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Elementary Teachers' Perceptions of Mathematics Instruction in Montessori and Traditional Classrooms

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Abstract

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Doctoral Study Submitted in Partial Fulfillment of

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Abstract

Students in grades 3 and 4 attending a traditional public elementary school in a northeastern state did not meet proficiency levels in mathematics as measured by the state's assessment system. Published reports indicated that students attending the Montessori programs were more proficient in solving math problems compared to students in traditional schools. However, researchers had not compared Montessori and traditional teachers' perceptions of teaching elementary mathematics. The purpose of this qualitative case study was to explore the perceptions of traditional and Montessori teachers regarding teaching basic problem-solving skills in mathematics. Koehler and Grouws' model provided the theoretical framework. Data collection included semistructured interviews with 6 traditional and 4 Montessori elementary teachers, field notes, and journaling. Data were analyzed using a coding scheme that incorporated the theoretical model's categories. Findings indicated that both groups of teachers reported that concrete (manipulatives) to abstract (pen and paper) learning was an effective approach to teaching basic math concepts and problem-solving skills. Social change will be realized when struggling elementary students in both Montessori and traditional settings begin to meet proficiency levels in mathematics and benefit from instruction that balances concrete and abstract learning skills. As such, students will be able to explore, develop, and become more actively engaged in learning math and problem solving in all elementary grades. The project deliverable, a position paper supporting the principal theme of concrete to abstract learning, may be used to promote effective instructional practices in mathematics, hence, positive social change.

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Dedication

I dedicate this doctoral study to God Who can do immeasurably more than all I can ask, think, hope, dream, or imagine. Because of God's faithfulness and grace, I have been able to complete this doctoral program, and so to Him I especially give thanks.

Acknowledgments

For the successful completion of this project study, I extend my thanks and appreciation to my chair, committee member, and URR for their scholarly guidance as well as to the local schools and teachers for their cooperation and participation. I also extend my gratitude to my mentors for their valuable support and to my family, friends, and colleagues for their steadfast love and prayers. Thank you all for your assistance and encouragement on this journey. We made it!

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Section 1: The Problem

Mathematics is a major subject in education and a critical part of elementary school academic preparation for children in the early stages of development (Morrison, 2011). Mathematics is also vital for college and career readiness. For U.S. students to compete globally in science, technology, engineering, and mathematics (STEM) careers, they must demonstrate competency in mathematics equal to or above their international peers (Aud et al., 2010; Lehman, 2013). The classroom teacher is the most effective tool for impacting student learning (Bigham, Hively & Toole, 2014).

In one school district in the Northeastern United States, two approaches are used for teaching elementary mathematics: Montessori education and traditional elementary education. Positive academic achievement outcomes may result from either approach (Jordan & Kaplan, 2007; Li & Yang, 2010; Ryniker & Shoho, 2001; Torrence, 2012). Montessori educators emphasize preparing students for applying mathematics in their lives and careers and building their confidence and interest in the subject (Chen, Mccray, Adams, & Leow, 2014; Morrison, 2011; Torrence, 2012). Although many researchers of Montessori programs maintain that these programs give students academic advantages, it has not been determined whether Montessori methods offer greater academic benefits than traditional methods in the core subject of mathematics (Reed, 2008).

The purpose of this project study was to explore the beliefs and instructional practices of Montessori elementary teachers and traditional elementary teachers regarding their perceptions of teaching mathematics. Findings may be used to improve the understanding of effective approaches to mathematics instruction in the elementary setting. Briley (2015) noted that teachers' beliefs may affect their instructional practices and may play a significant role in their classroom behaviors. A clearer picture of elementary Montessori and traditional teachers' perceptions and beliefs of math instruction was needed.

Lopata, Wallace, and Finn (2005) argued that Montessori and traditional education programs differ in terms of instructional practices and attitude of teachers. More research was needed to explore and compare the perceptions and instructional practices of teachers from both educational programs. Findings may expand the knowledge base and understanding of these differences and yield rich descriptions of mathematical approaches from the perspectives of Montessori and traditional elementary teachers.

In Section 1, the broad overview of the study is outlined, beginning with the introduction and background of the study. This section included the local problem, study rationale, and significance. In addition, definitions of terms, research questions, a literature review, and a summary are presented in this section.

Definition of the Local Problem

For the school years 2013 and 2014, third and fourth graders at a traditional elementary school (School A) did not meet the state proficient level in mathematics as measured by the Pennsylvania System of School Assessment (PSSA). A Montessori elementary school (School B) in the same district had positive results. Under the No Child Left Behind Act (NCLB) of 2001, Pennsylvania set the target percentage of students who must meet or exceed scores at the proficient level in mathematics at 35%. The state target for 2011-2012 was 67%, the target for 2012-2013 was 78%, and the target for 2013-2014 was 89%. By 2014-2015, every school was expected to meet the state standard of 100%.

School A was among the 25% of state schools that failed to meet its adequate yearly progress (AYP) goals during school years 2013 and 2014. As a result, the school's AYP status declined from the initial warning status, when the school was first identified, to Corrective Action II status, the lowest status. A school's status level changes from year to year depending on whether AYP goals are met. This problem was a concern for the district's school administrators, who searched for solutions. The administrators supported the idea that a variety of teaching methods should be explored to help students succeed, including a Montessori approach. A Montessori program was approved and implemented several years ago in another school (School B) in the district. Students from School B repeatedly scored at the proficient level on the state standards in math.

Rationale

Evidence of the Problem at the Local Level

Early mathematics proficiency is important for a student's future academic endeavors. If difficulties in learning mathematics are not remedied in the lower grades, these difficulties may result in students struggling in mathematics throughout their education (Chiu & Linn, 2012). The purpose of the study was to explore traditional and Montessori teachers' perceptions and classroom practices in teaching mathematics. Although students from the Montessori school had success, the perceptions and instructional practices of these teachers were unclear. In addition, it was not clear how these practices influenced students' math achievement compared with students from the traditional school. Findings from the study may provide the district with information to help all students achieve mathematics proficiency in concert with state and federal mandates.

Evidence of the Problem from the Professional Literature

Researchers have recognized that teachers' beliefs can influence how they approach teaching mathematics and use classroom resources (Polly et al., 2013). Polly et al. (2013) contended that more research is needed to explore teachers' instructional practices, their beliefs towards mathematics teaching, and student achievement. The elementary school is the foundation of mathematics education, and the straightest path to improving mathematics achievement in elementary schools is through improved teaching (Erhan, Akkoç, Ozmantar, & Demir, 2011; Fast & Hankes, 2010; McKinney, Chappell, Berry, & Hickman, 2009; Spelman & Rohlwing, 2013). Elementary school teachers take on an influential role in this regard (Hart & Swars, 2009; Lee & Johnston-Wilder, 2012). Lee and Johnston-Wilder (2012) stressed that for students to be efficient in learning mathematics, teachers must understand students' manner, process, or style of learning mathematics and adjust the approach according to students' needs.

Nurlu (2015) argued that math teachers should aim to empower students with mathematical knowledge and abilities that can be applied in everyday life. Strategies based on problem-solving approaches in teaching math help strengthen the student's ability to develop abstract, logical, critical thinking, and become efficient problem solvers (Nurlu, 2015). Gülteke, Tomul, and Korur (2013) argued that because the teacher's responsibility is to serve as the classroom manager, children are easily influenced by his or her actions, interactions, and emotional responses in the classroom environment. For these reasons, teachers' qualifications are crucial (Lee & Johnston-Wilder, 2012).

Definition of Terms

Constructivist theory: A self-directed building process of individuals' construction and integration of their understanding and meaning of concepts and knowledge that can be practically and correctly applied across a myriad of context and situations with personal relevance to both past and present experiences of the learner; the learning rate is different for everyone (Bas, 2012).

Mathematics: Learning tasks associated with numbers and measurement, algebra, geometry, statistics, and probability (Hsu, 2013).

Montessori education: An educational system developed by Maria Montessori based on the concept of independent self-correcting activities to stimulate a student's natural ability and intellect. The fundamental principle is that a pupil learns best through exploration, discovery, and creativity with the appropriate guidance (Holfester, 2014).

Montessori school: Any public or private special education, pre-K, K-12, or other learning institution that offers a Montessori-based curriculum to the students (Holfester, 2014).

Traditional learning: Subject-based instruction focused on competition and evaluation (Holfester, 2014). All public school teachers, including those at charter schools, must be state certified in concert with the NCLB requirement of staffing every classroom with a highly qualified teacher.

Significance of the Study

Understanding teachers' perceptions requires greater knowledge of the processes that have shaped their experiences. Little research has been conducted on the beliefs and practices on which traditional and Montessori teachers base their math concepts. The purpose of this study was twofold: (a) to explore the teachers' perceptions and beliefs regarding their current instructional practices in the area of teaching numbers and operations in mathematics, and (b) to understand how the Montessori and traditional practices differ in terms of approaches. The information gathered from this study may help form the basis for a meaningful interpretation of its results and lead to an understanding of effective approaches to mathematical operations in the elementary setting.

Researchers have examined teachers' beliefs and how they teach. Ambrose, Clement, Randolph, and Chauvot (2004) contended that teachers' beliefs play a major role in how mathematics are taught and learned. Golafshani (2013) argued that everybody believes mathematics is of great significance in education; however, many students are deficient in math skills, which suggests that changes are needed in the instructional approach teachers use to teaching mathematics. The premise is that two teachers can have similar knowledge and math backgrounds but approach teaching mathematics differently. For example, one teacher may teach mathematics with a problem-solving orientation while the other may have a more didactic approach. For this reason, I explored teacher's beliefs and practices regarding mathematics instruction using Koehler and Grouws' (1992) model.

Guiding/Research Questions

The following research questions guided this qualitative case study:

Research Question 1: What are teachers' perceptions and beliefs regarding their classroom practices in teaching basic math operations of addition, subtraction, multiplication, and division?

Research Question 2: How do teachers describe the practices they use to teach students basic problem-solving skills in math?

Research Question 3: How do the practices and beliefs of traditional teachers and Montessori teachers differ regarding teaching basic problem-solving skills in math?

Conceptual Framework

The conceptual framework for this study centered on factors influencing teacher beliefs and practices proposed by the model of Koehler and Grouws (1992). Koehler and Grouws argued that a teacher's beliefs have a powerful impact on the teaching and learning of mathematics. Their model consists of three factors: teacher knowledge, teacher beliefs, and teacher attitude. For the purpose of the current study, I focused on two factors: teachers' beliefs and attitudes about mathematics concepts and the teaching of mathematics. I chose to target teachers' beliefs and instructional practices not only to make the project more manageable but also because teaching is largely under the teacher's control.

Handal and Herrington (2003) stressed that some characteristics of teachers, such as educational background, math knowledge, and personality traits, are unalterable. However, teaching methods can be altered under supportive conditions.

Koehler and Grouws' model was reinforced by another researcher. Van Der Sandt (2007) supported Koehler and Grouws' model (1992) and stated that a teacher's beliefs about teaching mathematics can impact the child's attitude and achievement in mathematics. Van Der Sandt posited that how teachers perceive teaching mathematics includes factors such as whether they like or enjoy teaching math, the enthusiasm displayed, and confidence in their mathematics teaching abilities.

In addition to using the Koehler and Grouws' model (1992), I employed Piaget's theory (1972, 1977) to help understand and explain teaching approaches regarding how students learn math. Piaget was known for contributions to the knowledge of how students learn mathematics. Piaget's work has great significance for mathematics teachers, especially at the elementary school level (Tsafe, 2012). Piaget's theories of how children's minds work and develop are particularly important for teachers to keep in mind in their approaches to teaching mathematics. The teacher's task is to facilitate the child's learning and to act as a guide, which allows students to engage in the learning process (Ültanır, 2012).

Piaget (1972, 1977) viewed children's cognitive development as sequential stages in which their accomplishment is governed by their biological readiness and earlier experiences. These stages for school age children are (a) the preoperational stage (ages 27 years), (b) the concrete operational stage (ages 7-12 years), and (c) the formal operational stage (ages 12 years-adolescence). The preoperational stage generally covers children's cognitive development during the early elementary years and is evidenced by being able to deal with reality in symbolic ways. Children's thoughts in this stage are limited to centering, which is the inability to consider more than one characteristic of an object at a time. At this stage, children are not able to conceive the concept of conservation of number, volume, quantity, or space. The implication for mathematics teachers at this stage is that because logical/operational thinking has not commenced, attempting to convey to young children things they cannot experience through their senses can prove difficult for them to comprehend. During this stage, children should manipulate objects and engage in tasks involving problem-solving so that they can appreciate reality (Cable, 2014; Kefaloukos & Bobis, 2011; Piaget, 1972, 1977).

The second stage of development is the concrete operational stage. According to Piaget (1972, 1977), the stage covers ages 7-12 years. This stage is particularly important to primary school teachers as most elementary children function at this level (Ediger, 2012; Looijenga, Klapwijk, & de Vries, 2015). As it relates to math, this stage marks the beginning of what is called the *logico-mathematical* aspect of experience. For example, a child is shown two identical transparent containers filled with beads of equal amounts. Then the content of one of the containers is transferred to a different size and shape container. The child applied logic and transitioned to the next stage of cognitive development (concrete operational) if he or she understood that the amount of the content was the same regardless of the diameter of the new container. Once the content is returned to the initial container, the amount should be the same (Cable, 2014). The concrete operational stage is important for mathematics learning because many of the operations children carry out at this stage are mathematical in nature (Piaget, 1972, 1977). In the formal operational stage (12 years-adolescence), children can reason abstractly and hypothesize with symbols or ideas rather than needing objects or manipulatives in the physical world as a basis for their thinking (Piaget, 1972, 1977).

The implications of Piaget's developmental stages for mathematics teachers are that teachers need to take each stage into account and apply the appropriate teaching approach to avoid problems in the child's later stages. Mathematics teachers should make use of these approaches with thoughtfulness and consideration for the well-being of the child (Ewing, Foster, & Whittington, 2011; Tsafe, 2012).

Review of the Literature

The purpose of the current study was to investigate traditional and Montessori teachers' perceptions and beliefs regarding their classroom practices in teaching mathematics. Specifically, I sought to identify and understand the teachers' perspectives and beliefs regarding the difficulties their students face when solving mathematical problems in the four basic mathematical operations: addition, subtraction, multiplication, and division. In the review of the literature, I start with an overview and comparison of traditional and Montessori education. This is followed by a discussion of the literature on teachers' mathematical beliefs and instructional practices, student-centered mathematics teaching approaches, teacher-centered mathematics teaching approaches, a comparison of approaches, and mathematics learning outcomes for elementary learners.

Low mathematics achievement of students in U.S. schools compared to students globally continues to plague educators and school administrators (Reed, 2008). Some educators blame the math shortcomings on the overemphasis on procedural learning in traditional school settings. Elementary math instruction and curricula continue to attract educators, researchers, and curriculum designers because of their significance in the development of children's numeracy ability (Carl, 2014; Schwarts, 2009). The National Council of Teachers of Mathematics (NCTM) and current researchers consider number sense to be one of the foundational ideas in mathematics as well as the groundwork for the acquisition of formal mathematical concepts and skills (Baroody, Eiland, & Thompson,

2009; Jordan & Kaplan, 2007; Li & Yang, 2010; Sood & Jitendra, 2007; Tsao & Lin, 2011).

In this literature search, I used the Walden databases (e.g., ERIC, Proquest, EBSCO Host). I also used the public library and several books on Montessori education. Search terms used were *math*, *teacher beliefs*, *teacher perception*, *math difficulty*, *Montessori instruction*, *elementary education, and learning and teaching mathematics.*

Overview of Traditional Educational Instruction

The traditional approach to delivering instruction generally includes a teacher-centered instructional period. Lessons are presented in a whole-group format. Textbooks and workbooks are integral parts of this approach. Desks are arranged to support the whole-group teacher-centered format. Traditional teachers adhere to this approach regardless of the curriculum. Although traditional methods such as lectures from blackboards and students working from their seats have proven to be successful for some students, other students are not being helped by these methods (Elkind, 2012; Lockhorst, Wubbels, &van Oers, 2010; Marshall, 2006; McKinney et al., 2009).

Another instructional strategy often used but that does not necessarily promote conceptual understanding is the use of key words, such as stating that *in all* means addition. Dependence on key words can hinder the strengthening of abilities needed to problem-solve (Griffin & Jitendra, 2009). Strategies found to be constantly successful include using visuals and graphics for math problems, being explicit when exposing children to math concepts and principles, and employing peer group activities in instruction (Griffin & Jitendra, 2009; Li & Yang, 2010; McKinney et al., 2009; Morrison, 2011).

In the 21st century, there is a greater emphasis on mathematics as an academic/instructional subject in early childhood/pre-elementary education (Hachey, 2013). Many researchers agree that early childhood mathematics instruction, starting with pre-kindergarten, should cover the following five major content areas: numbers and operations, geometry, measurement, algebraic thinking, and data analysis (Chen & McCray, 2012; Stipek, 2013). Unless the fundamental systems/supports for effective mathematics instruction are instituted, designing standards, evaluating outcomes, and holding teachers accountable will not improve mathematics achievement (Ball, Themes, & Phelps, 2008; Deng, 2007; Stipek, 2013).

Overview of Montessori-Based Instruction

Montessori teaching accommodates an array of learners. Montessori-based curriculum caters to children of all demographics in public and private schools; inner city, rural, and urban magnet programs; low and high socioeconomic levels, and special needs populations (Holfester, 2014; Lillard, 2013). Montessori's core belief is to *follow the child*. In this philosophy, the teacher constructs an environment of materials and support that are responsive to the child's natural inclinations and allow the child to develop according to his or her own schedule and abilities (Lillard, 2013; Montessori, 1965, 1949). The aim was to allow the child to develop into a self-confident, self-motivated, and self-directed learner. The premise is that Montessori teachers operate within a system that respects each child as an individual learner. This included incorporating a cycle of experimentation with methods and materials, observations, and assessment of each child.

The Montessori philosophy reflects the early works of Maria Montessori.

Montessori, an educator and physician, believed that students' activities in the classroom were directly related to their intellects (Lopata et al., 2005). Montessori's key tenet was that as children interact with their external world, they pay attention to stimuli that are of interest to them and ignore others. In other words, children learn by actively constructing their learning experiences, and the Montessori teacher is responsible for carefully creating an environment to support children's learning (Montessori, 2006).

According to the American Montessori Society (AMS, 2015), quality Montessori education consists of the following elements:

- multiple ages in classes where older children are encouraged to serve as role models,
- a collection of age appropriate learning materials,
- qualified teachers from a Montessori teacher education program, and
- teachers serving as guides rather than givers of information.

The Montessori teacher prepares, manages, and maintains the classroom environment without being the focal point of the environment (Montessori, 2006). The absence of a teacher's desk, central backboards, and assigned student seating are noticeable elements of this pupil-centered setting. In the Montessori classroom, although the teacher maintains authority and directs the classroom community, teaching is not exclusive to the teacher. Students also teach and learn from each other.

The Montessori teacher is responsible for the prepared environment. The Montessori teacher's role is to provide an environment conducive to a child's natural physical, intellectual, and emotional growth (Montessori, 2006). This means that teachers must have sufficient knowledge of children's development as they move from being observers to experimenters (Elkind, 2012; Robinson, 2006; Schultz & Ravitch, 2013). Teachers must also present materials appropriate for the child's age and developmental growth based on clinical observation of the child and previous knowledge. The teacher then waits for the child to choose this material on her or his own and observes the child engaging with the materials and how the materials impact the child's thinking. Subsequently, the teacher prepares or controls the environment but does not control the child. The materials found in a preschool or elementary Montessori environment consist mainly of manipulative materials adapted by Montessori, who believed that materials or tools should hold a child's attention, be self-correcting, and cause some understanding.

Attributes that support lifelong learning include a focus on the student's early Montessori experience. In preschool and kindergarten, Montessori education focuses on building and strengthening a student's natural learning tools such as self-regulation, also referred to as self- or inner discipline, organization, and independence (Bagby & Sulak, 2012; Ervin, Wash, & Mecca, 2010). Self-regulation also includes elements of concentration and work ethic. Researchers indicated that a Montessori approach is effective in improving academic achievement among primary learners (Jordan & Kaplan, 2007; Kayili & Ari, 2011; Li & Yang, 2010; Torrence, 2012). Students often spend time participating in a variety of activities that can last for nearly 3 hours. Individual and group problem-solving tasks and other stimulating activities are at the core of learning. Older students are encouraged to assist and mentor younger students with projects and assignments. Teachers group the students according to common interests and experiences (Holfester, 2014). Concepts such as textbooks, homework, tests, test scores, rewards, and punishment are rarely included or applied (Holfester, 2014; Lillard, 2013).

Comparison of the Traditional and Montessori Approaches

There are several differences between a typical Montessori learning environment and a traditional school setting. Montessori (2006) described children in traditional classroom settings as students constantly seated at a desk without much movement compared to the Montessori atmosphere where students are free to explore and discover for themselves. Children in Montessori classrooms choose their tasks, sometimes after receiving suggestions from their teachers. According to the Montessori Children’s House (2016), both settings can provide positive learning outcomes for children; however, the mathematics teachers’ instructional approaches may be based on the teachers’ different learning principles. Some teachers support the idea that learners construct their own knowledge by selectively using objects in their environment. This means that some teachers use different methods to accomplish this goal. All methods should be constructed to help shape what children learn and how they view the future and the world around them.

According to the Montessori Accreditation Council for Teacher Education’s Accreditation Guide (2015), Montessori-targeted subjects for the early childhood level include each area of the classroom: practical life; sensorial; mathematics; language; and culture, which includes science, geography, cultural studies, physical education, music, and arts. The elementary level includes language arts, mathematics, sciences, social studies, arts, health and physical education, history, geometry, geography, biological sciences, physical sciences, and music. Demonstration and instructional practice using Montessori materials are inherent parts of the courses. Table 1 provides a summary of the differences between traditional and Montessori education.

Table 1

Comparison of Montessori Education and Traditional Education

Montessori	Traditional
Emphasis on cognitive and social development	Emphasis on rote learning and social behavior
Teacher has a guiding role; student self-discipline is built into the method and the environment	Teacher leads/controls the classroom; teacher generally establishes rules and is the primary enforcers of student discipline
Mixed-age groups	Same-age groups
Grouping promotes students teaching each other as well as collaborating with each other	Teaching is done primarily by teacher; collaboration is minimal
Student choses own work, the order of work/subject and sets own learning pace	Curriculum and class schedule structured for student; instruction pace is set by/depends on the group
Student discovers concept from self-teaching materials; student corrects own errors through feedback from self-correcting materials	Student is guided to concepts by teacher; student's work error is highlighted by teacher's feedback
Student is allocated as much time needed to complete work	Student generally allotted specific time for work as well as for specific activity
Learning is reinforced by student through repetition of work and individual success	Learning is reinforced by teacher through repetition
Individual instruction is the dominant way teacher presents lessons	Whole group/class instruction is the dominant way teacher presents lessons
Student may work anywhere in the room and move freely around the room; student may speak in the room without permission from teacher and without disturbing others.	Generally, seat is assigned by teacher; student is encouraged to participate during group lessons; teacher grants permission
Physical care of environment and self-care is incorporated in the program. Student may freely do these tasks	Teacher directed basic self-care instruction/activities—teacher assigns specific tasks for care of classroom environment

Note. Montessori Children's House (2016).

Montessori Mathematics Versus Traditional Mathematics

Montessori mathematics curriculum is logically connected and consistent. It begins with concrete representation of mathematical concepts and gradually advances into abstract mathematical ideas (Donabella & Rule, 2008). The Montessori hands-on approach to learning gives students the opportunity to work with the material, explore mathematical concepts for deep understanding, and make discoveries (Donabella & Rule, 2008; Ervin et al., 2010; Saracho & Spodek, 2009).

The differences and similarities in the way that Montessori and non-Montessori first- through third-grade students completed tasks involving place value concepts was studied by Reed (2008), who chose a non-Montessori school that used a common approach to learning that included mostly whole-class direct instruction, textbooks, workbooks, and limited use of manipulative materials past the first grade. Reed noted that there were many key distinctions between Montessori-based learning and traditional learning; however, the label non-Montessori or traditional for the comparison school was not meant to convey any judgment about the nature of the school, classrooms, or effectiveness of instruction. For this reason, Reed gave considerable attention to describing the various instructional methods.

The study focused on comparing students' understanding of place value concepts and abilities in a Montessori school (n = 47) and in a mostly traditional comparison school (n = 46) for a total of 93 students in Grades 1 through 3. Reed's (2008) findings revealed statistically significant differences favoring the Montessori students on conceptual tasks at all grade levels. However, no statistically significant differences were found on procedural tasks between the schools at any grade level.

Montessori education favors multiage classrooms. Therefore, students at the Montessori school in Reed's (2008) study were from the combined grades of 1 through 3. The traditional school contained two classrooms at each grade level with one teacher and 30 students in each room. All students from the Montessori school began as Montessori students at the kindergarten level or before.

Traditional Mathematics

Many researchers have focused on ways for delivering mathematics instruction in the traditional setting. In this setting, students are usually encouraged to verbally explain their thinking processes and openly exchange ideas with the teacher and the class. It is believed that this approach provides the foundation for a true understanding of mathematical concepts. However, there is disparity between teachers' actual classroom practice and research suggestions; teachers usually teach based on their own practical experiences (Cardetti & Truxaw, 2014; Hachey, 2013).

Elementary mathematics is divided into several categories. The five major content areas generally included in elementary mathematics are numbers and operations, geometry, measurement, algebraic thinking, and data analysis (NAEYC & NCTM, 2012). Large-scale published curricula are used in classrooms to address these content areas (Hachey, 2013). These curricula are sometimes fully or partially scripted, and curricula are changed periodically; Nonetheless, effective implementation depends on teachers' judgment (Schwarts, 2009; Stipek, 2013).

Missall, Mercer, Martinez, and Casebeer (2011) used the Test of Early Numeracy Curriculum-Based Measure (TEN-CBM) to measure early numeracy of kindergarteners and first graders to predict future performance. They also used it to determine longitudinal patterns in reference to third-grade state mathematics exam scores. Missal et al. concluded that the TEN-CBM's missing number (MD) and quantity discrimination (QD) tools, which measure number sense, were most foretelling of future math academic success.

In a study conducted by Jordan, Glutting, Dyson, Hassinger-Das, and Irwin (2012), high-risk, low-income kindergarten students were randomly placed in one of two intervention groups—a number sense intervention group and a language intervention group—or in a control group to test for the effectiveness of growth in number sense through small-group intervention (National Research Council, 2012). Three measures, the Number Sense Brief (NSB), the Woodcock-Johnson III Test of Achievement (WJ), and Bracken Basic Concepts Scale-3-Revised were used to assess number sense, general math achievement, and vocabulary. The number sense group demonstrated the most advancement of the three groups and showed the most meaningful results regarding number sense and math achievement in general. The other two groups showed no difference between them.

Developing the number sense of elementary students promotes flexibility and inventiveness in thinking; number sense correlates number, quantities, operations, and how they are related. Number sense is responsible in part for an adult's number representation and mathematical thinking (Bobis, 2007; National Research Council, 2012; Yang & Wu, 2010). Presently, mathematics difficulties are not stressed on the same level as reading difficulties (Fuchs, Fuchs, & Compton, 2012; Pimperton & Nation, 2010); accordingly, math interventions are not as commonplace as reading interventions (Jordan, 2007). Students are not generally screened for math difficulties early in their academic career or with the same regularity as they are screened for literacy difficulties in primary education. Literacy screening is commonly done as early as kindergarten (Jordan, 2007). Early screening for mathematical difficulties and early interventions are important factors in enhancing mathematics performance (Bryant et al., 2011).

A well-developed number sense is necessary for proficiency in arithmetic problem-solving (Chard et al., 2008). Number sense lack is at the root of many math difficulties (Jordan, 2007) and it is a predictor of mathematics difficulties in later years if not addressed in the lower grades (Chard et al., 2008; Erhan et al., 2011; Jordan, 2007). Characteristics of math difficulties as noted by the National Mathematics Advisory Panel include poor counting methods, slow mathematical fact recall, and inaccurate computation (Jordan, Kaplan, Ramineni, & Locuniak, 2009). A student with an advanced number sense would counter these mathematical difficulties.

In a 9-month study designed to evaluate the adequacy of a curriculum to develop the number sense of prekindergarten students, Baroody et al. (2009) recognized and maintained that number sense cannot be forced upon learners but must be gradually developed through such means as exploration and the implementation of informal intervention strategies. The Principles and Standards for School Mathematics (PSSM) published by the NCTM became an incentive for researchers to increase their focus on how math is taught and how math is learned. According to the NCTM, the PSSM is necessary for having mathematics learning communities that emphasize reasoning, problem solving, and conceptual understandings (McKinney et al., 2009). The straightest path to improving mathematics achievement is through improved teaching (Erhan et al., 2011; Fast & Hankes, 2010; McKinney et al., 2009). Mathematics researchers have found that math competence grows slowly if children are exposed to observing quantifiable relationships frequently and numerical relationships can be automatic and fluent (Howell & Kemp, 2010; Van Der Heyden & Burns, 2009).

Teachers' Mathematics Beliefs and Instructional Practices

The primary research question of this study was, "What are the beliefs and perceptions upon which Montessori and traditional teachers base their approaches to teaching math?" My discussion in the previous sections primarily focused on the basic principles of Montessori and traditional educational programs. In this section, I focus on teacher beliefs and practices of teaching mathematics in general and how these factors may influence student learning.

Teacher's beliefs regarding mathematics. Teacher beliefs are a weighty influence on their mathematics practices (Turner, Warzon, & Christensen, 2011). Polly et al. (2013) argued that teachers' belief systems are their perspective concerning adequate and effective mathematics learning and may be modified over time as teachers introduce new ideologies in their classrooms. Teacher beliefs was a key factor in Koehler and Grouws' (1992) model as beliefs greatly impact teaching. The key tenet is that teachers' beliefs sway how they use classroom resources and how they carry out mathematics curricula. However, to gain a full understanding, it is important to examine two popular approaches teachers use in mathematics teaching: student-centered and teacher-centered (Polly et al., 2013).

Teaching mathematics: Student-centered approaches. Constructivist theorists like Piaget argued that students consistently build their mathematical knowledge through their active involvement/experiences in mathematical encounters (Piaget, 1977; Ültanir, 2012). Through their experiences, students construct, reorganize, and arrange knowledge, which leads to positive academic outcomes in mathematics. Teachers leading student-centered classrooms give students the opportunities to invent their own learning by creating situations that drive them to expanding/developing their thinking in new ways. In student-centered classrooms active acquisition of knowledge is encouraged as opposed to students consistently receiving knowledge in a passive manner from the teacher (Polly, Margerison, & Piel, 2014). Therefore, student-centered classrooms are generally structured and organized differently than teacher-centered classrooms (Polly et al., 2014).

Researchers have linked Montessori teachers to the student-centered approach (Chattin-McNichols, 1992; Holfester, 2014). Montessori teachers give brief lessons and demonstrations. The objective is to introduce materials and their use along with accompanying exercises and then to passively guide the student through a student-centered inquiry period. The student-centered teacher allows and encourages students to self-construct their knowledge rather than gives knowledge to students. The teacher aids the students through open-ended questions. For the student-centered approach, teachers should view problem solving as a teaching practice. Traditional teachers support the idea that mathematics teacher's most important role is teaching basic computational facts (Polly et al., 2014). After computation proficiency is reached, problem-solving competencies are then considered as a mathematics objective to be taught to students (Polly et al., 2014). The use of strategies such as problem-solving to resolve computation problems results in a more concrete comprehension of mathematics (Polly et al., 2014). Teacher-centered and student-centered classrooms both use manipulatives.

However, in the teacher-centered classroom, particularly in the lower grades, the teacher generally directs their use, and the students follow the teacher. In student-centered classrooms, students are encouraged to engage with the manipulatives in a manner appropriate to them and that will cause them to arrive at answers. This means leading students into developing a more solid knowledge of mathematics (Polly et al., 2014). Children enter school with some knowledge of their surroundings and the world. Student-centered environments allow learners to construct or restructure previous knowledge when information is presented to them; students connect prior knowledge and the present information (Polly et al., 2014). In addition to using manipulatives, other activities common to student-centered classrooms include small group work, asking questions, and the continuous exploration of materials.

Teaching mathematics: Teacher-centered approaches. Teacher-centered approaches to teaching mathematics are more aligned with the traditional and non-constructivist oriented teachers. Common to this approach are elements such as structure, order, memorization, discipline, and accountability (Polly et al., 2014). Teachers believe in teaching subject content in the most direct, forthright, and time-efficient way; acquiring knowledge is considered more primary than acquisition process. This orientation allows putting forth more information in a shorter time period compared to the constructivist teaching practices (Polly et al., 2014).

For the teacher-centered classroom, the majority of class time is spent on instruction. Hence, teacher-centered teachers believe in instructing students in basic math facts before proceeding to the problem-solving process. For these teachers, students need to be grounded first in math facts for the problem-solving process to hold importance

(Polly et al., 2014).

In the teacher-centered classrooms, the teacher is recognized as the giver of mathematical knowledge. The teacher also assesses the knowledge presented to students. To the students, the teacher is responsible for the learning that takes place in the classroom (Polly et al., 2014). This view leads to a more structured classroom environment then a constructivist setting. Time allotted to mathematics exploration is minimum; essential core is the focus. Students are held to standards established by the teacher's expectations (Polly et al., 2014).

Teaching mathematics: A comparison of approaches. The crucial difference between student-centered and teacher-centered educators is that student-centered educators emphasize the process of learning mathematics and teacher-centered educators emphasize the mathematics knowledge acquired. This difference greatly influences each group's teaching practices. Research findings on these two groups have been inconsistent. Chung (2004) found no significant difference in the mean score of third graders taught by teachers in either setting, student-centered and teacher-centered. On a multiplication assessment, Smith and Smith (2006) found that third graders under student-centered practices had deeper mathematical understanding than fourth graders under teacher-centered practices. Polly (2008) indicated that student-centered practices for teaching mathematics lead to more academic success than teacher-centered approaches. More research is needed to further explore how teachers' beliefs and their instructional practices impact students' learning outcomes.

Mathematics Learning Outcomes for the Elementary Learner

Research indicates that the United States is lagging when it comes to identifying and preparing its best and brightest students for careers in STEM fields (National Science Board, 2010; Trinter, Moon, & Brighton, 2015). For students in the elementary grades, there are many games and manipulatives that students can use in traditional education and Montessori settings. However, it is unclear whether students are really applying those mathematics skills drilled in these games with the manipulatives (Swanson, 2014). The question then becomes: How can teachers determine whether students have successfully learned the math concepts? Swanson (2014) believes that learning takes place when children can apply their mathematics knowledge to the real world, they learn to collaborate, and they can solve problems on their own. Swanson claimed that this is key for successful STEM implementation. For these reasons, elementary teachers should seek ways to develop and implement instructions that teach abstract math concepts through more hands-on and inquiry-based activities (DeJarnette, 2012).

Trinter et al. (2015) conducted a qualitative study to determine the type of project based learning (PBL) instructional methods mathematics teachers used to help students achieve mathematics concepts. The aim was to help teachers identify gifted and talented students in STEM areas, particularly students from low socioeconomic and underserved areas. Trinter et al. argued that the teachers should take a closer look at the traditional approach of evaluating students' math competencies using standardized test scores. The researchers believed that PBL was necessary for engaging mathematically talented students (Trinter et al., 2015). PBL was characterized as an inquiry process focused on seeking resolutions to questions about things occurring in everyday life. Students were expected to actively participate by investigating, analyzing, and collaborating to solve problems. The teacher's role was to facilitate the process.

The study participants were three certified female second-grade teachers. The interviews were conducted via webcam with a focus on the teachers' perceptions of students' experiences with given math lesson approximately every other day. Teachers were asked about their perceptions of the experiences and mathematical performance of students in the PBL mathematics units (Trinter et al., 2015). The study findings suggested that when mathematics was shown to students in ways that were engaging and challenging for them, the students were then able to interact with the material in manners that provided insights on their mathematics potential. The general conclusion was that what students learn depends largely on *how* they have learned it (Trinter et al., 2015).

Implications

The literature reviewed for this study suggested that teacher beliefs and instructional practices of mathematics teaching are paramount to elementary students learning math in both Montessori and traditional education programs. Furthermore, the research revealed that teachers' beliefs about mathematics determined how they conducted their mathematics classes (Polly et al., 2014). Understanding mathematics teachers' beliefs and instructional practices may lead to improved student achievement in mathematics.

As such, several implications from the results of this study could be anticipated with the dissemination of a rich and detailed report on the teachers' beliefs and instructional practices in mathematics, which will include the production of a position paper that will be shared with the local school teachers, and school administrators. The position paper will address data collected during teacher interviews. Study results may benefit elementary school teachers, increase student achievement in mathematics, and contribute to the existing literature on teachers' beliefs and practices related to mathematics instructions in Montessori and traditional elementary school programs.

Additionally, this study has the potential to promote positive social change for students as teachers work to improve students' math skills. Mathematics is a critical part of academic preparation of the elementary school child or children in the early stages of development (Morrison, 2011). Educators place emphasis on preparing young students for the application of math in their world and careers, and on building their confidence and interest in the subject of mathematics (Morrison, 2011; Torrence, 2012).

Summary

In this review of the literature I identified few studies that focus on using Montessori materials or Montessori mathematics with students having mathematics difficulties. This lack of research provided encouragement for the study. It is clear the number sense of students with mathematics difficulties needs to be strengthened. The way math is taught and learned should be considered when addressing measures needed to assist students. As emphasized by NCLB, curriculum and teaching practices should be based in research (Agodini et al., 2009). Research methodology for the study is discussed in Section 2.

Section 2: The Methodology

The purpose of this qualitative study was to explore teachers' perceptions and beliefs regarding their educational and instructional practices in the area of teaching numbers and operations in mathematics and to understand how Montessori and traditional practices differ in terms of approaches. The findings from this study may improve the understanding of effective approaches to mathematics instruction in the elementary setting. The literature review revealed major differences between Montessori education and traditional education (Holfester, 2014; Polly et al., 2013); however, no studies were found that included a qualitative approach to exploring and comparing the beliefs and instructional practices of Montessori and traditional teachers regarding teaching mathematics at the elementary level. In this section, I described the research design, justification for using this approach, participants, instrumentation, data collection, data analysis, and results of the data analysis.

Research Design

In this study, I used the qualitative descriptive case study design to explore and understand the perceptions and beliefs of Montessori and traditional elementary teachers relevant to their practices of teaching basic mathematics concepts. Qualitative research is appropriate for uncovering the meanings individuals attach to an event, situation, or phenomenon (Merriam, 2002). Unlike quantitative designs, qualitative studies are more fitting when little is known about the topic and a relevant theory basis is inadequate (Yin, 2009). I did not choose a quantitative approach for the study because quantitative research requires asking specific questions with answers that are narrow and require a numeric value to analyze. By contrast, qualitative researchers rely on narrative phrases and words interpreted into themes (Merriam, 2002). Given the problem and purpose of this study, a descriptive case study design was most appropriate.

Yin (2009) defined case studies as empirical inquiries for understanding real-life phenomena in depth. Similarly, Merriam (2009) described the case study as a description and analysis of a phenomenon that calls for the researcher to concentrate on a single phenomenon (the case) or unit of analysis, which sets the design apart from other designs. Other qualitative approaches were not well suited to this study due to their focus on an individual life, lived experiences, or describing a group of people.

Participants

The ABC school district was selected for this study. The participants were 10 purposefully selected teachers from the ABC school district, four from the Montessori public elementary school and six from the traditional public school. The advantages of purposive sampling in case study research are the ability to select participants based on specific criteria and the potential to provide a balance between sampling (Yin, 2009). The inclusion criteria for the Montessori teachers were the following: (a) currently employed in ABC public school district, (b) hold a state certification or endorsement in Montessori education, and (c) currently teach in the Montessori school. For traditional teachers, the inclusion criteria were the following: (a) currently employed in ABC public school district, (b) hold a state teacher's certification in teacher education, and (c) currently teach in the traditional school.

The sample size of 10 was supported by previous qualitative research. Many qualitative researchers disagree on sample size; however, Merriam (2009) indicated that in qualitative research, interviewing should continue until saturation is reached, the point in continuous data collection when no new data or information is gained. Yin (2009) recommended at least six sources of interview evidence whereas Creswell (2009) recommended no more than four or five cases or three to five interviewees per case study.

Procedures for Gaining Access to Participants

Gaining access to the individual participants involved several steps. First, permission was sought from Walden's institutional review board (IRB) and obtained. Additionally, permission to conduct research was obtained from the school administrator in a signed letter of cooperation. The consent form that participants completed addressed the following:

- Their right to voluntarily withdraw from the study at any time.
- The central purpose of the study and the procedures to be used in data collection,
- A statement about protecting the confidentiality of the participants.
- A statement about the known risks and expected benefits associated with participation in the study.

The consent form was signed and acknowledged by participants. Participants were told from the onset that they could discontinue participation at any time during the study. Participants were ensured that their privacy would be maintained, and that all information gathered would remain confidential.

Ethical Protection of Participants

The study involved interviewing human subjects. Therefore, all relevant information was disclosed to ensure that participants were aware of their right to make informed choices (Lodico, Spaulding, & Voegtle, 2010). All decisions to participate were made voluntarily. An IRB application was submitted containing details of the study including population, data collection, and data analysis. The purpose of the study, the voluntary nature of the study, potential risks and benefits, confidentiality, and contact information were explained.

Measures Taken to Protect Participants

In compliance with the IRB, I took the appropriate measures to protect the rights and welfare of human research participants and their records. Any research project that involves human interaction has some degree of risk for participants. It is important for researchers to consider the risks and take the necessary actions to minimize these risks (National Institutes of Health [NIH], 2016).

There were no foreseeable or known risks presented in the study. However, even when risks are reasonable, no individual should participate in research without giving voluntary informed consent. I made appropriate disclosures and ensured that participants had a good understanding of the information and their choices, not only at the time of enrollment but throughout every phase of the research process. There was no known or anticipated conflict of interest, and I did not benefit financially from this research. The research was not conducted in my workplace or with clients or employees, and there were no affiliations or financial arrangements that posed potential conflict of interest.

Data Collection

I constructed the main instrument of this study as an interview guide with a few demographic questions and several open-ended questions aligned with the research questions. Open-ended questions allow participants to describe situations in a holistic manner while researchers actively listen, observe, and understand (Merriam, 2009). Interview questions were reviewed and approved by my committee members. The interview format was semistructured and face-to-face. Demographic data included field of education, current grade level, and years of service. According to McNamara (2009), the strength of the general interview guide is the researcher's ability to ensure that the same general area of information is collected from each participant. The following research questions guided this qualitative case study:

Research Question 1: What are teachers' perceptions and beliefs regarding their classroom practices in teaching basic math operations of addition, subtraction, multiplication, and division?

Research Question 2: How do teachers describe the practices they use to teach students basic problem-solving skills in math?

Research Question 3: How do the practices and beliefs of traditional education teachers and Montessori teachers differ regarding teaching basic problem-solving skills in math?

Data collected from participant interviews included direct quotes, opinions, and perceptions. Interviews were conducted face-to-face. Prior to beginning each interview, I introduced myself and described the purpose and potential benefits of the study. All information gathered during the interviews remained confidential. Interviews were digitally recorded and conducted at a convenient time and location for participants. The location was neutral and free from distraction to put the participants at ease and allow them to focus during the interviews. Participants were reminded that all data collected would remain confidential.

Role of the Researcher

As an educator, I sought to understand the perspectives of the interviewees and capture the relevance of the perceptions shared, which added depth and significance to the themes. In this study, there was no conflict of interest, and I did not benefit financially from this research. The term *conflict of interest in research* refers to situations in which financial or other personal considerations may compromise, or have the appearance of compromising, a researcher's professional judgment (NIH, 2016). Every effort was made to ensure that conflict of interest did not impact the study. The research was not conducted in my workplace or with clients or employees. Additionally, there were no supervisory relationships, affiliations, or financial arrangements that posed potential conflicts of interest that could undermine or compromise the validity of the research findings.

I am both state and Montessori certified, and I made efforts to ensure that bias was minimized, and preconceived ideas and knowledge did not interfere with data collection or analysis, a process call *bracketing* (see Creswell, 2012). Findings were presented in a narrative form and displayed in tables or charts. The aim was to provide rich and thick descriptions of the results.

Data Analysis

Data analysis in qualitative research is the process of organizing, reviewing, and interpreting the data for recurring patterns to determine the importance of relevant information. The data analysis for this study proceeded through the methodology ascribed by Yin (2009). Guided by Yin's method of inquiry, data analysis included transcribing recorded interviews, coding data, categorizing the coded data, and identifying the primary patterns and themes in the data, which involved several steps.

First, I prepared and organized the transcripts to gain a general understanding of the information obtained. This meant visually scanning the data, and reading and rereading all of the transcripts. Second, I developed a list of preliminary categories, a process called *coding*. Coding, as described by Merriam (2009), is the process of interacting with the data, raising questions about the data, comparing data, and reaching conclusions from knowledge generated from the data. The purpose of coding is not only to describe the data, but also to acquire new understanding of the events central to the study (Yin, 2009). This process resulted in generating a description of participants' perceptions as well as themes for analysis.

I coded and recorded data according to the constant comparative method until themes began to emerge. The constant comparative method involves breaking down the data into meaningful units and coding them to categories (Merriam, 2009). The interview data were coded to show my interpretation of what was stated in the interview. The codes were based on the participant responses and included explanation of math approaches, student-centered approaches, teaching-centered approaches, and student engagement. I sent a summary of the transcripts to participants by e-mail/mail for feedback and to check for accuracy. This ensured that I did not inadvertently misinterpret the information provided in the interviews.

Reliability and Validity

Reliability and validity are vital in data collection, analyzing results, and judging the quality of the study. Reliability for qualitative studies refers to the consistency and dependability of the data collected (Yin, 2009). Reliability involves trustworthiness, authenticity, and credibility. Credibility refers to the believability of the findings enhanced by evidence such as confirming the evaluation of conclusions with research participants and theoretical fit (Merriam, 2009). To ensure credibility, I began by reading and rereading the transcripts to describe and present the reader with a thick and rich description of the data.

Evidence of Quality

I used three data sources, a technique described as *triangulation*. The sources were the interviews, field notes, and self-reflective journaling. Interviews are a common form of case study research data collection (Hancock & Algozzine, 2006). An interview is essentially a conversation between a researcher and participants conducted for the purpose of gathering specific information (Glesne, 2011; Lodico et al., 2010; Merriam, 2009). The interviews allowed me to collect valuable information to answer the research questions and to obtain related personal information from participants. As soon as possible after each interview, I wrote brief handwritten field notes of what I experienced or observed during each interview. The notes mainly served as a reminder of the date, time, location, and any specific details. Although there were no unusual occurrences during the interviews, these details were helpful when reviewing the participant's responses in my reporting.

A final step was conducting a member check, a step taken to verify and authenticate the preliminary findings and get feedback from participants relevant to the accuracy of their statements during the interviews (Hancock & Algozzine, 2006; Merriam 2009). This process helped to alleviate the possibilities of misinterpretation of interviewees' comments, actions, and viewpoints (Merriam, 2009). It also helped to guard against researcher biases on the topic (Hancock & Algozzine, 2006). Bias can occur in the planning, data collection, analysis, and publication phases of the study. To minimize bias, I began a process of self-reflection by journaling. Understanding my personal bias allowed me to avoid potentially unethical behaviors that may have impacted the outcome.

Data Analysis Results

Data analysis for this project study included organizing, reviewing, and interpreting the data for recurring patterns (see Yin, 2009). The process included transcribing the recorded interviews, coding data, categorizing the coded data, and identifying the primary patterns and themes in the data. Table 2 shows the participants' demographic information.

Table 2

Demographics of Study Participants

Participants	Gender	Years teaching	Grade level	School
M001	F	10 years	3-6	Montessori
M002	F	5	K	Montessori
M003	F	15	PreK-K	Montessori
M004	F	5	K	Montessori
T001	F	14	1-2	Traditional
T002	F	12	2	Traditional
T003	M	15	2	Traditional
T004	F	16	K-6	Traditional
T005	F	28 years	K-6	Traditional
T006	F	32 years	K-6	Traditional

The participants included four Montessori teachers and six traditional teachers, all elementary teachers from the Northeastern United States. The *M* refers to Montessori, and the *T* refers to traditional. Elementary schools include prekindergarten through Grade 6. The years of teaching experience ranged from 5 to 32 years. All participants were assigned numbers (e.g., M001 and T001) to protect their confidentiality, and the school setting was not identified in reporting the demographics.

I designed the interview questions to obtain data that could answer the research questions. Teachers from both Montessori and Traditional schools were asked to respond to the same series of questions. The findings were generated from the analyzed face-to-face interview data. Excerpts from the interview responses of the teachers are presented in the following section.

Research Question 1

What are teachers' perceptions and beliefs regarding their classroom practices in teaching basic math operations of addition, subtraction, multiplication, and division?

Both groups of teachers were asked to explain their belief regarding how basic math principles should be taught at the elementary level. M001 believed that teaching basic math principles should be taught in "a concrete fashion and progress to the abstract." She believed that students learned basic problem-solving skills at an early age through role modeling and guided exploration. "As your little one learns to crawl, walk, talk, play, they are constantly observing and role modeling." M001 also believed that the most effective teaching method to help the child understand the fundamental of math equations and numbers was the Montessori method. "It starts with very concrete concepts and then moves to more abstract at a gradual process."

M002 believed that the student's problem-solving skills were developed through practice and exercising reasoning abilities through the manipulation of concrete materials in their environment as in real life situations. She explained:

> For example, during role call the teacher can pose a question such as "There are X number of students are in our class, so and so are not here today, how many students are in class today". Some students may count all students present in the classroom (counting forward, addition), others may count backwards (subtraction).

T001 believed that students should have hands-on opportunities in math to learn skills using concrete materials prior to applying skills to abstract concepts. Students need repetition and routine to master skills. She felt the students needed a specific procedure in order to learn problem solving skills. "The procedure should be modeled by a teacher, and students must follow the procedure systematically along with a teacher (until procedure is mastered) every time they encounter a problem-solving question."

Continuing, T001 said that she believed that once students understand the concrete reasoning behind equations and basic math facts, students should practice math facts regularly. "I have found that if students focus on a particular set of facts (ex. plus 1 facts or doubles facts), they are able to master them sooner than if all facts are randomly practiced together. I have found success using the Mastering Math Facts program (by Otter Creek) when using it in a smaller group setting." T001 said that when teaching addition, her students typically use counters (blocks, connecting cubes, counting bears, etc.) and/or a number line. I have also successfully used Touch Points to encourage counting on mentally (or invisibly on paper) rather than using fingers.

T004 believed that the most effective method is concrete materials and explicit instruction. Ex: 4+2=6 would be shown with some kind of rods and counters, and the materials would be manipulated to show the learner the operation. T004 claimed that she taught addition using many strategies. “Ex: Make a Ten would use 10 frames and counters to show Turn Around Facts. I use dominoes to explain.”

T004 admitted that she was not that familiar with Montessori. “My understanding is that it is student directed. In that case, my lessons are teacher directed in terms of concepts and skills. Once the lesson has been taught, some of what the students do is directed by them.” Continuing on, she said some students struggle with number sense. “This can seriously impede their work with numbers. Using materials and manipulatives is crucial for these students to have a concrete foundation of what numbers actually represent.” T004 noted that most of her students have a lack of number sense and a lack of exploratory problem solving. “It’s extremely important to have mastery. Number sense and operational mastery is the building block for future competency in higher order concepts and skills.”

T006 with more than 32 years of teaching experience believed students should be given opportunities to explore concepts, and with guidance draw conclusions. “Students should read, explain what they understand by using pictures, models etc.” T006 believed students need hands-on, concrete examples modeled consistently for them. She uses a variety of math exercises and materials to teach addition. “We discuss strategies for addition such as using counters, making a picture, using number bonds, using number line.” She reviews with students and if someone has difficulty, she will work with him or her to help them understand the concept.

Regarding students with difficulties in math, T006 believed that young students do not always understand the value of a number. “They need to have concrete interactions with numbers. For example, they need to not just say 7, but should count out 7 objects and discuss it and work with different numbers like this for a while, so they learn numbers in a concrete way. This would let them realize that 6 is greater than 5, so when you add 6+1, the sum cannot be 5. Also, they know the equal sign, but has not grasp it that 5+1=6 means 5+1 is the same as 6. T006 feels that the Montessori approach is similar to the approach in which students are expected to use their initiative and work at their comfort level.

Consistent with the other responses, T006 believed that mastery in numbers is the basic building block for the other areas. “We measure and write results in numbers and unit, and number base ten; so, students need to have a good grasp of number concepts both concrete and abstract.” T006 believed that many students lack comprehension of what they are expected to do. “They need to learn to analyze the information they have, use what is important, and answer the question they are asked, then explained what was done.”

In summary, the teacher’ perceptions and beliefs regarding their classroom practices in teaching basic math operations varied depending on the grade level and age appropriateness. All of the Montessori and Traditional teachers believed in the concrete to abstract learning approach. The key difference was that the Montessori teachers had no time restraints as oppose to the Traditional teachers. Both believed that modeling the steps were effective practices.

Research Question 2

How do teachers describe the practices they use to teach students basic problem solving skills in math?

Each group of teachers offered different examples and descriptions of their classroom practices. T002, said, “I believe that the most effective teaching method to help the child understand the fundamental of math equations and numbers is teaching them in a fun way.” The majority of the teachers believed that the most effective method was concrete and introducing real life problem solving skills (like going to the supermarket; something cost $2 and you have $3, giving them real situations to think about. M003 believed that problem solving skills work best when acted out. “This makes the problem a little more concrete for children. For example, “Joseph had 6 carrots. His friend brought him 3 more. How many does he have now?” This is a perfect spot for a role play! As Joseph gets his 6 carrots and has 3 more, he can really see that there are 9 total. We do this with addition, subtraction, multiplication, and fractions too”.

M001 said she used the bank game, the addition strip board, and addition with the short bead stairs as math exercises. She explained that each child’s brain is different and views things differently. “I find that when I am laying out the manipulatives or the bank game, I often have to change or adjust things, so children are looking at them from the right perspective. But for the most part I have had very little difficulty moving children through the progression.” In closing, M001 felt that children should not simply know the names and order of numerals, but they should understand and internalize the quantity of each numeral through repetition.

M002 said that she believed the most effective teaching method to help the child understand the fundamentals of math equations and numbers was a “step by step or sequential approach.” She explained that the teacher has to take into account the individual’s pace and ability; beginning with a concrete foundation. encouraging individual practice. Continuing, M002 claimed that in Montessori, specific materials are designed to teach specific skills. “You can use objects or manipulatives to demonstrate adding objects to other objects to create a larger amount; removing some objects away from a group of objects to demonstrate subtraction.”

T002 felt that teachers should model and teach students different strategies to solve math problems. T002 added, that students should then be given the opportunity to try the different strategies that work best for them. “These include manipulatives to use (concrete method) and slowly try to have students use mental math during abstract learning.”

T002 believed that the most effective teaching method to help the child understand the fundamental of math equations and numbers is teaching them in a fun way, not just drilling them math facts. T002 said she used several exercises and math materials in teaching. “In my classroom, we do the traditional way of solving math worksheets, but I also play “around the world” a game for students to beat other students if they know their math facts. She acknowledged that she did not know enough about Montessori to compare differences. She believed that most students have difficulties with problems solving in the area of numbers and operations because “they learned them without having any importance or relation to them. For example, if a student saw a dozen egg carton, they could have easily made the connection 6 in a row and another 6 in a row equals 12. So, 6+6 =12.”

T003 believed that basic math principles on an elementary school level should be taught through visual, tactile, and auditory methods. “Students learn in various ways and there must be interactive avenues to address their unique abilities.” She believed students are most successful learning basic math problem solving skills when they developing problem solving strategies. “One example would be to provide students with math vocabulary, which will enhance problem solving skills that would direct students through developing clues for finding solutions.” T003 said the most effective teaching method for helping children understand fundamental math equations and numbers is to use a ten frame with counters. She explained, “Children are given a ten frame and 11 red and yellow counters. Children place 5 red counters dots on top and 2 yellow on the bottom. She was not familiar with the Montessori approach.

Continuing on, T003 said a mastery of numbers and operations is critically important when it comes to answering complex 2 and 3 step word problems. Students’ difficulties arise when there are time constraints on their problem-solving math activities. “I believe at some point there is room for the rote approach to basic math facts. Math should be interactive at the elementary youth level. As students get older, some become “turned off” towards math if they found it boring. Teacher should bring more fun and excitement to math.” T003 believed that basic math principles should be taught using appropriate manipulatives/visuals to allow the learner to “see” the math so it is a concrete concept to them. In addition, the concepts should be scaffolded and reviewed to build on previous learning.

T004 believed that some students were better problem solvers than others, but everyone can learn strategies to help themselves. All should be allowed to explore using manipulatives/materials to have an understanding of what they’re working with. Also, students should be given “hacks” to work their way through a problem; Ex: the CUBES method for word problems. Circle the numbers involved. Underline the questions being asked. Box the math action words. Evaluate how to solve. Solve the problem. This kind of script gives all students something to guide them**.**

T005 believed that there is more than one way to learn. "Learning by rote does not necessarily accomplish the ends to the means, unless you understand the process of getting there." Also, T005 believed that children should be given the opportunity to explore ways to find the answer. She said:

> In my classroom, when a problem is given, the children can use different means in solving it, such as an addition problem 6+8. Children can count on their fingers, draw out the problem, use a number line, and count 8 up from 6. These are a few ways that they solve the problem.

Continuing on, T005 said the most effective teaching method is diversity. "Not every child has the same thought process, when it comes to solving a problem. I encourage my students to find which way helps them best. I let the students choose which method they prefer to learn." When we meet and discuss our answers, I tell the students even if they solve it incorrectly, it's ok."

T005 said she used a variety of exercises and math materials when teaching addition. "I use a daily Do Now to start off the day in my classroom. I pose a math question on the board. I even write out the numbers in word form so that the students take their time in really reading the problem." Mastery of numbers and operations was extremely important to T005. "They are the building blocks of which most mathematical processes are contrived. Multiplication would be nothing without addition, and the same can be applied to subtraction with division. "Those two operations (addition and

Subtraction) are the basis of our mathematical system. Without mastery, these building blocks crumble."

T005 believed that there can be many causes for a student's difficulty in problem solving. She said:

> You have to factor in variables: home life (no support when a child comes home after learning a new concept). It could be that parents don't know how to support them, or their feelings of inadequacies or not knowing what tools they have that could help them solve their problems.

T005 acknowledged that she had no knowledge of the Montessori approach and could not make a comparison.

In summary, the teachers' description of the practices they used to teach basic problem-solving skills in math varied. The Montessori and traditional teachers employed a variety of exercises and used math materials in teaching problem solving skills. Diversifying the approach and recognizing that students learned differently were key considerations in the teacher responses. Two teachers expressed that learning should be fun.

Research Question 3

How do the practices and beliefs of traditional education teachers and Montessori teachers differ regarding teaching basic problem-solving skills in math?

Three of the six traditional teachers said they had no knowledge of the Montessori programs. T001 said her understanding is that Montessori approaches are student-led. Continuing on, she said:

> When students show an interest in learning a skill or applying a skill to a real-life situation, then teachers will provide instruction. At my current school, I need to teach skills according to the curriculum. I am bound by assessment dates, so there is very little flexibility with when or how students learn.

M002, M003, and M004 had taught in traditional settings before and offered what they believed were differences in teaching basic problem-solving skills in math. M002 said, “In Montessori, the curriculum, its sequence, methods, principles, and materials are very stable for both teacher and students. The traditional approach there is always the possibility of major changes every school year.”

T001 claimed that students seemed to lack experience with using basic math concepts in their lives. “Therefore, problem solving questions seem new to them, so it is difficult for them to relate and solve.” It is essential that students have mastered basic number concepts, including the basic operations, because they are used in all parts of life regardless of what occupation the students intend to seek.

M001 believed that Montessori differed from the Traditional approach in that the Montessori approach is more hands on as opposed to memorization. “It also starts with very basic concepts that are concrete and then builds off of that as the concepts become more abstract. Children are able to see what numbers look and feel like.” M001 believed that the ability to have mastery in numbers and operations is very important. She claimed that she did not find many difficulties as the materials work quite well.

M002 claimed that in Montessori, the curriculum, its sequence, methods, principles, and materials are very stable for both teacher and student; whereas, in the traditional approach, curriculums are subject to change every school year. “What is important one year, may suddenly not be important the next year. Montessori is very stable, solid. It makes the child's progress clearer." She said that the child can see, understand the progression of his or her own ability.

M002 said that in Montessori children can work for an extended period of time during the day on an exercise. They can repeat the exercise/work as many times as needed. She believed that for children who start Montessori at an early age, the practical life exercises build up their concentration and help them complete multiple-steps exercises. For children who are new to the environment, the lack of focus/concentration can cause difficulty for them.

M002 expressed that parents should be encouraged to do some real-life math work with their children without making it look like school. "Make it very normal instead of trying to duplicate school. For example, hand me four onions and half of the carrots; set the dinner table for five people." M002 believed that mastery of numbers and operations is very important. She believed that "students need to truly understand numeration and operations in order to adequately progress in the subject." She believed that when a child has a solid foundation in the subject, then difficulties are minimized.

She stated,

> A child needs to know and internalize that, for example, "5" is a symbol that represents the quantity of 5 units and "5" is more than 4 units of something and is less than 6 units. In Montessori, this kind of concrete to abstract, hands-on approach is incorporated in the method. Children need to spend time internalizing these math concepts; not just memorizing the order of the numerals.

M003 believed that children should be taught fundamental skills using hands-on manipulatives. This makes the problem a little more concrete for children. M003 believed the most beneficial teaching method to help the child understand the fundamental of math equations and numbers is using concrete materials and role play are here. “Quantity is a very abstract principle, but we have wonderful materials to make it more concrete.” She said her first addition lesson is usually a small group lesson to illustrate the concept of “joining two sets together.” We use various sets of objects, like markers, blocks, rulers, etc., and act out “Autumn had three markers. Jasmine brought three more.”

M003 believed that Montessori is more concrete and allows the children to get appropriate practice because they associate the materials with the work/operations that are being asked. “We would typically use bead bars for addition and then penguins for subtraction. Since we are not using the same material to illustrate the concept, it helps the child to remember he is doing a different operation when trying to complete 6-4. We also use golden bead materials to help solidify the decimal system. “

M003 thought the causes of her students’ difficulties in the area of numbers and operations was that children with these difficulties never mastered the understanding of quantity and how numbers relate to one another.” I think once a child really masters quantity and number relations, he/she can carry out operations a bit easier.” A common difficulty was students struggling with numeral identification. She believed that children need a strong foundation and that the world is made up of numbers to problem solve.

M004 believed for elementary age children, concrete hands-on experience is needed to help them understand adding even like fraction. “I think concrete is very important for this age. We use the multiplication board to do some very basic things; even for division. She believed that basic problem-solving skills are learned using real-life problems, putting them into real life situations. An example is,

> We have 21 children in class today, two of them are absent so how many children do we have. John is not here, and Philip is not here, so let's take away one then it would be 20, and we take away another one and it will be 19. Also, another thing we can do, in the classroom we have lots of objects (frogs or pumpkins). They use them to do addition and subtractions.

M004 felt that Montessori has a very complete curriculum. "It is very step-by step, very sequential. I really, really like it because you work on one part, one level and know what comes next." Continuing, she said, "Montessori curriculum is really very helpful for the teachers to see where the child is and to help them where they are and help them move on to the next level." M004 believed that to have mastery in numbers and operations is very important. "If you cannot recognize the numerals, then you cannot do anything like addition or subtraction. Those are very basic skills they must have." T001 and M003 added the necessity of proficiency in the subject beyond the classroom and into everyday living as well as for students' future occupations.

All 10 teachers believed that concrete materials / manipulatives and teacher modeling are useful in helping students learn basic problem-solving skills. Montessori teachers discussed the importance of allowing learners to practice what was taught. Three Montessori teachers believed role playing also helped children. T002 also expressed that role playing is useful in helping students learn problem-solving skills.

T001, T003, T004, and T006 discussed how equipping students with various problem-solving strategies is helpful. T004 talked about strategies with mnemonics such as CUBES (circle, underline, box, evaluate, solve). “This kind of script gives all students something to guide them.” T001 also mentioned that students should “practice math facts regularly” and should do so by focusing on “particular set of facts; ex. Plus 1 or doubles facts” while T002 believe math fact drills should be minimized and teaching should be done in a “fun way” by “making it a game and by giving them group work to solve problems, of course with manipulatives”. T005 noted teacher should model with consistency and T006 believed in teaching various problem-solving methods and letting students choose their preferred methods. Montessori teachers emphasized a step by step sequential and gradual approach, using real-life situations, and tailoring to each student’s individual ability. M002 also stressed individual practice using the materials.

In summary, the ten teachers’ responses from both instructional settings ranged across the spectrum on their differences in beliefs and practices on how the basic problem-solving skills in math were taught. All 10 teachers believed that concrete materials / manipulatives and teacher modeling were useful in helping students learn basic problem solving skills. Montessori teachers and traditional teachers discussed the importance of role playing and allowing learners to practice what was taught. T002 also expressed that role playing is useful in helping students learn problem-solving skills.
A key difference noted was the expression of individuality specifically among Montessori teachers. The key phrase was “tailoring to each student’s individual ability” with a focus on individual practice using the math materials.

Summary of Interview Data Analysis

The overarching research question of this study examined both Montessori and traditional school teachers’ perceptions and beliefs regarding their classroom practices when teaching basic mathematic principles in the elementary setting. Excerpts and frequently occurring categories from both the Montessori and Traditional teachers' interview responses were compared and contrasted. From the analysis of the interviews emerged four themes: The four themes were: (a) concrete to abstract learning; (b) problem solving through manipulatives and visuals; (c) teacher role modeling; and (d) problem solving through applying real life situations.

Concrete to Abstract Learning

Concrete to abstract learning was the strongest of the four themes and was mentioned in teacher responses to at least five of the eight interview questions. All 10 participants believed the best way that basic math principles should be taught at the elementary level was through the concrete to abstract approach. The terms were mentioned at least 17 times. In the concrete stage, the teacher begins instruction by modeling each mathematical concept with concrete materials (Hauser, 2009). In other words, this stage is the "doing" stage, using concrete objects to model math problems.

In the abstract stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notation, and mathematical symbols to represent the number of circles or groups of circles. The teacher uses the basic operation symbols (+, –, x, /) to indicate addition, subtraction, multiplication, or division. This is the "symbolic" stage, where students are able to use abstract symbols to model problems (Hauser, 2009). Aligned with the views of the teachers, studies have shown that students who use concrete materials develop more precise and more comprehensive mental representations, understand mathematical ideas, and better apply math ideas to life situations (Hauser, 2009).

According to Piaget (1972, 1977), the stage covers ages 7 – 12 years. This stage is particularly important to primary school teachers as most elementary school age children function at this level (Ediger, 2012; Looijenga, Klapwijk, & de Vries, 2015). The concrete operational stage described by Piaget clearly supports the findings in the present study. As it relates to math, this stage marked the beginning of what is called the *logico-mathematical* aspect of experience. The concrete operational stage was deemed important for mathematics learning because many of the operations children carry out at this stage are mathematical in nature (Piaget, 1972, 1977).

The traditional teachers viewed math manipulatives as additional items to the assigned curriculum's text/work books and thereby utilized them as such. The Montessori teachers viewed the math materials as an integral part of the curriculum and not exclusive of each other. Aligned with Piaget, the findings showed that both groups of teachers exercised the use of concrete materials / manipulatives to facilitate students understanding when teaching mathematical concepts (Cable, 2014; Kefaloukos & Bobis, 2011; Piaget, 1972, 1977).

Problem Solving Through Manipulatives and Visuals

Manipulative materials are concrete models or objects that involve mathematics concepts. The most effective tools are ones that appeal to several senses, and that can be touched and moved around by the students. These are (not demonstrations of materials by the teacher). Heddens (1997) noted that the manipulative materials should relate to the students' real-world occurrences. All of the teachers discussed the use of manipulatives / concrete materials to convey mathematics concepts to the learner. Four traditional and two Montessori teachers emphasized the use of tactile / hands-on materials. T002 said, "Students should be given manipulative to use (concrete method) and slowly try to have students use mental math during abstract learning." T004 said, "Using materials and manipulatives is crucial for these students to have a concrete foundation of what numbers actually represent." T004 elaborated on the use of visuals "to allow the learner to *see* the math." M002 believed the use of manipulatives help learners to understand and internalize math concepts as they progress from concrete to abstract. The Montessori teachers expressed their preference of the Montessori materials to any other manipulatives.

Teacher Role Modeling

With regard to teacher role modeling, combined, this theme was mentioned nine times by participants in the context that teachers should model and teach students different strategies to solve math problems. This was a common concept mentioned by all the teachers. T006 said "students need hands-on, concrete examples modeled consistently for them." M001 said, "I believe that students learn basic problem-solving skills at an early age through role modeling and guided exploration. T001 said, "Students need a specific procedure in order to learn problem solving skills. The procedure should be modeled by a teacher, and students must follow the procedure systematically along with a teacher (until procedure is mastered) every time they encounter a problem-solving question."

According to the American Montessori Society (AMS, 2015), older children are encouraged to serve as role models, and teachers serve as guides rather than givers of information (AMS, 2015). The premise is that students also teach and learn from each other. The Montessori teacher prepares, manages, and maintains the classroom environment without being the focal point of the environment (Montessori, 2006).

The findings showed that teachers believed they significantly impacted their students' learning of mathematics. They believed themselves to be an integral part of the track to improving students' mathematics achievement. As explained by the Koehler and Grouws' (1992) model, teachers' beliefs impact their teaching extensively. For example, participant M004 believed the Montessori curriculum to be a "complete" curriculum and it was useful for both teacher and student. The curriculum guides the teacher in their work and the student as the learner.

Problem Solving Through Applying Real Life Situations

Applying math skill to real-life situations was mentioned by four of the Montessori teachers. They believed that basic problem-solving skills are learned using real-life problems, putting them into real life situations. The traditional teachers were more aligned with the non-constructivist teacher-centered approach to teaching mathematics. Collectively, the traditional teachers asserted the need to teach in a direct way due to being bound by assessment date, minimum flexibility allowed, and time constraints. They discussed the importance of teaching math facts as well as different ways or resources to teach it.

Swanson (2014) believed that learning takes place when children can apply their mathematics knowledge to the real world, they learn to collaborate, and they can solve problems on their own. Swanson claimed that this is key for successful STEM implementation. For these reasons, elementary teachers should seek ways to develop and implement instructions that teach abstract math concepts through more hands-on and inquiry-based activities (DeJarnette, 2012).

Consistent with the literature, the Montessori teachers were more aligned with the student-centered approach described by Holfester (2014). The Montessori teachers discussed the brief individual and small group lessons, generous amount of time students can use to practice and explore concepts introduced through the use of the Montessori self-correcting materials, as well as the teachers' role as guide. These characteristics were ascribed to the student-centered instructional approach as noted in the literature (Polly et al., 2014).

Project Deliverable as an Outcome of the Results

The Project Deliverable is a Position Paper on the topic of *Problem Solving in Math Using Concrete to Abstract Learning.* The position paper is centered around the outcome of the results supported by researched based literature on the topic. The intended audience is teachers who seek to explore effective teaching principles necessary to help students acquire and generalize math concepts and skills. Although the results of the initial case study indicated that teachers used several strategies to improve math learning at the elementary level, there was no conclusive evidence of superiority of one over several approaches that teachers should employ. The findings indicated that math involves several different components and the teacher approaches varied with respect to assisting elementary school students with difficulty grasping math concepts.

Summary

Section 2 outlined the research design method that was used to implement the study. I described the data sources and the way data were collected and analyzed. The section outlined the following: (a) population and sample, (b) instrumentation, (c) data collection procedures, (d) ethical considerations, and (e) the data analysis strategy and a summary of results.

Section 3: The Project

The project study included a position paper (see Appendix) based on the key findings from the research reported in Sections 1 and 2. The purpose of this position paper was to present detailed information to support teaching math concepts at the elementary level. Concrete to abstract learning emerged as the strongest of the four themes from the data analysis. In the concrete stage, the teacher begins instruction by modeling each mathematical concept with concrete materials (Hauser, 2009). Houser noted that this stage is the doing stage, using concrete objects to model math problems. In the abstract stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notation, and mathematical symbols to represent the number of circles or groups of circles. The teacher uses the basic operation symbols (+, –, x, /) to indicate addition, subtraction, multiplication, or division. This is the symbolic stage where students are able to use abstract symbols to model problems (Hauser, 2009).

My Position

To develop every student's mathematical proficiency, teachers should systematically integrate concrete and abstract concepts in classroom instruction at all grade levels, especially in the elementary setting. The key points of this position paper are the following:

- mathematics education should focus on enriching a child's inner resource and in developing life skills;
- a well-established math curriculum should be available to all students regardless of the school setting or type;

- in the concrete stage, the teacher begins instruction by modeling each mathematical concept with concrete materials (Hattie, 2012); and
- in the abstract stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notation, and mathematical symbols (Hattie, 2012).

Aligned with the position of The National Council of Supervisors of Mathematics Improving Student Achievement Series [NCSM], (2013), my position is that leaders and teachers can accomplish integrating the concepts of concrete and abstract learning in classroom instruction when they fully understand that manipulatives can be powerful learning tools in building mathematics skills at the elementary level. The key premise is that concrete models are essential tools for learning mathematics across all grade levels.

Goals

The goal of the position paper was to provide recommendations for mathematical instruction in the elementary setting to improve student achievement. The research findings suggested that both the traditional and Montessori teachers perceived four elements to be essential to teaching mathematics at the elementary level. The elements are (a) concrete to abstract learning, (b) teacher role modeling, (c) applying math to real-life situations, and (d) learning math through manipulatives and visuals. The element of concrete to abstract learning was the most prominent of the four themes. Although both groups of teachers emphasized concrete to abstract learning, the concept was expressed and used differently in each environment.

Rationale

The rationale for developing this position paper was not solely based on what students should know, but rather on the instructional approach that teachers in all settings should know and practice to help students achieve mastery of math concepts in the concrete and abstract approach. The Common Core State Standards indicated that concrete models are essential for learning mathematics at all grade levels (NCSM, 2013). Research suggested that students who use concrete materials develop more precise and more comprehensive mental representations, show more motivation, better understand mathematical ideas, and understand the applications in real-life situations (NCSM, 2013).

Review of the Literature

In this literature search, I used peer-reviewed articles from the Walden databases (ERIC, Proquest, EBSCO Host) and multiple Internet government sources. I also used the public library and several books on Montessori education. Search terms used were *concrete to abstract learning*, *mathematics*, *manipulative materials*, *abstract symbols*, *learning and instruction*, *concrete instruction*, *abstract instruction*, *mathematics*, *elementary education*, *hands-on learning*, and *Montessori materials.*

Concrete to Abstract Learning

The concept and practice of concrete to abstract learning in mathematics is common in U.S. elementary education. Concrete to abstract learning is a process of learning that begins with the modeling of abstract mathematical concepts with concrete materials using manipulatives to facilitate students' learning of mathematical principles, then gradually transitioning into modeling the concepts symbolically (Akinoso, 2015; Fyfe, McNeil, Son, & Goldstone, 2014). The practice was supported in the work of developmental theorists such as Piaget, Bruner, and Dale. Piaget (1970) emphasized that learners should physically interact with concrete representation. Bruner's (1966) three modes of cognitive development included a manner in which to present new concepts: enactive representation (concrete), iconic representation (pictorial or graphic), and symbolic representation (abstract). Dale (1969) maintained that more is retained by learners by what they do in contrast with what they hear, read, or observe, as indicated in the cone of experience.

Common Core States Standards for Mathematics

The Common Core States Standards for Mathematics was developed by the National Governors Association and the Council of Chief State School Officers and adopted by 42 states. The use of concrete models was emphasized in the standard for mathematics practice. The standards also suggested using concrete models in initial steps before progressing to other representational forms. The NCSM (2013) took the position that manipulatives must be integrated in classroom instruction to aid students in developing mathematical proficiency.

Mathematics concrete materials, also referred to as instructional manipulatives, include physical, virtual, and pictorial or graphic objects. Abstract materials involve symbolic representation (Swanson & Williams, 2014). Concrete materials promote thinking of objects, ideas, or concepts as specific items whereas abstract learning promotes generalization of principles (Fyfe et al., 2014; Swanson & Williams, 2014). The use of concrete materials in the learning of mathematics enables learners to construct their knowledge of abstract mathematical concepts. Concrete materials provide a practical context for the student during learning and prompt the need for action, physical or visualized, which can increase understanding or memory (Fyfe et al., 2014; Larkin, 2016; Moyer, 2001; Pouw, Gog, & Paas, 2014).

Belenky and Schalk (2014) cautioned that manipulatives that have irrelevant details may distract learners as such materials may draw attention away from their referent. Other researchers suggested the use of abstract materials and the avoidance of concrete materials. They argued that abstract materials promote the learner's focus on structure, eliminate the possibility of the learner dealing with unrelated details, and have a better likelihood of extensive application (Kaminski, Sloutsky, & Heckler, 2009; Uttal, O'Doherty, Newland, Hand, & DeLoache, 2009).

Research showed there are advantages and disadvantages to working exclusively in either a concrete or abstract framework for mathematics (Fyfe et al., 2014). Koblan (2016) investigated the impact that concrete experience (use of manipulative) together with abstract (traditional lecturing and exercises in books or on board) had on math achievement in seventh grade students with different learning styles in three different environments. The results indicated mixed outcomes. In one environment where mixtures of manipulatives and traditional methods were applied, achievement was not significant. However, when manipulatives were used, concrete learners showed higher mathematics performance than the abstract learners showed when only traditional methods were used. In another environment, concrete learners did not experience additional benefits due to increased number of manipulatives. In a third environment where only lecture and exercise-based activities were used, abstract learners performed better than concrete learners.

Concrete to Abstract Model

There are advantages and disadvantages in working exclusively in either concrete or abstract framework (Fyfe et al., 2014). Several variations of concrete to abstract learning mathematics instructional formats are employed in educational environments, and are generally aligned with techniques for solving math problems (Fyfe et al., 2014) beginning with concrete materials and gradually moving into the abstract. The technique combines the benefits of concrete and abstract materials while mitigating their disadvantages (Fyfe et al., 2014).

A common model of the concreteness fading technique is the concrete-representational-abstract (CRA) teaching sequence or concrete-representational-abstract instruction strategy (CRAIS). CRA is grounded in the works of notable theorists (Ding & Li, 2014; Flores, Hinton, Terry, & Strozier, 2014; Mancl, Miller, & Kennedy, 2012; Miller & Kaffar, 2011; Mudaly & Naidoo, 2015). It is used in the instruction of mathematics operations, computations, and other concepts for a wide spectrum of learners including students with autism spectrum disorder (ASD), students with developmental disabilities (DD), and general education students (Ding & Li, 2014; Flores et al., 2014; Mancl et al., 2012; Miller & Kaffar, 2011; Mudaly & Naidoo, 2015).

The CRA method involves three levels of instruction in which manipulatives are used on the concrete level and drawings are used on the representation level. The abstract level is often combined with integrated strategies. For instance, on the abstract level, computation involves learning a strategy, for example with a mnemonic, for solving problems using numbers only (Bruun, 2013; Flores, Hinton, & Burton, 2016). A sample strategy could be DRAW: (a) discover the sign, (b) read the problem, (c) answer or draw and check, and (d) write the answer (Bruun, 2013; Flores et al., 2016). The method or instructional sequence in teaching mathematics is said to work well with individuals, small groups, and whole groups (Akinoso, 2015; Mudaly & Naidoo, 2015).

Research findings on the CRA model have shown positive results. Flores et al. (2014) found that students with ASD made significant progress using the CRA and strategic instruction model when tested on three curricula-based measures. When two kinds of interventions, CRA and the designed regular method of teaching intervention for the particular school, were administered to two groups of randomly selected junior high school students (80 total). Kwaku Sarfo, Eshun, Elen, and Adentwi (2014) found that both groups improved significantly; however, the independent *t* test showed that the CRA was more effective in improving participants' performance in algebra and geometry.

Mathematical Tools/Materials/Manipulatives

Results of the current project study revealed that the elementary teachers were in concert with other researchers in their belief that concrete materials are essential for mathematics instruction. The teachers in the present study believed these materials were important tools that help guide students through the process of concrete mathematics to abstract mathematics learning. Mathematical tools, as defined by Van de Walle, Karp, and Bay-Williams (2013), are objects of any kind, illustrations, or pictures that are either representative of a concept or can be used to express the attributes, qualities, or features of that concept.

In the framework of concrete to abstract learning, concrete materials or manipulatives fall into two groups; physical and virtual (Van de Walle et al., 2013).

Physical materials/manipulatives are physical objects that can be held/handled and manipulated for instruction, demonstration, and discovery of mathematics concepts (Bouck, Satsangi, Doughty, & Courtney, 2014; Ding & Li, 2014; Van de Walle et al., 2013). Virtual manipulatives are digital expressions or animations of physical objects that can be manipulated on a screen by touching the screen, using a computer mouse, or using another digitalized interactive instrument such as the writing implement used with smartboards (Bouck et al., 2014; Shaw, Giles, & Hibberts, 2013; Van de Walle et al., 2013; Yildirim, 2016).

In the appropriate classroom environment, both groups of concrete materials, physical (hands-on) and virtual (computer base), can be beneficial (Ekmekci & Gulacar, 2015). Advantages to using virtual manipulatives include the availability of exploration opportunities, which differ from the experience of working or exploring with physical manipulatives. Virtual manipulatives also promote individual learning as the student is in control of the activity (Moyer-Packenham & Westenskow, 2013). Researchers credit both types of concrete materials with helping to develop children's mathematical understanding (McDonough, 2016; Tucker, Moyer-Packenham, Shumway, & Jordan, 2016) as well as for having positive effects on student achievement (Moyer-Packenham & Westenskow, 2013; Satsangi, Bouck, Taber-Doughty, Bofferding, & Roberts, 2016). However, when selecting mathematics manipulatives, consideration should be given to the developmental level of the user, the perceptual richness of the object, and the level of guidance during manipulative use (Carbonneau, Marley, & Selig, 2013; Laskin, 2016).

Moyer-Packenham et al. (2015) assessed learning performance and efficiency of

100 children ages 3-8 working with virtual mathematics manipulatives apps on iPads. Each group of children (preK, K, and Grade 2) responded differently to the apps, and some apps produced better results than others. In another study comparing the benefits of physical and virtual manipulatives used as tools of algebraic instruction in 30 intervention sessions for three students with learning disabilities, Satsangi et al. (2016) found that the students averaged 90% solving problems with both kinds of concrete materials; however, two of the students earned higher scores with the physical manipulatives.

Traditional and Montessori Teachers: Concrete Materials

Research indicates that the use of mathematical manipulatives during instructional time and during student classroom practice helps to convey mathematics concepts or ideas, solve mathematical problems, enhances number sense, and improve mathematics achievement (Pei-Chieh, Mao-Neng, & Der-Ching, 2013; Toptaş, Çelīk, & Tuğçe Karaca, 2012). The use of mathematics manipulatives is important to the concrete to abstract learning of mathematics (Toptaş et al., 2012).

In a study conducted by Toptaş et al. (2012) where 137 primary grade teachers in 25 elementary schools were questioned on their frequency of using mathematics manipulative (physical, illustration/drawing/picture, virtual), findings showed overall actual use of classroom manipulatives was not satisfactorily high. Toptaş et al. (2012) found this result to concur with literature that conclude that teachers' awareness of the importance of the use of manipulatives to aid mathematics learning were not in alignment with their actual practice of utilizing them. Toptaş et al. (2012) findings were contrary to the findings from this project study concerning the Montessori teachers and their classroom practice. This project study findings revealed high usage of mathematics material / physical manipulatives by Montessori teachers.

The Montessori curriculum, materials, and instruction are one integral unit of the Montessori method; instruction and materials are interconnected. The traditional teachers, as examples, consistently mentioned equipping students with strategies for solving mathematics problems modeled by teacher; the use of worksheets; knowing math facts. These scenarios are consistent with abstract learning. Both the traditional and Montessori teachers utilized “visuals” (illustrations/drawings/pictures) often; sometimes by default in the traditional classroom due to such inclusions in text books and workbooks. Use of virtual manipulatives did not have a strong presence in the Montessori classrooms. The traditional classrooms were more furnished than the Montessori classroom with technological equipment such as desktops and/or laptops. Neither groups mentioned utilizing virtual manipulatives for instruction.

Project Description

The project deliverable is a 15-page Position Paper entitled, *Concrete to Abstract Learning in the Teaching of Mathematics in Elementary Schools*. The position stressed in this paper is that in order to develop every student’s mathematical proficiency, teachers must systematically integrate the use of concrete and abstract concepts into classroom instruction at all grade levels, especially in the elementary setting. The overall aim is to improve student achievement in mathematics at the elementary level. The belief is that mathematics education should focus on enriching a child’s inner resource and in developing life skills. The key position is that teachers need to be knowledgeable, familiar, and comfortable with teaching mathematics using for students to receive the full benefits of the materials in the concrete to abstract learning progression (Golafshani, 2013).

The findings in the case study indicated that all teachers expressed the importance of concrete to abstract learning and how essential it is to their own classroom practice. However, consistent use of physical manipulatives was not evident with traditional teachers. I believe that this position paper may serve as a valuable resource on the topic for teachers and may be helpful in implementing concrete to abstract learning. Researchers suggest that professional development can strengthen teachers' implementation and in turn can translate into student academic success (Polly, 2015; Spelman & Rohlwing, 2013).

Potential Barriers

A potential barrier for the project may be the unwillingness of both traditional and Montessori administrators and teachers to buy-in / partake in the recommendations shown in the position paper. The study findings indicated that concrete materials / manipulatives are incorporated and utilized differently in the teaching and learning of mathematics. Successful implementation will require some adjustment from each group. It is therefore foreseeable that the position paper may be embraced partially or fully by some, but not necessarily by all.

Project Evaluation Plan

The type of evaluation planned for the project deliverable is outcomes based and was evaluated based on the guidelines established by Walden University EdD Project Study for a qualitative study. The final project will be evaluated by the Project Study

Chair and committee members. The following evaluation plan was established for the present position paper.

1. Meet standards of genre at a scholarly doctoral level.
2. Ensure immediate applicability to setting and problem
3. Use appropriate language for stakeholders or audience.
4. Include only original products

Project Implications

The key implication for this project study is positive social change. Teachers and school administrators must work to ensure that research-based recommendations are implemented in their individual school districts and states, regardless of the setting. Actions must be taken to create and sustain the classroom conditions and environment will enable every mathematics teacher and student to use manipulatives successfully. There should be some assurance that the school curriculum supports the use of manipulatives by their inclusion as an instructional tool on par with textbooks and other resources. Positive social change will be realized when all students from all demographics begin to understand math concepts leading to success. It is hoped that the information gathered from this study will help form the basis for a meaningful understanding of effective teaching approaches to mathematical operations.

Section 4: Reflections and Conclusions

The purpose of this project study was to explore practices of traditional and Montessori elementary teachers relevant to their perception to teaching mathematics. Section 4 includes my reflections on the process of organizing and carrying out this study and on my role as a scholar, practitioner, and researcher. Some gains resulting from this study included examples and recommendations for concrete to abstract learning in teaching mathematics. Details are reflected in the position paper. Suggestions for future research are also noted in this section.

Project Strengths and Limitations

The project deliverable is a Position Paper with a focus on concrete to abstract learning in the teaching of mathematics in elementary schools. During the development of this project, I identified two key strengths and limitations. The major strength of this position paper was that I was able to state my personal opinions and formulate my argument on the topic based on my case study. In addition, I was able to present sources and provide a reference list for readers who want to read more on the topic.

Noteworthy, this paper only included my viewpoint and perspectives on the topic. Due to time constraints, I was unable to administer a survey and obtain feedback from the intended audience of teachers and administrators. Therefore, the perspectives and opinions of others on the position paper were not obtained.

Scholarship, Project Development and Leadership and Change

The completion of this project study enabled me to acquire greater insight into how traditional and Montessori teachers' beliefs inform their mathematics instructional practices. This project afforded me the opportunity to engage with teachers and support them by observing, listening, recording, and valuing their perceptions. I was privy to these teachers' views about curriculum, mathematics methods and strategies, as well as frustrations and/or concerns regarding time constraints and mandates placed on them. It is my hope that upon the releasing and sharing the results of the study and the project, teachers will find both to be a genuine support and meaningful resource that informs their classroom best practices and assists them and the district in their goal to improve student mathematics achievement.

I chose this research project to assist teachers in the incorporation and implementation of concrete to abstract learning in their classrooms. The project was also selected to guide the district in providing appropriate and relevant backing to the teachers in the district's effort to improve student mathematics achievement. As an educator who has taught in both traditional and Montessori environments, I thought it was necessary to develop a project that supports both groups of teachers and promotes student achievement. Information furnished during the interviews made clear the need to encourage and aid teachers through a concrete to abstract sequence of instruction. This project provided an opportunity for me to produce research-based suggestions to address academic concerns for student achievement and teachers' instructional practices.

My scholarship in the field of education commenced with my initial course as a doctoral student at Walden University. Every subsequent course, professor, and residency challenged me in my academic pursuit and prepared me for this stage of the doctoral process. I used Walden's support resources such as the library and research center. My chairs guided me through communication, commentary, and feedback. The research designs and methodologies studied in the doctoral program were applied during the research portion of this study.

During data collection and analysis, I found interviewing the participants and observing and listening to their responses and later analyzing those responses to be an intriguing part of the process. It strengthened my resolve to become a part of the community of educational researchers assisting in creating positive change in education that can increase student achievement. The research process also taught me to embrace a deeper sense of empathy and understanding for the undertaking of the work, tasks, and practices of the teacher and his or her profession. New mandates, changing curriculum, and evaluation of teacher performance in the classroom present a need for relevant support for teachers and their classroom practices for teaching mathematics. This creates opportunities for research and leadership in teaching and learning.

Reflection on Importance of the Work

The processes of continuous observation, analysis, discerning, sorting, and evaluating data to formulate and support my decisions in the classroom and in other capacities in which I have served in private and public educational environments have been ongoing in a general and informal sense. Because of my scholarly training with Walden University, these processes have been scientifically ordered and streamlined. Achieving this doctoral degree will allow me to function as a formal and qualified researcher able to navigate these certified research processes in a professional manner to identify gaps, gather and analyze data, and recommend research-based solutions for social change in my field of education. This degree will enable me to add to the educational literature. It will also afford me the opportunity and responsibility to nurture and guide others in their higher educational goals.

As a result of this doctoral experience, I am committed to become a lifelong learner and to be more empathetic to the experiences of others. I believe that as the field of education continues to add more standards, mandates, and evaluation models to its array of resources for student achievement, teachers need to be included in the discussion and decision-making in a professional and meaningful way. I am excited to participate in such endeavors. The project development has strengthened my confidence to realize my ability to research problems and prepare and organize projects that could contribute to solutions and address the needs of my profession and its diverse membership.

Earning and applying this highest level of education positions me to effect change in my career field. The trend of individuals seeking to acquire and being granted entry into the ranks of educational influence and leadership as business entrepreneurs, financiers, owners, and policymakers without the requisite scholarship and licensure has created the need for credentialed and principled educators to restore proper and appropriate respect for professionalism and high esteem for sound, effective, and research-based policies. Unfortunately, some teachers in positions outside of the immediate classroom fall prey to becoming disconnected in their attitude and sensibility about real classroom experiences. My research at Walden University combined with my professional background and experience will prepare me to function in an advisory capacity to promote balance, fairness, and equity concerning all aspects of the field of education. Such efforts will reflect the purpose of social change to advance sustainable long-term approaches to promote student achievement.

Implications, Applications, and Directions for Future Research

The implications of this study include increased understanding of teachers' beliefs and perceptions related to instructional practices in the core subject of mathematics. Concrete to abstract learning is a research-based approach to teaching and learning rooted in educational theories and supported by the literature. Findings from the current study indicated its familiarity among teachers and use in mathematics classrooms. Although both groups of teachers in this study acknowledged the importance of concrete to abstract learning in the classroom setting, not all of the traditional teachers were able to apply it to their practice consistently with fidelity. The findings and the position paper developed in this study may provide support for these teachers.

Conclusion

The central focus of the position paper was to present ways to ensure that every student develops the foundation for learning mathematics, regardless of academic orientation. A wide range of strategies may be used for teaching mathematics, and some teachers may prefer one method over another. However, all teachers should provide students with multiple approaches for learning mathematics. Hattie (2012) noted that there is a balance between teachers talking, listening, and doing. The same holds true for students. Concrete to abstract learning provides a foundation around which teachers and students can talk, listen, and do.

To ensure that every student learns mathematics, using manipulatives is a critical part of the process. Teachers must continue to help students see the connections between manipulatives and mathematical concepts being taught. To ensure that research-based recommendations are implemented in their schools, teachers and school leaders must act to create conditions that will enable every mathematics teacher to use manipulatives successfully to promote positive academic outcomes.

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Appendix: Position Paper

Presented by Linda Kofa

Concrete to Abstract Learning in the Teaching of Mathematics in Elementary Schools

Abstract

The purpose of this position paper is to generate support for teachers and administrators on the topic of: *Concrete to Abstract Learning in the Teaching of Mathematics in Elementary Schools.* The overall aim is to improve student achievement in mathematics at the elementary level. *Concrete to Abstract Learning* emerged as a preferred teaching method among traditional and Montessori teachers in the project study. The position that is stressed in this paper is that in order to develop every student's mathematical proficiency, teachers must systematically integrate the use of concrete and abstract concepts into classroom instruction at all grade levels, especially in the elementary setting. The belief is that mathematics education should focus on enriching a child's inner resource and in developing life skills. In the concrete stage, the teacher begins instruction by modeling each mathematical concept with concrete materials. In the abstract stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notation, and mathematical symbols. This is the "symbolic" stage, where students are able to use abstract symbols to model problems.

POSITION PAPER

Concrete to Abstract Learning in the Teaching of Mathematics in Elementary Schools

Introduction

Mathematics is a major subject in education and is a critical part of elementary school academic preparation for children in the early stages of development (Morrison, 2011). The subject is instrumental in the development and progression of scientific and technological advancement (Akinoso, 2015). For U.S. students to compete globally in science, technology, engineering, and mathematics (STEM) related careers, they must demonstrate competency in mathematics equal to or above their international counterparts/peers (Aud et al., 2010; Lehman, 2013). Yet, as noted in the research component of this project study, many elementary students from public schools in Pennsylvania continue to struggle with mathematics concepts.

Many educators and school administrators agree that the problem of teaching mathematics in elementary schools is a great concern for all school districts as they search for solutions. Administrators support the idea that a variety of teaching methods should be explored to help students succeed (Chiu, & Linn, 2012; Jordan, 2007). Early mathematics proficiency is important for a student’s future academic endeavors. It may be argued that if difficulties in learning mathematics are not remedied in the lower grades, these difficulties may result in students struggling in mathematics throughout their education. The elementary school is the foundation of mathematics education and the straightest path to improving mathematics achievement in elementary schools is through improved teaching (Erhan, Akkoç, Ozmantar, & Demir, 2011; Fast & Hankes, 2010).

This position paper was prepared following a qualitative case study I conducted. The study included interviews conducted with traditional and Montessori elementary school teachers. The purpose of the study was to explore traditional and Montessori teachers' perceptions and beliefs of their classroom practices in teaching mathematics. The teachers were asked to respond to interview questions designed to address the problem and research question of the study. The data collected from these interviews resulted in important and notable results.

The results of the study indicated that both the traditional and Montessori teachers perceived four key themes: (a) concrete to abstract learning, (b) teacher role modeling; (c) applying math skill to real-life situations, and (d) learning math through manipulatives and visuals. The element of concrete to abstract learning was the most prevalent of the four themes. Both group of teachers expressed the need to provide students with concrete to abstract mathematics concepts in the instruction of basic mathematics operations. They also articulated how fundamental concrete to abstract learning informed their own classroom practice. As such, the focus of this position paper is concrete to abstract learning in the teaching of mathematics in the elementary school setting. I believe that the key issues identified in this position paper will enable educators to reflect on the revision of their math curriculum and consider its appropriateness within the broader context of teaching mathematics.

Concrete to Abstract Learning

The purpose of teaching through a concrete-to-abstract sequence of instruction is to ensure students have a thorough understanding of the math concepts/skills they are learning. When students with math learning problems are allowed to first develop a concrete understanding of the math concept/skill, they are more likely to perform/gain a better understanding of math concepts at the abstract level. The key premise is that as a teacher moves through a concrete-to-abstract sequence of instruction, the more likely the student will develop mastery of that skill. Based on the findings in this study, it is my position that mathematics education should focus on enriching a child's inner skills through teaching the concept of concrete to abstract learning.

The phrase "concrete to abstract" was heard frequently during the dialogue with both Montessori and traditional education teachers. However, the phrase can be rather ambiguous, especially to a new teachers or parents who may have little working knowledge of the concept and practices. Most Montessori teachers believe that by using concrete materials during the early, sensitive years, the child can learn the basic concepts of mathematics and language. The findings of the project study indicated that most teachers believed that by using concrete materials during elementary school, the child will learn the basic concepts of mathematics for life. It was clear from the Montessori teachers that students who used *concrete* hands-on learning materials and manipulatives were better able to understand abstract concepts.

The practice of concrete to abstract learning finds support in the work of notable theorists such as developmental psychologists Piaget (1972, 1977) and Bruner (1966). The famous, Jean Piaget (1972) proposed four distinct stages of human cognitive development. Based on his observations and research, he determined that each of these four stages of development was signified by the achievement of specific milestones. The concrete operational stage is the third developmental stage proposed by Piaget. As the name implies, the concrete operational stage of development can be defined as the stage of cognitive development in which a child is capable of performing a variety of mental operations and thoughts using concrete concepts. More specifically, children learn that just because an object changes shape or is divided into pieces, the object still retains certain important characteristics, such as mass or volume. According to Piaget, most children will enter this stage sometime around the age of seven and complete it sometime prior to age eleven.

Bruner's (1966) proposed three modes of cognitive development: Enactive representation (concrete), Iconic Representation (pictorial or graphic) and Symbolic Representation (abstract). Bruner's (1966) argued that modes of representation are ways in which information or knowledge is stored and encoded in memory. In contrast to Piaget's neat age-related stages, the modes of representation are integrated loosely sequential.

Historically, mathematicians are known for using a variety of tools, such as sliding rules, compass, and calculators to simplify doing mathematics. However, employing tools in the education setting requires paying special attention to certain pedagogical concerns (Borghi et al., 2017). All of the teachers, Montessori and traditional, believed the concrete to abstract learning approach is essential to elementary mathematics. The majority of them believed that the most effective way to illustrate this concrete learning is through offering examples of real life based problems. For example, "Joseph had 6 carrots. His friend brought him three more. How many does he have now?". Other examples included the use of currency in a supermarket and role playing the problems. In the United States mathematics leadership organizations, such as National

Council of Supervisors of Mathematics (NCSM) and multiple states adopted standards such as the Common Core States Standards for Mathematics (www.corestandard.org) which stress the use of concrete to abstract practices in the learning of mathematics to aid in students' proficiency in the subject.

Manipulatives

This section provides a discussion of the mathematical tools and manipulatives that may be used at the elementary level. The aim is to offer research based guidance about the use of manipulatives in the classroom. This information offers a start for teachers to examine their own practices and the ways in which manipulatives are used with children. Back (2013) suggested that although manipulatives are used in the elementary school setting to support teaching and learning they are not always used effectively. As such, some suggestions about how using manipulatives can be used to support children's mathematical thinking, reasoning, and problem solving are presented.

Bouck, Satsangi, Doughty, and Courtney (2014) described manipulatives as objects used to help better understand abstract mathematical concepts or properties. Concrete manipulatives are standard practice in mathematics education, especially for students with disabilities. Concrete manipulatives were proven to be effective in helping students with disabilities in solving problems related to area, perimeter, measurement, fractions, and word problems.

The NCSM (2013) issued a position statement on the use of manipulatives in teaching to improve student achievement. The council supports the idea that in order to develop every student's mathematical proficiency, educators must systematically integrate the use of concrete and virtual manipulatives into classroom instruction at all grade levels (NCSM, 2013). Historically, manipulatives have a solid research history in teaching mathematics. Manipulatives allow students to construct models for mathematical ideas that can be communicated to the teacher and other students. In addition to the ability of manipulatives to stimulate the learner, manipulatives can engage students and increase their interest in the study of mathematics. Students who are presented with the opportunity to use manipulatives report that they are more interested in mathematics (NCSM, 2013). A key premise is when teaching a math concept or skill, the teacher should describe and model it using concrete objects.

Research supports the regular use of manipulatives in classroom mathematics instruction. Building on the learning theory works of Piaget (1970) and Bruner (1966), in order for children to gain a deeper understanding and learn how to apply learning to new situations, a conceptual understanding that is grounded in direct experience with concrete objects is required. The role of the teacher is critical in helping students connect their manipulative or concrete experiences to essential abstract learning in mathematics.

Moyer (2001) posited that manipulative materials are visual and tactile objects designed to represent mathematical ideas that are abstract. Manipulatives provide hands-on experiences and can be manipulated by young learners. Practical tools used in the classrooms such as multilink cubes, place value counters, bead strings, place value cards, dice, and dominoes are considered manipulatives. This list is comprised of objects that children can pick up, touch, feel, and manipulate to help them work in the number system and understand math concepts.

Concrete to abstract methods involve three level instructional sequence in which manipulatives objects are used on the concrete level and drawings are used on the representation level. The abstract level is often combined with integrated strategies. On the abstract level, computation involves learning a strategy, for example with a mnemonic, for solving problems using numbers only (Bruun, 2013; Flores, Hinton, & Burton, 2016). A sample strategy could be DRAW; (a) discover the sign, (b) read the problem, (c) answer or draw and check, and (d) write the answer (Bruun, 2013; Flores et al., 2016). The method or instructional sequence is said to work well with individual students, small and whole groups, and is found to be as intuitive as it is predetermined in teaching mathematics (Akinoso, 2015; Mudaly & Naidoo, 2015). Research on the DRAW model has shown positive results (Flores et al., 2014).

There are advantages and disadvantages in working exclusively in either concrete or abstract framework (Fyfe et al., 2014). Several variations of concrete to abstract learning mathematics instructional formats are employed in present day educational environments. They are generally aligned to the 'concreteness fading' technique for mathematics (Fyfe et al., 2014) which begins with concrete materials and gradually fades into the abstract. In essence, the technique merges the benefits of concrete and abstract materials while mitigating their disadvantages (Fyfe et al., 2014).

Researchers agree that the use of concrete or manipulative materials promotes thinking of objects, ideas, or concepts as specific items; abstract learning promotes a more generalization of principles (Fyfe et al., 2014; Swanson & Williams, 2014). Making use of concrete materials in the learning of mathematics provides a means for learners to construct their own knowledge of abstract mathematical concepts; it makes available a practical context for the student during learning, and it prompts the need for action, physical or visualized, which can increase ones understanding or memory (Fyfe et al., 2014).

Abstract Learning

Problem-solving at the abstract level is solving problems without the use of concrete objects or without drawing pictures. Understanding math concepts at the abstract level requires students to solve problems using numbers and math symbols only. The abstract approach involves completing written math problems using paper and pencil. Many teachers refer to this approach is "doing math in the head." It is important that as a teacher move through a concrete-to-abstract sequence of instruction to use abstract numbers in conjunction with the concrete materials and representational drawings. This is especially important for students with special needs. This sequence of instruction promotes association of abstract symbols with their concrete and representational understandings.

Abstract Versus Concrete Learning

Abstract learning versus concrete learning takes into account that people learn or process information differently. That is, while some people learn in a particular way, others learn in a different way. These differences and variations in learning are natural and part of human nature. All people are born with a certain mindset which leads to individuals becoming either concrete or abstract learners. Both terms are different from each other and demonstrate people perceive things differently according to their way of learning and viewing things in the world around them (see Figure 1).

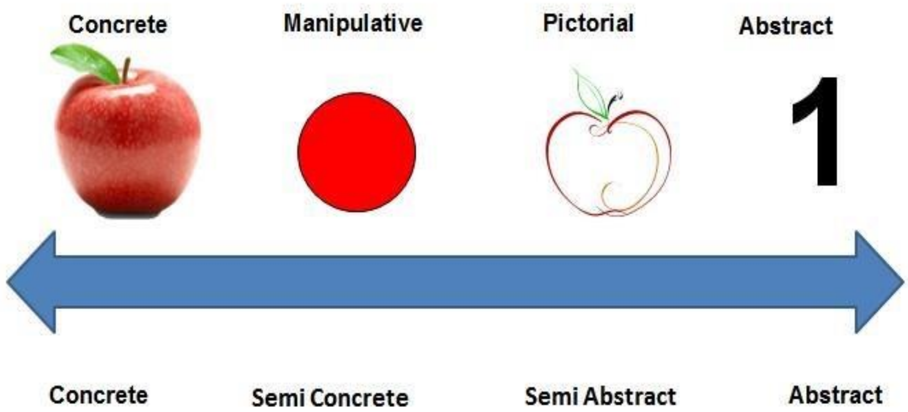

Figure 1: Depicts Concrete to Abstract Pictorial. (American Institutes for Research, 2004)

As Figure 1 suggests, concrete learning involves those things which are visible to the human eye. In the depiction above, the picture of the apple is obvious enough for anybody who is looking at the picture. A concrete learner considers and depends on the literal meaning of anything. On the other hand, the abstract learner pays close attention to the hidden meaning that often is not readily grasped or understood. Abstract learning involves a much deeper thought process and the learner finds multiple meanings of a single concept. For example, in Figure 1 the abstract learner sees the apple as the number 1 (one apple). The abstract learner seeks multiple solutions to a single problem and depicts hidden or underlying meanings in anything that exists (Akinoso, 2015).

Van Acker (2014) offered another description of concrete to abstract modeling described as Concrete/Representational(Pictorial) Abstract (CRA). The representational/Pictorial stage was viewed as the “seeing” stage using representations (pictures) of the concrete objects to help model a problem. In this model, the teacher transforms the concrete level to representations (often called the semi-concrete level) demonstrated in Figure 1. These can be pictures using circles, dots, or 2d images. Van

Acker (2014) described concrete as the "doing" stage to solve problems. The teacher usually begins by modeling the mathematical concepts with concrete materials. For example, the teacher will use place value blocks, money, connecting cubes, or tape measure/ruler.

Van Acker (2014) believed that teachers should follow the CRA sequence and not skip from concrete to abstract. The premise is that students need to proceed through all three levels to demonstrate that they understand a visual for the concrete model. If the sequencing of CRA is not followed, there is a chance that students will not have the understanding behind why they are doing something. In contrast, when the sequence of concrete, representational, abstract is followed, the students develop a mastery level of understanding of math concepts.

The final stage of learning math is the abstract or the "symbolic" stage. In this stage, the student use abstract symbols to model a problem such as 3x4=12. In this stage students have learned to create the equation in order to solve the problem. Students have developed the essential skills to apply what they have been taught in the earlier stages to solve the math problems.

Several key differences between abstract and concrete learning are noted:

- Abstract learning emphasizes the hidden or the intended meaning whereas concrete learning is direct, always literal, and straight to the point.
- Abstract learning requires deeper analysis whereas concrete learning remains on the surface.
- Abstract learning and concrete learning allow the individual to gain two different perspectives to arrive at a solution to the problem (Swanson & Williams, 2014).

In summary, the concrete to abstract learning process starts with the concrete phase which involves using concrete materials to represent mathematic concepts in order to assist students in the understanding the abstracts concepts. The process gradually progresses into the end phase, abstract. In this phase, symbolic representations of mathematics concepts are used (Akinoso, 2015). Mathematics concrete materials or instructional manipulatives are physical, virtual, and pictorial or graphic objects. The abstract is symbolic representation (Swanson & Williams, 2014).

Recommendations

The findings from the project study and literature review suggested that teachers from the Montessori school and traditional schools supported concrete to abstract learning in teaching math. However, it is unclear whether students fully understand the mathematic concepts behind the skills demonstrated by teachers at the elementary level as indicated by declining test scores. For this reason, I recommend the following actions for teachers:

- At the concrete level, re-teach the concept/skill using appropriate materials
- Provide opportunities for student to practice skills.
- Provide opportunities for students to explain how they got their math answers.
- Develop knowledge and skills necessary to properly implement the practices through professional development (Borghi et al., 2017).

Considerations should be given to the practices that involve using appropriate concrete objects to teach the math concepts/skills. These are items should be objects that students can see and feel. Teachers should use base-ten materials to build understanding of place value and number sense relationships. In addition, it would be helpful to teach students to draw simple representations of the concrete objects they use in problem solving. When students draw the objects, supports students' abstract understanding of the concept.

Conclusion

It is the position of this report that in order to develop every student's mathematical proficiency, regardless of school setting, teachers must systematically seek to integrate the use of concrete and abstract learning into classroom instruction at all grade levels. Concrete models are an essential tool for learning mathematics across all grade levels. Therefore, teachers should strive to support and sustain improved student achievement through the development of essential skills through professional development. This position can be accomplished when teachers and school administrators fully understand and build conceptual understanding of mathematics. Teachers must use research to guide the instructional use of manipulatives. It is important that all stakeholders seek to maintain a life-long commitment to provide equity and access for all learners.

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