prospective elementary teachers' mathematical CK (Browning et al., 2014). Common themes across many of the subgroups were a lack of research studies describing how MKT develops, how to improve MKT, and best practices in preparing future educators (Olanoff, Lo, & Tobias, 2014; Strand & Mills, 2014; Thanheiser et al. 2014). Others have provided a similar conclusion with related literature reviews, emphasizing the need for a clearer picture of how to develop MKT in future teachers (Hart, Auslander, Jacobs, Chestnutt, & Carothers, 2016; Hoover et al., 2016). There does seem to be activity towards addressing the gaps in the literature. Thanheiser (2015) provided evidence of PST learning and accompanying tasks for the math education community to review and refine. Also, a new working group has taken on the task of defining the role of elementary mathematics educators with a focus on the development of MKT in prospective elementary teachers (Welder, Appova, Olanoff, Taylor, & Kulow, 2016).

Implications

Findings from this study may lead to a positive social change in the form of curriculum revisions aimed at developing prospective elementary teachers' MKT and improving the mathematics instruction for their future students. Based on the findings, I developed a 16-week curriculum plan addressing weaknesses found in the current mathematics portion of the course to further prospective teacher's MKT. The resulting project may be shared with colleges of education and mathematics departments at other universities to improve curriculum related the development of elementary teachers with the goal of improving the quality of math instruction statewide. The project may also be modified to provide professional development training for in-service teachers. The professional development training may be used to provide in-service teachers with skills to improve instructional practices with the goal of increased student achievement.

Summary

The role of teacher education programs is to prepare teachers for the demands of teaching. The teaching of mathematics entails a unique blend of knowledge. Coursework for prospective teachers has the potential to improve MKT, especially when content and instructional strategies are presented in a blended format. However, more research is necessary to understand how MKT develops in prospective teachers. Therefore, the purpose of this study was to investigate a math and science strategies curriculum on prospective teachers development. Section 2 describes the research design and approach of the mixed methods study implemented to investigate how a math and science strategies course develops MKT in prospective teachers at the study site.

Section 2: The Methodology

Mixed Methods Design and Approach

The purpose of this sequential explanatory mixed methods study was to examine how prospective elementary teachers' MKT developed while enrolled in a math and science strategies course at a regional university. A mixed methods approach allowed this phenomenon to be investigated using both quantitative and qualitative methods. The intent of selecting a mixed methods design is to purposefully blend quantitative and qualitative methods to offer the best opportunity to understand the research questions (Burkholder, Cox, & Crawford, 2016; Johnson & Onwuegbuzie, 2004). The research questions for this study were descriptive and explanatory. RQ1 addressed changes in MKT after exposure to instruction and content in a math and science strategies course. RQ2a addressed teacher candidates' perceptions of the extent to which instruction and content presented in the strategies course changed their MKT. RQ2b explored prospective teachers' perceptions regarding how the instruction and content in the strategies course influenced their development of MKT. Studies that require quantitative and qualitative analysis should use a mixed methods approach (Collins and O'Cathain, 2009; Johnson & Onwuegbuzie, 2004). A mixed methods approach was suitable for this study since the research questions guiding the study required quantitative and qualitative analysis to offer an understanding of the development of prospective elementary teachers' MKT while enrolled in a math and science strategies course. To maximize the use of a mixed methods approach, I decided to adopt a sequential explanatory follow-up design for data collection and analysis.

Creswell and Clark (2006) defined various forms of mixed method approaches. This study assumed a mixed method sequential explanatory follow-up design with the collection of quantitative followed by qualitative data. The qualitative data served as the emphasis for the study, with the quantitative data acting in a supplemental role. Burkholder et al. (2016) described the differences between quantitative designs. This study used a quasi-experimental quantitative approach to describe changes in prospective teachers' MKT after exposure to a math and science strategies course. Teacher candidates' standardized MKT scores served as the dependent variable measured with pre- and posttest assessments given at the beginning and end of a math strategies course. Creswell (2014) suggested that one phase of a mixed methods study could be used to plan another stage. The intent of the study was to analyze the quantitative data before the qualitative phase for planning purposes. Since the quantitative data emphasized changes in CK specific to the teaching of elementary mathematics, additional measures were necessary to understand how the course influenced the development of the pedagogical components of MKT. Creswell and Clark (2011) stated that quantitative data could be used to inform interview participant selection. I intended to use the MKT data to inform the selection of participants for the qualitative phase based on positive and negative outcomes. However, due to a lack of volunteers, I interviewed all volunteers and modified the research design from a participant selection model to a follow-up model. According to Creswell and Clark (2006), sequential collection and analysis of quantitative data can be used to inform the qualitative stage. Therefore, the sequential collection and analysis of quantitative data was used to inform the design of interview questions during the qualitative phase of this study.

The sequential explanatory design was chosen with the potential for the qualitative data to provide a detailed explanation of changes in MKT and the descriptive findings of candidates' perceptions of the course during the quantitative phase. Ravitch and Carl (2016) maintained that a qualitative case study must explore a real life event. The study used a qualitative case study approach to interview volunteers using an open-ended format regarding perceptions of how the instruction and content presented during the course influenced their development of MKT. The interviews provided insight into instructional components of the strategies course that participants perceived were beneficial to supporting the development of MKT and any components that participants felt were missing as denoted by the MKT measures. Integration of the data sets occurred after analysis of the qualitative data was completed. According to Creswell (2014), the integration and interpretation of data types can be used to explain findings in more detail. The interpretation of the results in this study will focus on how the qualitative findings explain and extend the quantitative data.

Setting and Sample

The site for this study was a regional university in the south central region of the United States. According to a Title II report posted on the college's website, during the 2015-16 academic year, there were 623 teacher candidates enrolled in the college of education. Also, during the 2015-16 academic year, the college of education graduated 206 teachers across 11 majors, with 91 of those graduates earning certification to teach in elementary schools. The sample for the study was elementary education majors enrolled in the required math and science strategies course during the fall 2018 semester. The college of education offered two sections of the math and science strategies course during the 2018 fall semester, thus providing a sample of 36 adult elementary education teacher candidates. The sample was split across two sections of the course, each taught at a separate location within the university system and with different instructors. Site A had a sample size of eight teacher candidates and was taught by a veteran instructor who also served as the course coordinator. Site B had a sample size of 28 teacher candidates and was taught by a first-year instructor.

The sampling method for this mixed methods study used purposive techniques. Burkholder et al. (2016) defined a purposeful sample as the selection of participants based on their ability to answer a specific question. The purpose of the study was to investigate the development of MKT while exposed to a math and science strategies curriculum, so prospective teachers enrolled in the course served as a purposeful sample to quantitatively assess the development of MKT. Since inferential statistics were used to analyze changes in MKT scores, a power analysis was run to estimate sample size. Estimation of effect size was determined by examining effect sizes reported in studies using the same MKT assessment tool to compare changes in PSTs' MKT levels in similar math methods-type courses. The reported effect sizes in studies that used the same MKT assessment as this study ranged from small-medium to medium-large (Goodson-Epsy et al., 2014; Laursen, Hassi, & Hough, 2016; Matthews et al., 2010). Therefore, a medium effect size estimation was used to conduct a power analysis for a paired t-test given the potential sample size. An online power calculator (QFAB, 2018) revealed that a sample size ranging from 30 to 60 participants for a two-tailed paired t-test using an alpha of 0.05 and a medium effect size ($d = 0.5$) would yield power values between 0.75 and 0.96. Therefore, the participating sample of teacher candidates enrolled in the math strategies course during the fall 2018 semester served as an adequate sample.

Purposeful sampling was the planned sampling technique to be used throughout this study. Purposeful sampling is used to select participants by their ability to provide specific knowledge to answer research questions and address the purpose of a study (Patton, 2015). To explain the quantitative findings, purposive sampling was used to follow up on the results of the quantitative phase and interview individuals who volunteered to be interviewed. Teacher candidates who completed the pre- and posttest along with the course requirements were eligible to participate in the interview phase of the study.

Rubin and Rubin (2012) described saturation as exploring all perspectives to the point that additional inquiry fails to provide new information. To ensure that multiple perspectives were included, the original research design planned for 10 teacher candidates to be selected for interviews. Teacher candidates' raw scores on the Number Concepts and Operations MKT measure were to be used to divide the group into three subgroups. The groups would have been based off pretest scores and identified as high, medium, and low. Then, candidates were to be purposefully selected from each group to understand how MKT developed for teacher candidates at different levels of mathematical ability throughout the course. However, due to the fact that only three teacher candidates volunteered to participate in the study, I instead interviewed all volunteers in an attempt to ensure saturation was met during the qualitative phase.

Efforts were made from the beginning of the study to protect privacy, build relationships, minimize harm, and respect the experiences of all participants. One way to build relationships with participants and share how their privacy will be protected is through the use of an informed consent form (Dooly, Moore, & Vallejo, 2017; Ravitch & Carl, 2016). All teacher candidates enrolled in the strategies course were provided with a consent form at the beginning of the semester with detailed information about the study to make an informed decision about participating in the study. The consent form described the plan to protect the participants' privacy and how the data would be used to avoid harm. Participants were informed that no identifying information would be recorded or reported. I used codes in place of names to protect privacy and no information was shared with supervising instructors that identified participants by name or code. To build relationships with participants, I conducted interviews during times convenient to the teacher candidates' schedule. Member checks are commonly used to ensure researchers do not misinterpret data (Dooly et al., 2017; Kornbluh, 2015). Member checking was used to support the development of an objective and friendly relationship and avoid overinterpreting or misinterpreting data. The member checking process demonstrates respect for the experiences of each participant and ensures that the data was not misinterpreted in any way.

Sequential Data Collection Strategy

Quantitative Sequence

Instrument. A multiple choice assessment tool was used to collect data on changes in prospective teachers' MKT. The MKT measure Number Concepts and Operations Content Knowledge (The Learning Mathematics for Teaching [LMT], 2008b) assessment was developed to measure teachers' knowledge to solve mathematical problems, evaluate unique solution methods, and identify acceptable mathematical explanations. The items included in the 2008-piloted tests assess CCK and SCK with a limited number of items about KCS. The test authors consider the Number and Operations measure to be a CK test but maintain that the test provides a good overall look at a teacher's MKT (LMT, n. d.). The assessment tool also includes a post assessment survey. The survey is designed to collect participants' perceptions of the relationship between the MKT measures and the content and delivery of the course in which the participant was enrolled. The suite of instruments available from the LMT project is available after completion of mandatory training. Requests for training can be found on the LMT website. Completion of three online training modules is required before researchers earn the right to use MKT item banks and previously piloted assessments. Access was granted for the use of the LMT instruments for this study as I completed the necessary training modules (Appendix B).

Instrument relation to study. The items available by the LMT project provide insight into a teacher's MKT. The items were originally designed to assess in-service teachers' knowledge related to the evaluation of student work, using multiple representations, and anticipation of student errors (Hill et al., 2004). Though the items were designed for in-service teachers, many studies have used the MKT measures with pre-service teachers to assess changes in MKT levels similar to the use of the assessments with in-service teachers during professional development (Flake, 2014; Goodson-Epsy et al., 2014; Laursen et al., 2016; Matthews et al., 2010; Smith et al., 2012). The Number Concepts and Operations MKT measure was appropriate for use in this study as it provided details of changes in prospective elementary teachers MKT levels over time.

Instrument reliability and validity. This study used two forms of the Number Concepts and Operations Content Knowledge piloted in 2008. The reliability of the forms was determined using item response theory with indicated alpha values above .81 for each form using one and two-parameter models (Hill, 2007). Schilling and Hill (2007) used a validity argument approach to assess three assumptions of the MKT measures:

1. The items reflect teachers' mathematical knowledge for teaching and not extraneous factors such as test-taking strategies or idiosyncratic aspects of the items.
2. The domain of mathematical knowledge for teaching can be distinguished by both subject matter area and the types of knowledge deployed by teachers.
3. The measures capture the content knowledge that teachers need to teach mathematics effectively to students (p. 79).

Hill, Dean, and Goffney (2007) conducted cognitive interviews with teachers and found that the MKT measures supported the first assumption for content knowledge items but not items developed in the KCS domain. Modest evidence that MKT scores represent more than just CK was found through factor analysis (Hill et al., 2007; Schilling, 2007) and item response theory showed that teachers' scores could be distinguished from one another (Schilling, 2007). Hill, Ball, Blunk, Goffney, and Rowan (2007) correlated teachers' scores on MKT measures with videotaped classroom instruction and student achievement. Based on the validity findings, a few KCS items have been integrated into the content knowledge tests but the subdomain is not available in a stand-alone test.

Administration of MKT measures. Prospective teachers enrolled in the math strategies course took the Number Concepts and Operations Content Knowledge as a pre and posttest during the 2018 fall semester as part of the assigned curriculum. The test was administered in an online format and unidentified raw data was stored in an online database run by the researcher. The paired 2008 forms A and B were used during the semester with half of the participants taking form A as a pretest and form B as the posttest and vice versa. Each form takes approximately 20 to 25 minutes to complete. However, participants were given as much time as necessary to complete each test. Form A has twenty-eight items and form B has twenty-nine items. The LMT project does not allow publication of non-released MKT items, but released sample items have been provided for reference (LMT, 2008a). See Figure 2 for sample items.

Sample Content Knowledge item.

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	Yes	No	I'm not sure
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3
c) The number 8 can be written as 008.	1	2	3

Sample Knowledge of Content and Students item.

13. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students focused on particular difficulties that they are having with adding columns of numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

I)	II)	III)
1	1	1
38	45	32
49	37	14
+ 65	+ 29	+ 19
142	101	64

Which have the same kind of error? (Mark ONE answer.)

a) I and II

b) I and III

c) II and III

d) I, II, and III

Figure 2. Sample MKT items similar to the tasks found in the Number Concepts and Operations Content Knowledge test (LMT, 2008a).

Scoring and meaning of results. The LMT Project performed extensive validation tests on each of the items used on the different forms of the MKT assessment. Due to a varying degrees of difficulties between the questions and the fact that Form A had 28 items and Form B had 29 items, comparing raw scores between the two tests does not provide an accurate picture of a participant's change in MKT. To address the issue the LMT Project has provided scales to convert participants' raw scores to standardized scores based on teacher averages during validation testing. Standardizing the scores allows for comparisons to be made between a participants score on different forms while accounting for the difficulty differences between the forms. The standardized scales were created based on the large sample of teacher scores within the LMT database. An average teacher score was assigned a standardized score of zero, which equated to roughly 13 correct responses, out of 28 items, on Form A and 14 correct responses, out if 29 items, on Form B.

I obtained de-identified data from prospective teachers' pre and posttest scores on the MKT measures. Each teacher candidate was given a raw score representing the number of items answered correctly on each test. Raw scores were changed to standardized *z*-scores using a scale proved by the developers (LMT, n. d.). The standardized scores represented a teacher candidate's current MKT level. The LMT project (n. d.) maintains that teachers' scores on any of the MKT measures reflect their level of mathematical knowledge for teaching that is distinct from common mathematical knowledge. Responses to the post assessment survey were used to describe the extent to which participants' perceived the math strategies course changed their MKT.

Qualitative Sequence

Instrument. The MKT quantitative measures provided insight into changes in teacher candidates' specialized content knowledge, but did not explain how the course curriculum influenced MKT development. The second research question of the study concerns teacher candidates' perceptions of how the content and instruction of the strategies course helped develop MKT. Therefore, the perceptions of teacher candidates were necessary to answer the second research question. Qualitative data was collected from a series of individual interviews lasting approximately 30 to 45 minutes. Rubin and Rubin (2012) maintained that qualitative data from a case study was best collected after the event in question and should be recorded. Data was collected from audio-recorded interviews with teacher candidates at the conclusion of the math and science strategies course. Interviewing prospective teachers at the conclusion of the course provided the researcher with data that explained not only the findings from the quantitative data, but also provide an understanding of how the course content helped develop other subdomains of MKT.

I designed an interview protocol instrument (Appendix C) that utilized released MKT items (LMT, 2008a) to understand how prospective teachers' MKT developed beyond content knowledge. I designed the instrument to understand changes in the subdomains of SCK, KCS, and KCT. In the design of the instrument, I paid specific attention to participants' perceptions of how the strategies course utilized the construct of decentering to develop images of how students might think about math and knowledge of different activities to support an elementary student in the learning of mathematics. The instrument is sufficient for answering the second research question because the questions focus on participant's perceptions of how the curriculum and instruction within the course helped develop their MKT. According to experts in qualitative research, field notes are helpful when interpreting transcripts and reflective memos can be used to develop codes and themes (Ravitch & Carl, 2016; Rubin & Rubin, 2012; Saldaña, 2016). Throughout the interview process, I took notes to record observations to help clarify and enrich the transcription process. I also kept reflective memos to document the coding process and capture emerging findings.

Plan for interviews. A volunteer sampling method was used to identify interview participants from the larger sample based on volunteers. Since the teacher candidates enrolled in the math strategies course represented a purposive sample capable of answering the research question, then any volunteers willing to participate in interviews would be considered. In an attempt to reach saturation, all teacher candidates that volunteered for the study were interviewed. Interviews were scheduled to last for 30-45 minutes and took place either at the study site in a private meeting room or virtually using Zoom video conferencing software. All interviews were scheduled at times convenient to the participants.

Tracking data. Various forms of data were collected and cataloged for this study. Ravitch and Carl (2016) maintained that the organization of data should begin at the onset of the collection process. All files associated with the study were saved with a consistent naming process and saved in a password-protected folder on the researcher's personal computer. Additional folders were used to save information for individual interviews and coding analysis. During the interview process, I recorded observations in a notebook to enhance the identification of codes from the transcript. Saldaña (2016) suggested the use of page numbers and line numbers to assist the transcription and coding process. I transcribed all audio recordings verbatim with page numbers and line numbers for each line of text. The use of page numbers and line numbers were used as an indexing system during the first cycle of coding using in vivo coding methods. I used memos and visual representations to track codes and emerging themes. In vivo codes were transferred to an Excel spreadsheet for pattern coding. The codes were grouped into categories based on common characteristics and themes were created to identify patterns. Samples of transcript evidence, in vivo codes, and categories used to develop themes can be found in Appendix D.

Triangulation. Triangulation was built into the data collection plan. Ravitch and Carl (2016) stated that data triangulation occurs when multiple data sources are used across time. One way this study triangulated data collection was through the use of the post survey and interviewing every volunteer. Interviewing all volunteers provided in depth perspectives on the role of the course in developing MKT while the post survey results provided insight into the perspectives of every participant in the sample. The collection of different perspectives provided the opportunity for triangulation at the analysis stage.

Gaining access to participants. I was given permission to conduct the study at the local site by the administration. After gaining IRB approval from Walden University, I submitted for IRB approval at the study site. After gaining IRB approval at the study site, I then worked with the course instructors of the math and science strategies course to set up a time to gain access to teacher candidates enrolled in the course. I visited each section of the strategies course to recruit participants by sharing a brief description of the study and provided each teacher candidate with a copy of the consent form. After analysis of the posttest scores, teacher candidates that had volunteered for the interview phase were contacted via email to set up a convenient time to conduct the interview.

Role of the researcher. At the time of the study, I was employed as an adjunct instructor in the college of education at the study site. Before serving as an adjunct instructor, I taught secondary mathematics for fourteen years. The last ten years of service as a secondary math teacher were at a high school located in the same town as the university being studied. The potential existed for some of my former high school math students to be concurrently admitted to the college of education at the time of the study. The professional relationship carried over from past experiences may affect interview sessions with increase familiarity and comfort of the interviewee, but should not interfere with the purpose of the interview. Concerning the development of mathematical knowledge for teaching, I have also served as a clinical instructor for five pre-service teachers during their full field experience before graduation. Thus, I recognize a personal bias towards assisting in the development of MKT in future educators.

Data Analysis

Collection & Analysis

The purpose of this sequential explanatory mixed methods study was to examine how prospective elementary teachers' MKT develops while enrolled in a math and science strategies course at a regional university. The data collected during this study was analyzed sequentially, beginning with the quantitative assessment of changes in teacher candidates' MKT.

To address the first research question, inferential statistics were used to describe changes in prospective teachers MKT levels. Prospective teachers' raw scores on the MKT measures were collected for both pre and posttest administrations. The raw scores were converted to standardized scores and entered into SPSS for analysis. A paired t-test was conducted to determine any changes in the group's overall MKT levels after exposure to the strategies course.

Further analysis of the MKT data included the use of descriptive statistics to analyze the post assessment survey results. Ivankova, Creswell, and Stick (2006) maintained that in mixed methods designs steps should be taken to connect the quantitative and qualitative phases. The post survey results were used to answer research question 2a regarding teacher candidates' perceptions of how the course changed their MKT and to connect the two phases by informing the development of interview questions for use during the qualitative phase. The post survey consisted of 6-point Likert scale items. The results were exported to an Excel spreadsheet and the 6-point scale was reduced to a 3-point scale by combining two groups for each of the positive, neutral, and negative categories. The results were then converted to stacked bar graphs to get a graphical representation of the perceptions of the sample. This intermediate stage of the research process served to connect the quantitative and qualitative phases. The qualitative phase began with the identification of participants based on teacher candidates that volunteered. Three volunteers were interviewed individually for approximately 30 to 45 minutes.

I used manual coding techniques as described by Saldaña (2016) to analyze the qualitative data. To begin the coding process, audio files of interviews should be transcribed verbatim (Ravitch & Carl, 2016; Rubin & Rubin, 2012). To address research question 2b, I transcribed the audio files collected from each interview verbatim as soon as possible after the data collection to avoid loss of data. Each interview was transcribed and saved in a password-protected Microsoft Word document. I analyzed each interview after transcription using in vivo coding methods as described by Saldaña (2016). In the first cycle of coding, I looked for units of meaning related to the research question and framework and used participant's actual language from the transcript to apply codes. I then reviewed the transcript a second time to make sure all codes were applied correctly and to identify any missed codes before writing an analytic memo to summarize my interpretation of the interview.

The second cycle of coding identified categories and themes. After each interview was analyzed, I carefully copied and pasted statements and accompanying codes to a password-protected Microsoft Excel database for pattern coding. The first cycle codes were grouped into categories based on similar characteristics. Themes organize data with a phrase or sentence that identifies a pattern (Braun & Clarke, 2008; Saldaña, 2016). Therefore, I identified themes that explained the teacher candidates' perceptions of the influence of the strategies course on their MKT development and selected evidence from the transcripts to best support the findings.

Integration of Methods

I integrated the quantitative and qualitative findings to address the research questions fully. Ivankova et al. (2006) suggested that integration of the results of a mixed methods study should guide the development of the discussion of the outcomes of the study. I took multiple steps to integrate the quantitative and qualitative data in order to best describe the influence of the math strategies course on the development of MKT in pre-service teachers. First, the findings from the quantitative phase are presented to explain any changes in MKT level. Second, the findings from the post assessment survey are presented to explain results from the MKT assessment. Third, the findings from the case study are presented to explain the results from the quantitative phase. Lastly, interpretations of the grouped findings are shared to address the purpose of understanding how MKT develops in elementary pre-service teachers enrolled in a math and science strategies course.

Validity and Trustworthiness

Shenton (2004) suggested that qualitative researchers approach trustworthiness through various strategies to portray credibility, transferability, dependability, and confirmability. Credibility is achieved by presenting the actual experiences of participants and is supported through the use of triangulation and member checking (Shenton, 2004). I triangulated qualitative data from multiple sources and made use of member checking procedures. During member checking, I provided each interview participant with a summary brief of the findings for review to verify that I did not misinterpret the data during analysis. Rich description allows the reader to make an informed decision on the transferability and dependability of a study (Shenton, 2004). I have provided a rich description of the context of the study and the plan for data collection and analysis. Lastly, qualitative researchers confirm results by acknowledging their biases throughout the data collection process (Kornbluh, 2015; Ravitch & Carl, 2016; Shenton, 2004). I have reflected on any biases before conducting the study and used memos and field notes for continued reflection throughout the duration of the study.

Data Analysis Results

The data for this sequential explanatory mixed methods study was collected and analyzed sequentially. Figure 3 provides an overview of data collection and analysis of each phase of the study with the resulting product. The first phase focused on quantitative data collection using the MKT measures in the form of a pre and posttest and a post course survey. I obtained de-identified raw scores from each participant in an online password protected database. In the analysis of quantitative data, inferential statistics were used to determine changes in MKT scores over the duration of the course. Descriptive statistics were used to analyze participants' responses to the post course survey, which provided some insight into research question 2a. To connect the quantitative and qualitative stages, I devised questions for interviews and recruited volunteers that completed the pre and posttest assessments for interviews to provide further explanation of trends evidenced within the MKT assessment and post course survey.

The second stage was qualitative and served as the primary focus for this study. During the qualitative data collection, I interviewed volunteers for approximately 30 minutes using an interview protocol I developed based on released MKT items and Silverman and Thompson's (2008) framework. Audio of the interviews were recorded and transcribed for analysis. In vivo coding and thematic analysis were used to determine overall themes. Lastly, the results of both stages were integrated to allow the qualitative data to explain the findings from the quantitative stage.

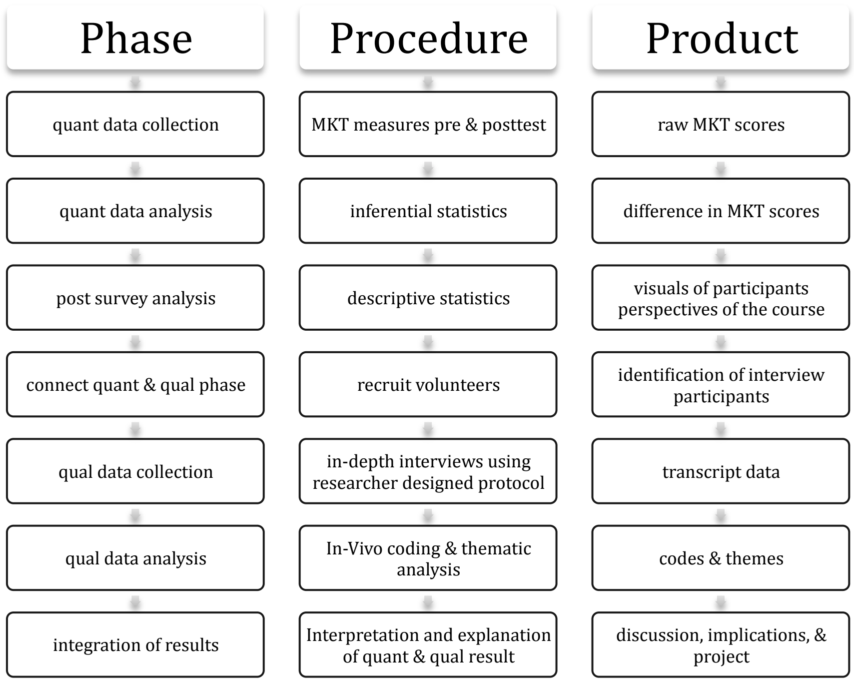

Figure 3. Overview of data collection and analysis plan.

Quantitative Findings

The MKT assessment portion of the quantitative phase of the study focused on answering RQ1.

RQ1: What is the difference between pre and posttest scores of elementary PSTs' MKT after taking a math strategies course?

$H_0 1$: There is no significant difference in terms of MKT between pre and posttest scores of PSTs after taking a math strategies course.

$H_a 1$: There is a significant difference in terms of MKT between pre and posttest scores of PSTs after taking a math strategies course.

Results and analysis of MKT measures. The results in Table 1 show the raw and standardized scores of each participant in this current study on the pre and post assessments along with the averages and standard deviations. On the pre assessment, 12 participants' scores were over one standard deviation from an average teacher's score. On the post assessment, 11 participants' scores were over one standard deviation from an average teacher's score. The MKT assessment is validated to assess changes in a teacher's content knowledge, not to provide an evaluation of any particular teacher's knowledge (LMT, n. d.). However, investigating trends within the participants' raw scores did provide insight into weaknesses within the MKT subdomains present in the sample.

I used Ball et al.'s (2008) definitions to code the MKT assessment items on both forms and verified the codes with a content expert knowledgeable in the MKT domains. Table 2 shows a breakdown of the number of CCK, SCK, KCS, and KCT items present on each form of the MKT assessment. Table 3 shows the percentage of items each participant got correct within each domain on the pre and post assessment.

Table 1

Teacher Candidate Scores on Pre and Post Assessments

Participant	Correct Responses		Standardized Scores	
	Pre	Post	Pre	Post
A_01	9	9	-0.85	-0.74
A_02	5	9	-1.59	-0.74
A_03	6	11	-1.32	-0.51
A_04	4	9	-1.79	-0.74
A_06	11	11	-0.38	-0.51
A_07	18	14	0.15	0.66
A_08	10	10	-0.68	-0.56
A_09	8	11	-1.03	-0.38
B_01	13	13	-0.02	-0.17
B_02	8	7	-1.03	-1.12
B_03	10	18	-0.56	0.66
B_06	7	13	-1.12	-0.17
B_07	5	1	-1.52	-2.49
B_09	14	14	0.15	-0.01
B_10	7	6	-1.12	-1.39
B_11	16	13	0.51	-0.17
B_12	10	4	-0.68	-1.74
B_13	10	9	-0.68	-0.74
B_16	9	6	-0.74	-1.39
B_17	4	5	-1.79	-1.52
B_18	7	4	-1.21	-1.74
B_19	7	9	-1.21	-0.74
B_20	10	7	-0.56	-1.21
B_21	10	7	-0.56	-1.21
B_22	8	5	-1.03	-1.52
B_23	13	8	-0.02	-1.03
B_24	8	13	-0.93	-0.17
B_25	12	8	-0.34	-0.93
B_26	11	9	-0.51	-0.74
B_27	11	12	-0.51	-0.20
M	9.37	9.17	-0.76	-0.78
SD	3.33	3.69	0.57	0.70

Table 2

Number of Items in Each Subdomain by Test Form

CCK Items		SCK Items		KCS Items		KCT Items	
Form A	Form B	Form A	Form B	Form A	Form B	Form A	Form B
12	16	9	9	4	3	3	1

Table 3

Participants' Percentage of Correct Items on Each Subdomain

Participant	Pretest	% CCK		% SCK		% KCS		% KCT	
		Pre	Post	Pre	Post	Pre	Post	Pre	Post
A_01	Form B	44%	25%	11%	44%	33%	25%	0%	33%
A_02	Form B	25%	25%	0%	44%	33%	25%	0%	33%
A_03	Form A	25%	38%	11%	22%	25%	100%	33%	0%
A_04	Form B	25%	42%	0%	22%	0%	25%	0%	33%
A_06	Form A	50%	38%	44%	33%	0%	67%	33%	0%
A_07	Form A	67%	69%	33%	56%	50%	67%	0%	100%
A_08	Form B	31%	33%	56%	33%	0%	50%	0%	33%
A_09	Form B	31%	33%	33%	56%	0%	25%	0%	33%
B_01	Form A	42%	38%	56%	44%	50%	67%	33%	100%
B_02	Form B	31%	33%	11%	33%	67%	0%	0%	0%
B_03	Form A	58%	50%	11%	78%	50%	67%	0%	100%
B_06	Form A	8%	44%	67%	56%	0%	33%	0%	0%
B_07	Form A	17%	6%	33%	0%	0%	0%	0%	0%
B_09	Form A	58%	50%	44%	67%	50%	0%	33%	0%
B_10	Form A	67%	25%	0%	22%	75%	0%	0%	0%
B_11	Form A	67%	50%	67%	33%	25%	67%	33%	0%
B_12	Form B	38%	33%	44%	0%	0%	0%	0%	0%
B_13	Form B	31%	25%	44%	44%	0%	50%	100%	0%
B_16	Form A	33%	25%	33%	11%	25%	33%	33%	0%
B_17	Form B	25%	33%	0%	11%	0%	0%	0%	0%
B_18	Form B	25%	17%	33%	11%	0%	0%	0%	33%
B_19	Form B	38%	50%	0%	33%	33%	0%	0%	0%
B_20	Form A	67%	25%	11%	22%	25%	33%	0%	0%
B_21	Form A	33%	31%	56%	22%	0%	0%	33%	0%
B_22	Form B	25%	25%	44%	11%	0%	0%	0%	33%
B_23	Form A	42%	38%	56%	22%	25%	0%	67%	0%
B_24	Form A	17%	50%	44%	33%	50%	67%	0%	0%
B_25	Form B	50%	42%	33%	11%	33%	25%	0%	33%
B_26	Form B	44%	33%	33%	44%	33%	0%	0%	33%
B_27	Form B	38%	58%	44%	33%	33%	50%	0%	0%

Looking at the participants' raw responses on both the pre and posttests and the data in Tables 2 and 3, it is clear that CK is a concern as 47% of the teacher candidates answered less than a third of the total CCK and SCK questions correctly. In general, the CCK and SCK scores on the pretest are low and less than half of the participants showed any improvement in their scores at the end of the course. The findings of low CK seem to confirm the findings of other researchers that show university level mathematics course, which is what this sample of teacher candidates has been exposed to, are ineffective in developing MKT (Qian & Youngs, 2016). Though limited in quantity, the questions regarding KCS and KCT also suggest areas of weakness as 83% of the teacher candidates answered less than one-third of the total questions correctly. The findings of Edelman (2017) support the idea that pre-service teachers need specific instruction to develop knowledge of students and teaching. To assess any changes to teacher candidates' MKT during the course, inferential statistics were used to determine the impact of the course curriculum and instruction.

Figure 4 shows that the pre and post assessments were moderately and positively correlated to one another ($r = 0.49$, $p < .01$). A paired-sample *t* test was conducted to investigate changes in MKT levels in teacher candidates enrolled in the math and science strategies course. There was no significant difference between the teacher candidates' pre and post MKT scores, $t(29) = 0.096$, $p > 0.9$, two-tailed. The effect size of the difference in the means (*mean diff* = .11, 95% CI [-.23, .26]) was small ($d = .02$). An online power calculator (Dhand & Khatkar, 2014) revealed that a sample size of 34 participants for a two-tailed paired t-test using an alpha of 0.05 and a medium effect size (*d* = 0.5) would yield a 0.8 power value. To detect an effect of 0.02, a study would need a sample size of 19,625 participants. Therefore, based on the small effect size and sample size, I am unable to reject the null hypothesis. There is no significant difference in MKT between pre- and posttest scores of PSTs after taking a math strategies course. Therefore, it is safe to conclude that the math and science strategies course had essentially no effect on the development of MKT in enrolled participants.

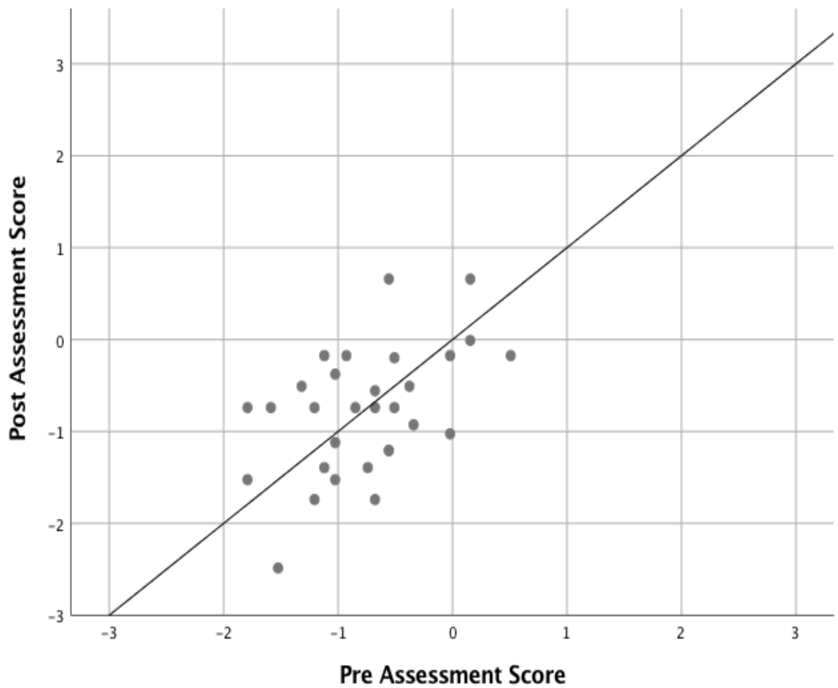

Figure 4. Scatter plot showing positive correlation between teacher candidates' pre and post assessment scores.

Results and analysis of post assessment survey. The post assessment survey portion of the quantitative phase provided some insight into research question 2a dealing with participants' perceptions of the math strategies course.

RQ2a: To what extent did elementary PSTs perceive their math strategies course changed their MKT?

Descriptive statistics were used to analyze the post assessment survey results to inform the development of follow-up questions during the qualitative phase and provide further explanation of the quantitative findings. Each survey question was presented as a 6-point Likert scale item. Selected questions are presented along with descriptive information including number of responses, mean response value, and standard deviation of responses. In addition to informing the interview process, the survey results also provide insight into the non-significant results found in the paired *t*-test above.

Figure 5 shows that 53% of teacher candidates who took the MKT assessments felt strongly that the questions reflected knowledge teachers should know in order to teach math. However, 73% of teacher candidates felt that the course did not prepare them to answer the questions. Since the assessments focused mostly on CCK and SCK, it seems teacher candidates perceive that the course does not include content that supports the development of key understandings of math concepts (Silverman & Thompson, 2008). Thus, many elementary pre-service teachers perceive the math strategies course did not support changes in their MKT. This supports the findings related to the MKT assessment where no changes to CK were observed. The lack of CK development should be recognized as a problem as according to Silverman and Thompson, developing deep understanding of content is an essential component of developing MKT in teachers.

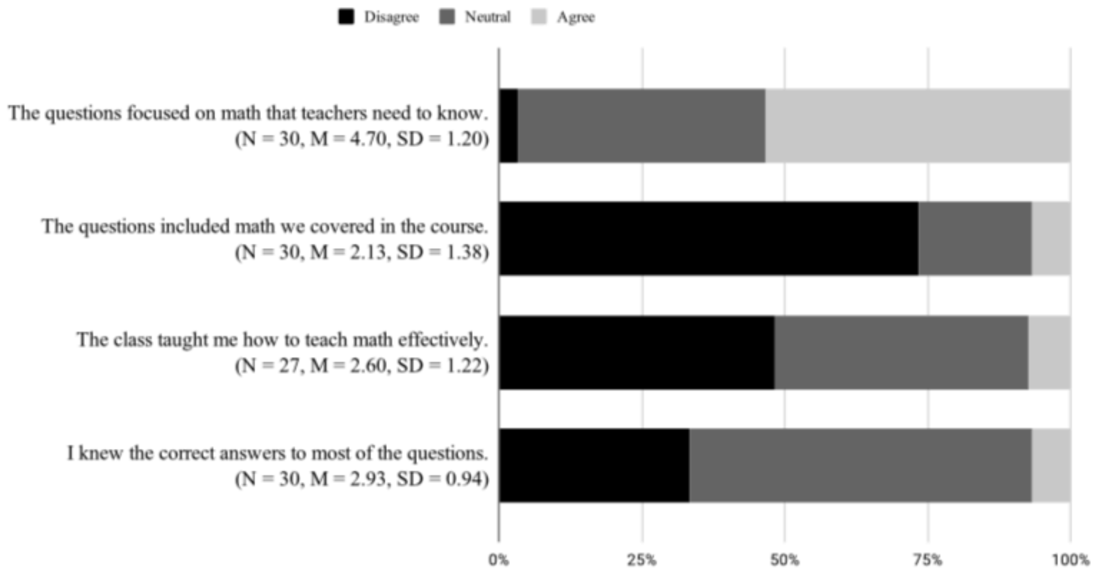

Figure 5. Teacher candidates' agreement to statements about the MKT questions and course.

Further reflecting on Silverman and Thompson's (2008) framework, MKT is developed in teachers when they construct models of student thinking, build connections between multiple models, and develop an understanding of what activities and conversations can advance student development. Figure 6 shows how teacher candidates perceived time was spent during the course on various math related activities that support Silverman and Thompson's construction of MKT model. The teacher candidates' perceptions suggest that activities that align with Silverman and Thompson's model were not emphasized in the math and science strategies course. For example, 63% of participants did not feel their instruction included samples of student work, which could develop SCK and KCS through investigating errors in thinking, learning to provide explanations, and anticipating student thought (Goodsen-Epsy et al., 2014; Son & Lee, 2016; Thanheiser, 2015). Also, 63% of participants responses suggest no exposure to videos of teaching or reviewing cases, which could support the development of KCT through learning how to sequence learning experiences or planning to address student errors (Ineson, Voutsina, Fielding, Barber, & Rowland, 2015). Also, 63 to 70% of participants express that they did not solve math problems or practice explaining math during the course. Exposing teacher candidates to math tasks has been shown to develop CK and PCK (Ghousseini, 2017; Thanheiser, 2015). The lack of time spent on the activities presented in Figure 6 could explain some of the weaknesses recognized in overall CK found in teacher candidates' scores on the MKT assessments.

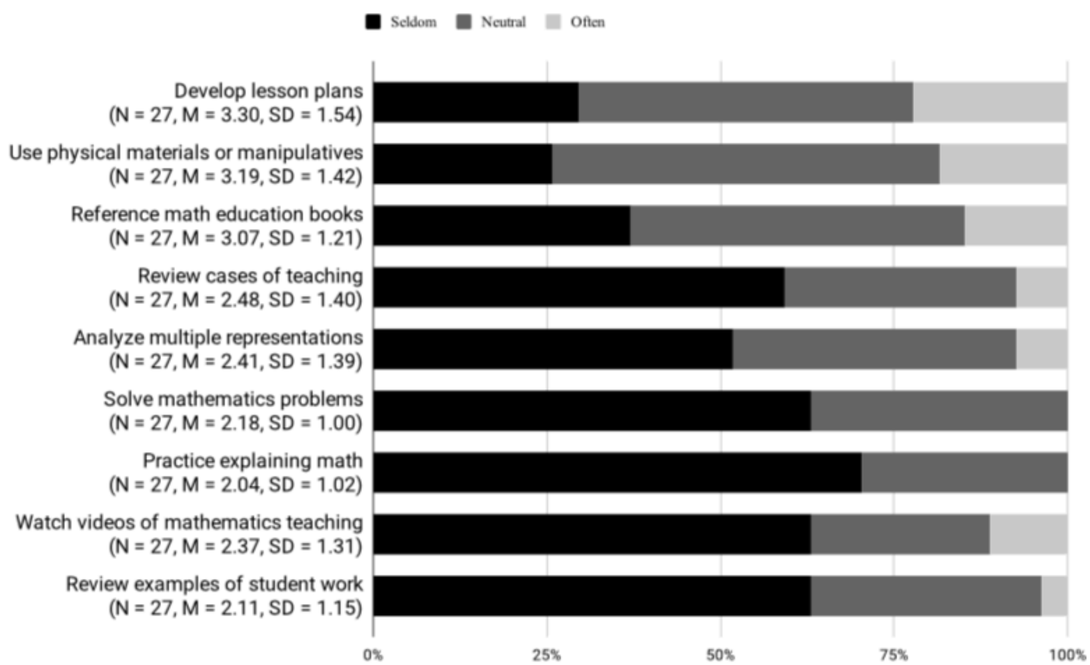

Figure 6. Teacher candidates' perceptions of time spent participating in math related activities during course.

Figure 7 shows that teacher candidates did not feel like the course developed their ability to teach math. For example, 54% of participants perceived that the course had no influence on developing their KCS, specifically, their knowledge about how student think about math. Likewise, 61% of teachers felt that the course failed to provide them with strategies to teach diverse populations of math learners. The lack of perceived instruction about teaching strategies suggests a lack of development of the KCT subdomain. The responses in Figure 7 suggest that the course did not address either the KCS or KCT subdomains and might account for the large percentage of missed questions within those subdomains on the assessments.

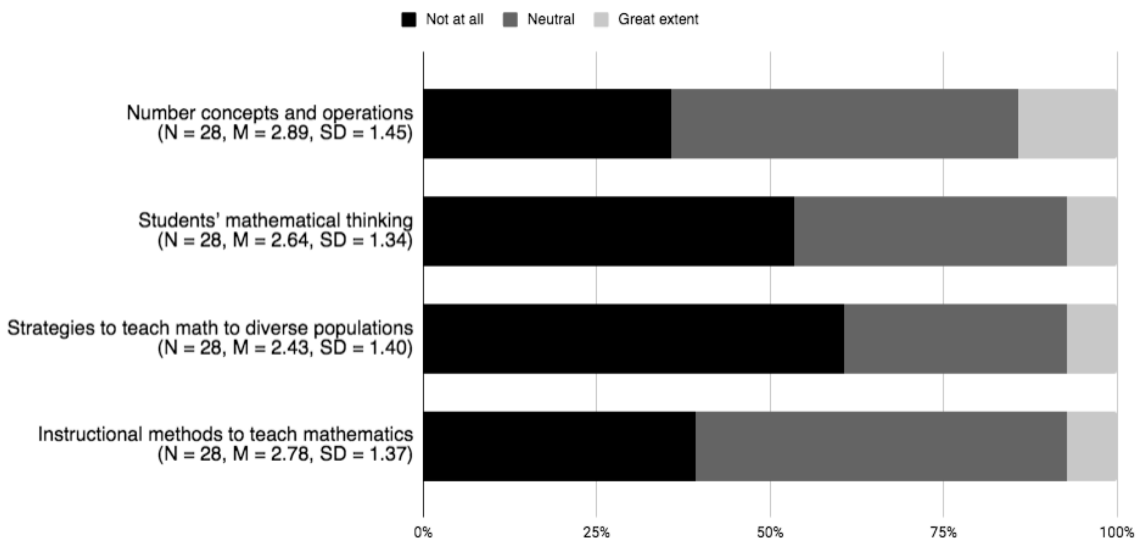

Figure 7. Teacher candidates' perceptions of how content and instruction provided in the course helped develop their knowledge in various areas.

Summary of quantitative findings. Descriptive analysis of teacher candidates' raw scores revealed areas of weakness across the MKT domains. Inferential analysis showed that the MKT levels of the elementary teacher candidates enrolled in the course did not change during the semester. Descriptive analyses from the post assessment survey provided insight into the teacher candidates' perceptions about the content and instruction of the math strategies course. The teacher candidates' perceptions confirmed the inferential analysis that the course did not have a positive impact on their MKT development and that the course curriculum did not include content shown to have a positive impact on developing MKT.

Qualitative Findings

The qualitative phase of the study focused on explaining the information gained from the post course Likert survey to help answer RQ2b.

RQ2b: In what ways do elementary PSTs perceive the content and instruction in their math strategies course influenced their development of particular MKT components?

Results and analysis of open-ended interviews. Three pre-service teachers enrolled in the math strategies course volunteered to participate in open-ended interviews to share their perceptions of how the math strategies course influenced their development of MKT. Two of the participants, Teacher Candidate A and Teacher Candidate B, were enrolled in the course taught by a first-year instructor at Site B. Teacher Candidate C was enrolled in the course at Site A taught by a veteran instructor. Thematic analysis of the interview transcripts revealed three themes related to the teacher candidates' perceptions of their preparation to teach mathematics at the elementary level (Appendix D). The participants indicated that the math strategies course had minimal influence on their development of MKT. However, participants expressed that MKT components were addressed in their Geometry and Measurements course. Lastly, after completing the math strategies course, pre-service teachers conveyed feelings of anxiety towards math and a sense of unpreparedness to teach.

Theme 1: Pre-service teachers perceived that the content and instruction of the math strategies course had minimal influence on their development of MKT. The teacher candidates felt that the content and instruction in the course did not address any of the MKT components directly. When initially asked about their experiences in the course, the participants expressed that strategies were a focus at Site B but not at Site A. Teacher Candidate A stated:

> this has been, I don't know how to say this without being super negative. It's been one of the worst classes I've ever had… I've dreaded going every week, and I'm not sure if it's just because our professor it's her first year teaching at a college level. But I mean, in my opinion, we have not actually learned one thing this entire semester about how to teach math strategies.

Teacher Candidate B shared similar feelings when asked if their experience in the strategies course was positive, "No, not really, the professor is nice. Um, we, I mean, I haven't learned a whole lot of actual strategies in it." Teacher Candidate C was enrolled in the course at Site A and had a slightly more positive experience. Teacher Candidate C conveyed that learning about different ways to solve math problems was helpful, but wished that the course had provided more strategies and direction of how to use different strategies.

> It's been interesting because I've grown up doing a set of, you know, one set of strategies, you know, that, you know long division. That's all you did. And so in this class, I've learned that there's different strategies to get the same problem and are to solve the same problem. And so that has really opened up my, you know,

> kind of my tool chest so that I have different strategies. So if you know one student doesn't learn, you know, a particular way, I have another strategy that I can teach them, so I don't have to teach all my students to one set strategy. I have, you know, a multitude of strategies to teach from. So that's something that's really been beneficial.

In an attempt to understand how the math strategies course addressed MKT components I asked a series of questions related to different subdomains. Participants were asked to explain any content about representing mathematical ideas, exploring errors in student thinking, examining unusual approaches students might use or providing mathematical explanations to students to investigate the influence the course had on SCK. The math strategies course seemed to lack activities devoted SCK. At both sites, participants maintained that activities highlighting student thinking, such as samples of student work, were not used in the class as noted by Candidate C's statement, "No, we didn't have any of that." After reviewing the released MKT item about exploring errors in student thinking, Candidate A expressed, " I mean, there's no actual teaching of anything like this going on in there." Since the participants could not recall activities that directly reflected the domain the researcher probed further to see if they could think of any activities that were similar.

At Site B the participants stated that they had a small exposure to student errors in thinking during an assignment that required them to tutor an elementary-aged student. Candidate A shared, "We did a tutoring activity. Umm, so we kind of had that opportunity to do that umm, once." Candidate B’s agreed, “Yeah, we did have one or two assignments where we had a math tutoring activity, and I worked with a student…. but it was nothing like this... it is pretty basic. I, I just really helped him with his homework...". Candidate B's explanation revealed that the tutoring activity hinted at using some of the components within the SCK domain, albeit at a low level. The activities used within the math strategies course seem to lack in providing teacher candidates opportunities to recognize errors in students thinking, practice providing mathematical explanations, and investigate unusual approaches to mathematics. While asking Candidate B if they could recall practicing any of the SCK components during the course, they replied, "I hoped I would, but it didn't really come up." Teacher Candidate C pointed out that the course activities did not focus on SCK but the textbook did provide exposure to the SCK components.

> We talked about it [recognizing student errors] a little bit in class. But I got more from the textbook part of it. Because it was more, you know, explicit more, more laid out, than the lecture was I felt like. [If I didn't have the textbook] then I wouldn't have gotten much I don't think. I would have gotten a little, but not, not as thorough.

To explore the influence the math strategies course had on KCS, participants were asked to explain any content that helped them understand how students may think about math or how students learn math. Each of the candidates expressed that the idea of investigating student thought was not present within the course. Candidate B's response captured the collective response well, "Um, to anticipate their thinking, not necessarily. But it did push me to be prepared for their different kinds of thinking I guess...." When asked to explain how the course prepared them for different kinds of student thinking Candidate B referred to the math journal, or interactive notebook, that they kept throughout the course. A math journal was used during the course to document different strategies and collect lesson plans shared during the course.

> Well within that project that we had, that interactive notebook, um, there were different, I guess, concepts that we had to have within it. And so I was kind of an overachiever, and I wanted to, like, make sure that I had different ways of, like, addressing different problems.

Candidate B perceived the math journal assignments were related to the KCS domain. However, further probing revealed evidence that the candidate may have been confusing the collection of different strategies to solve a problem with understanding and anticipating how students may think about math. Candidate B described the assignments as “being able to find different concepts…. I could have had manipulatives or just different things like that I could help students like if they are kinesthetic learners or visual learners”.

Candidate C's comments demonstrated a closer alignment with the KCS domain. Candidate C maintained "Math journals help because they prompted us to find where assessments had been done." Candidate C explained that in their research to complete an assigned task for the math journal they used the textbook and a website where a math specialist shared an assessment of student thinking.

> Honestly, the book helped me a lot because I researched that because that's something that when I was going through our journal, our math journal, I came

> across a website that did an assessment on place value. And it interested me because, you know, I've seen students have difficulty of, you know, with this same thing in second grade when I was trying to help them figure this out and so, you know, I looked in the book to see how I could teach those strategies to students. Reading the book chapter, it really emphasized, okay, If a student does this why? If they can do all this other stuff, why can't they do this? And I think this was even one of the examples or something really close to it. The book though broke it down enough to where I could understand, you know, okay, this is where a kid could misinterpret this. And so, it gave you different strategies to use to assess them, but also to show them how to correct the errors in their thinking.

At Site B, Teacher Candidate B maintained that “we haven’t used it at all” when referring to the course textbook. Teacher Candidate A indicated that they did use the textbook at the very end of the course, but not in a way that they deemed effective. The manner in which the textbook was utilized may have added to both participants’ negative experiences in the course.

> I think we haven't even, our textbook, our textbook we haven't even opened.... assigned something from chapter one, two, and three, to do the last week of class in the math concepts book. I mean, it's, and it's just, we're just looking at case studies and like answering writing a discussion board about it.

The textbook for the course seems like a valuable learning tool that did have the potential to develop MKT. However, the participant's responses show that any influence on their MKT development most likely came from their efforts to review the textbook.

Therefore, the textbook may not have been used effectively to develop teacher candidates' MKT intentionally.

Interestingly, based on the released MKT items and the brief descriptions of the subdomains provided by the researcher, the candidates were able to assess the lack of content related to the MKT components. Overall, the participants say that the course mainly focused on creating an interactive notebook that collected strategies, or lesson plans, that they shared weekly with their peers during class. The activities identified by the participants seemed to suggest that the course focused mostly on different ways to solve problems without a focused exploration of difficulties students might have, how students might think about math concepts, or how to use strategies to support student learning. For example, Candidate C shared, "I just wish that we could have delved into them [strategies] a little more deeply...We were introduced to different strategies but, when to use those strategies was not really expressed." Even with a lack of evidence in the math strategies course, participants were able to identify content from another course that they feel did positively influence their MKT development. For example, Candidate B shared, "I feel like my, umm, my Geometry Measurements course taught me more about methods than this one." The Geometry and Measurement course is taught within the mathematics department at both sites with multiple instructors.

Theme 2: Pre-service teachers perceived that the content and instruction of a Geometry and Measurements course had a positive influence on their development of MKT. Before investigating how participants perceived the math strategies influenced their development, each interviewee was asked to share their other math experiences. All of the interview participants expressed that a Geometry and Measurement course was a transformative experience for them as future teachers. Candidates A and B shared the same professor and revealed that the efforts of the professor were instrumental in growing their confidence to teach math. Candidate A stated:

> In the geometry and measurement class that like by far, that was one of my favorite classes at [Study Site] period. Yeah. It was because it was Professor A teaching and she is, I mean, phenomenal. Like I've never enjoyed, I mean, it took me all the way to get to college, until I enjoyed math class and it was because of her. Yeah, by far one of my favorite classes I've ever had. She made it incredibly enjoyable; I don't even know how to explain it, she's wonderful.

Candidate B shared:

> My geometry and measurements course, I took that here, and that was one of the best math classes I've ever taken. And the professor was really awesome because I wouldn't consider myself amazing at math, and she really gave her students that confidence that they knew what they were doing and she knew what she was teaching in order for them to be successful….she made sure to go around and just have conversations with us and like talk to us about our strengths and our weaknesses in math and really just assuring us that we're capable of doing it, and we're capable of teaching it, just a lot of encouragement, and I think that's what a lot of people appreciate about her.

The comments of teacher candidates A and B suggest that the actions of their instructor had a positive influence on their confidence with math. When asked to expand on how the Geometry and Measurement course prepared them to teach, Candidate C shared, “In the geometry, they did model how we could teach that or those concepts to students, which was very beneficial.” Candidate A asserted that the course helped develop a deeper understanding of content by engaging the candidates in solving math problems while considering the thoughts of elementary-aged students. Candidate A’s remarks suggest that the content and instruction of the Geometry and Measurement course included activities that aligned with SCK, KCS, and KCT as teacher candidates were exposed to common errors, student thinking, and how to use strategies.

> But, you know, she would explain to us and walk us through like this is something you could use in a third-grade classroom, and this is how you would use it. You know, and she would walk us through using it and then she's like, now you use the manipulative. So it was like us actually doing it. Like, at the collegiate level and then we would take it down, and she's like now, like, let's work on some problems like your students would. So we were actually doing them ourselves. Which that was like, oh, this makes sense to me, you know. Like I'm, I'm, she would essentially help us think like an elementary student, and she was like this is, these are common mistakes that these students make and this is why. I was like, oh, hey, like, that makes sense. So there was a lot of that that we could actually understand like, why they, like why they think this way. It's like that makes sense to me.

The instructors of the Geometry and Measurement courses seem to have presented content that aligns with multiple components of MKT and Silverman and

Thompson's (2008) suggestion to have teachers practice decentering to develop an understanding of how their learners may approach math. Whether the instructors intended to address MKT development is unknown. However, the interview participants recognized that the efforts of the instructors in the Geometry and Measurements course to model strategies to teach and investigate student thinking were related to the MKT components that this study was investigating. Even though each of the teacher candidates felt that the Geometry and Measurements course was a positive experience, their comments did reveal a weak area in their overall math preparation in regards to MKT.

The KCT subdomain seems to be a major weak area in the elementary teacher preparation curriculum within the college of education. The participants revealed that nowhere in their preparation did they recall intentional discussions of sequencing math tasks to support learners or the use of videos to connect content, strategies, and pedagogical decision-making.

Theme 3: Teacher candidates express unpreparedness to teach math and a desire for additional training. The interview participants shared varying degrees of unpreparedness when it comes to math. Teacher Candidate A shared how math has always been a source of struggle and that they are anxious about the thought of teaching math.

> Math has always been, it's not been my strong suit. I've been super intimidated by math and everything about it. And the thought of teaching it is really intimidating to me...I feel like most of the elementary education majors, you know, they really don't like math or they all, we all feel the same way it is intimidating.

Teacher Candidate C seemed less anxious about their math abilities but expressed a strong desire to help their future students build confidence when learning math and a willingness to take responsibility to improve as a teacher.

> I'm not too concerned with my math knowledge because I know I'm willing to learn and I'm willing to put the thought and the work into planning and looking for all those questions students are going to ask. And I'm going to be prepared. Because, I have to be. Because I want them to know more when they graduate, or when they, you know, move on to the next grade. I want them to be prepared.

Similarly, Teacher Candidate B shared a desire to help their future students build confidence like teachers had done for them in the past.

> Um, I would love to be able to give my students, the confidence that other teachers gave me in math because I know that ultimately, there's going to be at least one student in my class that is going to feel… that's not going to have great self-esteem with math, and I want to be able to give it, give them that confidence within the course. Not just saying, oh, you can do it, but like being able to back it up with like different methods, that's going to work for them.

In regards to their math anxiety or that of their future students, the teacher candidates expressed a concern that they were unprepared to teach math and suggested that the education curriculum include more math strategies coursework. For example, Teacher Candidate B stated, “I wish that we actually learned more strategies from our professor and how we can incorporate it into the classroom." Teacher Candidate C said, “I don't feel like I've learned enough of what I need to have, to be in the classroom."

Teacher Candidates A had similar feelings and even suggested separating the course from the science content.

> Okay, um, man. This was, it was a class I was really looking forward to, to be honest with you, because like I said, like math has always been, it's not been my strong suit....I think it needs to be its own separate course, um, I just think that would be helpful. I really do.

Teacher Candidates C also suggested that the strategies course needed to be separated into individual math and science courses.

> Combining those two into one strategies class. Was like, come on, really. You're, you're training these teachers to go out and, and make a difference. But you combine two of the hardest concepts together and skim over the top of it, and you wonder why students are doing so poorly in school. It's a no-brainer, in my opinion. So, I think separating the classes would enable the professor to dive more deeply into it to cover things that should be covered. Okay, you have students that are not understanding. Okay, why? Here are some misconceptions. Okay, why are they misconceptions? Okay, how do you fix those misconceptions to improve? I just feel like they're combining so much into one class that it’s a disservice.

Teacher Candidate A also expressed a willingness to take additional coursework to prepare themselves better to teach math and plans to seek math related professional development after they start teaching full-time.

> Just teaching, teaching teachers how to be confident and what they're teaching and why they should be confident. I think it because I know it's an important class and

> if I have the opportunity if it gets changed, to go back and like take it, like I know I would. And that's the thing is, even if I've been teaching for a few years, because I am so intimidated by math, that would be you know professional development, I would be excited about. Because it's something that I want to know how to teach and I want to know how to teach it well. Because, I feel so confident in other areas I want to be able to get kids excited about math then they don't go through and be intimidated like I was all throughout my years, you know. Like I said before, it took me till college to be excited about something all because of my teacher because she loved it. She was confident her ability to do it. And that reflected on her teaching, and it reflected the way that you know I felt about math going out. So I think that that would be important to be able to implement that in the future class.

Integration of quantitative and qualitative findings. Inferential analysis of the MKT assessments showed that the MKT levels of teacher candidates enrolled in the math and science strategies course did not change over the course of the semester. Descriptive statistics conveyed that many candidates enrolled in the strategies course did not feel that the course incorporated components that would support the development of their MKT. Thematic analysis of participant interviews revealed that teacher candidates felt that the math strategies course did not influence their MKT development due to the fact that course content did not align with the MKT components. Specifically, interview participants shared that the course incorporated little to no opportunities to interact with student thinking or how to sequence learning experiences to advance student learning, similar to those represented in Figure 6. The interview participants' responses provide insight into why the whole class responses to the survey in Figure 5 suggest that the course did not address the types of question on the MKT assessment. Based on the interviews, it seems that a lack of content focused on the MKT domains related to students and teaching may contribute to the lack of changes in MKT development.

Participants were able to identify teacher actions within a Geometry and Measurements course that did align with the MKT components and that participants felt did have a positive influence on their development as teachers. However, the teacher candidates that were interviewed expressed a lack of confidence to teach math and a desire for additional instruction in how to teach math. The feelings of unpreparedness and perceived lack of learning opportunities related to MKT components seems to explain the low percentage scores across the MKT subdomains in Table 3. Section 3 presents a 16-week curriculum plan for a separate math methods course taught within the college of education designed to specifically address the findings of this study.

Section 3: The Project

Introduction

The purpose of this study was to investigate how the curriculum in a math and science strategies course developed MKT in prospective elementary teachers at a regional university. The findings presented in Section 2 of this study show that the current curriculum had little to no influence on teacher candidates' MKT levels and teacher candidates desire additional preparation to teach math. Based on the findings of this study, I developed a revised 16-week curriculum plan to develop MKT in prospective elementary teachers in a standalone math methods course (see Appendix A).

The goal of the proposed project is to provide instruction that supports the development of future teachers' MKT. The overall goal of the curriculum plan is to develop teacher candidates' confidence with mathematics so they can successfully develop, implement, and evaluate learning experiences that support student learning. Supporting course goals include deepening teacher candidates' understanding of elementary math content, elementary student thinking, and pedagogical approaches to support learner development. The proposed curriculum plan includes a course syllabus, course alignment matrix, and course calendar. The design of the curriculum considered state academic math standards, standards for teaching, and research-based teaching practices as a foundation for the course. Also, evidence from the findings in Section 2 and Silverman and Thompson's framework for developing MKT were used as a guide to ensure the curriculum intentionally addressed each of the four MKT subdomains: CCK, SCK, KCS, and KCT.

Rationale

This study focused on the problem that it was unclear how the curriculum within a college of education at a regional university developed MKT in prospective elementary teachers. When it comes to math instruction, a teacher's MKT level has been shown to have a positive impact on their instruction and their students' achievement (Hill & Charlambous, 2012; Hill et al., 2013; Sahin, 2015) while mathematical self-efficacy also has the potential to negatively influence instructional quality (Holzberger, Philipp, & Kunter, 2013). As states have adopted more rigorous math standards and national organizations such as the NCTM have emphasized research-based teaching practices, the demands of a teacher's MKT have increased (Hill et al., 2013; NCTM, 2014). The findings of this study showed that the current curriculum within the math and science strategies class does not develop MKT in PSTs. Similarly, teacher candidates at the study site expressed feelings of unpreparedness when it comes to teaching math in the elementary classroom. Therefore, the development of a curriculum plan was chosen to address the problem identified during data analysis.

The proposed curriculum plan addresses the problem in multiple ways. First, the curriculum plan provides teacher candidates the opportunities to develop deep conceptual understanding of elementary math concepts through problem-solving activities. Problem-solving tasks have been shown to develop prospective teachers' conceptual understanding (Aydin & Ozgeldi, 2017), which can be connected to the CCK and SCK subdomains. Second, the curriculum plan addresses a lack of KCS learning experiences by providing PSTs with opportunities to analyze student thinking. Barth-Cohen, Little, and

Abrahamson (2018) showed having PSTs analyze videos of students working math tasks supported the development of the prospective teachers' abilities to anticipate, notice, and interpret student thinking. Therefore, video analysis tasks have been included in the curriculum plan. Reflective journals and analysis of videos of teaching have been effective at developing KCT in prospective teachers (Amador, 2017; Livy, Muir, & Downton, 2017). Therefore, the curriculum plan addresses weaknesses in the KCT subdomain by including components that allow teacher candidates to practice sequencing and selecting tasks through reflective journals and video analysis. To ensure that the proposed curriculum plan appropriately addressed the problem identified at the study site, a review of the literature was conducted to identify best teaching practices for elementary math teachers and design instruction to develop MKT in PSTs.

Review of the Literature

This section provides a synthesis of the research literature directly related to the findings of this study and the influence of teacher education coursework on the development of MKT. It includes an overview of the types of tasks used during teacher education coursework that had positive influences on MKT development. The influence of coursework on PSTs' math anxiety and self-efficacy is also discussed based on the findings in theme 3 of the qualitative analysis.

Peer-reviewed journals provided the resources necessary to reach saturation for this literature review. Several databases available through the Walden University Library provided access to current literature, including Education Source, ERIC, SAGE Journals, and Thoreau Multi-Database Search. Search terms included: *mathematical knowledge* *for teaching*, *pre-service teachers*, *math anxiety*, *math self-efficacy*, *content knowledge*, *pedagogical knowledge*, *cognitively guided instruction*, *video analysis*, *teaching*, *student thinking*, *mathematics*, and *elementary*.

Project Genre

Curriculum aimed at developing prospective teachers' skills has the potential to positively impact a teacher's actions and beliefs in their future classrooms. Many prospective elementary teachers are intimidated by math and have low self-efficacy when it comes to teaching math concepts (Itter & Meyers, 2017). Education courses focused on the teaching of mathematics have the potential to change PSTs' mathematical knowledge (Suppa, DiNapoli, & Mixell, 2018; Stockero, Rupnow, & Pascoe, 2017) and beliefs toward the teaching of mathematics (Bahr, Monroe, & Eggett, 2014; Gonzalez-DeHass, Furner, Vasquez-Colina, & Morris, 2017; Jao, 2017; Jong & Hodges, 2015; Looney, Perry, & Steck, 2017). The findings of this study suggest that the current math strategies curricula is in need of a revision to best support the development of prospective teachers' knowledge to teach mathematics and increase teaching efficacy beliefs.

Recognizing a need for improved methods coursework, Goodson-Epsy et al. (2014) redesigned coursework to encourage prospective teachers to develop SCK and KCS. Goodson-Epsy et al. reported findings of improved CK and increased teacher efficacy in prospective teachers after using instructional materials designed around the National Assessment of Educational Progress (NAEP) assessment. Goodson-Epsy et al. reported that the learning modules focused on active learning strategies, analyzing assessment results, analyzing student work samples, and assessment items. Larkin (2016) reported on course redesign that improved mathematical content knowledge, pedagogical knowledge, and teacher's confidence. Larkin described changes made to the design of the course including delivery methods and levels of student autonomy which increased PSTs' engagement and confidence within the course. The two examples provided suggest that a well-designed course has the potential to positively influence prospective teachers' preparation to teach mathematics. Based on this evidence and the findings of this study, a curriculum plan was chosen as an appropriate project genre to solve the identified problem.

Project Content

Silverman and Thompson (2008) said that MKT grows when teacher candidates develop a deep understanding of math content, models of how students may understand content, and images of appropriate activities to support students' development of similar mathematical ideas. The findings of this study suggest that the math strategies curriculum did not include learning experiences that supported prospective teachers' development in MKT domains associated with Silverman and Thompson's MKT development framework. Therefore, the proposed curriculum plan emphasizes activities shown by research to positively influence the development of the SCK, KCS, and KCT domains. Strategies that will be discussed include the use of problem-solving tasks, video analysis, samples of student work, simulations or rehearsals, and written reflections. The goal is that by incorporating these types of activities, prospective teachers will increase their MKT levels and improve their confidence to teach mathematics.

Problem-solving tasks. The first component in Silverman and Thompson's (2008) model deals with content knowledge. Weak CK may contribute negatively to a teacher's self-efficacy. For example, Rushton, Hadley, and Stewart (2016) identified multiplication fact fluency was related to teaching efficacy in prospective teacher candidates. To overcome gaps in content knowledge, researchers regularly recommend the use of problem-based tasks in math methods coursework (Livy & Downton, 2018; McGalliard & Wilson, 2017; Son & Lee, 2016; Whitacre, 2018). Problem-solving tasks have been shown to develop prospective teachers' conceptual and procedural understanding (Aydin & Ozgeldi, 2017; Ghousseini, 2017) and reduce math anxiety (VanDerSandt & O'Brien, 2017). Along with introducing tasks, researchers suggest designing curriculum to focus on building connections between different representations of math concepts.

Yee Lai and Clark (2018) developed a framework for identifying pre-service teachers' SCK and suggested that coursework include work focused on procedural and conceptual explanations along with visual and non-visual representations of math concepts. Similarly, Son and Lee (2016) maintained that prospective teachers need more opportunities to explore different mathematical representations and interpret student thinking. Based on the findings of this literature review, the proposed curriculum plan has incorporated problem-solving tasks that require teacher candidates to work collaboratively to explore math concepts and engage in mathematical discourse to examine and interpret the thinking of their peers and build connections.

Videos of teaching and learning. The second component in Silverman and Thompson's (2008) model deals with student thinking about mathematics. Hallman-Thrasher (2017) pointed out that prospective teachers need exposure to coursework that provides the opportunity to analyze student thinking, practice anticipating student thinking, and rehearse responding to student thinking. Researchers have explored the use of videos of students solving math tasks and teachers interacting with students on the development of prospective teachers' noticing of student thinking and teacher actions (Amador, 2017; Barth-Cohen et al., 2018; Ineson et al., 2015; Santagata, Yeh, & Mercado, 2018). The general consensus is that analyses of videos to teaching and learning help prospective teachers develop their noticing skills. Also, Mongillo and Boeke (2016) added that the use of videos of model teaching had a positive influence on teacher candidates' self-efficacy. The use of video seems like a valuable tool in teacher preparation, however, some researchers suggest that additional supports provide added benefits.

Teacher educators use observation protocols to assist video analysis. McDuffie et al. (2014) designed video analysis activities with an observation protocol in a mathematics methods course. McDuffie et al. found that prompts helped prospective teachers noticing of interactions of between students and teacher. Van Es, Cashen, Barnhart, and Auger's (2017) findings were consistent with other research in that the use of video supported teacher candidates' noticing of student and teacher actions, but like McDuffie et al., they suggest that candidates may need multiple cycles of observation and analysis with structured guidance. The findings of this study revealed that videos of student thinking or models of teaching were not included in the math strategies course. Therefore, the curriculum plan includes opportunities for prospective teachers to construct models of student thinking and develop an understanding of how to implement activities to support student learning through the use of video with the support of observation protocols similar to McDuffie et al. (2014) and Mongillo and Boeke (2016).

Student work samples. Samples of student work have been used to help prospective teachers understand student thinking and plan future instruction, which are components of the KCS and KCT subdomains. Edelman (2017) maintained that pre-service teachers need more instruction focused on developing KCS and KCT. Teacher Educators have researched the influence student work samples have had on developing educators' skills to plan and sequence whole group discussion around a math task (Livy et al., 2017; Tyminksi et al., 2014). Talanquer, Bolger, and Tomanek (2015) used samples of student work to identify what components of student thinking prospective teachers noticed. Talanquer et al. suggest that using samples of student work may be more beneficial than video or classroom observations as the distractions of classroom management are removed. Samples of student work seem to have the potential to positively influence prospective teachers KCS and KCT. Therefore, samples of student work have been included in the proposed curriculum plan, often in the form of case studies to provide models of teacher pedagogical decision-making and student thinking.

Simulations and rehearsals. The KCT domain defines the work a teacher does to advance student learning such as selecting and sequencing appropriate activities and leading conversations about concepts. Ghousseini (2017) argues that teacher educators should use simulations of teaching to support prospective teachers' development of MKT. Averill, Drake, Anderson, and Anthony (2016) used teaching rehearsals to improve pre-service teachers ability to orchestrate math discussions. Teaching rehearsals provide teacher candidates an opportunity to plan and implement a learning experience with their peers (Amador, 2017; Dotger, Masingila, Bearkland, & Dotger, 2015). Amador had prospective teachers record simulations of teacher actions and student thinking in small groups and share their videos with peers for review and development of noticing skills. Khalil, Gosselin, Hughes, and Edwards (2016) reported that mixed reality simulations improved prospective teachers' reformed-based teaching skills. Based on these results, the curriculum plan includes opportunities for teacher candidates to practice teaching skills through rehearsals and simulations.

Reflection. Researchers and teacher educators suggest that teacher candidates must engage in reflection of their learning experience (Felton & Koestler, 2015; Singh & Mabasa, 2015). McGalliard and Wilson (2017) documented that pre-service teachers benefit from reflection, especially in regards to their reasoning and how they are incorporating prior knowledge of mathematics. Cross and Bayazit (2014) reported on the use of double journal entries to help teacher candidates see the relationship between best practices discovered within course readings and practices they observe during field experiences. Based on the findings of this study, the curriculum plan has included double journal reflections (Cross & Bayazit, 2014) in an attempt to help teacher candidates reflect on their growth and build connections between theory and practice.

Project Description

A 16-week curriculum plan was developed to improve the mathematical preparation of elementary teacher candidates at the study site. The project is designed to replace the current 3-credit hour math and science strategies course with a standalone 2-credit hour math strategies course. The goal is to provide the results of the project study to the study site along with a proposal for course revision.

Necessary Resources, Existing Supports, and Potential Barriers

Resources and supports already exist at the study site to assist the implementation of this project. Since the study site is a college of education, routine reviews of curriculum are common. The administration is supportive of evidence-based recommendations to improve the preparation of teachers at the site. Therefore, all of the resources and supports are in place to assist in taking the proposed curriculum plan through the processes necessary to revise the current course.

A potential barrier to the project may come in the form of finding qualified faculty members to teach the proposed course. The knowledge required of teacher educators to prepare future teachers of mathematics is under-researched (Beswick & Goos, 2018). However, preliminary evidence does suggest that the work of mathematics teacher educators is complex and requires a knowledge base that extends beyond that needed by K-12 teachers of mathematics (Castro & Li, 2014; Muir, Wells, & Chick, 2017). Based on the findings of this study, it is unclear if the knowledge of the instructors was sufficient to develop prospective teachers to teach mathematics. A future area of research may be to assess the MKT levels of the instructors assigned to the math strategies courses. There are two potential solutions to the barrier of qualified instructors. One solution is to provide current faculty members opportunities to attend professional development sessions to improve their understanding of preparing mathematics educators and MKT. Another solution would be to conduct a search for a new faculty member experienced in mathematics education.

Proposal for Implementation and Roles of Stakeholders

A timeline for proposal has been incorporated into the Evaluation Matrix found in Appendix A. The proposal for implementation includes three phases. Year 1 of the timeline includes Phase 1, which will focus on planning and validation of developed curriculum. During Phase 1, college faculty members will work with program administrators to design curriculum components and verify with peer reviewers that the work meets quality standards. Phase 2, implementation, will start in the second year of the project. During Phase 2, program administrators will assess if the curriculum is being taught with fidelity and will begin to collect data on participant satisfaction and changes in MKT. The third phase will include the evaluation of the project during years 3-5. During Phase 3, program administrators, college faculty, clinical faculty, and district partners will evaluate pre-service teachers' ability to demonstrate effective instructional practices and impact of improved instruction on student achievement for graduates of the program.

Project Evaluation Plan

The evaluation of the revised math strategies curriculum for pre-service elementary teachers will follow an objectives-based approach. Objectives-based evaluation designs use pre-determined objectives to guide data collection (Spaulding, 2014). Program administrators at the study site will use formative data during the early phases to measure how well the project is meeting the intended goals. By gathering data during the planning and early implementation phases, program administrators will be able to provide necessary feedback to improve the implementation of the program to ensure that the program meets the stated objectives (Chen, 2015; Spaulding, 2014). A logic model can be found with the project in Appendix A that describes activities, outputs, and roles of stakeholders at various stages of the evaluation plan.

The goal of the revised math strategies curriculum is to provide pre-service elementary teachers an opportunity to develop MKT. The course goals and objectives address the findings of this study that pre-service teachers need to build confidence to teach math and more exposure to curriculum that develops the ability to recognize students' mathematical errors, provide mathematical explanations, anticipate student thinking, and select appropriate tasks to build mathematical understanding. The project evaluation goals will focus on evaluating new curriculum materials, fidelity of delivery of curriculum, satisfaction of stakeholders, and impact of instruction on student achievement. An overview of the evaluation plan, including objectives, questions, and data collection timetable can be found with the project in Appendix A.

The proposed evaluation will engage program administrators, college faculty, partner districts, clinical faculty, and pre-service teachers. Since the program is new, all identified stakeholders will be involved in the planning stage and design of the evaluation objectives (Chen, 2015; Spaulding, 2014). After the goals of the program are defined, select stakeholder groups will be tasked with the development of training materials. Various stakeholder groups will also be involved in the collection of data to measure validation of trainings, fidelity of delivery, participant satisfaction, and final outcomes.

Project Implications

The proposed curriculum plan has potential to lead to positive social change at the local site because it was developed to address the specific gap revealed by the analysis of data from this study. The project proposes curriculum revisions aimed at weaknesses revealed in prospective teachers MKT. The curriculum plan also addresses the lack of instructional activities backed by research shown to develop MKT in prospective teachers. The project has the potential to provide opportunities for prospective teachers to build confidence in teaching mathematics and develop stronger MKT levels to help their future students be successful in math. At the local level, the project has the potential to improve the quality of elementary teacher candidates produced when it comes to MKT.

On a larger scale, other universities may be positively influenced by the project. The evaluation plan incorporates peer reviewers from other universities. Sharing the design and development with other universities provides the potential for impact beyond the study site. Other universities may use the information to improve their teacher preparation programs. The partnership also has the potential to improve instructional practices with the goal of increased student achievement on a state level.

Section 4: Reflections and Conclusions

This section includes a discussion of the strengths and limitations of the proposed project, recommendations for alternative approaches to address the problem, and a reflection of my growth as a scholar while conducting this study. Also, I have included a description of the potential impact on positive social change at the study site and identified directions for future research. Lastly, a summary of the project is included.

Project Strengths and Limitations

MKT describes a teacher's understanding of mathematical concepts and the teaching of those concepts. To be successful teachers of mathematics, elementary teachers must have sufficient levels of CK and PCK. A lack of PCK negatively influences a teacher's practice, resulting in decreased levels of student achievement (Baki & Arslan, 2016; Hill et al., 2012; Leong et al., 2015). Developing the CK and PCK of prospective teachers is essential to promote the successful teaching and learning of mathematics. Therefore, it is critical that teacher education programs ensure that curriculum requirements intentionally address MKT development.

A strength of the proposed curriculum plan is that it intentionally focuses on the blending of CK and PCK in teacher training. The blending of content and instructional practices during teacher preparation has been shown to support the development of MKT more than addressing content in isolation (Auslander et al., 2016; Hoover et al., 2016; Son & Lee, 2016). Silverman and Thompson (2008) said that a teacher's MKT develops when they develop a deep understanding of content, create models of student thinking, and build ideas of instructional activities that support learner development. The proposed curriculum plan engages teacher candidates in the exploration of mathematical concepts with a focus on student thinking and instructional decisions or strategies to advance student thinking. I have included various evidence-based practices in the curriculum plan to support the development of MKT in prospective elementary teachers. However, limitations to the project do exist.

Two limitations of the project to be considered are qualifications of faculty and continued teacher support post-graduation. The knowledge to prepare K-12 teachers of mathematics is complex and requires a unique understanding of CK and PCK (Castro & Li, 2016; Muir et al., 2017). To be implemented successfully, the study site will need to ensure that either a qualified instructor is assigned to the course or be willing to seek out necessary training to prepare instructors. Another limitation of the proposed curriculum plan is that the project only addresses one course within the teacher preparation program. Achinstein and Davis (2014) said that new teachers need mentoring and content-focused mentoring is needed given the increased focused on educational content standards. One math methods course focused on all of the K-6 math content standards is not sufficient to fully prepare a teacher for the demands of teaching on their own. Regardless of the quality of preparation received, most new teachers will need some sort of additional support in their early years. Mentoring and additional training seems like potential options that the study site might be able to assist in supporting in-service teachers.

Recommendations for Alternative Approaches

Based on the work of this study, I have two recommendations for alternative approaches. One alternative approach to addressing the problem at the study site would be to investigate MKT changes and teacher candidate perceptions in math content courses. Based on the interview participants' statements about the Geometry and Measurements course, there may be some valuable information learned from the content and instruction within the math content courses to further aid the revision of the math strategies course.

Another alternative approach would be to reconsider the focus of the local problem. The current study focused on the problem of MKT development in prospective teachers. However, an alternative approach might consider the MKT of instructors in the math strategies course and potentially even the math content courses. Investigating the MKT levels of course instructors may provide additional insight regarding why MKT levels either change or do not change over the course of time for prospective teachers enrolled in the program.

Scholarship, Project Development and Evaluation, and Leadership and Change

Completing this project study has developed my skills as a scholar, practitioner, and project developer. I have learned how to use appropriate literature to identify a gap in practice, define a problem, and design research questions. Through the research process, I have expanded my understanding of how teacher educators can best support the development of future teachers. I have already noticed the impact of learning on evidence-based teaching practices during the research process within my own teaching practices with prospective teachers. As an adjunct instructor, I have worked at adapting some of the strategies to my current courses to help support learner development. As a project developer, I have learned how difficult a task it is to ensure that a course not only aligns with necessary standards but also includes appropriate assessments to document course objectives were met. Also, I have learned how the use of program evaluation by the teacher preparation provider is necessary to ensure that the proposed curriculum plan is implemented with fidelity and determine if goals are met. I will be able to use the knowledge gained from this experience to be a source of social change as I transition into my future role as a teacher educator, researcher, and leader.

Reflection on Importance of the Work

CK alone is insufficient to support the effective teaching of mathematics. Teacher training often falls short of developing mathematical pedagogical knowledge (NCTM, 2014; Teuscher et al., 2015). The findings of this study provide insight into the influence the curriculum and instruction had on developing MKT. MKT represents a teacher's knowledge of content and pedagogy; however, how prospective teachers develop MKT remains unclear. The work of this study is important because it not only confirmed a gap in practice, but also collected data to inform curriculum change and provided a few insights into how MKT might develop. The proposed curriculum plan is important because it incorporates evidence-based practices shown to develop MKT in prospective teachers. Future research on the influence the curriculum plan has on prospective teachers' mathematical knowledge has the potential to provide valuable insight into the current body of literature on MKT development.

Implications, Applications, and Directions for Future Research

My study has potential to impact positive social change at the organizational level. The teacher preparation program at the study site will be impacted because it encourages the reflective review of curriculum and instruction to prepare teacher candidates. The proposed curriculum plan has the potential to impact all future elementary teacher candidates through the improvement of their mathematical preparation to teach K-6 students. Curriculum improvements in the area of mathematics may encourage other revisions across the departments within the college.

I recommend that the study site conduct future research into the role of field experiences in the preparation of teachers. Jackson et al. (2018) stated that prolonged field experiences had a positive influence on PST's CK, confidence, and perceptions of struggling students. Investigating the potential of a math clinic, similar to the reading clinic already run by the study site, may help support teacher candidates' development of MKT.

Conclusion

Teacher preparation programs must be intentional about developing MKT in prospective elementary teachers. If teacher preparation curriculum fails to develop MKT in prospective teachers, then the potential exists for negative student achievement outcomes across the state as teachers may use ineffective teaching practices (Baki & Arslan, 2016; Leong et al., 2015). If they have not already done so, teacher preparation programs should assess the influence of current curriculum requirements on the development of MKT in their teacher candidates. The proposed project can be replicated and modified for similar teacher education programs to improve the preparation of teachers and positively impact student achievement in mathematics.

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Appendix A: Project

Proposed Math Strategies Course Syllabus

1. **ELED XXXX, CRN: XXXXX, 2 Credit hours**

2. **Instructor Information**
   E-mail:
   Office:
   Office Hours:

3. **Course Delivery Method:** Face-to-face.

4. **Class Days / Meeting Times:**

5. **Course Prerequisites and/or Co-requisites**
   45 credit hours and 2.5 GPA. Intermediate computer skills are expected.

6. **Catalog Description**
   A methods course focused on developing prospective teachers' knowledge to teach mathematics in an elementary setting. Course content will focus on the P-6 math content as defined by the math standards set by the State Department of Education. Teacher candidates will develop their understanding of math concepts through problem solving while also exploring ways that P-6 learners approach mathematical tasks and ways teachers can use different mathematical tasks and strategies to support understanding.

7. **Course Purpose / Goals**
   The purpose of this course is to help prospective teachers explore how P-6 students think and learn mathematical concepts. The overall goal of the course is to develop teacher candidates' confidence with mathematics so they can successfully develop, implement, and evaluate learning experiences that support student learning. Supporting course goals include deepening teacher candidates' understanding of elementary math content, elementary student thinking, and pedagogical approaches to support learner development.

8. **Student Learning Outcomes**
   The learning objectives for this course align with the eight effective teaching practices outlined by the National Council of Teachers of Mathematics (NCTM, 2014) and the InTASC Model Core Teaching Standards (CCSSO, 2011). At the conclusion of this course, teacher candidates will be able to:

   1. Establish goals for the mathematics that students are to learn. [InTASC 4e, 7a, 7b]

2. Implement tasks that promote problem solving and mathematical reasoning. [InTASC 4c, 4d, 5b]
3. Engage students in making connections between different mathematical representations. [InTASC 4a, 5f]
4. Facilitate mathematical discourse between students and teacher. [InTASC 3b, 4b, 5d, 8f]
5. Pose purposeful questions to assess and advance student understanding. [InTASC 4e, 8i]
6. Develop students' procedural fluency through conceptual exploration of math concepts. [InTASC 4c, 4d]
7. Provide students opportunities to struggle with mathematical ideas. [InTASC 4e, 7c]
8. Elicit and use evidence of student thinking to inform instructional decisions. [InTASC 6a, 6b, 6i, 7d, 7f]

9. Instructional Methods / Strategies

Instructional strategies include, but are not limited to: active learning strategies, expository-discussion, demonstration, modeling, peer learning, independent learning, group activities and problem based learning. Blackboard will be used to post assignments, announcements, quizzes, grades and to communicate with students. Candidates will apply computer skills and related technologies in both a team approach and individual responsibilities.

10. Learning Outcome Assessment Methods

This course is composed of learning modules with a variety of assessment types. The assessments are embedded within each module and include, but are not limited to, *in-class assignments, quizzes, discussion boards, journal entries, demonstrations, lesson plans, presentations, fieldwork, and competency assessments.* Also, a scoring rubric is available for each evaluation. See the course calendar for specific due dates for each assessment.

Competency Assessments

The College of Education ensures high quality teacher education graduates through implementation of a competency-based curriculum as required by the State. Initial level candidates (undergraduates) will demonstrate their competency through course-required assessments that meet the General Competencies for Licensure and Certification (2016). Failure to achieve an acceptable level of assessment for ALL elements of the required competency-based assessments for this course will result in a grade of "F" for the course. The 2016 General Competencies for Licensure and Certification are available on the Accreditation and Accountability webpage of the Office of Educational Quality and Accountability.

11. Instructional Materials

a. The approved textbook for this course is ***Elementary and Middle School Mathematics: Teaching Developmentally*** by Van de Walle, Karp, & Bay-Williams (10th Edition)
b. Students must have access to a computer and Internet outside of class time in order to successfully complete this course. Several computer labs are available for student use in the library, the Business & Technology building and the Education building.
c. COE Education majors will be required to use their Chalk and Wire accounts.

12. Grading Scale

Grades for this course are assigned according to the following scale:

90%-100% of Total	A
80%-89% of Total	B
70%-79% of Total	C
60%-69% of Total	D
59% and below of Total	F

13. Class and Instructor Policies

Details of the instructor policies and expectations are also explained in the Learning Contract. The Learning Contract is available on Blackboard, and students will submit a signed copy to the instructor during the first week of class. Candidates are expected to complete all assignments, read the assigned materials, participate in discussions and team decision-making and follow through with decisions, complete assignments, and contribute to the classroom assignments. Candidates are expected to complete every assignment, activity, test and administrative requirement of the College of Education if they are to receive a grade in the course.

All students enrolled are expected to exhibit professional attitudes. Any form of academic misconduct will not be tolerated (see Academic Policies). Missing more than three face-to-face meetings, or three or more late arrivals and/or early departures, will result in the lowering of the overall course grade by one letter grade. It is the student's responsibility to get any information, materials, or assignments missed from their absence.

Any project, quiz, or assignment not turned in could result in a full course letter grade lowered. Assignments missed during face-to-face classes due to absences will not be allowed to be made up for credit. As a general rule, assignments submitted 2 days late will be assessed a 20% penalty and an additional 10% for each day, up to 7 days. Assignments received more than 7 days late will not be accepted for course points. It is the instructor's discretion to determine if late course assignments will be accepted or provided credit.

14. Diversity Statement

One of the goals of education is to provide an equal opportunity for all students to learn. Diversity and global education is an approach to teaching and learning based upon democratic values and beliefs. This approach strives to ensure a safe, welcoming, and inclusive classroom environment for students of all races, ethnicities, sexual orientations, gender identities/variances, ages, religions, language backgrounds, economic classes, and ability statuses. As such, students are encouraged to use language, communications, and basic interaction techniques that are respectful, inclusive, representative, and culturally appropriate. Faculty will strive to establish classes, coursework, and activities that respect the diverse backgrounds of participants.

Tentative 16-Week Course Calendar

Week	Session	Topic	To Read/ To Do	Assignments Due
Module 1: The Teaching of Mathematics				
Week	Session	Topic	To Read/ To Do	Assignments Due
1	A	Course Introduction, Syllabus	**Read:** Chap 1 in Van de Walle **Do:** Reflection 1: My math experience **Do:** Double Journal Entry	
	B	Teaching Mathematics in the 21st Century	**Read:** Chap 2 in Van de Walle **Do:** Math teacher observation & interview (*due by end of Week 8*) **Do:** Double Journal Entry	Reflection 1: My math experience
2	A	What it Means to Know and Do Math	**Read:** Chap 3 in Van de Walle **Do:** Double Journal Entry	
	B	Focusing on Problem Solving	**Read:** Chap 4 &5 Van de Walle **Do:** Reading Quiz 1 **Do:** Problem Solving Lesson Critique **Do:** Double Journal Entry	Reading Quiz 1
3	A	Planning & Assessment	**Read:** Chap 6 in Van de Walle **Do:** Double Journal Entry	
	B	Equitable Mathematics for All Students	**Read:** Chap 7 in Van de Walle **Do:** Reading Quiz 2 **Do:** Double Journal Entry	Problem Solving Lesson Critique
Module 2: Developing Number Concepts				
Week	Session	Topic	To Read/ To Do	Assignments Due
4	A	Developing Number Sense	**Read:** Chap 8 in Van de Walle **Do:** Double Journal Entry	Math Talk 1
	B	Meanings for the Operations	**Read:** Chap 9 in Van de Walle **Do:** Double Journal Entry	Math Talk 2
5	A	Developing Fact Fluency	**Read:** Chap 10 in Van de Walle **Do:** Double Journal Entry **Do:** Reading Quiz 3	Math Talk 3 Reading Quiz 3
	B	Whole Number Concepts	**Read:** Chap 11 in Van de Walle **Do:** Double Journal Entry	Math Talk 4 Place Value Demonstrations
Module 3: Exploring the Basic Operations				
Week	Session	Topic	To Read/ To Do	Assignments Due
6	A	Strategies for Addition & Subtraction	**Do:** Implement a Math Task with K-6 students (*due by end of Week 12)*	Math Talk 5
	B	Strategies for Addition &	**Do:** Reading Quiz 4	Reading Quiz 4

Week	Session	Topic	To Read/ To Do	Assignments Due
		Subtraction		
7	A	Micro-teaching Simulations	**Read:** Chap 12 in Van de Walle **Do:** Double Journal Entry	Individual Math Lesson Simulation
	B	Strategies for Multiplication & Division	**Do:** Select & Prepare task for Peer Simulation	Math Talk 6
8	A	Strategies for Multiplication & Division	**Do:** Reading Quiz 5	Reading Quiz 5
	B	Micro-teaching Simulations	**Read:** Chap 13 in Van de Walle **Do:** Double Journal Entry	Individual Math Lesson Simulation
		Module 4: Exploring Algebra & Fractions		
Week	Session	Topic	To Read/ To Do	Assignments Due
9	A	Algebra in the Elementary Grades	**Do:** Select & Prepare task for Peer Simulation	Math Talk 7
	B	Algebra in the Elementary Grades	**Do:** Select & Prepare task for Peer Simulation	
10	A	Micro-teaching Simulations	**Read:** Chap 14 in Van de Walle **Do:** Double Journal Entry	Individual Math Lesson Simulation
	B	Developing Fraction Concepts	**Do:** Case Study Teaching & Learning Analysis	Math Talk 8
11	A	Developing Fraction Concepts	**Read:** Chap 15 in Van de Walle **Do:** Double Journal Entry	Case Study Teaching & Learning Analysis
	B	Developing Fraction Operations	**Do:** Select & Prepare task for Peer Simulation	Math Talk 9
12	A	Developing Fraction Operations	**Do:** Reading Quiz 6	Reading Quiz 6
	B	Micro-teaching Simulations	**Read:** Chap 16 & 17 in Van de Walle **Do:** Double Journal Entry	Implement a Math Task with K-6 students
		Module 5: Investigating Rations, Geometric Patterns, & Data Analysis		
Week	Session	Topic	To Read/ To Do	Assignments Due
13	A	Exploring Ratios & Proportions	**Read:** Chap 18 in Van de Walle **Do:** Double Journal Entry	Math Talk 10
	B	Developing Measurement Concepts	**Read:** Chap 19 in Van de Walle **Do:** Double Journal Entry	
14	A	Exploring Geometric Concepts	**Read:** Chap 20 in Van de Walle **Do:** Reading Quiz 7	Reading Quiz 7 Math Talk 11
	B	Micro-teaching Simulations	**Do:** Analyze Teaching & Learning Observation	
15	A	Developing Concepts of Data Analysis	**Read:** Chap 21 in Van de Walle **Do:** Double Journal Entry	Math Talk 12
	B	Exploring Concepts of Probability	**Do:** Final Reflection: My growth as a mathematician	Final Reflection: My growth as a mathematician
16	A	Final		Final Assessment

Proposed Course Alignment Matrix

Course Objectives (CO) [NCTM & InTASC Alignment]				
1. Establish goals for the mathematics that students are to learn. [InTASC 4e, 7a, 7b] 2. Implement tasks that promote problem solving and mathematical reasoning. [InTASC 4c, 4d, 5b] 3. Engage students in making connections between different mathematical representations. [InTASC 4a, 5f] 4. Facilitate mathematical discourse between students and teacher. [InTASC 3b, 4b, 5d, 8f] 5. Pose purposeful questions to assess and advance student understanding. [InTASC 4e, 8i] 6. Develop students' procedural fluency through conceptual exploration of math concepts. [InTASC 4c, 4d] 7. Provide students opportunities to struggle with mathematical ideas. [InTASC 4e, 7c] 8. Elicit and use evidence of student thinking to inform instructional decisions. [InTASC 6a, 6b, 6i, 7d, 7f]				
Module	**Module Objectives [CO Alignment]**	**Assessments [Module Objective]**	**Learning Materials**	**Activities/ Instructor Notes**
Module 1: The Teaching of Mathematics	1. Investigate connections between learning theories and effective teaching practices. [CO 1-8] 2. Determine if math tasks promote problem solving and procedural fluency. [CO 2 & 3] 3. Explain ways to engage students in mathematical discourse. [CO 4 & 5] 4. Design lessons focused on mathematical inquiry. [CO 2, 3, & 7] 5. Differentiate between summative and formative assessment. [CO 8] 6. Differentiate instruction to meet needs of all learners. [CO 1]	Journal Reflections [MO 1, 2, 3, 4, 5, 6] Problem Solving Lesson Critique [MO 2, 4] Math Teacher Observation & Interview [MO 1, 2, 3, 4, 5, 6] Reading Quizzes [MO 1, 2, 3, 4, 5, 6]	Van de Walle Textbook Chapters 1-6 NCTM Effective Teaching Practices & Beliefs [Handout] NCTM Case Studies Videos of Teaching & Learning from Van de Walle text	Demonstrate Effective Teaching Practices Problem Solving: • Model use of math tasks Facilitate Discourse • Model Number Talks Recommended Reads: Mathematical Mindsets (Boaler, 2016) Cognitively Guided Instruction (Carpenter et al., 2014) Math Talks (Parrish, 2014)

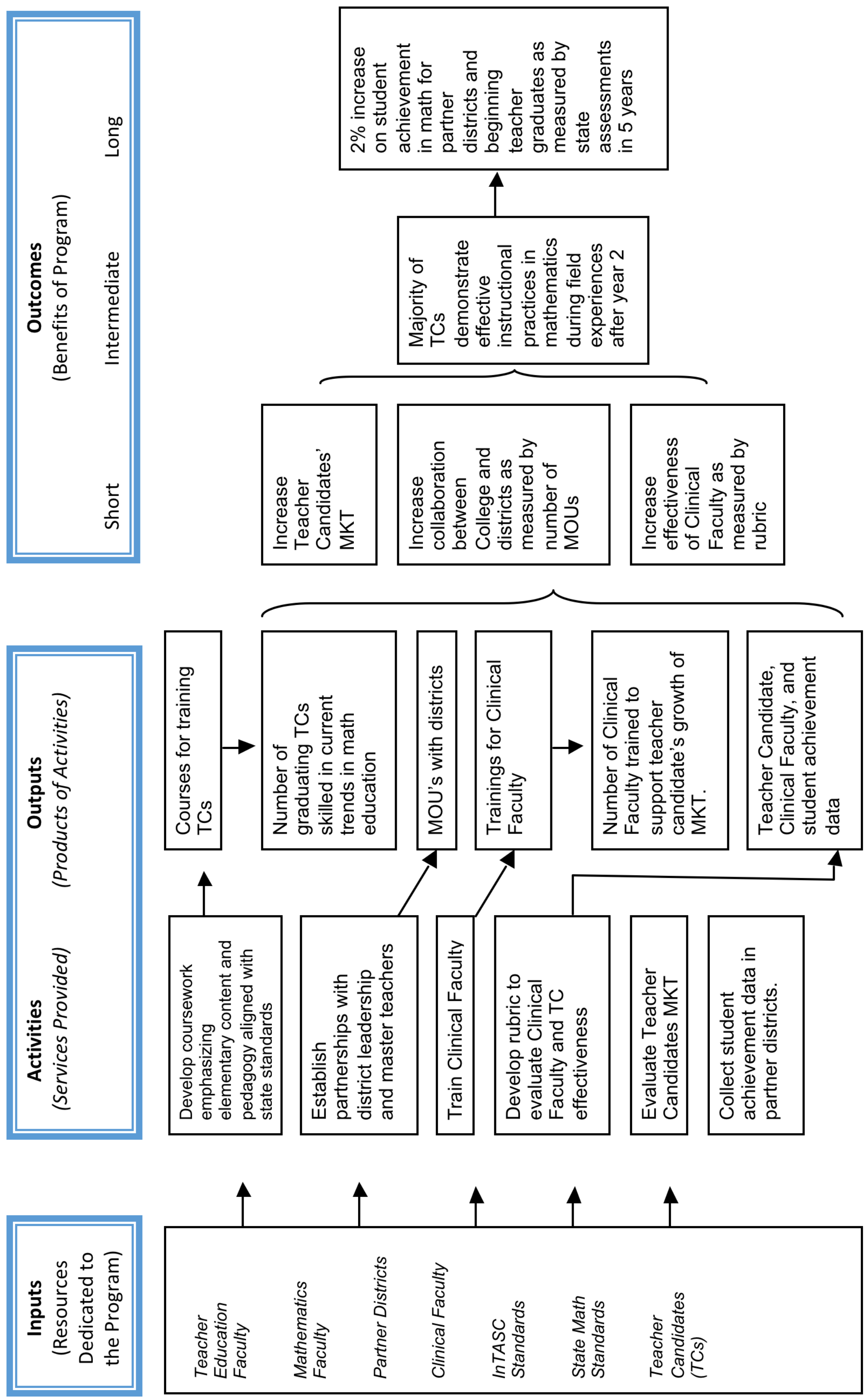
Module	Module Objectives [CO Alignment]	Assessments [Module Objective]	Learning Materials	Activities/ Instructor Notes
Module 2: Developing Number Concepts	1. Demonstrate how to develop counting skills. [CO 2] 2. Explain how students can apply the properties of the operations as strategies. [CO 1, 3, & 6] 3. Describe approaches to develop fact fluency. [CO 2 & 3] 4. Justify an effective approach for reinforcing fact fluency. [CO 2 & 3] 5. Demonstrate how to develop student's skills in place value through the use of base-ten models. [CO 2, 3, & 6]	Journal Reflections [MO 1, 2, 3, 4, 5] Reading Quiz [MO 1, 2, 3, 4, 5] Lead Math Talk Simulation [MO 1] (*on-going throughout course)* Paired Place Value Base-Ten Demonstrations [MO 5]	Van de Walle Textbook Chapters 7-10 NCTM Case Studies Videos of Teaching & Learning Samples of Student Work Base-ten manipulative	Demonstrate Effective Teaching Practices Problem Solving: • Model use of math tasks • Engage candidates in exploration of number concepts using problem solving tasks Facilitate Discourse • Model Selecting & Sequencing to lead discussion about math tasks Show videos of student thinking about operations using different strategies [MO 2] Use samples of student work for candidates to practice identifying strategy use and to recommend interventions [MO 2, 4]

Module	Module Objectives [CO Alignment]	Assessments [Module Objective]	Learning Materials	Activities/ Instructor Notes
Module 3: Exploring the Basic Operations	1. Explain multiple strategies for addition and subtraction with multidigit numbers. [CO 2, 3, 6, & 7] 2. Identify a variety of models and recording approaches for multiplication and division. [CO 2, 3, 6, & 7] 3. Show a strategy to teach one of the basic operations. [CO 1, 2, 3] 4. Demonstrate ways to teach estimation as a way to develop flexibility and recognize reasonable answers. [CO 2 & 3] 5. Analyze student thinking about basic operations. [CO 8] 6. Explain ways to engage students in mathematical discourse. [CO 4 & 5] 7. Design lessons focused on mathematical inquiry. [CO 2, 3, & 7]	Journal Reflections [MO 1, 2, 4] Reading Quiz [MO 1, 2, 3] Lead Math Talk Simulation [MO 6] (*on-going throughout course)* Individual Lesson Simulation [MO 3, 4, 7] Analyze Teaching and Learning with Observation Protocol [MO 5] Implement a Task with K-6 students [MO 5]	Van de Walle Textbook Chapters 11-12 NCTM Case Studies Videos of Teaching & Learning Samples of Student Work Manipulatives	Demonstrate Effective Teaching Practices Problem Solving: • Model use of math tasks • Engage candidates in exploration of operations using problem solving tasks Facilitate Discourse • Model Selecting & Sequencing to lead discussion about math tasks Show videos of student thinking about operations using different strategies [MO 1, 2, 5] Use samples of student work for candidates to practice identifying strategy use and to recommend interventions [MO 2, 4]

Module	Module Objectives [CO Alignment]	Assessments [Module Objective]	Learning Materials	Activities/ Instructor Notes
Module 4: Exploring Algebra & Fractions	1. Illustrate how to infuse teaching of patterns and functions into K-6. [CO 2 & 3] 2. Show examples of fraction models. [CO 2, 3, 6, & 7] 3. Demonstrate strategies to teach comparing fractions conceptually. [CO 2, 3, 6, & 7] 4. Demonstrate a process for teaching fraction operations with understanding. [CO 2, 3, 6, & 7] 5. Connect whole-number multiplication and division to fractions with meaningful contexts. [CO 2, 3, 6, & 7] 6. Explain ways to engage students in mathematical discourse. [CO 4 & 5] 7. Analyze teaching and learning of mathematics. [CO 8]	Journal Reflections [MO 1, 2, 3, 4, 5] Reading Quiz [MO 1, 2, 3, 4, 5] Lead Math Talk Simulation [MO 6] (*on-going throughout course)* Individual Lesson Simulation [MO 1, 2, 3, 4, 5] Analyze Teaching and Learning with Observation Protocol [MO 7] Problem-Solving Investigation-Individual Presentation [MO 3]	Van de Walle Textbook Chapters 13-17 NCTM Case Studies Videos of Teaching & Learning Samples of Student Work Manipulatives	Demonstrate Effective Teaching Practices Problem Solving: • Model use of math tasks • Engage candidates in exploration of operations using problem solving tasks Facilitate Discourse • Model Selecting & Sequencing to lead discussion about math tasks Show videos of student thinking about operations using different strategies [MO 1, 2, 5] Use samples of student work for candidates to practice identifying strategy use and to recommend interventions [MO 2, 4]

Module	Module Objectives [CO Alignment]	Assessments [Module Objective]	Learning Materials	Activities/ Instructor Notes
Module 5: Investigating Geometric Patterns & Data Analysis	1. Explain how fractions are related to decimals and percents. [CO 2 & 3] 2. Illustrate research-based methods to teach proportional reasoning. [CO 2, 3, 6, & 7] 3. Explain development of area, volume, and measurement models. [CO 2, 3, 6, & 7] 4. Describe best model for teaching elapsed time. [CO 2 & 3] 5. Analyze strategies for teaching geometric concepts. [CO 1 & 2] 6. Describe appropriate ways for students to analyze and represent data. [CO 1, 2, & 3] 7. Explain ways to engage students in mathematical discourse. [CO 4 & 5] 8. Analyze teaching and learning of mathematics. [CO 8]	Journal Reflections [MO 1, 2, 3, 4, 5, 6] Reading Quiz [MO 1, 2, 3, 4, 5, 6] Lead Math Talk Simulation [MO 7] (*on-going throughout course)* Individual Lesson Simulation [MO 2, 3, 4] Analyze Teaching and Learning with Observation Protocol [MO 5, 8]	Van de Walle Textbook Chapters 18-21 NCTM Case Studies Videos of Teaching & Learning Samples of Student Work Manipulatives	Demonstrate Effective Teaching Practices Problem Solving: • Model use of math tasks • Engage candidates in exploration of operations using problem solving tasks Facilitate Discourse • Model Selecting & Sequencing to lead discussion about math tasks Show videos of student thinking about operations using different strategies [MO 1, 2, 5] Use samples of student work for candidates to practice identifying strategy use and to recommend interventions [MO 2, 4]

Logic Model: Evaluation of Revised Math Strategies Curriculum



Revised Math Strategies Curriculum Evaluation Matrix

Type of Objective	Evaluation Objective: Question(s)	Stakeholders	Data Collection Tools	Timeline: Data collection	Timeline: Dissemination of Information
Capacity-Intent	By year two of the program, faculty members will have designed 100% of the new mathematics coursework for prospective elementary teachers (PSTs) enrolled at the college.	Program administrators and college faculty members.	Surveys and interviews.	Year 1 Month 9	Formative data at stakeholder meeting at the end of Year 1. Summative data at the end of the 5-year period.
	Are all materials ready for implementation?				
Validation	By year two of the program, program administration will verify that math coursework designed for PSTs aligns with InTASC standards and meet quality standards set by peer reviewers.	Program administrators, college faculty, and college faculty at different sites.	Surveys, observations, checklists, and interviews.	Year 1 Month 11	Formative data at stakeholder meeting at the end of Year 1. Summative data at the end of the 5-year period.
	To what extent do the courses cover all of the skills addressed by the InTASC standards? To what extents do peer reviewers from similar colleges consider the courses to be well designed?				
Activity Fidelity	By the end of year two of the program, program administration will assess the quality of instruction and fidelity of delivery of the program curriculum.	Program administrators, college faculty, and college faculty at different sites.	Surveys, checklists, and interviews.	Years 2-5 Each semester.	Formative data at stakeholder meeting at the end of each semester. Summative data at the end of the 5-year period.
	To what extent are the courses being delivered as designed?				

Participant Satisfaction	At the conclusion of each semester, program administration will document PST's satisfaction of the math coursework. Are PSTs satisfied with their math preparation? What do PSTs feel is missing from their math courses?	Program administrators, college faculty, and PSTs.	Surveys, focus groups, and interviews.	Years 2-5 Twice, once each semester.	Formative data at stakeholder meeting at the end of each semester. Summative data at the end of the 5-year period.
Intermediate Outcomes	After completing the course, program administration will assess PST's ability to demonstrate effective instructional practices during field experiences. What behaviors are PSTs demonstrating based on the program training? What practices are PSTs struggling to implement within the classroom?	Program administrators, college faculty, clinical faculty, and PSTs.	Surveys, checklists, site visits, observations, focus groups, and interviews.	Years 3-5 During PST's field experiences.	Formative data at stakeholder meeting at the end of each semester. Summative data at the end of the 5-year period.
End Outcomes	Five years after program implementation, program administration will assess the impact PST training has had on student achievement in math within partner districts and classrooms of graduates. Did changes to curriculum support student achievement at clinical sites? Has student achievement increase at least 2% for beginning teachers that graduated from the program?	Program administrators, partner districts, and beginning teacher graduates.	Current and archival student achievement data from state math assessments.	Ongoing Years 2-5	Formative data at stakeholder meeting at the end of each semester. Summative data at the end of the 5-year period.
Sustainability	At the conclusion of each year, program administration will determine the financial need to sustain each component of the program. To what extent can the elementary math program be sustained given current funding? What external sources of funding are in place?	Program administration and college faculty.	Interviews and letters of support.	Ongoing	Formative data at stakeholder meeting at the end of each semester. Summative data at the end of the 5-year grant period.

Appendix B: Permission Granted for Use of MKT Measures

Gmail

Jason Proctor <[redacted]@gmail.com>

Teacher Knowledge Assessment System (TKAS) - Training Approved

1 message

noreply@qualtrics.com <noreply@qualtrics.com> Tue, Jan 16, 2018 at 6:24 PM
Reply-To: [redacted]@umich.edu
To: [redacted]@gmail.com

Dear Jason Proctor,

You have been approved for LMT-TKAS online training. Here is your login information:

Your email address is: [redacted]@gmail.com

Your password is: [redacted]

Please save this information in a safe place for future reference.

Thank you for using the TKAS system!

Gmail

Jason Proctor <[redacted]@gmail.com>

Teacher Knowledge Assessment System (TKAS) - Trained User Information

1 message

noreply@qualtrics.com <noreply@qualtrics.com> Thu, Jan 18, 2018 at 2:01 PM
Reply-To: [redacted]@umich.edu
To: [redacted]@gmail.com

Dear Jason Proctor,

You are receiving this email as confirmation of your completion of the mandatory modules for the LMT/TKAS online training.

To access the measures, please go to the TKAS website at https://az1.qualtrics.com/apps/harvard-tkas. From TKAS, you can access the measures to download and use as paper and pencil forms, or you may choose instead to create an assessment plan for administration through TKAS.

Your access code is [redacted]

Your email address is [redacted]@gmail.com

Your password is the one you were given when you were approved for training or whatever you reset it to, if you reset it.

Please save this information in a safe place for future reference.

Appendix C: Teacher Candidate Interview Protocol

I. Introduction

A. Thanks for agreeing to take part in this interview. The information gathered from this interview will be used to develop an understanding of how the curriculum presented in the math and science strategies course develops knowledge in prospective teachers to teach elementary mathematics. This interview will be recorded to ensure accuracy. I assure you that all forms of identification will be removed from the data to protect your identity and privacy. At any time in the interview if you do not wish to answer a question or want to discontinue the conversation, feel free to do so.

1. Will you please review this consent form? (Inform participant they can keep this for their records).
2. Do you have any questions before we start recording and begin the interview?

II. Demographics

A. To begin, I would like to ask some general questions about you and your math background.

1. Could you please confirm your name and email contact information?

2. How long have you been a student at the university?

3. What degree are you currently seeking?

4. What grade level of student are you most interested in teaching?

5. What was the highest level of math you took in high school?

6. What math courses have you taken at the university level?

7. Could you tell me about your experience in the math and science strategies course?

8. What stood out to you as the major focus of the math portion of the strategies course?

9. Could you describe the different types of knowledge you think you might need to teach math in an elementary classroom?

The next portion of the interview will focus on specific types of knowledge used when teaching mathematics. I will present some open-ended prompts (Hill et al., 2004) of classroom scenarios and ask you to reflect on your experience in the math strategies course.

III. Specialized Content Knowledge

A. Consider the knowledge needed by the teacher in this prompt to recognize errors in student thinking:

> You are working individually with Bonny, and you ask her to count out 23 checkers, which she does successfully. You then ask her to show you how many checkers are represented by the 3 in 23, and she counts out 3 checkers. Then you ask her to show you how many checkers are represented by the 2 in 23, and she counts out 2 checkers. What problem is Bonny having here?

1. How would you describe the content or instruction presented in the strategies course influenced the development of your ability to recognize student errors?

2. In what ways do you feel the course developed your understanding of mathematical topics?

IV. Knowledge of Content and Students

A. Consider the knowledge a teacher may need to anticipate student approaches to particular problems similar to this prompt:

> Mr. Garrett's students were working on strategies for finding the answers to multiplication problems. Which … strategies would [he] expect to see some elementary school students using to find the answer to 8 x 8?

1. Please describe any components of the strategies course that helped you develop a sense of how students may understand math content?

2. In what ways do you feel the strategies course helped develop your ability to anticipate different student approaches to math?

3. In what ways do you feel the strategies course helped you anticipate common student errors?

V. Knowledge of Content and Teaching

A. Lastly, consider the knowledge a teacher might need when selecting, designing, or sequencing tasks to promote student learning.

(Lower Elem Prompt)
To introduce the idea of grouping by tens and ones with young learners, which of the following materials or tools would be most appropriate? (Circle ONE answer.)

a) A number line
b) Plastic counting chips
c) Pennies and dimes
d) Straws and rubber bands
e) Any of these would be equally appropriate for introducing the idea of grouping by tens and ones.

(Upper Elementary Prompt)
Ms. Williams plans to give the following problem to her class:

Baker Joe is making apple tarts. If he uses ¾ of an apple for each tart, how many tarts can he make with 15 apples?

Because it has been a while since the class has worked with fractions, she decides to prepare her students by first giving them a simpler version of this same type of problem. Which of the following would be most useful for preparing the class to work on this problem?

a) Baker Ted is making pumpkin pies. He has 8 pumpkins in his basket. If he uses ¼ of his pumpkins per pie, how many pumpkins does he use in each pie?

b) Baker Ted is making pumpkin pies. If he uses ¼ of a pumpkin for each pie, how many pies can he make with 9 pumpkins?

c) Baker Ted is making pumpkin pies. If he uses ¾ of a pumpkin for each pie, how many pies can he make with 10 pumpkins?

1. Can you describe how your knowledge of different types of activities to support student learning developed during the strategies course?

2. What components of the course were helpful in developing your ability to identify, select, and sequence different teaching strategies?

VI. Closing

A. I know your time is valuable, so I want to thank you again for taking the time to participate in this interview. Thank you for sharing such valuable information. Before we end the interview,

    1. Is there anything else that you would like to add about your experience in the strategies course or in general about your preparation to teach elementary mathematics?

Thank you again for your time. As I analyze the data, I may reach out to you to verify that my interpretations of your responses are portrayed accurately. If you have any questions, feel free to contact me.

Appendix D: Sample Transcript Coding & Thematic Analysis

Table D1

Sample In-Vivo codes

Codes	Source	Transcript Evidence
"walk us through" "could use in a third grade classroom" "like your students would" "think like an elementary student" "mistakes students make" "makes sense"	Candidate A	"Um, I think like just doing it, you know, we would, um we would, obviously it was modified for the college level. But, you know, she would explain to us and walk us through like this is something you could use in a third grade classroom and this is how you would use it. You know, and she would walk us through using it and then she's like, now you use the manipulative. So it was like us actually doing it. Like, at the collegiate level and then we would take it down and she's like now, like, let's work on some problems like your students would. So we were actually doing them ourselves. Which that was like, oh, this makes sense to me, you know. Like I'm, I'm, she would essentially help us think like an elementary student, and she was like this is, these are common mistakes that these students make and this is why. I was like, oh, hey, like, that makes sense. So there was a lot of that that we could actually understand like, why they, like why they think this way. It's like that makes sense to me. "
"haven't learned a lot" "learning how to teach students different methods" "haven't used it at all"	Candidate B	"No not really, the professor is nice. Um, we, I mean, I haven't learned a whole lot of actual strategies in it. It's a methods course. And so we're supposed to be, essentially, learning how to teach our students different methods of math in the classroom and a lot of the course has just been doing lesson plans that, you know, we might not even need for our, for our grade level that we're wanting to teach. Like I could come in, and I could come into class with a grade... pre algebra lesson plan, whenever I really want to teach third grade, and it's not really going to help me. It's just something that was a really quick and easy to find. Also, we have the math book that we haven't used it at all. We also have the scientific and we haven't use that at all."

Table D2

Themes with grouped code categories

Teacher candidates perceived that the content and instruction of the math strategies course had minimal influence on their development of MKT.	Teacher candidates perceived that the content and instruction of a Geometry and Measurements course had a positive influence on their development of MKT.	Teacher candidates express unpreparedness to teach math and a desire for additional training.
Perception of math strategies course	Perceptions of Geometry & Measurement course	Perception of math and readiness to teach
Failure to develop MKT	Influence of Instructor (Professor A)	Math Anxiety
Intrinsic motivation	Support of MKT development	Suggestion for course revision
Support of MKT development	Impact of instruction related to MKT	Future desires as teacher