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Agent-Based Overlapping Generations Modeling for Educational Policy Analysis

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Abstract

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MA, University of Southern California, 1986

BS, National Chengchi University, 1984

Proposal Submitted in Partial Fulfillment

of the Requirements for the Degree of

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Abstract

Educational systems are complex adaptive systems (CAS). The macroeffects of an educational policy emerge from and depend on individual students' reactions to the policy. However, educational policymakers traditionally rely on equation-based models, which are deficient in reflecting the work of microbehaviors. Using inappropriate tools to make policies may be a reason why there were many unintended educational consequences in history. A proper methodology to design and analyze policies for complex educational systems is agent-based modeling (ABM). Grounded in the theories of CAS and computational irreducibility, ABM is capable of connecting microbehaviors with macropatterns. The purpose of this study was to contribute to the application of ABM in educational policy analysis by constructing an agent-based overlapping generations model with hypothesized inputs to qualitatively represent the environment of the Taipei School District. Four research questions explored the effects of Taipei's 2016 student-assignment mechanism and its free tuition policy on educational opportunity and school quality under different assumptions of students' school-choice strategies. The simulated outputs were analyzed using descriptive statistics and paired samples *t* tests. The findings, which could hardly be revealed by traditional models, showed that the effects were complex and depended on students' strategies along with the number of choices students were allowed to make; the assignment outcomes for elite students were robust to the mechanism, and the free tuition policy worsened school quality. Although exploratory, these findings can serve as hypotheses and a guide for Taipei's policymakers to collect empirical data in evaluating their 2016 mechanism and tuition policy.

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Dedication

This study is dedicated to my late father, Tsun-Chuan Wang. He never talked much. He guided me with his character and deeds. In my heart, he still does.

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Chapter 1: Introduction to the Study

Overview

Education systems are complex adaptive systems (CASs), in which the distribution of educational opportunity and school quality emerge from the interactions of students with the environments including admission policies. One of the most salient features of CASs is emergence; that is, aggregate properties emerge from the decentralized interactions among the rationally bounded and adaptive constituents, which follow simple behavioral rules and do not have any properties similar to the aggregate properties (Flake, 1998; Holland, 2006; Squazzoni, 2012). The classical methodology to study macrolevel issues of CASs is equation-based modeling (Borrill & Tesfatsion, 2011; Gilbert, 2004). However, equation-based modeling cannot easily reflect the connections between the behaviors of microconstituents and the macropatterns (Heckbert, Baynes, & Reeson, 2010). This weakness makes classical equation-based modeling a questionable tool to assess an educational policy that involves students' behaviors in its process of influencing macrophenomena.

To change the aggregate properties of a CAS, "we must understand how the aggregate behavior emerges from the interactions of the parts" (Holland, 1992, p. 20). Individual behaviors may change to respond to the change of policy, which in turn changes macrostructures of interest. Therefore, it is not possible to deduce or predict the future states of a CAS *a priori* purely from the study of its structural characteristics, as the classical equation-based analytical models are designed to do (Laughlin & Pines, 2000; Wolfram, 2002). Using the wrong tools to design policies for CASs may be a reason that history shows a myriad of cases where policies produced unexpected and undesired consequences (Farmer & Foley, 2009; Groff, 2013; Maroulis et al., 2010).

The approach to studying CASs advocated by many scholars, including John Holland, Duncan Foley, and Leigh Tesfatsion, is agent-based modeling (Farmer & Foley, 2009; Holland, 2006; Borrill & Tesfatsion, 2011). Agent-based modeling (ABM) is a bottom-up approach to studying the micro-, meso-, and macroevolution of a CAS by programming the constituent agents' behavioral and interaction rules as well as their environment and letting the aggregate patterns grow through simulations (Axelrod, 1997; Chen, Yang, & Yu, 2011). ABM's capability to connect micro- and macrobehaviors makes it a better design than classical mathematical equations to study the dynamical macroscopic problems in CASs (Chen, Chang, & Du, 2012; Epstein, 1999). While it has been extensively applied to policy analysis in many CASs, particularly in the fields of economics, management, and environmental studies, ABM is still new in the field of educational research (Chen et al., 2011; Maroulis et al., 2010). Few agent-based models are constructed to simulate distribution patterns in educational systems. The agent-based model built in this study contributes to the development of ABM in educational policy analysis.

The model is an overlapping generations model because there are three generations of students (grades 10 -12) in each simulation year, and younger generations make their school choices based on school qualities emerging from senior students' performances. I constructed this model to represent the Taipei Senior High School District (the Taipei School District) qualitatively and to simulate the possible effects of

Taipei's 2016 senior high school admission policies on the distributions of educational opportunity and school quality. This agent-based simulation model is capable of running experiments under different explicit hypotheses and showing the link between emergent aggregate outcomes and individual behaviors. This study demonstrates to policymakers and stakeholders that ABM can facilitate a disciplined and informative public discourse on educational policies that have complex effects on the current and future generations.

In the following section, I discuss the background of Taipei's senior high school admission reform. I argue the lack of proper analysis of its policy implications in the section of problem statement, followed by the sections in which I describe the purpose, research questions, nature, conceptual framework, delimitations, assumptions, and limitations of this study. This chapter ends with a discussion of the significance of this study in agent-based educational research in general and research on school admission policies in particular.

Background

Since the 1980s, Taiwan's income inequality has been rising; the ratio of the income earned by the top five percent to the income earned by the bottom five percent has reached 96:1 (Chu, Chow, & Hu, 2015). Various educational reforms, in addition to new social and economic policies, have been tried to tackle the inequality problem. One of them was the massification of post-secondary education during the 1990s and the 2000s, which has resulted in more than 90% admission rate at Taiwan's universities since 2005 (Chou & Wang, 2012). However, the massification of higher education does not improve income equity; it reproduces the socioeconomic class stratification (Chu et al., 2015). The elite, most of whom are richer, attend elite universities while the poorer attend lower ranked universities with much fewer resources. Moreover, since Taiwan's elite universities are all public and most public universities rank higher than the private universities, on average, richer students pay less than poorer students. What makes the situation worse is the shortage of demand for higher-educated labors due to the economic downturn in the recent decade, forcing those non-elite university graduates to compete for jobs with reduced starting salaries (Chou & Wang, 2012).

In 2014, Taiwan's government started another large-scale educational reform, the 12-Year Basic Education Program (the 12-Year Education Reform), to tackle the inequality problem in postsecondary education (Ministry of Education, 2013). This reform aimed at equalizing educational opportunity and school quality in high school education. The strategies of the reform were to extend the free-tuition policy to all students in both public and private senior high schools and to overhaul the senior high school admission mechanisms in all school districts (Ministry of Education, 2013). In Taiwan, many school districts rely on the private senior high schools to provide enough seats for all students. For example, in the Taipei School District, the private schools provide about 50% of the places needed to accommodate all children in senior high school ages (15 – 18). Those who attend private schools are generally poorer financially and academically than those in public schools; therefore, the government had provided full tuition subsidy for those who attended private vocational high schools (Ministry of Education, 2013). This reform extended this free tuition policy to all high school students. The government claimed that the extended free-tuition policy could ease the financial burden on poorer people and thus promote equal educational opportunity. The government also overhauled the student-assignment mechanisms to change the school systems from tracking (assigning students to schools according to their performances) to mixing in the belief that a mixing system promotes equal educational opportunity in comparison to a tracking system. This belief has been supported by many studies (Ferreira & Gignoux, 2014; Van de Werfhorst & Mijs, 2010). If school quality is measured by students' scores, then mixing also promotes equality of school quality.

Unfortunately, the 2014 admission mechanisms of all school districts were heavily criticized for their complicated student prioritization rules and the creation of justified envy (Lin, 2014). Justified envy occurs when a higher ranked student loses the seat in his or her preferred school to a lower ranked student (Abdulkadiroglu & Sonmez, 2003). The abundant occurrence of justified envy infuriated many parents of higher performing students, who were relatively affluent and politically influential. Although no parents complained about the free tuition policy, some scholars criticized this expensive policy for its crowding-out effect on the programs that could directly benefit low-quality schools and low-performing students (Sheu & Chang, 2014). Those scholars also argued that this policy is unnecessary because Taiwan had a more than 93% of senior high school attendance rate before the implementation of this policy (Sheu & Chang, 2014). Compromising to the political influence of the richer parents, almost all school districts in Taiwan, including the Taipei School District, revised their admission mechanisms in 2015 toward the original system of sorting students by academic performance, while holding the free-tuition policy unchanged.

As the capital of Taiwan, Taipei has the most competitive school district and contains the top-ranked senior high schools in Taiwan. The Taipei School District includes students from both Taipei City and its satellite cities (Suburban Taipei). Taipei's policymakers have to face the criticisms from the most politically influential parents in Taiwan, who believe that mixing will lower the quality of the top-ranked schools. With a total of 70 thousand senior high school applicants each year, the Taipei School District took a more aggressive step to change its admission mechanism than many other school districts in 2015, disregarding the central government's objection (The Central News Agency, 2014). The Taipei School District reported that its device reduced a great deal of justified envy in 2015 (Taipei City Government Department of Education, 2015). This statement seemed to imply that the Taipei School District has moved more back to the original tracking system.

In 2016, Taipei's policymakers changed their student-assignment mechanism again to solve two problems caused by the prioritization rules in the mechanism: too many ties (too many students having the same priorities) and justified envy (New Taipei City Government, 2015b). Justified envy is viewed as a problem only when a society believes in a school system of tracking rather than mixing. This social belief is exactly what the 12-Year Education Reform was designed to cope with. How well the capital city reaches the goal of the 12-Year Education Reform serves as an index to the performance of this reform. Regardless, Taipei's policymakers seem to focus more on complaints about justified envy than the goal of the reform.

History has provided many examples of well-intended educational policies creating adverse outcomes. One example is the cream-skimming effect observed in some school voucher systems, in which better-performing students transfer to private schools, leaving lower-performing students in public schools, which further lowers the quality of the public schools and fails the purpose of school vouchers (Nechyba, 2003; Tabarrok, 2013; Walsh, 2009). Another example is the U.S. No Child Left Behind (NCLB) policy, which was intended to improve student performance majorly by holding schools accountable for having students achieve the proficient level of state assessment (WGBH educational foundation, 2014). NCLB's unintended consequences were identified and commonly recognized before there was any consensus about its effect on school quality and closing student achievement gap. NCLB's side effects include: focusing only on absolute scores without recognizing student's achievement growth, evoking states to lower their assessment standards, and driving schools and teachers to concentrate only on high-stakes subject areas and short-term test-preparation skills (Berliner, 2009; Dee & Jacob, 2011; Groff, 2013; Jennings & Sohn, 2014; Ladd & Lauen, 2010; Murnane & Papay, 2010; The White House, 2014). Because of the unintended consequences, one reform usually calls for another to fix them, such as the Every Student Succeeds Act signed by President Obama to replace NCLB on December 10, 2015 (U.S. Department of Education, n.d.).

South Korea's High School Equalization Policy (HSEP) is another example. Initially implemented in 1974 to reduce school inequality and shadow education, HSEP randomly assigned students to high schools, whether public or private, (Byun, Kim, &

Park, 2012). By comparing the socioeconomic statuses of high school students in the HSEP and non-HSEP areas, Byun et al. (2012) concluded that HSEP had reached the goal of reducing school inequality. However, the activities of shadow education (cram schools) kept increasing in HSEP areas in comparison to non-HSEP areas (Byun et al., 2012; C. J. Lee, H. Lee, & Jang, 2010). If the cost parents spent on cram schools is considered, whether HSEP reduces educational inequality is still under debate (Lee et al., 2010). In addition to its controversial effects, HSEP was criticized for depriving students' right to choose schools, restricting private schools' autonomy, making teachers hard to teach in mixed-ability classrooms, and lowering average student performance (Byun et al., 2012). Therefore, some areas including Seoul have abolished or revised the original HSEP, allowing more school choices and private school autonomy (Byun et al., 2012).

The 12-Year Education Reform, NCLB, and HSEP are only some examples in the large pool of the educational policies that received criticisms for their unintended effects much earlier than their goal achievements could be confirmed. The effects of educational policies depend on how students react to the policies. Inability to foresee and bring into consideration of students' heterogeneous reactions might be a common cause of those unanticipated outcomes. The reductionist models commonly used to analyze the educational policies, if the policies were ever analyzed, might also constrain researchers and policymakers from exploring students' heterogeneous behaviors and possible policy outcomes emerging from students' interactive behaviors. For tractability purpose, reductionist models impose strict assumptions on agents, such as homogeneity and profitmaximization (Epstein, 1999; Macal & North, 2010). These restrictions make the reductionist models, such as equation-based models, hard to deal with the issues in human CASs, where macropatterns emerge from the interaction of the heterogeneous, boundedly rational, adaptive agents between themselves and with the environment.

The policy implications of an admission mechanism design are complex because the design involves anticipating how students will react to a new environment, and the strategic behaviors of human beings are complicated (Roth, 2002). These complications may prevent analyzing the design analytically, but computational simulations can overcome these complications (Axtell, 2000; Roth 2002). Taipei's policymakers claimed that they had used the method of simulation to test their revised admission mechanism based on students' school-choice lists in the previous year (New Taipei City Government, 2015b). However, the result simulated in this way could hardly be used to understand the potential macroimpacts of the new mechanism because students probably would submit different school-choice lists under the new mechanism. When people change their behaviors, they also change the structural relations in the environment (Lucas, 1976). Therefore, as the Lucas critique states, if a new policy may cause a change in people's decision rules, it is inappropriate to predict the policy implications just based on historical data (Lucas, 1976). Not using the proper method to design their new admission policies, Taipei's policymakers were accused by the parents of treating their children as experimental subjects for the 12-year Education Reform (Center for Educational Research and Evaluation, 2016).

Problem Statement

Educational systems are CASs. The macroscopic effects of a school admission policy, as well as many other educational policies, emerge from the behaviors and interactions of the heterogeneous students and the policy. However, the methods used in the mainstream educational policy analysis and Taipei's new admission policy design tend to ignore the impact of individual behaviors. Using the wrong methodology to make educational policies may be one of the reasons why many policies, including Taiwan's 12-Year Education Reform, produced unintended effects or failed to accomplish their goals.

Many scholars contend that the better way, or even the only way, to study CASs and their related policies is to build and run computer-based models and observe the simulated aggregate results (Borrill & Tesfatsion, 2011; Holland, 2006). The computer-based modeling that many complex systems scholars advocate is ABM, because of its capability to connect microbehaviors and macropatterns and its flexibility to run scenario analysis (e.g., Borrill & Tesfatsion, 2011; Chen 2016; Chen et al., 2012; Duffy, 2006; Epstein, 1999; Farmer & Foley, 2009). ABM is a bottom-up approach that generates macropatterns by programming the behavioral and interaction rules of individual agents and letting aggregate patterns grow through simulations (Axelrod, 1997; Chen et al., 2011). Even if how agents will behave in a new environment is unknown or uncertain, ABM is flexible to perform what-if analysis and investigate the possible best and worst macroeffects of a new admission policy. Because of ABM's advantages in studying

CASs, Maroulis et al. (2010) urged researchers to apply ABM to educational policy analysis to understand "not only what works but also how and why it works" (p. 39).

ABM has been extensively applied in many fields, particularly in economics, management, and environmental studies (Chen et al., 2011). Unfortunately, up to today, only a few agent-based models have been constructed to analyze macroeducational policies (e.g., Harland & Heppenstall, 2012; Maroulis et al., 2014; Millington et al., 2014). None has been applied to evaluate Taipei's 2016 admission policies either. With the advancement of the ABM technique and computing speed, it is time to solidify the application of ABM to educational policy analysis and to explore the impact of Taipei's new admission policies on educational equality.

Purpose of the Study

The purpose of this study was to answer Maroulis et al.'s (2010) call to contribute to the complexity methodology for educational policy analysis. I built an agent-based overlapping generations model to reflect the qualitative properties of Taipei's senior high school system. Although ABM is a quantitative methodology, I built an agent-based model to explore and gain qualitative insights into the impacts of Taipei's 2016 admission policies (the Taipei mechanism and the free-tuition policy) on the distributions of educational opportunity and school quality. I also compared the results with those under the following four prevalent student-assignment mechanisms: serial dictatorship, deferred acceptance, the Boston mechanism, and the Chinese parallel mechanism. I simulated the policy effects under different assumptions of students' school-choice strategies and the number of school choices a student is allow to make. A total of 60 scenarios, described in Chapter 3, were simulated under each student-assignment mechanism.

Research Questions

To have an in-depth understanding of the equality effect of Taipei's 2016 admission policies, I collected and analyzed the simulated data to answer the following research questions:

- Does the Taipei mechanism help equalize educational opportunities?
- Does the Taipei mechanism help school qualities converge upward?
- Does the Taipei mechanism, combined with the free-tuition policy, help equalize educational opportunities?
- Does the Taipei mechanism, combined with the free-tuition policy, help school qualities converge upward?

Theoretical Framework

The theories underpinning ABM are complex adaptive systems (or complexity theory) and Wolfram's (2002) theory of computational irreducibility. *Complex adaptive system* is a term to describe not only a kind of system but also a paradigm to study this kind of system, although there is still no universal definition of CAS. Holland (1992) argued that a CAS has three characteristics: (a) evolution through adaptation, (b) emergence of aggregate behavior, and (c) agents' abilities to anticipate. Epstein (1999) posited that the agents in a CAS have the following features: heterogeneity, autonomy, explicit space, local interactions, and bounded rationality. From their descriptions, a CAS can be described as a system comprising heterogeneous, autonomous, adaptive, self-organizing, boundedly rational agents, through whose interactions with each other and their environment the system's aggregate properties emerge (Epstein, 1999; Holland, 1992).

The most salient characteristics of CASs are emergence and adaptation. Emergence implies that the aggregate behaviors are different from the components' behaviors, and thus aggregate behaviors are not simply the sum of the parts (Archer & Smeins, 1991). Adaptation means changing to fit the environment; adaptation makes a CAS continuously evolve and rarely stay in a constant state (Holland, 1992; Keshavarz, Nutbeam, Rowling, & Khavarpour, 2010). With these two features, CASs are categorized as Wolfram's (2002) Class 4 systems, which are computationally irreducible (Borrill & Tesfatsion, 2011; Crockett, 1993; Darley, 1994). Wolfram (2002) posited that a system or program is computationally irreducible when the only way to know its future state is to simulate its evolution step by step, and this computational work cannot be reduced by using any set of equations. The conventional reductionist paradigm or equation-based models cannot study the features of CASs easily (Heusser, Scheffer, Neumann, Tauschel, & Edelhäuser, 2012). Therefore, Borrill and Tesfatsion (2011) argued that computer modeling, particularly ABM, is the only way to understand CASs.

An educational system consists of subsystems of nested hierarchies, including schools, classrooms, educators, administrators, teachers, students, and parents (Keshavarz et al., 2010). Its subsystems are autonomous, heterogeneous, adaptive, and self-organizing, even though some of their activities are subject to governmental regulations (Burns & Knox, 2011; Keshavarz et al., 2010). Its aggregate properties, such as admission distribution and school quality, emerge from the interactions among its subsystems. Therefore, educational systems are CASs. Research on educational systems requires complexity methodologies. ABM is the right approach to understanding the emergent properties of educational systems (Maroulis et al. 2010). The aim of this study was to join the pioneering work of applying ABM to exploring the mesoscopic and macroscopic impacts of Taipei's new admission policies on Taipei's senior high school system.

Nature of Study

The purpose of this exploratory study was to understand qualitatively how school admission policies would affect educational opportunity and school quality in the Taipei School District from a CAS perspective. Although agent-based simulation per se is a quantitative research, the parameters in this agent-based model were not calibrated to the real data. Therefore, this study was exploratory. The simulated outcomes provide qualitative, rather than quantitative, insights into the properties of Taipei's 2016 admission policies.

Educational opportunity has been measured by the proxies of either educational resources or the efficacy of the resources since the Coleman Report (Coleman et al., 1966). The proxy for school quality used in most studies is either school resource or student performance (Ladd & Loeb, 2013). Therefore, studies related to educational opportunity and school quality largely call for quantitative research designs. The classical quantitative methodology used for this type of macroeducational policy analysis is equation-based modeling or statistical regressions, which cannot easily study CASs' properties of emergence and adaptation (Chen, 2015). On the other hand, ABM, a quantitative computational approach, allows researchers to program the behaviors of individual agents and let the macrophenomena emerge through the interaction of the agents. This simulation methodology not only addresses CASs' properties but also shows the linkage between microbehaviors and macrophenomena (Chen, 2015; Epstein, 1999; Macal & North, 2010).

Educational opportunity and school quality are aggregate properties emerging from the interactions of the hierarchical agents in a complex educational system. ABM can be used to explore the dynamics of the distributions of educational opportunity and school quality under different behavioral assumptions. ABM's ability to perform simulations of behavioral scenarios is essential in analyzing the design of a new educational policy that involves anticipating agents' behaviors in a new environment, such as the Taipei mechanism. The reason is that how students will react to a new policy is usually unknown. Although human-subject experiments can help acquire behavioral knowledge, experimentation is often constrained due to factors like cost, time, space, human fatigue, and ethical issues (Chen, 2015; Roth, 2002). New admission policies, such as the reformed Taipei admission policies, often fall into this difficult-to-experiment category and involve many significant uncertainties. In this situation, a methodology easy to perform behavioral scenario analysis is critical (Hedstrom & Ylikoski, 2010). Therefore, ABM is the right methodology to conduct this exploratory research and to investigate the possible impacts of Taipei's 2016 admission policies in the complex Taipei School District.

In this study, I built an agent-based overlapping generations model to simulate and explore the outcomes of Taipei's 2016 admission policies (the Taipei mechanism and the free tuition policy) under different assumptions of student behaviors and admission mechanism designs. There are three types of entities (students, schools, and the governmental authority) and two neighborhoods with different average incomes in the model. The submodel of matching process contains five matching mechanisms, including the Taipei mechanism. In each simulation, student agents were assigned to schools through the matching process, and the scores of the admitted students were updated by a formula reflecting the effect of peer networks. During the simulations of 300 combinations of student behavioral rules and policy settings, the program automatically collected the simulated data representing educational opportunity and school quality. I then analyzed and compared the collected data by using descriptive statistics and paired samples *t*-tests to answer the research questions.

Definitions

Agent-based modeling (ABM): is a bottom-up simulation approach characterized by modeling the behavioral and interaction rules of a system's constituents and letting the collective phenomena of interests emerge through simulations (Axelrod, 2005; Gilbert, 2004; Macal & North; 2010).

Complex adaptive system (CAS): refers to a system consisting of heterogeneous, autonomous, adaptive agents, through whose interactions with each other and their environment complex systemic properties that are different from those of the agents emerge (Epstain, 1999; Gell-Mann, 1994; Holland, 1992).

Computational irreducibility: is a concept claimed by Stephen Wolfram (2002), which mainly states that there are computationally irreducible questions in nature and in human societies that cannot be answered by any mathematical shortcut but by simulating the evolution of the system step by step.

Deferred acceptance: refers to the student-proposing deferred acceptance mechanism first discussed in the literature of school choice by Abdulkadiroglu and Sonmez (2003), which is a version of deferred acceptance originally designed by Gale and Shapley (1962). The section of matching mechanism in Chapter 2 describes the algorithm of this mechanism in detail.

Equality of educational opportunity: is defined from the input point of view as that all junior high school graduates have the same opportunity to attend each of the senior high schools in an educational system, regardless of their family income. This definition is in line with the policy intention of Taiwan’s 12-Year Education Reform (Ministry of Education, 2013).

Equality of school quality: is defined from the output point of view as equal seniors’ mean scores across all schools (Ladd & Loeb, 2013).

Exploratory modeling: is to model a system and perform computational experiments on the model under various assumptions and hypotheses to explore the implications of research questions (Bankes, 1993). Exploratory modeling allows researchers to conduct research through computational simulations without waiting for the collection of all facts.

Principle of maximum entropy: is a technique to select a probability density distribution that does not imply any assumptions while satisfying all of the limited known information (Penfield, 2014). Given the mean and the standard deviation, the distribution selected under the principle of maximum entropy is normal distribution (Garrett & Fisher, 1992).

Risk aversion level: refers to the extent to which a student will make a school-choice strategy to prevent the worst eventuality (Klijn, Pais, & Vorsatz, 2010).

Serial dictatorship: is a matching mechanism, in the context of school choice, that allows students, in the sequence of their priorities, to choose their schools. Under this mechanism, a student is assigned to his or her top choice of schools that still have seats before students who have lower priority than this student are assigned (Pathak, 2011).

The Boston mechanism: refers to the centralized admission mechanism used by the Boston School District to assign students to schools before 2005 (Abdulkadiroglu, Pathak, Roth, Sonmez, 2005). The process of this mechanism is described in the section of matching mechanism in Chapter 2.

The Chinese parallel mechanism: is the admission mechanism used in some Chinese provinces and municipalities to assign students to universities. The section of matching mechanism in Chapter 2 describes the algorithm of this mechanism in detail.

The free-tuition policy: refers to Taiwan's new education policy that provides free tuition to students enrolled in senior high schools, whether the school is public or private (Ministry of Education, 2013).

The Taipei mechanism: is the centralized admission mechanism used by the Taipei Senior High School District in 2016 to match its students and schools (New Taipei City Government, 2015b). See the section of matching mechanism in Chapter 2 for the operational details of this mechanism.

Assumptions

Analysis for a new policy often involves significant uncertainties simply because the society never experiences the policy. Thus, a model for such analysis unavoidably has an exploratory nature and contains various assumptions and hypotheses (Bankes, 1993). I made the assumptions in this exploratory agent-based overlapping generations model by reference to the literature and my observations in the Taipei School District and summarize the significant assumptions underpinning the model below. The section of model description in Chapter 3 provides a more detailed explanation of the model assumptions.

In generating student agents' school preferences, I assumed that student agents had moderately to highly correlated school preferences for higher ranked schools. Chang (2011) found that all students in Taiwan prefer higher quality schools to lower quality schools; however, when low-quality schools are considered, distance is relatively more important than quality. Most Taiwan students use publicly recognized school ranks as the index for school quality (Chang, 2011; Lu, 2012; Shao, 2015; Yan, 2015). Therefore, I assumed that student agents had moderately to highly correlated school preferences for higher ranked schools and preferred nearby lower ranked schools to far-away lower ranked schools. As to financial subsidy, only low-income students consider it an influential factor in making their school choice decisions (Chang, 2011). Therefore, I further assumed that without the free-tuition policy, students with bottom 50% family income would not choose fee-paying private schools.

Studies have found that students do not always report their school preferences as their school choices; instead, students use strategies in response to the admission mechanism to make their choices, aiming at improving their chances of attending more preferred schools (Chen, Jiang, & Kesten, 2015; Pathak, 2011). In Taiwan, a commonly advised strategy found on the internet states the following: (a) Students should refer to each school’s past admission information, in comparison with their own scores or ranks, to form the list of possible schools; (b) students should choose their schools from their lists of possible schools; (c) students should arrange the order of the selected schools in a way that they are confident to be admitted to a preferred school while gambling for the admission to a more preferred school (e.g., Sun, 2015; Zhang & Wang, 2015). Similar advice is also circulated in mainland China (e.g., H. Wang, 2015; R. Wang, 2015; Song, 2015). I designed two behavioral strategies founded on the above advice for the student agents in the model to generate their school-choice lists. In this internet era, it is very likely that admission candidates all learn about the popular strategy and use it in their school-choice decision making. Therefore, when the two school-choice strategies were simulated, I assumed that all student agents used the same strategy. For comparison purpose, I also run simulations under the assumption that student agents used heterogeneous strategies to make their school choices.

By applying the central limit theorem, I assumed that students' performances were normally distributed. I also assumed that a high-performing student usually performed well in all subjects and vice versa. The literature shows that the students' performances in Taiwan are positively correlated with their incomes, and the income of Taiwan follows a lognormal distribution (Chou & Wang, 2012; Hojo & Oshio, 2012; Pinkovskiy & Sala-i-Martin, 2009). Therefore, I assumed that new admission candidates' log family incomes and scores formed a multivariate normal distribution and could be randomly generated from this distribution.

In generating students' high school scores, I assumed that students' socioeconomic statuses and personal factors remained the same throughout their high school years and that the peer effect was the only factor that influences students' scores in high schools. Since the 1966 Coleman report, numerous studies have shown that once students' socioeconomic statuses are controlled, schools contribute little to the explanation of the difference in student performance, although school factors, mainly the composition of peers, do have different effects on different groups of students (e.g., Coleman et al., 1966; Burke & Sass, 2013; Jennings, Deming, Jencks, Lopuch, & Schueler, 2015; Hojo & Oshio, 2012). However, it is not clear in the literature how students are affected by their peers. Salgado, Marchione, and Gilbert (2014) argued that peer effect emerges when students learn from their friends in the network formed by the students and their peers. Salgado et al. (2014) assumed that socioeconomic status, gender, and performance were the factors that determine whether a network would be formed and found that most students have high tolerance toward performance difference, but different groups might have different levels of tolerance toward gender and socioeconomic status. By reference to Salgado et al.'s study, I assumed that only the scores of the high school students whose family incomes were within the tolerance level of their peers would be influenced by the performance of their peers.

Scope and Delimitations

The purpose of this study was to contribute to the development of ABM in educational policy analysis. The agent-based model constructed in this study focuses on the centralized admission processes in school-choice systems and the designs of students' decision rules which includes the consideration of students' geographical and socioeconomic differences. I used this model to explore the possible distribution results emerging from the Taipei mechanism and the free tuition policy under different behavioral assumptions. Since many the real data were not available, this study was exploratory in nature. The input parameters and simulated outcomes were not calibrated with real data.

I followed the prominent KISS (keep it simple, stupid) principle in the social simulation field to construct this model. Under this principle, a model should be kept as simple as possible; more complexity is added to the model only if required (Axelrod, 1997; Barth, Meyer, Spitzner, 2012). This principle helps researchers to understand every element added into the model, which in turn helps researchers to analyze a surprising emergence observed in the simulations (Axelrod, 1997). This principle also helps other researchers to extend the model in a new direction (Axelrod, 1997). As a result, this model only has two levels of hierarchy (schools and students). Therefore, it could not replicate the process of peer effect modeled by Salgado et al. (2014), which needs the network structures in the level of classrooms. Nevertheless, this model design was enough to show the qualitative influence of peer effect on high school scores. Neither did this model contain other school-level factors that might affect high school scores as argued by some scholars, such as teacher quality (e.g., Burke & Sass, 2013).

I set the values of the parameters in this model to represent qualitatively the environment and culture of the Taipei School District, where students compete for highly ranked schools. In CASs, different initial conditions often result in different patterns of system evolutions (Borrill & Tesfatsion, 2011). Therefore, the simulation results of this study could not be generalized to other educational systems with different cultures.

Limitations

ABM is a bottom-up approach, which starts with the design of individual agents' behavioral rules and let global patterns emerge from agents' interactions. This approach relaxes the assumptions needed in most top-down approaches, such as fixed macrostructures, rationality, optimization, and homogeneity (Epstein, 1999; Macal & North, 2010). However, researchers usually have less microinformation needed for an agent-based model than macroinformation needed for a top-down model. Consequently, microspecifications must be hypothesized to construct an agent-based model. In this model, I inferred agents' school-choice strategies from the literature and my observation in the Taipei School District. Without the support of experimental or empirical evidence, the behavioral strategies designed in this model remain as candidate explanations for the simulated macropatterns, even if these simulated macropatterns can correspond to real data collected in the future. The causal relationship between the hypothesized microspecification and the emergent macrostructure cannot be established simply because the microspecification can generate the macrostructure because other microspecifications may also have the same explanatory power (Epstein, 1999). Nevertheless, if the simulated macropatterns match the data, then the behavioral rules programmed in this model serve as a reasonable causation hypothesis and can guide empirical data collection (Bankes, 1993; Epstein, 2008). If otherwise, then this study provides the information that this particular set of behavioral rules may not be a good hypothesis for future empirical research. Therefore, with the flexibility to perform what-if analysis, an exploratory agent-based model like the one in this study can reveal possible outcomes based on what we know and help make an informed policy decision, even if it cannot predict the exact quantitative results of a new policy.

Significance of Study

Most models for macroeducational policy analysis are equation-based. Equation-based models are weak in explaining the relations between microlevel behaviors and macrolevel patterns. However, the global effects of educational policies often depend on students' reactions. To know the micro-macro relations is essential in educational policy analysis because the effects of those policies often depend on how students react to the policies. Educational research needs an alternative tool to provide information about the micro-macro relations, and ABM is the right approach for this need (Maroulis et al., 2010; McClelland, 2014). Unfortunately, ABM is still new in the field of educational research.

Few agent-based models have been built for educational policy analysis. The model in this study enriches the application of ABM to educational policy analysis.

This model was the first agent-based model to qualitatively represent the environment of the Taipei School District, including the operational details of its 2016 admission mechanism (the Taipei mechanism) and the free tuition policy. This model was also one of the pioneering agent-based overlapping generations (OLG) school-choice models to simultaneously observe students' school-choice behaviors and macroeducational phenomena under various real-world matching mechanisms. OLG design is necessary when older generations' behaviors affect the overlapping younger generations' decision making. In many educational systems, like the one in Taipei, older students' performances affect the reputations and rankings of the schools, which in turn affects younger generations' school preferences and school choices (Allen & Burgess, 2013; Lu, 2012; MacLeod & Urquiola, 2012). Therefore, to include the OLG design is essential to study the effects of admission policies in these educational systems, whether the effects of interest are microscopic, mesoscopic, or macroscopic.

Most simulation models for school choice research have the assumptions that students always report their school preferences as their school choices without adapting to the change in matching mechanism. However, literature has shown that students use strategies to make their school choices, which may not be the same as their school preferences (Chen et al., 2015; Pathak, 2011). To model students' strategical behaviors, researchers must distinguish students' school preferences from students' school choices. Chen, Wang, and Chen (2017) were the first to make this attempt. However, in their model, student agents' only consideration is score. I significantly expanded Chen et al.'s (2017) design of students' preferences and school-choice strategies by including the consideration of distance and family income, which is in line with the findings in literature and observations. This more realistic design of student preferences and strategies helps explore the possible outcomes of the Taipei mechanism, especially when there are still no empirical data to know its consequences. The model description in Chapter 3 states in detail all assumptions, uncertainties, and calculations in this model, which helps other researchers to replicate the simulation results or run further scenario analysis under their perceived reasonable assumptions. Therefore, this model provides not only the right tool for the analysis of Taipei's admission policies but also a platform for rigorous discussions on these policies.

Summary

Educational systems are CASs, where aggregate patterns emerge from the interactions of individual agents. When the effect of an educational policy depends on how students behave in a new environment, it is essential for policymakers to understand the relation between macrophenomena and microbehaviors. However, the classical tools used to analyze macroeducational policies, such as equation-based modeling, cannot easily link macropatterns with microbehaviors. These classical models also were built upon the assumption of fixed system structures, violating the Lucas critique (Lucas, 1976). Using the wrong tools may be one of the reasons why there are full of cases where educational policies produced unexpected and undesired consequences.

Education researchers need new tools to analyze policies in complex educational systems. The complexity tool advocated by many CAS scholars is ABM, a bottom-up simulation approach that generates global patterns by simulating the interactions of individual agents (Farmer & Foley, 2009; Holland, 2006; Borrill & Tesfatsion, 2011). ABM allows researchers to understand not only the micro-macro connection but also the dynamic evolution of a system. ABM is also flexible to perform scenario analysis, which is of particular importance when how people will react to a new policy is uncertain or unknown (Roth, 2002; Schieritz & Milling, 2003). However, ABM is still new in the field of education research. Only a few models have been built to analyze macroeducational policies. Therefore, the purpose of the agent-based OLG model in this study was to contribute to the development of ABM in educational policy analysis. Since I built this model to represent the qualitative aspects of the Taipei School District, this study also helps to understand qualitatively how Taipei's new high school admission policies affect the distribution of educational opportunity and school quality, in comparison to other prevalent matching mechanisms.

Having evoked great objections, Taipei's high school admission mechanism, which was a part of Taiwan's 12-Year Education Reform, has been modified twice since its inauguration in 2014. The policymakers claimed that they had simulated their 2016 student-assigning mechanism (the Taipei mechanism) in their decision-making process. However, they used students' school-choice lists in the previous year to run the simulation, ignoring the Lucas critique. Furthermore, the government did not analyze the possible effects of the new mechanism on students in different performing and income quartiles, unable to know whether the new revision would reach or deviate from the Reform’s goal of enhancing the equalities of educational opportunity and school quality. The simulation results help Taipei’s policymakers and stakeholder to have a better understanding of the possible effects of Taipei’s new admission policies on students in different income and performing groups. Therefore, this study facilitates inclusive and informative public dialogues on Taipei’s admission policies.

The next chapter contains a comprehensive literature review. It starts with a review of the theoretical framework of ABM, followed by a discussion of the concepts and measures of equality of educational opportunity and equality of school quality. A large part of Chapter 2 is devoted to a thorough review of commonly used matching mechanisms in school admission processes. The chapter concludes with a discussion of the current development of ABM in the field of educational research.

Chapter 2: Literature Review

Introduction

ABM is a computational approach to modeling a system composed of autonomous agents and to studying the aggregate patterns emerging from agents' interactive behaviors. This approach is an attempt to respond to traditional methodologies' inadequacy to deal with CASs (Gilbert, 2004; Tesfatsion, 2003). The origin of ABM can be traced to cellular automata of John von Neumann and Stanislaw Ulam in the 1950s (Chen, 2012). A cellular automaton is a theoretical self-reproducing machine that consists of "cells" as its building blocks (Chen, 2012; "John von Neumann's," 2010). The states of the cells will change according to the previous states of their neighboring cells under predetermined rules; the dynamics of the cellular automaton exhibited by the cells' state changes are similar to the self-reproduction or evolution process of life ("John von Neumann's," 2010). The concept of cellular automata is fundamental in the fields of artificial intelligence and artificial life (Chen, 2012; "John von Neumann's," 2010). The expansion of ABM did not take off until the rapid growth of computational power in the 1990s (Gilbert, 2004). A growing number of disciplines have now adopted ABM to study complexity problems (Chen et al., 2011). Nevertheless, it is still new to most educational policymakers and researchers and has rarely been applied in educational policy research. Therefore, this chapter begins with a review of the concepts underpinning ABM, followed by the general structure of an agent-based model and a brief comparison between ABM and equation-based modeling.

ABM is used to construct autonomous agents and their environment, followed by simulating the interactions of the agents and letting macrolevel phenomena emerge (Axelrod, 2005; Gilbert, 2004; Macal & North; 2010). This bottom-up simulation methodology naturally integrates knowledge regarding individual behaviors and macrostructures (Maroulis et al., 2010). Modelers must review the knowledge on both micro- and macrolevels to construct a sensible model and draw meaningful insights from the simulation results. In this study, I built an agent-based OLG model to explore the systemic impacts of school admission policies on educational opportunity and school quality. Therefore, this chapter continues with a review of two strands of the literature in education: (a) educational equality and school quality, and (b) school choices and admission mechanisms. A review of the former defines and operationalizes the measurement of the macrophenomena that I focused on in this study while a review of the latter helps to construct agents' behaviors and the matching mechanisms that I used in the model of this study.

Many studies have used equation-based models to investigate the impacts of school choices on educational equality and school quality. A review of their mixed findings provides a foundation for how to discuss the ways in which ABM can complement a study on aggregate effects of education policies. A handful of attempts has been made to apply ABM to explore the linkages between students' behaviors and the admission results in school-choice systems (e.g., Chen et al., 2017; Harland & Heppenstall, 2012; Maroulis et al., 2014; Millington et al., 2014; Wang, Chie, & Chen, 2017). These agent-based educational models are relatively simple in comparison to the agent-based models in other fields, such as economics and finance. However, they serve as the stepping stones to construct a comprehensive agent-based educational model that is general enough for theoretical and exploratory discussions and flexible enough for research on a specific educational system. This study is an additional stepping stone for such a model. Therefore, a review of the extant agent-based educational models is necessary to understand what these pioneering models have accomplished. The review also concludes this chapter.

Literature Search Strategy

With the development of information technology in recent years, a literature search is no longer limited to certain databases or libraries. Google Scholar now consolidates almost all library databases in the whole world and provides direct links to the articles in the libraries that the users have access to. Many articles are free through Google Scholar. All the references in this chapter were collected through Google Scholar and downloaded either directly from Google Scholar or from the libraries of Walden University and National Chengchi University. Some articles related to Taiwan’s education systems cited in this chapter are written in Chinese.

The original keywords used to search articles were *complex adaptive systems, computational irreducibility*, *agent-based, equation-based modeling, system dynamics, general equilibrium, educational equality, educational inequality*, *school quality, school choice, matching mechanism,* and any combination of the above keywords. I also used the snowball strategy to extend the search by reviewing the references in the articles found from the above keyword search.

As to the literature related to ABM, I did not limit the search to any specific timeframe so that I could have a thorough understanding of ABM's history and its underpinning concepts. I also conducted an exhaustive search for all extant agent-based school-choice models because these models were the ground on which I built this model. As to the two strands of the literature on education mentioned in the previous section, I focused on articles published in or after 2010, but the review includes some seminal works in these fields, such as the initial argument about school choice made by Friedman (1955) and the first discussion of school matching mechanism by Abdulkadiroglu and Sonmez (2003).

Complex Adaptive System and Computational Irreducibility

ABM is grounded in the concept of complex adaptive system (Macal & North, 2010). The term *complex adaptive system* describes not only a kind of system but also a new paradigm of thinking versus the conventional reductionist paradigm (Dodder & Dare, 2000). Reductionists reduce a phenomenon into parts and focus on the analysis of the parts, holding the view that the whole is equal to the sum of the parts and the whole can be understood by understanding the working of its parts (Green, 2001; Holland, 2006). Researchers with the view of CASs challenge reductionist thinking by claiming that the whole is greater than the sum of the parts because CASs have emergent global macroscopic properties that are different from the properties of the components (Archer & Smeins, 1991). Emergent properties can be seen only when the systems are studied as a whole (Archer & Smeins, 1991). The advocates of CASs have provided myriad examples of emergent properties in nature and in human societies to support their views, including the immune system's ability to distinguish self from intruders, the formation of a supply network in an economy, swarm intelligence of ant colonies, and school health conditions (e.g., Holland, 1992; Keshavarz et al., 2010; Zimmerman, Lindberg, & Plsek, 1998).

CASs became a paradigm during the 1980s with the establishment of the Santa Fe Institute in New Mexico, in the United States. The Santa Fe Institute was founded and joined by many key figures in this new thinking, including George Cowan, Nobel laureate Murray Gell-Mann, Nobel laureate Kenneth Arrow, and John H. Holland (Dodder & Dare, 2000). It has been playing a leading role in developing an interdisciplinary platform and modeling methodologies to discuss problems in CASs (Dodder & Dare, 2000; Santa Fe Institute, 2014). This new paradigm has now been adopted by many researchers in various disciplines, including economics, epidemiology, management, technology, and ecology (Bale, Varga, & Foxon, 2015; Levin et al., 2013). However, there still has not been a universal term or definition for CASs (Keshavarz et al., 2010). Some researchers use the term "complexity theory" to mean CASs, and some use both terms interchangeably (The Health Foundation, 2010). As to the definition, while Holland (2006) defined CASs as "systems that have a large numbers of components, often called agents, that interact and adapt or learn" (p. 1), Nobel laureate Murray Gell-Mann referred to the "agents" in Holland's definition as CASs (Gell-Mann, 1994). This discrepancy in definition may be because CASs usually have a nested hierarchy; that is, a system's components (the agents) are themselves smaller-scaled CASs (Keshavarz et al., 2010; Pathak, Day, Nair, Sawaya, & Kristal, 2007). In this study, I adopted Holland's definition because it is now adopted by the majority of the ABM society.

Characteristics of Complex Adaptive Systems

Although researchers in the field of CASs may have different opinions about the terminology and the precise definition of such systems, they seem to agree that the most distinguishing characteristics of CASs are emergence and adaptation. In CASs, macroscopic, or collective, properties emerge from the simultaneous and parallel interactions of more primitive parts with each other and with their environment (Choi, Dooley, & Rungtusanatham, 2011; Damper, 2000; Holland, 1992, 2006; Levin et al., 2013). The constituents in a CAS are mostly heterogeneous; an example is human beings. Even if the massive heterogeneous individuals all follow a few simple behavioral rules, or *schemata* as termed by Gell-Mann (1994), their interactions typically still form nonlinear aggregate patterns and thus make the system complex (Choi et al., 2011).

Emergent properties, or emergent system behaviors, do not follow individual agents' behaviors (Newman, 2011). Emergent properties cannot be predicted simply from the knowledge about the individual agents; they are irreducible to the sum of the properties of the constituents (E. P. Odum, H. T. Odum, & Andrews, 1971). The whole is more than the sum of the parts (Hiance, Doogan, Warren, Hamilton, & Lewis, 2012).

Birth rate, for example, is not an emergent property because it is just the summation of individual births in a period expressed in the percentage format (Odum et al., 1971). On the other hand, wealth inequality observed in many free economies is an emergent property. This macrophenomenon emerges from individuals' self-interested economic activities; it cannot be investigated only by studying individuals' economic behaviors. Another example is a stock market crash, which is a nonlinear property emerging from the interactions of heterogeneous stock market players, who typically pursue their own maximum profits (Levin et al., 2013). Emergence results from the interactions of the self-organized constituent entities in the system without any central control. Because of this nonlinear, irreducible, emergent feature, it is not easy to use equations derived from the conventional reductionist thinking to study CASs (Heusser et al., 2012). Therefore, researchers have explored new approaches, such as simulation modeling, to deal with the emergent properties of CASs (Cioffi-Revilla, 2013).

The second distinguishing feature of CASs is agents' abilities to adapt to the problems and changes in their environments (Holland, 1992). All complex systems show the feature of emergence while CASs show not only the characteristic of emergence but also the characteristic of adaptation (Newman, 2011). Some examples of adaptation in social CASs are children adapting to school lives, parents adapting to new job requirements, and firms adapting to technological changes.

Adaptation involves the concepts of fitness and anticipation (Holland, 1992; Newman, 2011). Agents anticipate the consequences of their reaction options in comparison to competitions and choose the best available behavioral rules or strategies that help them fit the changes in their environments (Holland, 1992; Newman, 2011). The process of adaptations can be briefly described as follows: (a) agents store their input-output experiences where inputs are their behaviors and outputs are the results of these behaviors; (b) from their input-output experiences, agents identify perceived patterns or regularities; (c) agents compress these perceived patterns to form schemata for actions; and (d) after receiving feedbacks regarding the outcomes of the schemata, agents replace some schemata with new ones or enhance the standings of some schemata with respect to other competitive ones (Arthur, 1994; Gell-Mann, 1994). Adaptation in social CASs is a continuous process because agents' interactions change the environment, which in turn causes agents to adapt to fit the environmental changes (Keshavarz et al., 2010).

In social CASs, adaptation by no means implies that agents always have complete knowledge or information to choose the strategies that can maximize their interests as assumed in classical economics (the Homo Economicus assumption); instead, agents mostly act with bounded rationality (Arthur, 1994; Heckbert et al., 2010). There are several reasons that agents cannot exercise perfect rationality in CASs: (a) human rationality can cope with complexity only to a certain level; (b) agents have no way to know but guess other agents' actions in the interactive situations which agents frequently encounter; and (c) to acquire complete information to make unboundedly rational decision is costly, while less-expensive heuristics often provides adequate solutions (Arthur, 1994; Conlisk, 1996). Since adaptation is mostly made with bounded rationality, it can succeed or fail. Cioffi-Revilla (2014) argued that a successful adaptation needs to go through a series of processes: (a) the agents are aware of the need to adapt, (b) the agents have the intent to adapt, (c) the agents have the capacity to adapt, and (d) the agents can overcome various challenges to implement the adaptation. Since agents in

CASs are heterogeneous and autonomous, their adaptation decisions and results are also heterogeneous (Epstein, 1999; Heckbert et al., 2010).

Because of the continuous adaptation of the agents and continuous emergence of global phenomena, CASs can hardly reach the state of optimality, if optimality can even be defined for CASs (Holland, 1992). Sometimes, a CAS may seem to stay at local optima, but the stays are usually only temporary for live systems (Holland, 1992). Morrison (2008) explained that “Change, disequilibrium and unpredictability are *requirements* for survival: a butterfly that flies only in a straight line is soon eaten.” (p. 20). However, classical equation-based computational models, such as computable general equilibrium models, concentrate on optimal fixed points and assume representative agents. These models do not seem to be very useful to deal with CASs. Instead, ABM, by allowing agents to self-organize their activities, is a better candidate when heterogeneity, bounded rationality, and adaptation are involved (Heckbert et al., 2010; Epstein & Axtell, 1996; Lansing, 2002).

Computational Irreducibility

Computational irreducibility is a concept claimed by Stephen Wolfram (2002), who argued that there are computationally irreducible questions in nature and in human societies that cannot be answered by any mathematical shortcut but must be analyzed by simulating the system directly (Wolfram, 2002). From the study of cellular automata, Wolfram concluded that systems could be categorized into four classes. Class 1 systems refer to systems that will always have the same fixed or repetitive final patterns regardless of their initial settings. Class 2 systems grow into several different final states, depending on their initial settings, but these states all appear to have either repetitive patterns or nested structures. Class 3 systems produce random or chaotic final states, and their initial conditions have long-lasting effects on the evolutions of the systems. Class 4 systems are systems between order and chaos.

In Class 4 systems, the cells organize themselves to form localized structures that move around and sometimes interact with each other to form a cascade of new complicated patterns. In the transition between patterns, the systems seem to be in chaotic states but far from chaos. Waldrop (1993) named this state “edge of chaos”. Cioffi-Revilla (2014) described Class 1 to Class 4 cellular automata as stable, oscillating, chaotic, and complex, respectively. While mathematical equations (computational shortcuts) can be used to describe the behaviors of Class 1 and Class 2 cellular automata, Wolfram claimed that systems belonging to Class 3 and Class 4 are computationally irreducible. There is no way to know the future states of Class 3 and Class 4 systems with any computational shortcuts; the only way is “to simulate each step in their evolutions explicitly” (Wolfram, 1988, p. 187). Figures 1 shows the graphical examples of the four classes.

Cellular automata contain some popular models that demonstrate the emergence of complex macrolevel patterns from microlevel interactions of agents who follow simple behavioral rules (Chen, 2012). These popular models are used by social scientists to describe the properties of CASs (Chen, 2012). Like Wolfram’s Class 4 systems, the patterns in social CASs expand, contract, suddenly collapse, and transform into new patterns. With this concept in mind, it is not surprising to see some large companies suddenly suffer enormous loss, or some ancient nations fell apart abruptly (Crockett, 1993). Because social CASs have the characteristics as shown in Class 4 cellular automata, many researchers concluded that social CASs are computationally irreducible (Borrill & Tesfatsion, 2011; Corckett, 1993). The traditional models using differential or difference equations can hardly find out how these systems behave and evolve. The only way to solve the problems in CASs is to use computational simulations, preferably ABM, to compute every step of the systems' possible evolutions (Borrill & Tesfatsion, 2011).

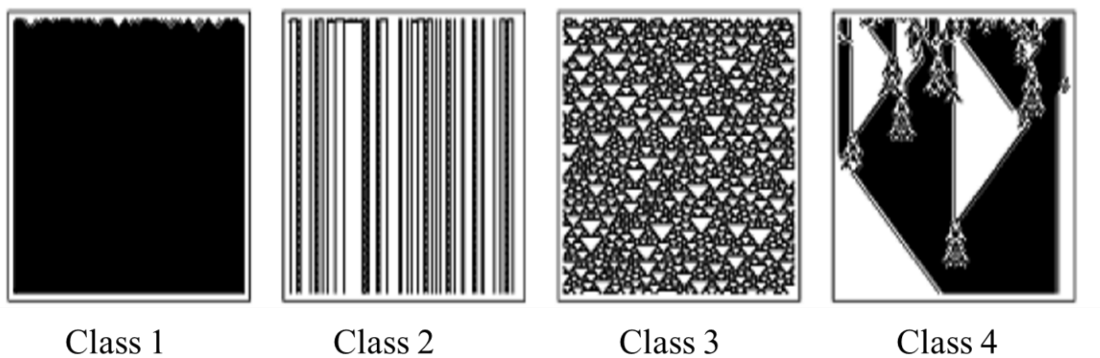

Figure 1. Graphical example of each class of cellular automata. This figure is adopted from Wolfram, 2002, p. 231.

Educational Complex Adaptive Systems

Education systems involve learning, pedagogical strategies, resource allocations, and interactions between and within all levels of hierarchy from policymakers, school administrators, teachers, to students and parents. Education systems are CASs comprising nested CASs (Burns & Knox, 2011; Groff, 2013). A school district has schools as its subsystems while each school has classrooms as its subsystems; meanwhile, school districts, schools, and classrooms are all CASs (Burns & Knox, 2011).

Take schools as an example. A school is composed of administrators, teachers, students, and their parents, all of whom are of different properties and constantly interact with each other. The constituents of a school are autonomous even though laws, culture, and ethos strongly affect their actions (Keshavarz et al., 2010). In addition to autonomy, schools exhibit the characteristics of CASs: learning, adaptation, and emergence (Keshavarz et al., 2010).

The ethos of a school is an example of an emergent phenomenon. It emerges from the intertwining effects of many factors including principal's leadership style, teachers' morale, student's learning environment, parent's participation, and so on (Mason, 2008). To change the negative ethos of a school, it is often not enough to implement just one intervention but interventions at many levels, including a job policy to improve students' family incomes, a school leadership reform to stimulate teachers' commitment, a curriculum transformation project, a plan to develop a more efficient learning environment for students, and a project to encourage parental involvement (Mason, 2008).

Educational interventions cannot be blindly copied from one system to another because autonomous agents in a different system may react to the same intervention differently. Policymakers should estimate stakeholders' possible reactions in designing a new educational policy. Knowing not only what agents will do but also why they do it is essential to have a better estimate of agents' behaviors in a new environment (Lemke & Sabelli, 2008). The findings of educators, psychologists, sociologists, economists, and political scientists, to name a few, shall all be referenced to have a better knowledge about how students may respond to a new educational intervention. Even if students' reactions are all the same, different initial settings of a CAS may generate different interaction patterns and thus different emergent macrophenomena (Haggis, 2008).

Although educational systems exhibit all the features of CASs, educational policymakers often pass legislation based on linear cause-effect models, ignoring the nonlinear features of CASs (Groff, 2013). Many complexity researchers have urged educational researchers and policymakers to apply the concepts of CASs and CASs tools to studying educational issues and policies (e.g., Groff, 2013; Maroulis et al., 2010). The concept of CASs requires researchers to study not only the variables in a CAS but also the dynamic interactions between the variables in a holistic view (Radford, 2008). The complexity thinking also requires researchers to expect possibilities and interdisciplinary collaboration (Groff, 2013; Keshavarz et al., 2010). If complexity thinking and simulation were applied to the decision-making process of, for example, California's mandated statewide class-size reduction in 1996, the supply shortage of qualified teachers might have been considered a priori (Maroulis et al., 2010; Sklar, Davies, & Co, 2004). Similarly, the reactions of the states, schools, and teachers to the NCLB policy might have been simulated beforehand if complex systems tools were applied in the policy analysis.

From the viewpoint of CASs, building an agent-based simulation model is the right way, or even the only way, to analyze the design of new educational interventions. As early as 2004, researchers attempted to construct agent-based models representing hierarchical educational systems for educational policy analysis (e.g., Sklar et al., 2004).

In 2008, Lemke and Sabelli began to define the conceptual framework of a complex education system model. They argued that modelers should consider what levels of hierarchies to be included in the model (students, teachers, classrooms, schools, district, or up to states and central government), how these levels are related, the variables that may drive changes, and the scale of the model. These considerations will guide the collection of data needed for model construction (Lemke and Sabelli, 2008). As to the modeling tool, Lemke and Sabelli found ABM promising for the study of the dynamic changes in a complex education system.

The above pioneering works do not seem to lead the trend toward agent-based educational research. Up to today, the application of ABM to educational policy analysis is still in the exploratory stage, not to mention a fully calibrated agent-based educational model capable of real-world quantitative prediction. The construction of an agent-based educational model capable of prediction requires multidisciplinary collaboration among educators, agent-based modelers, behavioral scientist, design economists, to name a few, and intensive feedbacks between data collection, testing, and simulation (Farmer & Foley, 2009; Yu, 2015). More efforts are needed to produce a full-fledged agent-based educational model. This study contributes to the continuing efforts needed to realize that goal.

Agent-Based Modeling

ABM is a simulation approach characterized by modeling individual agents' behaviors and interactions, from which, complex, dynamic macropatterns emerge (Axelrod, 2005; Gilbert, 2004; Macal & North; 2010). In an agent-based model, the aggregate phenomena “grow” from the individual agents’ interactions. Therefore, ABM is a bottom-up modeling approach, in contrast to equation-based modeling approaches, such as computable general equilibrium modeling and systems dynamics, which model the macrostructures of a system directly without addressing too much of its microfoundation (Parunak, Savit, & Riolo, 1998).

Epstein (1999) argued that ABM is a generative approach to social science. ABM does not accept the law of excluded middle, which accepts a hypothesis as long as its negation is derived to be false. To claim the existence of a linkage between a microspecification and a macroemergence, an agent-based modeler must generate the emergence from the microspecifications, or the modeler does not explain the emergence (Epstein, 1999). A famous saying of Epstein (1999) is: “If you didn’t grow it, you didn’t explain its emergence” (p. 43).

The logic behind Epstein’s claim that ABM is a generative approach is abduction (Richiardi, Leombruni, Saam, & Sonnessa, 2006). Abduction is a type of logic introduced by Charles Sanders Peirce; different from deduction and induction, it is “the process of forming explanatory hypotheses” (as cited in Frankfrut, 1958, p. 593). By generating a causal relationship between a set of input and outputs, ABM provides an explanatory hypothesis, which is to be verified by way of induction, with the aid of deduction (Frankfurt, 1958). Without using the term “abduction”, Axelrod (1997) regarded agent-based simulation as the third way of doing science, different from deduction and induction. Like deduction, agent-based simulations start from a rigorous set of assumptions. However, unlike deduction, agent-based simulation is not to prove theorems; instead, it is to produce data to be analyzed inductively (Axelrod, 1997). Unlike induction, the data generated by agent-based simulation is from a rigorous set of rules rather than empirical observations (Axelrod, 1997). Therefore, the data generated from agent-based models will not have the problems of missing data or uncontrolled variables as happened in social science experiments and observations (Axelrod, 1997).

The basic elements of an agent-based model are agents, agents' interaction rules, and agents' environment (Macal & North, 2010). An agent is an identifiable autonomous individual situated in an environment where it interacts with other agents (Macal & North, 2010). Each agent has a state consisting of its attributes, and its state may change over time. Agents' attributes may include, for example, gender, socioeconomic status, location, preference, performance score, ability to adapt (ability to change behavioral rules), and goals to achieve. Agents can be heterogeneous in attributes and behaviors (Macal & North, 2010). Additionally, agents are endowed with behavioral protocols and mechanisms to interact with other agents in their spatial neighborhoods or social networks. Agents' behavioral rules usually are rather simple, reflecting the behavioral patterns of humans, who mostly follow norms, habits, and protocols (Macy & Willer, 2002). Agents' environment refers to the space where agents perform activities. The environment may simply be used to tell the locations of the agents or may be constructed in a way to constrain agents' actions (Macal & North, 2010). Once an agent-based modeler programs the three elements, all the modeler needs to do is to execute the model, which generally runs in discrete time (in steps), and see how the observations emerge from agents' interactions (Axtell, 2000). Figure 2 shows a conceptual example of an agent-based model in which the environment simply is used to show agents' positions.

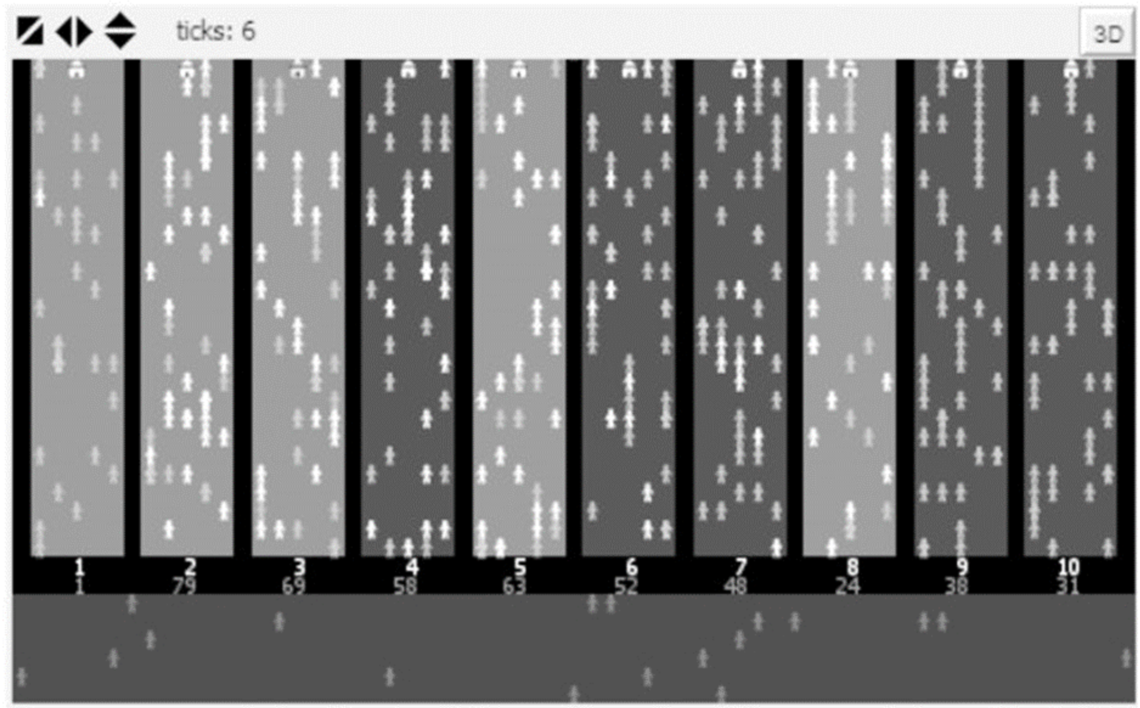

Figure 2. A snapshot of an agent-based simulation in this study. The 10 rectangles represent the 10 schools. The agents within each school are the newly admitted. The agents below the rectangles (outside of the schools) are those who are unassigned to any school.

Advantages of doing ABM

ABM as a right tool for CASs. ABM is a right methodology to study complex systems and complex adaptive systems because its design embeds the features of these systems: heterogeneity, decentralization, explicit space, interactions, bounded rationality, and emergence (Borrill & Tesfatsion, 2011; Epstein, 1999; Macal & North, 2010). As in the real world, agents can be heterogeneous in every attribute whether it is a character, background, or decision rule. There is no need to assume representative agent or to group agents into several homogeneous groups as in classical equation-based models or system dynamics. All agents can be autonomous in making decisions without a central control although they may be constrained by social norms or institutions endogenously generated by agents' interactions in earlier steps (Epstein, 1999; Macal & North, 2010). Agents interact with other agents according to their social networks that are explicitly presented in the computational environment (space) and may be changed endogenously when the artificial system evolves. Agents do not need to be rational with full information or use optimizing strategy as assumed in equation-based models for mathematical tractability. Agents can only have bounded information and bounded computing power as most people in the real world (Epstein, 1999; Macal & North, 2010). In an agent-based model, the macropatterns emerge from the interactions of individual agents. Therefore, ABM allows researchers to link and map microbehaviors to macroperformance, which is a function difficult to achieve by equation-based models. (Chen, 2015; Epstein, 1999; Macal & North, 2010).

Equation-based modeling focuses on equilibria while ABM allows researchers to collect and analyze the entire dynamical data in a process, not just equilibria if equilibria exist (Axtell, 2000). Continuously changing and adapting is how a CAS survives; a CAS will soon move out of an equilibrium even if there is an equilibrium, or the CAS will probably die (Holland, 1992). Therefore, the focus of a CAS study shall be on dynamics rather than equilibria. Although the approach of system dynamics is also used to handle the dynamics of a system, it is under the assumption that the system structure is fixed

(Schieritz & Milling, 2003). Under system dynamics, adaptation is either unconsidered or assumed to have no effect on system structure (Schieritz & Milling, 2003). On the contrary, ABM is flexible in handling adaptation either through learning at the individual level or through the reproduction of fitter individuals at the population level (Schieritz & Milling, 2003).

ABM also can easily handle dynamic networks, both spatial and social, because networks (interaction topologies) are embedded in the programming of agent interactions (Macal & North, 2010). On the other hand, equation-based models, including system dynamics, have difficulties reflecting the function of networks, not to mention network dynamics (Axtell, 2000). Since the interactions in complex social systems are mostly non-linear, it is extremely difficult to deduce its emergences into equations, which makes ABM the only existent candidate to study global properties of complex social systems (Axelrod, 1997; Borrill & Tesfatsion, 2011; Macal & North, 2010). Borrill and Tesfatsion (2011) further argued that it is not only impractical but also impossible theoretically to study social systems by using equation-based models because social systems are Wolfram's Class 4 systems, which are computationally irreducible. Therefore, the only option to understand the macroproperties of social systems is to build and run agent-based models representing these systems and then observe the simulation results (Borrill & Tesfatsion, 2011).

ABM as a complement to and an extension of human-subject experiments. ABM can also be viewed as a complement to human-subject experiments (Duffy, 2006). Experimenters face more constraints on what they can do than agent-based modelers

(Duffy, 2006). For example, the cost and the size of the lab may constrain the number of human subjects in an experiment. Another example is that human fatigue may limit the time length of an experiment. That is why many agent-based models have been calibrated with the experimental data to understand the laboratory findings or to scale up the size of a human-subject experiment by, for example, increasing the number of artificial agents or the periods of treatment (Chen, 2015). Nowadays, the findings in ABM even inspire experimenters to redesign their human-subject experiments so that they can examine the findings in ABM (Chen, 2015). ABM allows researchers to lift the constraints in human-subject experiments and create an artificial system in which parameters and variables can be changed one at a time to build the causal relationships between microinputs and macrooutputs (Epstein, 2008; Montes, 2012). These simulated causal relationships may not be available in literature yet and thus can serve as hypotheses for future data collection and empirical testing (Epstein, 2008; Montes, 2012). Therefore, ABM is not only a tool to represent or imitate a system based on experimental findings or other empirical data but also a methodology to discover new questions and new theories (Axelrod, 1997; Chen, 2015; Epstein, 2008). Indeed, as argued by Cioffi-Revilla (2013), the major scientific contribution of ABM is its ability to run computational experiments on complex social systems because too often, it is impossible to conduct human-subject experiments on the social systems of interest for either scientific research or policy analysis.

Limitation of ABM

Although ABM is created to tackle the problems in CASs, it is not to say that agent-based models have better prediction power than classical models. It is because we may not have enough knowledge to know how the agents in a CAS will adapt to changes and, in turn, affect the network and structure of the system (Hazy, 2012; Holland, 1992). Nevertheless, as George E. P. Box's famous quote says "Essentially, all models are wrong, but some are useful" (Box & Draper, 1987, p. 424). When there are significant uncertainties, ABM is much more flexible than equation-based modeling to perform scenario analysis and provide critical information needed to make policy decisions (Bankes, 1993; Lansing, 2002).

Although each simulation run is sufficient to build the possible causal relationship between microspecification and macrostructure, one single run is not enough to answer the question about how robust the result is (Axtell, 2000). Multiple runs of simulation with different initial settings and parameters are needed to test the robustness of the result, which takes a lot more computational power than analytical or numerical resolutions of equations (Axtell, 2000). Fortunately, this problem has become manageable due to the rapid development of computer technology.

Even when a microspecification in an agent-based model can "grow" the macropatterns of interest, the microspecification can only be viewed as one hypothetical explanation, because other microspecifications (individual behavioral rules) might also have the same explanatory power (Epstein, 1999). How to find alternative microspecifications and determine the right explanation often challenges agent-based modelers (Epstein, 1999). Agent-based modelers usually need to collaborate with other researchers to conduct transdisciplinary research because ABM intrinsically needs to program the features of agents and their environments, which inevitably are multi-disciplinary (Epstein, 1999). Sometimes, it is challenging to establish an interdisciplinary research team. However, agent-based research is not the only research that requires transdisciplinary collaboration. Social science researchers have long called for interdisciplinary research because it is impossible to decompose human activities clearly into separate disciplines. In comparison with other disciplines in social sciences, agent-based social science naturally promotes interdisciplinary teamwork.

Educational Equality

Throughout human history, education has played a major role in determining an individual's life chances (Green, 2011). People with higher education tend to have more earnings, better health, and longer lives (Colclough, Kingdon, & Patrinos, 2010; Green, 2011; Spasojevic, 2010). Another worldwide phenomenon is that people born in families with higher socioeconomic status tend to have better educational performance than those born in families with lower socioeconomic status (OECD, 2010). Family socioeconomic status, education, and personal well-being seem to be reciprocally related. A general belief is that education, especially higher education, is the key to moving upward for most individuals (Attewell, 2010; Hojo & Oshio, 2012). Therefore, when income disparity and health disparity are expanding globally in most countries, many scholars focus on the issue of educational inequality, in hopes of finding education policies that can eventually reduce the gap in well-being (Corak, 2013; Ortiz & Cummins, 2011).

The Organization for Economic Co-operation and Development (OECD, 2010) suggested three types of policies to ameliorate educational inequality: (a) additional assistance to lower-performing schools and students as well as disadvantaged students, (b) higher evaluation standards for all students, and (c) inclusion of all marginalized students in mainstream schools and regular classrooms. As to which specific policies to apply, OECD admitted that it depends on the situation of each region. Whether or not following OECD's suggestions, the governments in different countries and regions have made various policies and reforms to tackle the problems of inequality of school quality and educational opportunity. Examples are NCLB in the United States, HSEP in South Korea, and Taiwan's 12-Year Education Reform.

Whether reducing educational inequality will certainly result in reducing income and health inequality is questionable. The reason is that the distribution of well-being is a complex phenomenon, emerging from the interacting influences of many factors, including the social, cultural, and economic structure of a labor market (Thompson & Simmons, 2013). For example, a meritocratic labor market where professional interns receive extremely low or no pay may scare students from middle- and poorer-class families away from pursuing those professional careers. A labor market that does not have enough demand to absorb graduates from higher education will keep the graduates from finding the fitted jobs (Thompson & Simmons, 2013). Societies desiring to tackle the problem of well-being inequality shall consider multifaceted interventions that are relatively more beneficial to lower-income families, alongside policies that reduce educational inequity (Corak, 2013).

In any case, educational achievement is a powerful driver of upward social mobility and intergenerational earning mobility (Attewell & Lavin, 2007). To reduce the gap in well-being, policies reducing educational inequality undoubtedly play a significant role. However, before the evaluation of a policy aiming at reducing educational inequality, the definition of educational equality and its measures must be determined. Unfortunately, there has not been a universal answer ever since the seminal survey *Equality of Educational Opportunity* by Coleman et al. (1996), commonly known as the Coleman Report. Educational equality usually refers to equality of educational opportunity, which is a tradition inherited from the Coleman Report. However, as explained by Coleman (1968), there is no single concept of equality of educational opportunity. Moreover, because the global trend has shifted the responsibility of education to the public sector, the concept of educational opportunity is now intertwined with that of school quality. Depending on researchers' perspectives, the concepts of equality of educational opportunity and equality of school quality may be the same, enclaved in the other, or overlapping. Even if the concepts can be clearly defined, their measures may not be able to fully capture the concepts. The reasons include the involvement of many interacting factors, the long-lasting effects of schooling, the limitation of methodology, and the difficulties in collecting data (Borman & Dowling, 2010; Jennings et al., 2015; Ladd & Loeb, 2013). Because of the complexity of definition and difficulty in measurement, it is necessary for each study to state clearly the operational definitions of the concepts as well as their measures.

Equality of Educational Opportunity

Originally, the US society perceived equality of educational opportunity as providing students with equal school resources (Coleman, 1968). The idea has evolved to include the discussion of this issue from the effect side of school resources (Coleman, 1968). The Coleman Report provides five definitions of equality of educational opportunity to accommodate the different perspectives of this issue. These five definitions are: (a) same tangible school inputs, such as per student expenditures and quality of teachers that can be measured quantitatively; (b) same student composition of schools; (c) same intangible school inputs, such as teacher's morale, teacher's expectation of students, and average student's learning attitude; (d) same school results given the same student background and abilities; and (e) same school results given different student backgrounds and abilities (Coleman et al., 1966). The former three are related to school input; the latter two, effects of schooling. Coleman (1968) explained that there was no evidence to show that policies based on the definitions in (a) and (b) could improve school's effects. The definition in (c) does not offer where to stop and how relevant these factors are for school quality. The definition in (e) is impossible to achieve by any policy unless children can be free from the influence of their unequal family environments (Coleman, 1968). Therefore, the Coleman report focused on providing information related to the definition in (d). However, Coleman (1968) also argued that if the focus of the policies is on the definition in (d), then the constant gap between different student groups would be considered acceptable, which again would not be acceptable in a society where the purpose of schooling is to prevent the disadvantages in children's family environments from impeding their achievements in adult life.

Up to today, the definition of inequality of educational opportunity still depends on the purpose and limitation of a study; the same applies to the measure (Ladd & Lauen, 2010). Since the social trend is to hold schools responsible for reducing the impacts of family differences, most of the studies on inequality of educational opportunity focus on measuring the portion of student achievements accounted for by students' family background (Ferreira & Gignoux, 2014). If there is perfect equality in educational opportunity (that is, no achievement difference is caused by family background), then the mean achievements of students from all types of family backgrounds shall be the same (Ferreira & Gignoux, 2014). The proxies for student achievements commonly used are regional or national test scores, Program for International Student Assessment (PISA) scores, the highest level of education attained, and earnings (e.g., Brunello, Fort, & Weber, 2009; Ferreira & Gignoux, 2014; Pfeffer, 2008).

In this study, I defined equality of educational opportunity from the input point of view; that is, all junior high graduates have the same opportunity to attend any senior high school, regardless of their residences, backgrounds, or test scores. This definition is in accord with the intention of Taiwan's 12-Year Education Reform, which was to change the high school entrance system from ability tracking to mixing. There is an equality of educational opportunity if the mean family income of the admitted students in each school is the same.

Equality of School Quality

To discuss school quality is to discuss how much value a school can add to the well-being of a student, a community, or a country. Since well-being is a property emerging from the interaction of many factors, how to measure a school's value-added has long been an issue among scholars, especially when some benefits of schooling cannot be seen immediately. The common proxies for school quality are measures of resources, measures of school process, and measures of student achievements, corresponding to the viewpoints of input, the effectiveness of the inputs, and the outcome of the inputs, respectively (Ladd & Loeb, 2013). Spending per student, teacher-to-student ratio, and teacher's years of teaching or certification are examples of measures of resources. However, the measurement of the quantities of resources cannot catch the quality and effectiveness of the resources because two schools with the same amount of resources may not have the same quality due to different resource processes (Ladd & Loeb, 2013). Evaluation of teacher's practice in a classroom is an example of measures of school process. Nevertheless, the evaluation itself is difficult because it is costly and hard to standardize the evaluators' rating practice (Ladd & Loeb, 2013). Therefore, some researchers turn to student outcomes to measure school quality. Student's scores and educational attainment are examples of the measures of student outcomes. This approach is not without problems because test score or educational attainment only represents a portion of contents learned in schools that will benefit student's future life (Ladd & Loeb, 2013). Even if test score is a good indicator of student's future outcome, it is still difficult to calculate a specific portion of test result attributed to schooling even with today's most sophisticated model (Ladd & Loeb, 2013).

With all these criticisms in mind, when the discussion of school quality focuses on educational outcome, test score is still appropriate to be the proxy for school quality, even if it is not the best. The reason is that test score reflects whether a student attains a specified outcome, which in this case is more important than whether the outcome is accurately attributed to schools (Ladd & Loeb, 2013). However, it may not be practical to require schools to make every student have equal test performance because students' performances also depend on their individual motivation, aptitude, and family influences (Ladd & Loeb; Tsai & Yang, 2015). A more reasonable proxy for school quality is the average test score, which can also serve as an index for schools to examine their input efficiency and adequacy (Ladd & Loeb, 2013). Therefore, in this study, I chose seniors' average score in a school as the proxy for school quality.

Factors Affecting Student Performance

Student performance is a complex phenomenon, emerging from the interaction of many factors, including, but not limited to, student's attitude and attributes, family background, teacher's quality, school resources, and peer effect. Due to the complexity of the interactions, it is not easy to find all of the influencing factors and their magnitudes, even after researchers' tremendous efforts in this regard. Additionally, methodology remains as an issue half a century after the Coleman Report, regardless of the advancement of statistical technologies. What and how the factors affect student performance is still under debate.

The 1966 Coleman report's main finding was that after controlling students' socioeconomic status, schools contribute little to the difference in student performance, although schools do have different effects on students with different socioeconomic statuses (Coleman et al., 1966). The most influential school factor is peers. Disadvantaged students benefit more from being mixed with peers having strong educational supports while advantaged students' performance is hardly affected by peers with lower socioeconomic statuses (Coleman et al., 1966). Teacher's quality also has some effects, but the effects are more on disadvantaged students than on advantaged students (Coleman et al., 1966). Over the decades, these findings are still largely supported by voluminous empirical studies (Burke & Sass, 2013; Dearden, Ferri, & Meghir, 2002; Hanushek, 1989; Jennings et al., 2015). However, the findings on the size of peer effect across different ability groups are mixed. Burke and Sass (2013) argued that the loss experienced by higher performing students in a class with an increased percentage of very low-performing peers is more than the gain received by a very low-performing student in a class with an increased percentage of high-performing peers. On the other hand, Carman and Zhang (2012) found that peer effect is significant on middle-performing students but insignificant on students at both ends.

The empirical findings on the factors influencing Asian students' performance also largely conform with those of the Coleman Report. Using the data from the 2007 Trends in International Mathematics and Science Study (TIMSS), Hojo and Oshio (2012) showed that individual and family factors are the keys to student's performance in the top 5 mathematics-performing Asian countries (Taiwan, South Korea, Singapore, Hong

Kong, and Japan). School resources have very limited impact, but peer effect, measured by the average peer scores, has a significant influence on student performance, which implies that ability-sorting will further improve the performance of higher-performing students and exacerbate the inequality of educational outcomes (Hojo & Oshio, 2012). By investigating the science achievement of the 8th-grade students in Taiwan, Tsai and Yang (2015) again confirmed the significance of family and individual factors on student performance; as to school-level factors, while school ethos contributes to student performance significantly, school resources play a little role. Using the 2006 PISA Hong Kong sample for their research, Sun, Bradley, and Akers (2012) also confirmed the significance of the individual and family factors. As to the school factors, Sun et al. (2012) argued that the socioeconomic status composition of the student body and the length of instruction time both contribute to the variance of science performance among the 15-year-old students in Hong Kong. Sun et al. (2012) did not measure peer effect directly. However, they argued that the significance of the student socioeconomic status composition of a school implies the significance of peer effect because socioeconomic status and peer performance are positively and highly correlated (Sun et al. 2012).

Although most empirical studies worldwide have shown that school factors have much less influence on student performance than individual and family factors, many educators still believe in the power of school education to reduce socioeconomic inequality. As argued by Hojo and Oshio (2012), the findings could only be interpreted as the inefficiency of the current schemes for school education; they did not mean that school education could not be improved. Other scholars argued that test score, as used in most studies, cannot fully reflect the impact of school education on students' future well-being; other measures must be used to have a better understanding of the effect of school education (Dearden et al., 2002; Jennings & Sohn, 2014).

Regardless of the mixed opinions about the overall school effect, the statement that peers influence a student's performance is generally accepted in the literature. However, how the peers influence student performance or what the process of peer effect is cannot easily be seen through statistical regression, which is mostly used to study peer effect. Lomi, Snijders, Steglich, and Torlo (2011) argued that students tend to have a performance similar to the average performance of their friends. Therefore, Salgado et al. (2014) assumed that peer effect is through networks; they further assumed that students form networks based on how much they can tolerate the differences in performance, gender, and family background (represented by father's occupation in their model). They then used an agent-based model to simulate students' fifth-grade math scores based on their actual third-grade math scores to find the best-fit parameters of the three variables. They found that across the 22 schools in their data, students all had high tolerance towards performance difference; what made the network formation mechanisms different between schools were the different levels of tolerance in gender and family background. Their simulated parameters could generate students' fifth-grade math scores with 90% accuracy, which indicates that network is a candidate explanation of how peer effect works (Salgado et al., 2014).

Educational institutions may also have effects on student performance. Compared to comprehensive education, tracking (sorting students into academic or vocational tracks) in secondary schools magnifies inequality in performance by family background (Ferreira & Gignoux, 2014; Van de Werfhorst & Mijs, 2010). Tracking may further increase future socioeconomic differences if those who attend vocational schools have less chance or tendency to receive tertiary education (Ferreira & Gignoux, 2014; Van de Werfhorst & Mijs, 2010). In Taiwan, students in secondary schools were mainly tracked by test scores. Since Taiwan students' scores are positively correlated with their socioeconomic statuses, tracking by test scores largely resulted in a sorting of students by socioeconomic status (Liu & Chen, 2010). Ferreira and Gignoux (2014) also suspected a strong relationship between the tracking and high inequality in math performance in Taiwan. These empirical studies support the intention of Taiwan's 12-Year Education Reform to change the system from tracking to mixing.

School Choices

School choice is a concept initially discussed in Milton Friedman's (1955) seminal article "The Role of Government in Education." Friedman argued that the governmental action of administering or operating educational institutions is not justified, even though the governmental action of financing the primary and secondary education and requiring all educational institutions to meet minimum quality standards is justified. Friedman (1955) believed that parents should have the freedom to send their children to the schools that meet their needs; school vouchers can achieve this objective by increasing the variety and quality of educational institutions through inter-school competition for students. Nowadays, in addition to school vouchers, various designs of school choices have emerged, including charter schools, open enrollment, education savings accounts, tax-credit scholarships, and individual tax credits and deductions (Cunningham, 2013).

Advocates of school choices argued that school choices help mitigate socio-economic segregation between schools (Pathak, 2011). Under the traditional neighborhood school system, only the rich can afford to live in the neighborhood of better schools, and better schools tend to attract richer people and further contribute to the rising of the neighboring housing prices. With school choices, better schools are no longer the privilege of the rich, and thus a society with less educational inequality can be achieved (Pathak, 2011). However, the discussion about whether and how to implement school choices has never stopped because the empirical studies have mixed conclusions about the benefits of school choices claimed by the advocates (Musset, 2012). Taking open enrollment as an example, by analyzing the data from a lower-income urban school district, Hastings, Neilson, and Zimmerman (2012) found that open enrollment improves participants' test scores significantly. On the other hand, Choi and Hwang (2014) analyzed the empirical data of test scores after the implementation of open enrollment policy in Seoul, Korea, 2010 and found that those exercised school choices and enrolled in private schools experienced a significant improvement in scores but at the cost of score deterioration among students who remained in public schools. Since the overall average score did not change, Choi and Hwang (2014) argued that what school choice brings about is not productivity enhancement through competition as claimed by the advocates but private schools' cream-skimming better-performing students from public schools.

Another example of school choice is voucher systems. Wolf et al. (2013) examined student graduation rate and performance under Washington DC's school voucher program and concluded that voucher participants who attend private schools have higher graduation rate but no higher performance, except reading, than their counterparts in public schools. Lindbom (2010) reviewed the empirical studies on the impacts of government-funded private schools, mostly established after the 1990s' Swedish large-scale voucher reform, and concluded that Sweden experiences marginal positive effect of school choice on student performance and limited negative effect on segregation, after controlling the effect of residential segregation. However, Lindbom (2010) could not rule out the possibility that the much higher residential segregation experienced in Sweden after the school voucher reform was indeed caused by the new voucher system.

After reviewing the studies published in the 2000s, Musset (2012) summarized the effects of school choices as follows: (a) there is only weak or no correlation between school choices and student performance; and (b) there is a positive correlation between school choices and student segregation in terms of socioeconomic status and academic performance. Nevertheless, Lindbom (2010) argued that it is dangerous to generalize the findings of school-choice effect because the impact of a school-choice program is context-dependent; the success of school choices highly depends on their forms, sizes, implementations, and social environments. Regardless of the effect of school choices, many societies worldwide, including the Taipei School District, recognize students' rights to choose their own schools and provide students with various forms of school choices.

Factors influencing School Choice Decisions

Researchers also have different opinions about the factors influencing parents' and students' school choice decisions. A key premise underpinning the idea that school choice helps improve student performance and educational equity is that academic performance is the main determinant of all parents' school-choice decisions (Musset, 2012; Burgess, Greaves, Vignoles, & Wilson, 2014). However, empirical data showed that advantaged parents are more likely than disadvantaged parents to exercise school choices, which suggests the possibility of heterogeneous factors that influence school-choice decisions (Musset, 2012). Researchers have found that in choosing schools, parents consider not only academic performance but also travel distance and school socioeconomic composition (Burgess et al., 2014; Musset, 2012). On the one hand, Burgess et al. (2014) argued that although less affluent parents weigh distance a little higher than more affluent parents, generally speaking, all parents' consideration priorities are similar: high academic performance, school socioeconomic composition with less poor student percentage, and distance. On the other hand, Dronkers and Avram (2010) made a cross-national analysis and argued that students (parents) in different countries have different considerations in choosing schools; some even weigh religious, ethnic, and socioeconomic factors over performance.

The mixed findings in the empirical studies on the impact of school choice and the factors influencing students' school choices reflect the complexity of educational systems and the heterogeneity in constituent agents' decision rules. How students are admitted depends not only on how students make their school choices but also on the implemented matching mechanism. A change in matching mechanism will also change students' strategies in making school choices. These complex and adaptive interactions among constituents and institutions make classical reductionist techniques hard to study the effects of school choices. The agent-based approach used in this study provides a new way to study complex school choice effects.

Factors considered by students in Taiwan. Taiwan students consider both school quality and transportation cost, in importance order, in making their high school choices (Chang, 2011; Chen, 2007). However, middle- and low-performing students give more consideration to transportation cost than higher-performing students (Chang, 2011). The reason may be that Taiwan students are prioritized by schools mainly based on their academic performance in the admission process; when students feel that their academic performance is so low that only low-quality schools will admit them, school quality no longer matters and distance becomes relatively important (Chang, 2011). Chang's (2011) argument could be inferred that Taiwan students have similar preferences for high-quality schools but prefer nearby low-quality schools to far-away low-quality schools. Tuition is a factor of school choice only among low-income students (Chen, 2007). I referred to the above findings to design students' school preferences and school-choice rules in this model.

Matching Mechanisms

The core of school-choice designs is matching mechanism (assignment procedure), which determines how to assign students to schools. Matching mechanisms have long interested mathematicians and economists because they are essential to a well-functioning market (Roth, 2002). When a free market fails to produce a satisfying matching result, a centralized clearinghouse is usually established to implement the matching procedures, such as the establishment of the National Resident Matching Program in 1952 to solve the problem of matching medical interns to hospitals (Roth, 2008; Roth & Peranson, 1999). The matching mechanisms in school choice programs have also received researchers' wide attention since the pioneering work of Abdulkadiroglu and Sonmez (2003). Abdulkadiroglu and Sonmez (2003)'s discussion of student assignment mechanisms led to a series of school-choice mechanism reforms, such as New York City's 2003 reform and Boston's 2005 reform (Pathak, 2011). The 2003 reform in New York City (NYC) successfully reduced the percentage of students not being assigned to a school in their choice lists from 30% to 3% (Zweifel, 2009). Since the matching mechanism in an admission policy is critical to the policy's success, it is important to have a clear understanding of how the matching mechanism implemented in an admission system works.

A matching mechanism can be one-sided (either the demand side or the supply side) or two-sided (Pathak, 2011). In a one-sided mechanism, the matching is determined according to the preferences and rankings (order of choices) of the agents on the autonomous side, while the agents on the other side are only objects (Pathak, 2011). In a two-sided mechanism, agents in both sides are autonomous and can have their own preferences and rankings, even though some agents' priorities may be controlled by law; for example, schools may be required by law to have special treatments for disadvantaged students and reserved seats for students within the walk zone (Abdulkadiroglu & Sonmez, 2003; Pathak, 2011). It is not unusual for both types of mechanisms to coexist in a school-student matching system.

In literature, the following three criteria usually are used to evaluate a matching mechanism: Pareto efficiency, stability, and strategy-proofness (or strategy immunity). Pareto efficiency in school choice usually is defined from the perspective of students' welfare as follows: Pareto efficiency is reached when it is impossible to assign a student to his or her more preferred school without assigning another student to his or her less preferred school (Abdulkadiroglu & Sonmez, 2003). Stability is to pair a student and a school in a way that no students and schools that are not paired prefer to be paired with each other; that is, there is no blocking pair (Gale & Shapley, 1962; Roth, 2008). A stable matching algorithm will assign a student to his or her preferred school before assigning those who have lower priorities than this student to the same school; that is, a stable matching algorithm eliminates justified envy (Abdulkadiroglu & Sonmez, 2003). Capable students tend to play complicated ranking games when they feel that, if they do not rank higher their less preferred schools in which they have higher priorities, they may waste their top choices and end up of being assigned to even worse schools (Abdulkadiroglu & Sonmez, 2003). A matching mechanism is strategy-proof if students' best strategy to choose schools is to rank the schools according to their true preferences

(Roth, 2008). These three criteria, Pareto efficiency, stability, and strategy-proofness, may not be compatible. A stable algorithm may not be a Pareto-efficient one and vice versa; stability or Pareto-efficiency does not equal strategy-proofness either (Abdulkadiroglu & Sonmez, 2003; Roth, 2008). The empirical evidences show that stability is critical to the success of a matching policy (Roth, 2008). However, in practice, policymakers usually have other considerations and do not always choose a stable mechanism. The Taipei mechanism is an example.

In the following paragraphs, I discuss six matching mechanisms: serial dictatorship, deferred acceptance, the Boston mechanism, top trading cycles, the Chinese parallel mechanism, and the Taipei mechanism. The first four are widely discussed in the literature. The Chinese parallel mechanism is a matching mechanism used to be adopted by many Chinese provinces and municipalities to admit university students. The Taipei mechanism is the one adopted by the Taipei School District in 2016.

Serial Dictatorship

Under this mechanism, the student with the top priority receives his or her top choice, the student with the second priority receives his or her top choice among the schools that still have seats, and so on (Pathak, 2011). I demonstrate this mechanism in the following example.

Example 1. There are 4 students, S = {s1, s2, s3, s4} and 3 schools, C = {c1, c2, c3}. Every school has 1 seat and prioritizes the students in the following order: s1 $\succeq$ s2 $\succeq$ s3 $\succeq$ s4, where the symbol $\succeq$ means “superior to”. The students’ school choices are as follows:

s1: c1 ≻ c2 ≻ c3,

s2: c1 ≻ c2 ≻ c3,

s3: c2 ≻ c1 ≻ c3,

s4: c2 ≻ c3 ≻ c1.

The symbol ≻ means "preferred to." Under serial dictatorship, Student s1 has the first right of choice and receives her first choice, School c1. Then, s2 has the right of choice and receives her second choice, c2, since c1 has no more empty seat. Next, s3 receives her third choice, c3, while s4 is rejected by all schools because there is no more empty seat remained in the system. The matching result, $M_{SD,}$ is presented as below:

$$M_{SD} = \begin{pmatrix} s1 & s2 & s3 & s4 \\ c1 & c2 & c3 & -- \end{pmatrix}.$$

Serial dictatorship is considered strategy-proof (manipulation-free) and Pareto efficient; it is stable only when the school agents prioritize the students in the same way (Abdulkadiroglu & Sonmez, 2003; Pathak, 2011). This mechanism was chosen for Chicago's selective high schools admission reform in 2009 (Pathak & Sonmez, 2013). However, the number of choices a student could make in this reform was less than the number of participating schools, which made this mechanism subject to strategy manipulation (Pathak & Sonmez, 2013).

Deferred Acceptance

The school-student matching mechanism adopted by NYC and Boston in the 2000s was a stable mechanism design called student-proposing deferred-acceptance algorithm (Roth, 2008). This mechanism is a version of deferred acceptance originally published by Gale and Shapley (1962). In this mechanism, students are temporarily assigned to their first-choice schools, and the schools temporarily accept them according to the priority order of the students in the schools up to the schools' capacities. Those who are rejected by their first-choice schools are assigned to their next-choice schools, and the schools reselect among the previously accepted students and the newly assigned students according to their priority orders. This process continues until all students exhaust their choice lists (Pathak, 2011). Example 2 explains this algorithm.

Example 2. The conditions here are the same as in Example 1. The process is as follows:

Step 1: All students are temporarily assigned to their first choices. The schools temporarily accept those with the highest priorities and reject the rest. Therefore, c1 temporarily accepts s1 and rejects s2; c2 temporarily accepts s3 and rejects s4.

$$M_{DA1} = \begin{pmatrix} s1 & s2 & s3 & s4 \\ c1 & \text{--} & c2 & \text{--} \end{pmatrix}.$$

Step 2: Those who are not assigned go to their second choice. Each school compares the priories of the students assigned to it in step 1 and step 2, temporarily accepts those with the highest priorities and reject the rest. Therefore, s2 goes to c2 and s4 goes to c3. Since s2 has higher priority than s3, c2 accepts s2 and rejects s3. Since s4 is the only student assigned to c3, s4 stays in c3.

$$M_{DA2} = \begin{pmatrix} s1 & s2 & s3 & s4 \\ c1 & c2 & \text{--} & c3 \end{pmatrix}.$$

Step 3: The student who has not been assigned in this step is s3. So, s3 goes to his next choice, c1. Since s1 has higher priority than s3, c1 keeps s1 and rejects s3. Therefore, the assignments remain the same as M_{DA2}.

Step 4: Now s3 goes to his next choice, c3. Since s3 has higher priority than s4, School c3 accepts s3 and rejects s4.

$$M_{DA4} = \begin{pmatrix} s1 & s2 & s3 & s4 \\ c1 & c2 & c3 & -- \end{pmatrix}.$$

Step 5: The student who has not been assigned in this step is s4. So, s4 goes to his next choice, c1. Since s1, the student who is currently assigned to c1, has higher priority than s4, s4 is rejected by c1. The process ends here because s4 exhausts his choice lists.

$$M_{DA} = \begin{pmatrix} s1 & s2 & s3 & s4 \\ c1 & c2 & c3 & -- \end{pmatrix}.$$

The above M_{SD} is the same as M_{DA} because the students have the same prioritized orders in all schools in the above two examples. That is, when all schools prioritize students in the same way and the number of choices is not limited, deferred acceptance is, in fact, the same as serial dictatorship (Chen et al., 2015; Pathak, 2011). Deferred acceptance is stable and eliminates justified envy. It is also strategy-proof as long as the number of students' choices are not limited. Because of these advantages, deferred acceptance is the most popular mechanism in school-choice reforms (Chen et al., 2015).

The Boston Mechanism

The Boston mechanism refers to the mechanism adopted by the Boston city before the 2005 reform (Abdulkadiroglu et al., 2005). While deferred acceptance and serial dictatorship emphasize student's performance, the Boston mechanism emphasizes student's choice. This mechanism works as follows. First, each school considers only those students who list it as their top choice and assigns seats to the students according to their priority orders until no more space left or no more students left. Second, each school that still has space considers those unassigned students who list it as their second choice and accepts the students in the same way as in the first step. This process continues until no school has any seat left or all students have been assigned (Pathak & Sonmez, 2013). The acceptance in each step is final. Example 3 explains this mechanism.

Example 3. The conditions here are the same as in Example 1. The process is as follows.

Step 1: All students are assigned to their first choices. Since each school has only one seat, each school accepts the student with the highest priority in that school and rejects the rest. Therefore, c1 accepts s1 and rejects s2; c2 accepts s3 and rejects s4. The acceptance is final.

$$M_{BM1} = \begin{pmatrix} s1 & s2 & s3 & s4 \\ c1 & \text{--} & c2 & \text{--} \end{pmatrix}.$$

Step 2: s2 and s4 go to their second choices, c2, and c3, respectively. Since c2 is full, s2 remains unassigned. The assignment of s4 to c3 is final. Since no more seat is left, the process stops here.

$$M_{BM} = \begin{pmatrix} s1 & s2 & s3 & s4 \\ c1 & \text{--} & c2 & c3 \end{pmatrix}.$$

Under the Boston mechanism, it is possible that if students do not list a school as their top choice, they will lose the seat in that school to a student with lower priority. That is, the Boston mechanism easily creates justified envy. Often, students must manipulate their submitted choices in a way different from their true preferences to secure places in the schools that they like more. Therefore, the Boston mechanism is neither strategy-proof nor stable (Chen et al., 2015). Nevertheless, even though it is heavily criticized for its strategy complexity and creation of justified envy, the Boston mechanism and its similar versions are still widely used in school admissions (Chen et al., 2015). In China, the traditional college admission mechanism (the sequential mechanism) is equivalent to the Boston mechanism; a newer mechanism called the Chinese parallel mechanism adopted to replace the sequential mechanism by many provinces still has the attributes of the Boston mechanism in its design (Chen et al., 2015).

The Boston mechanism may not be efficient when students manipulate their choices (Chen et al., 2015). However, when students have very similar ordinal preferences and schools have coarse or no priorities over students, the Boston mechanism may be more Pareto-efficient than deferred acceptance in ex-ante welfare

(Abdulkadiroglu, Che, & Yasuda, 2011). Therefore, Abdulkadiroglu et al. (2011) suggested that governments reconsider their decision on switching the Boston mechanism to deferred acceptance because justified envy for better-performing students means more chances for low-prioritized students, who are usually disadvantaged students, to attend better schools.

Top Trading Cycles

The mechanism of top trading cycles was attributed to David Gale by Shapley and Scarf (1974). Abdulkadiroglu and Sonmez (2003) were the first to apply this mechanism to school-choice design. This mechanism works as follows. Each student points to his or her top choice, and each school points to its top-priority student. This process will form at least one cycle. The students in the cycle are assigned to the schools that they point to and leave the process. Each remaining student points to his or her next choice, and each school that still has space points to the remaining student who has the highest priority in that school. Also, there is at least a cycle, and the students in the cycle are assigned to the schools to which they point. This process ends when all students are assigned, all schools have exhausted space, or all students have used up their choice lists. The top trading cycles mechanism is considered Pareto efficient and strategy-proof, but not stable (Abdulkadiroglu & Sonmez, 2003). This mechanism has been widely discussed in the literature since its first introduction in 2003. However, it was not implemented in any real-world school system until 2012 by New Orleans's Recovery School District (Vanacore, 2012). Example 4 shows how this mechanism works.

Example 4. There are 3 students, S = {s1, s2, s3} and 3 schools, C = {c1, c2, c3}. Each school has only one seat. The students' school-choice lists are:

s1: c2 ≻ c1 ≻ c3,

s2: c1 ≻ c2 ≻ c3,

s3: c1 ≻ c2 ≻ c3.

The students' priorities in each school are:

c1: s1 ≥ s3 ≥ s2,

c2: s2 ≥ s1 ≥ s3,

c3: s3 ≥ s1 ≥ s2.

Step 1: s1 points to c2, his top choice; c2 points to s2, its top priority student; s2 points to c1; c1 points to s1. Now the first cycle is formed as shown in Figure 3 (left cycle), and Students s1 and s2 are assigned to Schools c2 and c1 respectively.

$$M_{TTC1} = \begin{pmatrix} s1 & s2 & s3 \\ c2 & c1 & -- \end{pmatrix}.$$

Step 2: s3, who has not been assigned, points to his next choice that still has seats, which is c3; c3 points to his top priority student who has not been assigned, s3. Now the second cycle is formed as shown in Figure 3 (right cycle). Since all students are assigned, the process ends here.

$$M_{TTC} = \begin{pmatrix} s1 & s2 & s3 \\ c2 & c1 & c3 \end{pmatrix}.$$

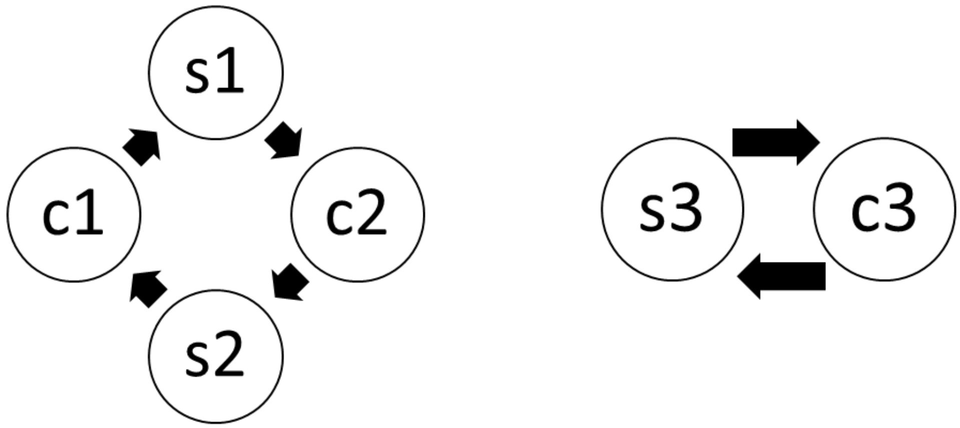

Figure 3. Top trading cycles explained in Example 4. The left cycle is formed in Step 1; the right cycle is formed in Step 2.

The Chinese Parallel Mechanism

Before 2003, all provinces and municipalities in China used the Boston mechanism to admit university students (Chen et al., 2015; Zhu, 2014). As in other places where the Boston mechanism is adopted, many Chinese parents complained about justified envy, and media never stopped reporting the news that some elite students were unadmitted simply because of the way they ranked their choices (Zhu, 2014). Therefore, more and more provinces and municipalities changed their matching mechanism from the Boston mechanism to the parallel choice algorithm, or the Chinese parallel mechanism as named in the literature, to alleviate the problem of justified envy.

The feature of the Chinese parallel mechanism is to insert deferred acceptance (or serial dictatorship since all Chinese universities prioritize students in the same way) into the Boston mechanism. Under the Chinese parallel mechanism, each choice in the Boston mechanism becomes a class of choices with several choices within each class. For example, the Chinese parallel mechanism used in Beijing's 2014 university admission process had two classes of choices: Class 1 and Class 2. Students were allowed to submit two choices in Class 1 and three choices in Class 2. Deferred acceptance (or serial dictatorship) is employed within each class while the Boston mechanism is implemented between the classes. Choices within the same class are considered parallel, and thus this mechanism is named the parallel choice algorithm. The following is an example of the Chinese parallel mechanism.

Example 5. There are 4 students, S = {s1, s2, s3, s4} and 4 schools, C = {c1, c2, c3, c4}. Every school has one seat and prioritizes the students in the same way as follows: s1≥ s2 ≥ s3 ≥ s4. Each student is allowed to have two choices in Class 1 and one choice in Class 2. The students' school choices are as follows.

s1: (c1 ≻ c2), c3;

s2: (c1 ≻ c2), c3;

s3: (c2 ≻ c1), c3;

s4: (c2 ≻ c3), c1.

Step 1: Deferred acceptance is applied to the choices in Class 1. s1 goes to his top choice in Class 1, which is c1. s2 goes to c2. Since the schools listed in s3's Class 1 are full, s3 is not assigned. s4 goes to c3. Because all students run out of their Class 1 choices, the process for Class 1 ends here, and the assignments are final.

$$M_{CP1} = \begin{pmatrix} s1 & s2 & s3 & s4 \\ c1 & c2 & \text{--} & c3 \end{pmatrix}.$$

Step 2: Those who remain unassigned after the process for Class 1 go to the process for Class 2. In this example, only Student s3 has not been assigned. The choice of Student s3 in Class 2 is School c3. However, c3 has no seat left. Therefore s3 is rejected by c3. Since all choices in all classes have been processed, the whole process stops here. The final assignment for the four students is the same as in M_{CP1}:

$$M_{CP} = \begin{pmatrix} s1 & s2 & s3 & s4 \\ c1 & c2 & \text{--} & c3 \end{pmatrix}.$$

According to the experimental results, Chen et al. (2015) argued that the performance of the Chinese parallel mechanism lies between those of deferred acceptance and the Boston mechanism in terms of stability and manipulability. Students are most likely to use strategies under the Boston mechanism, followed by the Chinese parallel mechanism and then deferred acceptance. Deferred acceptance is more stable than the Chinese parallel mechanism, and the Chinese parallel mechanism is more stable than the Boston mechanism (Chen et al., 2015). The Chinese parallel mechanism can become the Boston mechanism if only one choice is allowed in each class. The Chinese parallel mechanism can also become deferred acceptance if all choices allowed are in Class 1.

The empirical data show that the Chinese parallel mechanism indeed reduces justified envy in China. However, some scholars criticized that the shift from emphasizing choices to emphasizing test scores in the matching process gives students even more study pressure to have better test scores and causes students to focus only on high-stakes subjects, which jeopardizes a balanced learning (Xin, 2008). This mechanism also caused some lower-ranked universities but with highly praised special programs hard to find good students for those special programs (Xin, 2008). The Chinese government has decided to reform university admission system again in 2017 to tackle these problems but insisted in upholding the policy of transforming all university matching mechanisms to deferred acceptance nationwide (State Council, 2014).

The Taipei Mechanism

I named the 2016 admission mechanism adopted by the Taipei School District the Taipei mechanism. There are two major features in this mechanism. The first is the inclusion of the spirit of the Boston mechanism in deferred acceptance by assigning points to each student's choice and adding this "choice score" to the composite score, which is used by each school to prioritize students. 36 points are assigned to student's first five choices; 35, the sixth to the tenth choices; 34, the eleventh to the fifteenth choices; 33, the sixteenth to the twentieth choices; 32, the twenty-first to the thirtieth choices. No points will be assigned after the 30th choices. Whether or not achievable, the intent of this design was to encourage students to focus on their own interests rather than blindly follow school ranks. This feature was more salient in its original 2014 design. In 2014, the point was decreased per each consecutive choice; 30 points were assigned to the first choice; 29, the second, and so on. In 2015, it was revised to assign decreasing points to each consecutive group of choices to mitigate many parents' objections to the gaming feature embedded in this design (School Year 104's Committee for Exam-free Admission to Taipei District Senior High School (CEFA), 2015). The second feature is to convert students' raw scores to coarse-grained scores. This design of blurring scores aims at reducing students' study pressure under the assumption that students care about every point of grade; the less distinctive the grade is, the less pressure students have. Under this design, a student's raw exam scores (1-100 percentage scores from the Comprehensive Assessment Program for Junior High Students) are converted to 7-scale scores. A rough conversion from 1-100 percentage score to 1-7 scale score is shown in Table 1.

Table 1

Conversion of Raw Score to 7-Scale Score under the Taipei Mechanism

	Raw score = x						
	$x < 41$	$41 \leq x < 61$	$61 \leq x < 71$	$71 \leq x < 84$	$84 \leq x < 90$	$90 \leq x < 94$	$x \geq 94$
7-scale score	1	2	3	4	5	6	7

Note. The conversion is approximate to the average conversion rate of all subjects calculated based on the press release issued by Research Center for Psychological and Educational Testing (RCPET, 2015).

Under the Taipei mechanism, a school prioritizes students based on their composite scores. The composite score is the sum of the scores of the following three categories: tests, school choice, and diversity learning (Taipei City Government Department of Education, 2015). Each student has to take the tests administered by Taipei's Comprehensive Assessment Program for Junior High Students. There are six tests in the program: Chinese, mathematics, English, sociology, science, and Chinese composition. All tests are measured on the scale of 1-7 as illustrated in Table 1, except that Chinese composition is measured on the scale of 0-1 (0.1, 0.2, 0.4, 0.6, 0.8, and 1). Hence, the maximum score a student can receive is 36 in this category. As illustrated in the previous paragraph, the maximum choice score is also 36. The diversity learning score refers to student's performance in art, physical education, and community service. A student can also receive a maximum of 36 points in this category. Therefore, the maximum composite score a student can receive is 108. If a tie happens in the prioritization process, the following scores are compared in a consecutive order to break the tie: (1) the diversity learning score, (2) the total test score, (3) the choice score, (4) Chinese test score, (5) math test score, (6) English test score, (7) sociology test score, (8) science test score, and (9) Chinese composition score. Once students are prioritized in each school, deferred acceptance is employed to allocate students to schools (Taipei City Government Department of Education, 2015). Example 6 explains the features of the Taipei mechanism in a simplified version.

Example 6. There are 4 students, S = {s1, s2, s3, s4} and 4 schools, C = {c1, c2, c3, c4}. 36 points are assigned to students' first choice; 35, the second; 34, the third; 33, the fourth. Every school has one seat. Table 2 shows the students' raw exam scores, converted 7-scale scores, diversity-learning score, and their school choices.

Student s1's composite score in School c1 is 78 (7 for the 7-scale score + 35 for the diversity learning score + 36 for the choice score); s1's composite score is decreased by 1 in each consecutive choice of schools. Table 2 shows each student's composite score and priority in each school.

Table 2

Example 6 - Student's raw exam score, converted 7-scale score, diversity-learning score, and school choices

Student	Raw exam score	7-scale score	Diversity-learning score	School choices
s1	95	7	35	c1 ≻ c2 ≻ c3≻ c4
s2	93	6	35	c1 ≻ c2 ≻ c3≻ c4
s3	85	5	36	c1 ≻ c3 ≻ c2≻ c4
s4	75	4	36	c3 ≻ c4 ≻ c2≻ c1

Table 3

Example 6 – Student's total score and rank in each school

	Composite score				
	s1	s2	s3	s4	Student's ranking
c1	78	77	77	73	s1 ≥ s3 ≥ s2≥ s4
c2	77	76	75	74	s1 ≥ s2 ≥ s3≥ s4
c3	76	75	76	76	s3 ≥ s4 ≥ s1≥ s2
c4	75	74	74	75	s4 ≥ s1 ≥ s3≥ s2

In Schools c1 and c4, although s2 and s3 have the same composite score, s3 is prioritized higher than s2 because s3 has a higher diversity-learning score than s2. Similarly, s4 is ranked higher than s1 in School c4 because s4 has higher diversity-learning score than s1. In School c3, s1, s3, and s4 have the same composite score.

Since s3 and s4 have higher diversity-learning score than s1, s3 and s4 are ranked higher than s1. s3 is ranked top in School c3 because s3 has higher test score than s4.

Once the students' priorities in each school are determined, deferred acceptance is applied to match the students.

Step 1: All students are temporarily assigned to their first choices. The schools temporarily accept those with the highest priorities and reject the rest. Therefore, c1 temporarily accepts s1, and c3 temporarily accepts s4. Students s2 and s3 are temporarily unassigned.

$$M_{TM1} = \begin{pmatrix} s1 & s2 & s3 & s4 \\ c1 & \text{--} & \text{--} & c3 \end{pmatrix}.$$

Step 2: Those who are not assigned go to their second choices. Each school compares the priories of the students assigned to it in step 1 and step 2, temporarily accepts those with the highest priorities, and rejects the rest. Therefore, s2 temporarily goes to c2, s3 takes over the seat of s4 in c3, and s4 is now unassigned.

$$M_{TM2} = \begin{pmatrix} s1 & s2 & s3 & s4 \\ c1 & c2 & c3 & \text{--} \end{pmatrix}.$$

Step 3: s4 goes to his next choice, which is c4. Since s4 has no competitor in c4, s4 stays in c4. Now, all students receive their seats. The process ends here.

$$M_{TM} = \begin{pmatrix} s1 & s2 & s3 & s4 \\ c1 & c2 & c3 & c4 \end{pmatrix}.$$

Under the Taipei mechanism, more than 90% of the students can receive the maximum diversity-learning score (The Central News Agency, 2014). The inclusion of this category seems to be more for the purpose of promoting balanced learning than for the purpose of prioritizing students. Therefore, students' priorities (ranks) in each school are primarily determined by their choice scores and test scores. Assigning scores to student's choices is an unconventional design and puzzles many parents who argue that it is ridiculous to assign decreasing scores to school choices because it is like punishing students for their bad choices (Zheng, 2015). Whether this unconventional design can better reach the purpose of mixing students than other mechanisms is undiscussed in literature; it would be answered by agent-based simulations in this study.

Summary

Most researchers used set-theoretical models to study the stability, strategy immunity, and Pareto efficiency of a new school-choice mechanism in equilibriums (e.g., Abdulkadiroglu et al., 2003; Abdulkadiroglu, Pathak, & Roth, 2009; Kamada & Kojima, 2014; Pathak & Sonmez, 2013). It is not until recently that a few researchers have used general equilibrium models and simulation models to explore the impacts of a school-choice mechanism on distribution equality (e.g., Calsamiglia, Martinez-Mora, & Miralles, 2014; Hafalir, Yenmez, & Yildirim, 2013; Hatfield, Kojima & Narita, 2015; Maroulis et al. 2014). However, the existing simulation models in this line either employ historical data as students' choices or randomly generate students' choices by assuming that students truthfully report their school preferences as their school choices without employing strategies (e.g., Abdulkadiroglu et al., 2009; Erdil & Ergin, 2008; Hafalir et al., 2013). Roth and Ockenfels (2002) warned that a small change in mechanism design could induce a significant change in agents' behaviors and interactions. This change, in turn, may result in dynamic changes in macropatterns, such as the distribution of admissions and distribution of school quality. Therefore, it is improper only to consider agents' truth-telling behaviors or assume static system structures. A promising alternative to study the effects of school-choice mechanisms on educational equality is ABM. Agent-based simulation is flexible in assuming agents' behaviors and allows researchers to investigate the dynamic impacts of a new admission policy. In the next section, I discuss the pioneering agent-based models applied to school-choice studies.

Agent-Based Models of Education Systems

ABM is still a new research tool in the field of education. In literature, not many researchers have used ABM to study educational macrophenomena or analyze macroeducational policies. To the best of my knowledge, there are only five agent-based models published in peer-reviewed journals addressing macroeducational phenomena in school-choice systems. In this section, I compare these models to have a better understanding of the current development of ABM in this field.

Maroulis et al. (2014) wrote an agent-based model in NetLogo (Wilensky, 1999) to study the aggregate effect of an open-enrollment policy on student achievement by calibrating the initial setting of their model to reflect the patterns of the student and school distributions in Chicago School District. Maroulis et al. (2014) argued that whether school choice can improve student achievement depends on the portion of parents participating in the school-choice program, higher performing schools' capacities, and the survival rate of new, higher value-added schools. Harland and Heppenstall (2012) hypothesized seven common-sense rules to choose schools and tested them by adding one rule at a time to their model to find a set of rules that best explains the patterns of student allocation in Leeds, UK. In Harland and Heppenstall's (2012) model, the rule of closest distance alone could explain 50% of the student allocation in Leeds; all seven rules combined could explain 60% of the student allocation. Millington, Butler, and Hamnett (2014) used their model, also written in NetLogo, to replicate the geography of educational inequality in London, UK. In this region, popular schools have more percentage of higher-performing students than less popular schools, and the students of popular schools live closer to their schools than the students of less popular schools. Millington et al. (2014) argued that one of the following two conditions must be met to generate such geography: (a) parents must have various aspirations and abilities to move to the neighborhood of higher performing schools, and (b) schools have different value-adds. Chen et al. (2017) focused their NetLogo model on the operational algorithms of serial dictatorship, the Boston mechanism, and the Chinese Parallel mechanism and used the assignment result of serial dictatorship as the baseline to investigate the distributional effects of the above three mechanisms. Wang et al. (2017) extended Chen et al.'s (2017) model to compare Taipei's 2015 student-assignment mechanism with the Chinese Parallel mechanism for their abilities to mix students.

Regarding model structures, the models of Maroulis et al. (2014), Harland and Heppenstall (2012), and Millington et al. (2014) showed the geographic distribution of student agents and considered the distance between a school and a student in forming students' school choices. In addition to spatial differences, the student agents' backgrounds in the above three models were heterogeneous in at least one aspect. Maroulis et al.'s (2014) student agents were heterogeneous in race, gender, socioeconomic level, and school performance. Harland and Heppenstall's (2012) student agents had heterogeneous backgrounds in religious belief, gender, and social status. Millington et al.'s (2014) student agents had different aspirations to attend higher performing schools; if the student agents were affluent and had high aspiration, they would move closer to the schools of their choices one period before the start of the admission process to improve their chances of attending those schools. As to the structure of school agents, all of the above three models had school agents different in value-added and location. Harland and Heppenstall's school agents also differed in types, such as Catholic schools, same-sex schools, and regular public schools.

In comparison to the above three models, the models of Chen et al. (2017) and Wang et al. (2017) were relatively simple; none of them have geographic features. Their school agents were only different in fixed ranks, and their student agents only had the state variables of scores and school preference lists. However, these two models contained the operational algorithms of more than one matching mechanism, while the other three only employed one mechanism. Millington et al. (2014) claimed that the matching mechanism they used was Gale-Shapley's deferred acceptance, but the matching algorithm described in their latest model document published at the website of OpenABM is more like the Boston mechanism. Neither Maroulis et al. (2014) nor Harland and Heppenstall (2012) explicitly stated the matching mechanism used in their models. From their model descriptions, I inferred that the Boston mechanism was used in both models.

Regarding agent's behaviors, Millington et al.'s (2014) model allowed student agents to have heterogeneous aspiration to attend higher performing schools, so the agents weigh school distance and school performance differently. Similarly, in Maroulis et al.'s model, school choice was a function of school performance and school distance. Each student agent weighed these two factors differently. In Harland and Heppenstall's model, only the more affluent student agents considered school performance in making their school choices. The student agents in the above three models were all truth-tellers; that is, students report their school preferences as their school choices without applying any strategies. On the other hand, in addition to the truth-telling strategy, students in the models of Chen et al. (2017) and Wang et al. (2017) used strategies to report their school choices which might be different from their school preferences.

As to student prioritization, Maroulis et al's (2014) school agents randomly prioritized their student applicants. Millington et al.'s (2014) school agents prioritized students based on their distances to schools. Harland and Heppenstall's (2012) school prioritization rule was the most sophisticated among the three; the schools prioritized the student applicants not only based on distance but also based on gender and religion. On the other hand, the schools in the models of Chen et al. (2017) and Wang et al. (2017) only used scores to prioritize students.

I consider both Maroulis et al.'s (2014) and Millington et al.'s (2014) models OLG models because in their simulations, there were several generations of agents coexist during a period and older generations' performances affected younger generations' school-choice decisions. Both models used the grades of the older student generations as the proxy for school performance, which in turn was considered by younger student generations in making their school-choice decisions. Once student agents enter schools, both models calculated their performances based on a fixed formula calibrated to a set of empirical data without considering how student agents' performances would be affected by their peers.

Another agent-based educational model that deserves attention is Salgado et al.'s (2014) hierarchical model, which contains the levels of student agents, student networks within classes, and classes. As described in the section of educational equality, the purpose of their model was to test whether students' social networks could explain peer effect and subgroup achievement differences within classes. Among all models discussed in this section, Salgado et al.'s model is the only model having more than two levels of hierarchy. Although Salgado et al.'s model pre-assigns students to classes and does not contain a matching process, it was a pioneering mesoscopic model that provided middle-level processes to link the micro- and macrophenomena in a complex educational system.

In summary, although the current agent-based models of educational systems are still simple in comparison to agent-based models developed in other fields, such as economics, their simulation results have generated interesting policy implications that may not be easily detected by equation-based models. For example, Maroulis et al.'s (2014) model showed that the treatment effect of a school-choice program might become diminishing with the increase of participants' emphasis on school quality. This information alerts policymakers to the possible disproportional effects of a school-choice program in complex educational systems. Salgado et al. (2014) argued that their agent-based model can provide a reasonable causal explanation of peer effect, while traditional equation-based models, such as its counterpart of a multilevel model, generally lack this ability. With more improved models, ABM is foreseen to further benefit the field of educational policy analysis (Harland & Heppenstall, 2012). In this study, I enhanced the design of the matching mechanisms and agents' behavioral rules in existing agent-based models with an agent-based OLG school-choice model to contribute to the ongoing development of the agent-based models for educational policy analysis.

Conclusion

Founded on the concepts of CASs and computational irreducibility, ABM is a right methodology to study complicated educational systems because it naturally embeds the features of CASs: emergence, adaptation, heterogeneity, bounded rationality, and dynamics (Epstein, 1999; Macal & North, 2010; Maroulis et al., 2014). However, to the best of my knowledge, only five agent-based models have been published to study macroeducational phenomena. In this chapter, I describe and compare these five models in detail because they served as stepping stones to construct the model in this study which contains enhanced real-world matching mechanisms and students' behavioral rules. I also review in detail the processes of the six real-world matching mechanisms included in this model: serial dictatorship, deferred acceptance, the top trading cycles mechanism, the Boston mechanism, the Chinese parallel mechanism, and the Taipei mechanism.

The literature has mixed findings on how students prioritize schools in their preference lists. I adopted Chang's (2011) findings to construct students' preferences because Chang's research was one of the few empirical studies that focused on

Taiwanese students. Since there is no strong evidence in the empirical literature to explain students' school-choice strategies, I referred to Chen et al.'s (2017) hypotheses and my observation in the Taipei School District to construct the behavioral strategies in this study. An advantage of ABM is that it is flexible to do what-if analysis to accommodate data shortage (Montes, 2012). This study has run simulations with different strategic scenarios.

Many researchers agree that student performance is influenced largely by individual and family factors and only modestly by school factors (Burke & Sass, 2013; Dearden et al., 2002; Hanushek, 1989; Jennings et al., 2015). Among the influential school factors, peer effect is the most prominent one (Burke & Sass, 2013; Carman and Zhang, 2012; Coleman et al., 1966). In constructing the module to adjust the scores of high school students, I referred to the above findings and Salgado et al.'s (2014) assumption that peer effect is through the networks formed by students themselves.

The purpose of this model was to answer the research questions of whether the Taipei mechanism and Taipei's free-tuition policy help equalize educational opportunity and school quality. There is no consensus about the definition of educational opportunity or school quality in the literature; it all depends on the purpose and limitation of the study (Ladd & Lauen, 2010). In this study, I chose the definition of educational opportunity from the input point of view and that of school quality from the output point of view. I defined equality of educational opportunity as equality of freshmen's average family incomes across the schools. This definition is in line with the intention of Taiwan's 12-Year Education Reform. I defined equality of school quality as equality of seniors' mean scores across the schools, as suggested by Ferreira and Gignoux (2014) as well as Ladd and Loeb (2013).

In Chapter 3, I discuss the model in detail, including the design concepts, parameters, variables, processing, computational algorithms, and data analysis plan. The discussion also includes the issues of model verification and validation.

Chapter 3: Research Method

Introduction

The purpose of this study was to contribute to the development of agent-based educational modeling for educational policy analysis. I built an agent-based OLG model to qualitatively reflect the environment of Taipei's senior high school system, including its 2016 admission mechanism (the Taipei mechanism) and the free tuition policy. I collected the simulated data to answer the research questions regarding whether Taipei's 2016 admission policies could help equalize educational opportunity and school quality. This chapter includes a description of this model according to the overview, design concepts, and details protocol (the ODD protocol) designed by Grimm et al. (2010) and the rationale for the model design.

Research Design and Rationale

The classical methodology to study school admission mechanisms is set-theoretical models (e.g., Abdulkadiroglu et al., 2003; Abdulkadiroglu, Pathak, & Roth, 2009; Kamada & Kojima, 2014; Pathak & Sonmez, 2013). These models are not proper to investigate the complex effects of admission policies or useful for understanding the relative consequences of policies for different groups of students (Hafalir et al., 2013). Researchers have long called for using ABM to conduct research in this line (e.g., Maroulis et al., 2010). The few existing agent-based school-choice models also have demonstrated that ABM is more flexible to handle heterogeneous student agents (Chen et al., 2017; Harland & Heppenstall, 2012; Maroulis et al., 2014; Millington et al., 2014;

Wang et al., 2017). Therefore, I chose ABM to study admission policies in an environment qualitatively representing the Taipei School District.

Most of the few agent-based school-choice models assumed that students were truth tellers, who submit their school preferences as their school choices. The literature has shown that students use strategies corresponding to the matching mechanism to make their school choices (Chen et al., 2014; Pathak, 2011). To assume truth-telling is ineffective for admission policy analysis. Although Chen et al. (2017) and Wang et al. (2017) have started the attempt to distinguish students' preference from choices and model students' strategic behaviors, the factor considered in their designs was too simple to reflect the real-world observations. Therefore, I enhanced their design by adding the factors of distance and family income, which complied with the findings in the literature and my observation in the Taipei School District.

An admission mechanism is essential to school choice systems; it determines how students are assigned to school. Most existing agent-based school-choice models only used the Boston mechanism to assign students to schools (e.g., Harland & Heppenstall, 2012; Maroulis et al., 2014; Millington et al., 2014). Although the Boston mechanism might still be the dominant mechanism to assign students to schools in the United States and some other countries, it has been replaced with deferred acceptance or other mixed mechanisms in many admission reforms globally. Interestingly, abiding by Taiwan's 12-Year Education Reform, the Taipei School District has brought back the spirit of the Boston mechanism by mixing it with the original deferred acceptance to form its admission mechanism since 2014. It seems that none of the mechanisms mentioned above would be rid of easily. Therefore, an agent-based model for educational admission systems should be able to process not only the matching mechanism of interest but also the commonly used matching mechanisms to provide an in-depth understanding of the complicated effects of a matching mechanism in comparison with those of the other real-world mechanisms. Wang et al. (2017) have programmed the operational algorithms of serial dictatorship, the Boston mechanism, deferred acceptance, the Chinese Parallel, and Taipei's 2015 admission mechanism in their NetLogo model. I adopted their program codes to process the former four mechanisms and wrote the code for Taipei's 2016 admission mechanism (the Taipei mechanism) because the Taipei School District overhauled the school prioritization rules in its 2015 admission mechanism in 2016. With all the above improvements, this agent-based simulation model could process five real-world student-assignment mechanisms in each simulation run and thus allowed a better analysis of the effects of Taipei's 2016 admission policies (the Taipei mechanism and the free tuition policy) on educational equalities.

I made this model an OLG model because in the Taipei School District, students' school preferences and school ranking are highly correlated, and school ranking are largely determined by seniors' performance (Shao, 2015; Stanley1986, 2014). Since older generations' behaviors will affect the overlapping younger generations' decision making, OLG design is necessary.

One of the advantages of ABM is its flexibility to run scenario analysis. This feature is important in policymaking because how students may react to a new educational policy is usually unknown or uncertain. I simulated 300 scenarios with different combinations of students' strategic behaviors, the number of choices, and type of mechanism with and without the free tuition policies. The simulated results provide policymakers and stakeholders with a range of plausible policy outcomes and thus facilitate informative admission policy decisions.

The independent variables in this study were the parameters that defined students' behavioral rules and admission policies. The control variables were the parameters that defined the attributes of the entities in this model. I describe these parameters in the section of model description below. The dependent variables were the outcomes generated from simulations. Agent-based simulations generate rich microscopic, mesoscopic, and macroscopic outcomes. It can trace the activity and status of each individual agent and record the evolution of meso- and macrophenomena of interest at every time step. The outcomes that I was interested were the distribution of the mean freshman family income and the distribution of the mean senior score, which are the proxies for the distribution of educational opportunities and distribution of school quality, respectively. I used descriptive statistics and paired samples *t*-tests to compare the simulated outcomes of all mechanisms and answered the research questions of whether Taipei's new admission policies help equalize educational opportunity and school quality.

Model Description

Under ABM, a system is modeled as a collection of autonomous agents that interact with each other and with the computational world within which the agents reside (Borrill & Tesfatsion, 2011). Researchers then set the parameters to determine the system's initial conditions and let the model run to collect and analyze the micro-, meso-and macrodata generated from the simulations. Grimm et al. (2006) suggested the ODD protocol to clearly and rigorously document a model's purpose and design so that a third party can replicate it. After reviewing the uses of the ODD protocol, Grimm et al. (2010) modified some terms used in the original ODD protocol to better fulfill its purpose and for the users to better understand and adopt the protocol. The ODD protocol is also advocated by the Network for Computational Modeling for SocioEcological Science (CoMSES Net), which maintains the website of OpenABM to promote the methodology of ABM and to serve as a node for agent-based modelers to share models and exchange ideas. Therefore, I adopted the 2010 ODD protocol to describe the model in this study.

Under the 2010 ODD protocol, a model should be described with three consecutive components: overview, design concepts, and details. The context and the general structure of a model should first be described (overview), followed by the general concepts supporting the model design (design concepts) and then the technical details (details). The overview section is further divided into three parts: (a) purpose; (b) entities, state variables (variables that characterize an entity), and scales; and (c) process overview and scheduling. The section of design concepts includes the following 11 elements: (a) basic principles underlying the model's design, including concepts, theories, and hypotheses; (b) emergence of phenomena from individual agents' behaviors; (c) adaptation, (d) objectives of the act of adaptation; (e) learning that can change the adaptive traits of individual agents; (f) prediction involved in agents' decision making; (g) sensing the information in the environment; (h) interaction among the agents; (i) stochasticity applied in the processes; (j) collectives formed by individual agents; and

(k) observations collected from model simulations. However, any of these elements not included in a model can be omitted. The last section, details, includes the three subsections: (a) initialization of each simulation, (b) input data, and (c) submodels for the processes (Grimm et al., 2010).

In the following sections, I describe the agent-based OLG model of this study in the format guided by the 2010 ODD protocol.

Overview

Purpose. The purpose of this model is to understand qualitatively how Taipei's new high school admission policies affect the distribution of educational opportunity and school quality under different behavioral scenarios, in comparison with other matching mechanisms including serial dictatorship, deferred acceptance, the Boston mechanism, and the Chinese parallel mechanism. This model is an OLG model because in each simulation run, there will be three generations of student agents in the same time and the older generation's performance will affect the school choice decisions of the younger generation. This model is an example of how ABM can facilitate the micro-,meso-, and macroanalysis of educational policies, especially in a centralized school admission system.

Entities, State Variables, and Scales. This model contains three types of entities: schools, student agents, and the governmental authority. The schools refer to 3-year senior high schools covering grades 10-12. Each school agent has the following attributes (state variables): location, admission capacity, type (private or public), the lowest score of the admitted freshmen, and the lowest rank of the admitted freshmen.

The government works as a clearinghouse and determines the high school admission and tuition policies, including the school-student matching mechanism, the level of information to be released, the number of school choices each student can make, and the tuition policy. These educational policies are the exogenous factors of the model. They are determined before the start of each run and remain constant during the run. Student agents are characterized by the following state variables: scores, location, school preference, school choice, and family income. There will be three student cohorts in each simulation year: the cohort of candidates for high school freshmen, the cohort of high school sophomores, and the cohort of high school seniors.

Space is implicitly included in the model. There are two neighborhoods in the model (Neighborhood 1 and Neighborhood 2) to represent the two neighborhoods in the Taipei School District. Neighborhood 1 represents Taipei City and has higher average household income, higher average student performance, and better high schools than Neighborhood 2, which represents Suburban Taipei. In the model, there are a total of 10 schools which are distributed in a way that resembles qualitatively the distribution of school ranks in the Taipei School District in 2015 (Sleepcat615, 2015). The proportions of the total school capacity in these two neighborhoods are similar to those in Taipei City and Suburban Taipei in 2015 (sleepcat615, 2015). Table 4 describes the distribution of the 10 schools. The identification of each school represents its initial rank. For example, School #1 is initially ranked the highest; School #10, the lowest. The distribution of the schools in this model reflects the fact that on average, the schools in Taipei City are ranked higher than the schools in Suburban Taipei. School #3 represents a fee-paying private school which lower-income students will not choose if there is no free-tuition policy. The locations of the student agents also represent their admission statuses. The admission candidates stay outside of the schools (see Figure 2 for illustration.) Once they are admitted, they move into the locations of the schools that they are admitted to. Those who are rejected by all schools leave the system.

Table 4

School Distribution

	Neighborhood 1	Neighborhood 2
School ID	School #1	School #4
	School #2	School #6
	School #3	School #7
	School #5	School #9
	School #8	School #10
Total Capacity percentage	50%	50%

Note. The spatial distribution of the 10 schools in the model qualitatively represents the distribution of school ranks in the two neighborhoods of the Taipei School District (Taipei City and Suburban Taipei). School #3 represents a fee-paying private academic school. Each school's ID represents its initial rank in the system.

Each simulation run proceeds in discrete time steps for 33 steps. Since most high school districts process admissions once a year, each time step represents 1 school year.

Process Overview and Scheduling. At the beginning of each year, freshmen become sophomores, sophomores become seniors, and seniors graduate and leave the system. Then, a new set of admission candidates (new student agents) enters into the system. While some of the new candidates stay in Neighborhood 1 and some stay in Neighborhood 2, all stay outside of the 10 schools. Admission candidates receive raw scores and family incomes from a multivariate normal distribution, form their school preference lists, and use strategies to make their school choices. Each school prioritizes the students based on the rules set by the admission policy. The governmental authority (clearinghouse) matches the schools and the students according to the matching mechanism determined by the admission policy. The students who are admitted move to their high schools and stay in the same schools until graduation. Those who are not admitted leave the system. After the admission process, the system updates the scores of the new sophomores and seniors, records the lowest scores and the lowest ranks of the freshmen each school admits, and calculates the standard deviations of the distribution of the mean freshman family incomes and the distribution of the mean senior scores across the schools. These 2 standard distributions are the measures of the inequalities of educational opportunity and school quality, respectively, in the computer environment. Figure 4 illustrates the process flow of the model.

Design Concepts

Basic Principles. The design of student agents' behaviors was based on the observations made in the Taipei School District and the empirical findings in the literature. The surveys show that Taiwan students prefer higher quality schools to lower quality schools and use school rank as the index for school quality (Chang, 2011; Lu, 2012; Shao, 2015; Yan, 2015). However, when schools' qualities are too low, quality becomes relatively irrelevant, and students prefer the nearby low-quality schools to the far-away low-quality schools (Chang, 2011). That is, Taiwan students have highly correlated preferences for higher-ranked schools and prefer nearby lower-ranked schools to far-away lower-ranked schools.

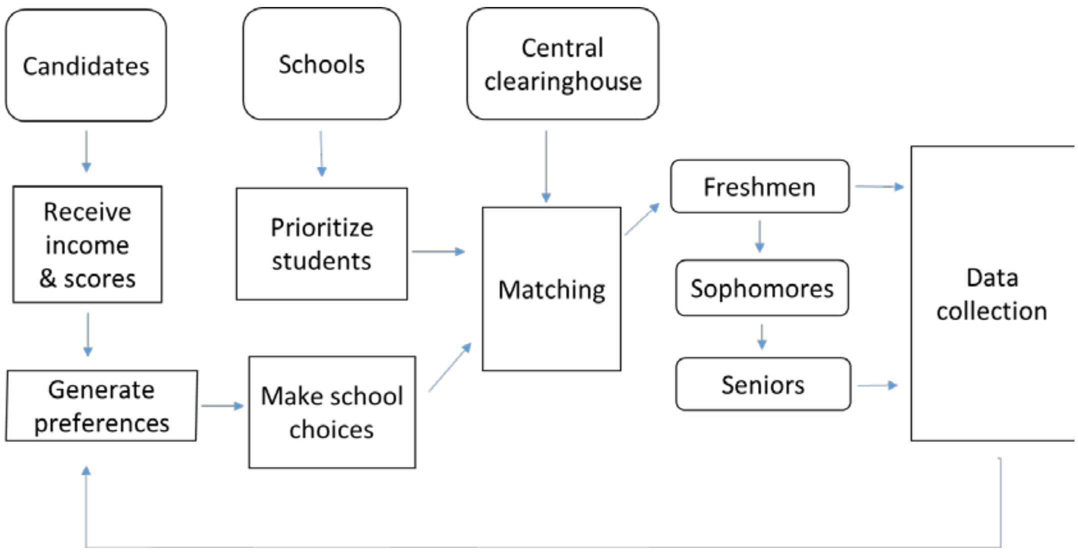

Figure 4. Process overview of the model. Schools prioritize students according to the admission policy. Candidates submit their school-choice lists based on their preferences and strategies used. The clearinghouse matches students and schools according to the matching mechanism determined by the policy. After the matching process, freshman cohorts emerge as well as the admission distribution. Next year, the freshman cohorts will become the sophomore cohorts, and the sophomore cohorts will become the senior cohorts. At the end of each school year, the model updates school information and collect data needed for analysis.

Students do not always report their school preferences as their school choices. If how students rank their schools affect their admission results under a mechanism, students will respond to that mechanism and use strategies to form their school-choice lists in a way that they think will maximize their chances to attend their most preferred schools possible (Chen et al., 2015; Pathak, 2011). A strategy commonly taught to Taipei students states as follows: (a) Student should first review each school and its features to form a preference list; (b) student should collect each school's past admission information, such as the lowest accepted rank or score, in comparison to student's own rank or score, to predict the chance of being admitted to each school and form a list of possible schools; (c) students should understand the admission mechanism and choose schools mainly from the list of possible schools; (d) student should form the school-choice list in a way that student is confident to be admitted to a school while gambling for the admission to a more preferred school (Sun, 2015; Zhang & Wang, 2015). A similar strategy is also widely disseminated in mainland China (e.g., H. Wang, 2015; R. Wang, 2015; Song, 2015). There are three strategies in this model. Strategy #2 and Strategy #3 reflect the above commonly advised strategy (see the subsection of details below). In this information age, it is very likely that students all learn about and use the same strategy to form their school choices. Therefore, when Strategy #2 or Strategy #3 is simulated, all student agents use the same strategy to make their school choices. The design of Strategy #1 was based on the assumption that student agents are truth tellers and choose schools according to their own considerations (see Strategy #1 in the subsection of details below).

Without tuition subsidy, it is unlikely for lower-income students to choose expensive private schools (Chen, 2007). Therefore, student agents in the bottom 50% income group will not choose School #3, the fee-paying private school, if there is no free-tuition policy. All student agents are boundedly rational. Like students in the Taipei School District, student agents in the model have limited information about their competitors' scores, ranks, and school-choice decisions.

At the beginning of each simulation run, the ranks of the schools are in the following order:

School #1 ≥ School #2 ≥ …. ≥ School #10,

where the symbol "≥" means "superior to." The distribution of the school ranks in this model qualitatively represents the distribution of school ranks in the Taipei School District in 2015, where most schools in Taipei City were ranked higher than those in Suburban Taipei. In Taiwan, high schools are ranked mainly based on the percentage of their graduates who attend elite universities (Shao, 2015; Stanley1986, 2014). The higher the seniors' scores, the better chances they have to attend the elite universities. Generally speaking, if a school's average senior score is higher, the percentage of its graduates who attend elite universities will be higher. Therefore, the ranks of the schools in this model will be adjusted according to their seniors' average scores in the simulation process.

Income in Taiwan, like those in many other countries, follows a lognormal distribution (Pinkovskiy & Sala-i-Martin, 2009). As in other Asian countries, Taiwan students' performances are positively correlated with their incomes (Hojo & Oshio, 2012). Based on the central limit theorem, it is reasonable to assume that student agents' scores are normally distributed. It is further assumed that a high-performing student, in average, performs well in all subjects and vice versa. Under all the above assumptions, candidate agents' log family incomes and their subject test scores form a multivariate normal distribution in this model.

After the admission (matching) process, student agents stay in their assigned schools for the entire high school years (Grades 10-12). These high school student agents' scores are calculated based on the following two assumptions: (a) The individual and family factors are constant throughout the agents' existence in the model, and (b) the influence of schooling is mainly through peer effect as evidenced in many studies (e.g., Coleman et al., 1966; Burke & Sass, 2013; Hojo & Oshio, 2012; Jennings et al., 2015). The design of peer effect is based on Salgado et al.'s (2014) hypotheses that peer effect is through the formation of network and that the level of tolerance towards the difference in family background plays an important role in the network formation. Only those who can join the peers will be influenced by the peers.

Emergence. After the matching process, a new freshman cohort emerges in each school, and a new admission distribution emerges.

Adaptation, Objectives, Learning, and Prediction. Candidates for admission pursue the objective of attending their most preferred schools possible. They use the information released to them to predict which schools they are confident to be admitted to (their possible schools) and adapt strategies to form the best school-choice lists possible. Since each candidate for admission will be in the matching process only once, this activity is considered a one-shot game. Thus, no learning behavior is programmed in the candidate agents' decision process. However, learning is implied in the peer effect assumed in the calculation of the scores of the high school student agents.

Sensing. All admission candidates know each school's quality, the lowest score and rank of the students admitted to each school, and, as long as the admission policy allows, their own scores and ranks without any information seeking process. Once candidate agents form their school-choice lists, the governmental authority receives them without the need to go through any transmission process either.

Interaction. In the real world, admission candidates talk to their peers and advisers about how the tests went and how to form school-choice strategies. Students also talk to high school students they know to seek advice. Even so, admission candidates can hardly have the same school-choice lists because their scores are hardly the same. However, in this information age, it is very likely for all of them to use the same decision strategy as described in the previous paragraph of basic concepts. Therefore, when Strategy #2 or #3 is employed, all candidate agents will use the same strategy to make their school choices (see the submodel of student's school choice list below.)

Stochasticity. In the Taipei School District, 36% of the admission candidates reside in Taipei City; 64%, Suburban Taipei (Taipei City Government Department of Education, 2016). In the model, the admission candidates created each year are placed randomly in Neighborhoods 1 and 2 according to the above ratio. Each admission candidate's log family income and subject test scores are randomly generated from a multivariate normal distribution. The estimated parameters to generate incomes and scores are described in the subsection of details below and listed in Table 5. Candidate agent's preference list is generated from a Zipf distribution, Zipf (α, n), in conjunction with the distance consideration for low-quality schools. The parameter α depicts how correlated the candidate agents' preferences are and n is the number of the remaining schools to be ranked (see the subsection of details below).

Collectives. Each year, after the matching process, candidate agents move to their individual schools, and the freshman cohort of each school emerges. The old freshman cohorts formed last year become the sophomore cohorts; the sophomore cohorts become the senior cohorts; the senior cohorts graduate and leave the system.

Observation. For model testing, individual candidate agents' admission results and individual senior agent's scores were recorded and observed. For model analysis, only the macro- and mesodata were recorded and observed, e.g., the freshman cohorts' average incomes, the senior cohorts' average scores, the top 10-percent candidates' admission status, and the admission status of the students in the bottom income quartile.

Details

The model, written in NetLogo 5.3 (Wilensky, 1999), can be downloaded from the website of OpenABM. The direct link is https://www.openabm.org/model/5521/version/1/view. I also attach the code in Appendix A.

Initializations. At each initial state, the model created 10 schools, 5 in Neighborhood 1 and 5 in Neighborhood 2 (see Table 4). These 10 schools were located in the same places throughout the simulation runs, and all schools were of the same admission capacity. I set each school to provide 100 admission seats; therefore, there were a total of 1,000 admission seats for the candidates each year. This level of capacity represents a 100% admission opportunity rate, as in the Taipei School District, because exactly 1,000 admission candidates were created each simulation year in all simulation runs. At each initial state, the schools were ranked in the following order: School #1 ≥ School #2 ≥ … ≥ School #10. The lowest score accepted by each school was set to be 0. The lowest ranks accepted by the schools were all set to be the lowest rank of all students, which was 1,000 in the simulations.

Similar to the residence distribution of the admission candidates in the Taipei School District, 36% of the candidate agents live in Neighborhood 1; 64%, Neighborhood 2.

The mean and standard deviation of the family income in Neighborhood 1 are TWD1,575,000 (around USD47,700) and TWD873,000 (around USD26,400), respectively, approximating to the mean and standard deviation of the 2014 household income in Taipei City (Taipei City Government Department of Budget, Accounting and Statistics, 2015). The mean and standard deviation of the family income in Neighborhood 2 are TWD1,147,000 (around USD34,700) and TWD574,000 (around USD17,400), respectively, approximating to those of the 2014 household income in Suburban Taipei (New Taipei City Government, 2015a). These values are constant throughout all simulations.

The mean and standard deviation of all students' entrance exam scores in the Taipei School District were roughly calculated to be 54 and 23, respectively (RCPET, 2015). Based on this information, I set the mean scores of the admission candidate agents in Neighborhood 1 and Neighborhood 2 to be 65.00 and 47.50, respectively, to qualitatively represent the positive correlation between income and score. I set the standard deviations of the scores in both neighborhoods to be 23, which was the same as that in the whole Taipei School District. Without empirical data to support the above information, the above score figures were arbitrarily assumed. In Taiwan, the entrance exams have been systematically designed to properly reflect students' performances. Therefore, the score means and standard deviations were constant throughout all simulations.

Each candidate agent had five subject test scores. The log family income and student's five subject scores formed a 6-dimensional multivariate normal distribution. I set the correlation coefficient between any two standardized variables in this multivariate normal distribution, *Rho*, to be .80, which reflected qualitatively a moderate to high correlation between score and family income as empirically found in the literature (e.g., Hojo & Oshio, 2012; Tsai & Yang, 2015).

I set the alpha in the Zipf distribution used to generate admission candidates' school preference lists to be 3. This value helped the model to produce moderately to highly correlated preferences for higher-quality schools among the candidates as observed in the Taipei School District. The parameter *Tolerance* determines which high school student will be affected by the peers, and the parameter *PeerEffect* determines the magnitude of the peer effect. I set the value of the parameter Tolerance to be .58, which means that only those high school student agents whose family incomes are within (1 $\pm$ 58%) of the average family income of their peers would be affected by their peers. I adopted this value from the estimated average tolerance level calculated by Salgado et al.

(2014). I also assumed the value of PeerEffect to be .58, denoting that for each point of the peers' mean score higher (lower) than the student's score, the student's score would be increased (decreased) by .58. This value was a direct adoption of the regression coefficient on peer's mean score calculated by Hojo and Oshio (2012) to predict Taiwan students' math score. Without empirical evidence in the literature, the values of Tolerance and PeerEffect were arbitrary, just to reflect the quality of peer effect. Table 5 summarizes the parameters and their values at the initial state. These values were constant in the simulations.

At the initial state of each simulation, the admission policies, given exogenously, determine the matching mechanism, the number of school choices allowed, the level of information to be released, and the tuition policy of that simulation. The parameter *sort-extra-choice* is an additional design to reflect candidate agents' risk aversion attitude. When *sort-extra-choice* = yes, candidate agents have a lower risk-aversion level; when *sort-extra-choice* = no, candidate agents have a higher risk aversion level. Table 6 shows the 300 combinations of the values of *sort-extra-choice,* the types of strategy, the mechanisms, and the values of the parameters related to the admission and tuition policies. I simulated each combination 30 times to make statistical analysis feasible.

Input Data. This is an exploratory model to simulate the impact of admission and tuition policies. No input or data file from external sources were used, except that I referred to the data from Taipei's Comprehensive Assessment Program for Junior High Students in 2015 and the income statistics for the Taipei area to set the values of the means and standard deviations of candidate agents' scores and family incomes (see the previous paragraph of initializations).

Table 5

Parameters at Initialization

Parameter	Description	Value
Students	Number of students	1,000
Capacity	School capacity	100
Score-Mean-1	Mean test score of candidates in Neighborhood 1	65.00
Score-Mean-2	Mean test score of candidates in Neighborhood 2	47.50
Score-SD	Standard deviation of candidates' test scores	23.00
Income-mean-1	Average household income in Neighborhood 1 (representing Taipei City)	$1,575,000.00
Income-mean-2	Average household income in Neighborhood 2 (representing Suburban Taipei)	$1,147,000.00
Income-SD-1	Standard deviation of household income in Neighborhood 1	$873,000.00
Income-SD-2	Standard deviation of household income in Neighborhood 2	$574,000.00
Rho	Correlation coefficient between scores and log family income and among scores	.80
Alpha	α of the Zipf distribution used to generate school preference list	3.00
Tolerance	Family income difference tolerance rate	.58
PeerEffect	Coefficient of peer effect	.58

Table 6

Combinations of Policy Parameters and risk aversion level in Simulations

Parameters	Serial dictatorship/deferred acceptance	Boston mechanism	Chinese parallel mechanism	Taipei mechanism
Free tuition?	Yes, No	Yes, No	Yes, No	Yes, No
Number of Choices	2, 4, 6, 8, 10	2, 4, 6, 8, 10	(1, 1), (2,2), (3, 3), (4, 4), (5,5)	2, 4, 6, 8, 10
Strategy used?	#1, #2, #3	#1, #2, #3	#1, #2, #3	#1, #2, #3
Sort-extra-choice?	Yes, No	Yes, No	Yes, No	Yes, No

Submodels. There are a total of 6 submodels in the program as described in the following paragraphs.

Candidate's log family income and scores. Each new candidate agent's log family income and five subject test scores are randomly selected from a 6-dimensional multivariate normal distribution. To simplify the process, I use Derde and Massart's (1984) technique of sampling from the standardized multivariate normal distribution and then converting the sample points to their original units. In the standardized multivariate normal distribution, its variable z_i has a mean of 0 and a standard deviation of 1. Its variance-covariance matrix coincides with its correlation matrix Γ,

$$\Gamma = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & \rho_{14} & \rho_{15} & \rho_{16} \\ \rho_{21} & 1 & \rho_{23} & \rho_{24} & \rho_{25} & \rho_{26} \\ \rho_{31} & \rho_{32} & 1 & \rho_{34} & \rho_{35} & \rho_{36} \\ \rho_{41} & \rho_{42} & \rho_{43} & 1 & \rho_{45} & \rho_{46} \\ \rho_{51} & \rho_{52} & \rho_{53} & \rho_{54} & 1 & \rho_{56} \\ \rho_{61} & \rho_{62} & \rho_{63} & \rho_{64} & \rho_{65} & 1 \end{bmatrix}$$

where ρ_{ij} is the correlation coefficient between z_i and z_j. In this model, I set ρ_{ij} to be .8. z_1 is the variable for the standardized log family income; z_2 to z_6 are the variables for the standardized subject test scores. The unstandardized means and standard deviations of the family income and the five subject test scores are listed in Table 5. Note that the means and standard deviations of family incomes and scores in Neighborhood 1 are different from those in Neighborhood 2. The mean, μ, and variance, σ^2, of log family income can be calculated as below:

$$\mu = \ln\left(\frac{m^2}{\sqrt{v+m^2}}\right);$$

$$\sigma = \sqrt{\ln\left(1+\frac{v}{m^2}\right)}$$

where v and m are the variance and mean of family income, respectively. Therefore, the μ in Neighborhood 1 is 14.14; the μ in Neighborhood 2 is 13.84. The σ in Neighborhood 1 is .52; the σ in Neighborhood 2 is .47.

Once the six values, represented by the vector z_k, are randomly sampled from the standardized multivariate normal distribution, they are converted to their original units, represented by the vector x_k, by using the following formula:

$$x_{ik} = \sigma_i z_{ik} + \mu_i.$$

Note that x_{1k} is student's log family income. It then is converted to student's family income, $y = \exp(x_{1k})$.

Under the Taipei mechanism, each subject test score is coarse-grained into a 7-scale score (Table 1), and schools must use students' 7-scale scores to prioritize students.

Under all other mechanisms, students' five subject test scores are summed up, and schools use the summed scores to prioritize students.

Candidate's school preference list. Each year, the ten schools are ranked according to their quality represented by the average scores of their senior students, and this ranking is assumed to be publicly recognized. Student agents have similar preferences for higher-ranked schools. Each student's preference for the top seven schools is generated in sequential order from a Zipf distribution, i.e., $x \sim \text{Zipf}(\alpha, n)$, where n = the number of the remaining schools to be selected and x is the rank of the remaining schools ($x = 1,...,n$). In this study, I set α to be 3. To set $\alpha = 3$ is to depict the moderate to high correlation between candidates' school preferences. Therefore, Zipf(3,7) (Figure 5) is used to determine the most preferred school of a student agent, Zipf(3, 6) is used to determine the second preferred school, and so on.

To determine the preference for the bottom three schools, student agents first group them into neighborhood schools and non-neighborhood schools, and the program will prioritize the schools in the same group by using the Zipf distribution. The following is an example of how a student's preference list is determined.

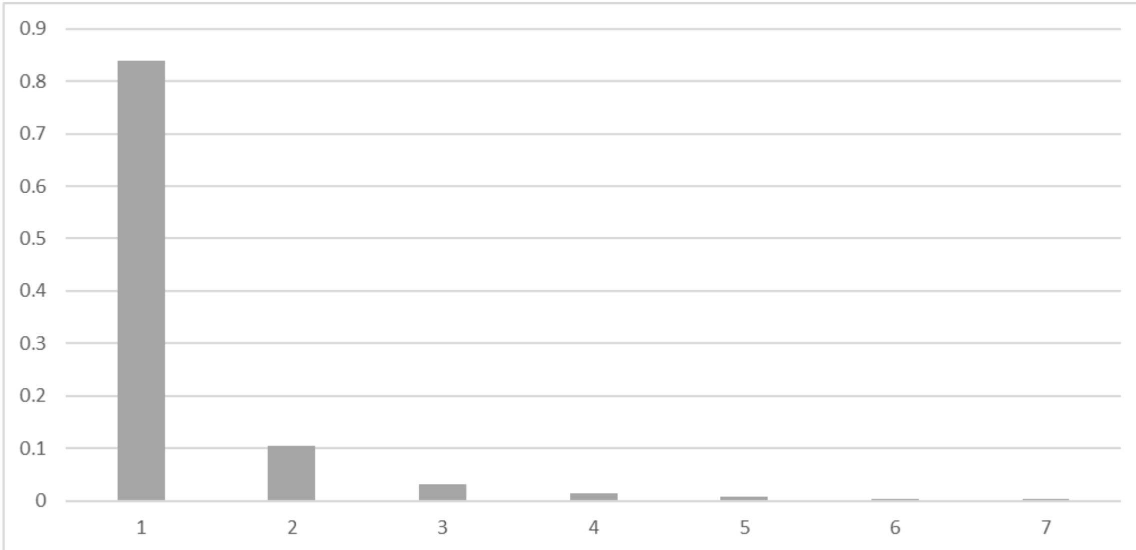

Figure 5. The probability mass function for Zipf (3,7). The top-ranked school has an 83.81% probability to be selected as a student's most preferred school under Zipf(3,7).

Example 7. Assuming that the schools are ranked as in the initial setting; that is, School #1 ≥ School #2 ≥ … ≥ School #10. Candidate *i* lives in Neighborhood 2. After randomly selected from the Zipf distribution, Candidate *i*'s preference for the top seven schools is:

$P_{it} = \{2, 1, 3, 4, 5, 6, 7\}$

in which School #2 is the candidate's most preferred school. For the bottom three schools, Candidate *i* first ranks the schools in Neighborhood 2 (Schools #9 and #10). The program randomly ranks these two schools from the Zipf distribution, Zipf (3, 2), and the result is: $P_{ic} = \{10, 9\}$. Now, the only school to be ranked is School #8. Apparently, School # 8 is Candidate *i*'s least preferred school. As a result, Candidate *i*'s complete preference list is as follows:

$P_i = \{2, 1, 3, 4, 5, 6, 7, 10, 9, 8\}$.

I use the above preference list in later paragraphs to explain the simulation process.

If there is no free-tuition policy and Candidate *i* is from a bottom 50% family, Candidate *i* will not consider School #3 at all. In this case, Candidate *i*'s preference list is adjusted to be the following:

$P_{i,-3}$ = {2, 1, 4, 5, 6, 7, 10, 9, 8}.

Candidate's school choice list. Candidates' school-choice decisions depend on the strategies applied. There are three heuristic strategies for the candidate agents to adopt. Strategies #2 and #3 are the mimics of the commonly advised strategy described in the previous subsection of design concepts. Strategy #1 is a truth-telling strategy as commonly assumed in the literature on school choice and is used for comparison purpose. I simulated each scenario listed in Table 5 with all of the three strategies to explore how student agents' behaviors would change the outcomes of an admission policy.

Under Strategy #1, candidate agents are truth tellers, who arrange their school choices in the same order as their school preferences. Therefore, when students are allowed to choose 10 schools, their school-choice lists are the same as their preference lists. When the number of choices allowed is less than 10, students each form a strategy that fits his or her risk aversion level and other considerations. The application of the principle of maximum entropy reflects candidate agents' heterogeneous considerations. That is, under Strategy #1, students randomly select the schools from their preference lists up to the number of choices allowed. Therefore, Strategy #1 is not a single strategy but a combination of many strategies. The selected schools will then be sorted according to their orders in candidates' preference lists because candidate agents are truth tellers.

For example, if the number of choices allowed is 4, a possible school-choice list of Student *i* in Example 7 is C*i* = {2, 5, 7, 10}.

Under Strategy #2, candidate agents consider their own test scores and all information released by the government to make their choice lists. They first compare their total raw scores, $q_{i,t}$, with the lowest total raw scores of the students admitted to each school last year, $r_{j,t-1}$, to form two groups of schools: the possible group, PG, and the impossible group, IG. PG contains all schools whose $r_{j,t-1}$ are lower than the student's $q_{i,t}$, while the $r_{j,t-1}$ of the schools in IG are at least the same or higher than the student's $q_{i,t}$.

$$PG = \{ j : q_{i,t} > r_{j,t-1} \}, \quad IG = \{ j : q_{i,t} \leq r_{j,t-1} \}$$

Refer to Example 7. Candidate *i*'s PG and IG may be as follows: PG = {7, 10, 9, 8}; IG = {2, 1, 3, 4, 5, 6}. If Candidate *i* is from a bottom 50% family and there is no tuition subsidy, then Candidate *i*'s PG and IG are as follows: PG = {7, 10, 9, 8}; IG = {2, 1, 4, 5, 6}.

Then the candidate agent begins the school-choice process by first selecting schools from PG up to the number of choices allowed. If the school-choice list is not full, the student agent will strategically select the schools from IG until the list is full. Again, I apply the principle of maximum entropy to reflect agents' heterogeneous considerations to form the strategies to select the schools from IG; that is, the schools in IG will be randomly selected to fill the school choice list. If *sort-extra-choice* = yes, candidates have a low risk-aversion level and are willing to take the risk of wasting their top choices in order to gamble for the chance of attending their more preferred schools; that is, candidate agents will sort all of the selected schools according to their preference lists. If *sort-extra-choice* = no, candidate agents have a higher risk-aversion level and will simply put those impossible schools selected at the end of their school-choice lists. Refer to Example 7. Using Strategy #2, Candidate *i* may have the school-choice lists under the influence of the parameter *sort-extra-choice* as shown in Table 7.

Table 7

Effect of sort-extra-choice on Candidate i's School Choice List

	Number of choices allowed	Serial dictatorship/deferred acceptance/Boston mechanism/Taipei mechanism	Chinese parallel mechanism
Sort-extra-choice = yes	4	{7, 10, 9, 8}	{7, 10}, {9, 8}
	10	{2, 1, 3, 4, 5, 6, 7, 10, 9, 8}	{3, 7, 10, 9, 8}, {2, 1, 4, 5, 6}
Sort-extra-choice = no	4	{7, 10, 9, 8}	{7, 10}, {9, 8}
	10	{7, 10, 9, 8, 2, 1, 3, 4, 5, 6}	{7, 10, 9, 8, 3}, {2, 1, 4, 5, 6}

Note. Refer to Example 7 and assume Candidate *i*'s PG = {7, 10, 9, 8} and IG = {2, 1, 3, 4, 5, 6}. When there are only 4 choices allowed, Candidate *i* selects schools only from PG. When the number of choices is 10, Candidate *i* selects schools first from PG and then from IG. If *sort-extra-choice* is yes, Candidate *i* sorts all selected schools according to his preference list. If *sort-extra-choice* is no, Candidate *i* simply puts the schools selected from IG behind the schools selected from PG.

Strategy #3 is the same as Strategy #2, except that the information of score rank, instead of score, is used. That is, candidates compare their own score ranks with the lowest score rank of the students accepted by each school last year to determine their PG and IGs. From the example in Table 7, one can see that when *sort-extra-order* = True and the number of choices = 10, the choice lists generated by Strategy #2 and Strategy #3 under SD, DA, TM, and BM are the same as students' preference lists. When a student's choice list is the same as his or her preference list, the strategy used by the student is called the truth-telling strategy.

Matching. After receiving all candidates' school-choice lists, the clearinghouse matches the schools and the candidates according to the matching mechanism regulated by the policy. The model processes the following five mechanisms for each simulation run: serial dictatorship, deferred acceptance, the Boston mechanism, the Chinese parallel mechanism, and the Taipei mechanism. To increase the computational speed, I have simplified the process of the Taipei mechanism without jeopardizing its quality in the following aspects: (a) Instead of grouping every 5 choices, this model groups every 2 choices in computing a student's choice score, i.e., 35 points for the 1st and 2nd choices, 34 points for the 3rd and 4th choices, 33 points for the 5th and 6th choices, and so on; (b) since almost all students can receive the full diversity learning score, this category is not functional in distinguishing students' performance and thus omitted in the computation; and (c) since the maximum score for Chinese composition is only 1, this score is relatively unimportant and is also omitted in the process. Therefore, the composite score of the Taipei mechanism in this model is composed of the choice score (a maximum of 35 points) and the five subject test scores (a maximum of 35 points total). In the real-world, the maximum choice score under the Taipei mechanism is 36. It is changed to 35 in this model to maintain the same proportion of these two scores in the composite score.

After the matching process, candidates who are admitted move to their individual schools, and each school updates the record of the lowest score and the lowest rank of its admitted freshman students. The candidate agents who are not admitted to any school leave the system.

High school student's score. The scores of the high school students are their original scores adjusted by the peer effect. A student's total raw test scores will be adjusted by the peer effect only when the student's family income is within the tolerance range of the peer.

$$q_{ij,t+1} = \begin{cases} q_{ij,t} + \left(SU_{j,t} - q_{ij,t}\right)P, & \text{if } (1-T)FU_{j,t} < f_{ij,t} < (1+T)FU_{j,t} \\ q_{ij,t}, & \text{otherwise,} \end{cases}$$

where $q_{i,t}$ is the score of Student i in School j in year t, $q_{ij,t+1}$ is the student's score in year t+1, $SU_{j,\,t}$ is the mean score of the student's cohort in School j in year t, P = PeerEffect, T = Tolerance, $FU_{j,\,t}$ is the mean family income of the student's cohort in School j in year t, and $f_{ij,\,t}$ is the student's family income in year t. In this model, both PeerEffect and Tolerance are .58. The score adjustment is made at the end of each high school year for freshmen, sophomores, and seniors. After the senior students' scores are updated, the mean senior score in each school is calculated, and the schools are re-ranked according to the mean scores of their senior students.

Data collection. At the end of each matching process, the program automatically collects the following macro- and mesolevel data: (a) the mean freshman family income in each school, (b) the standard deviation of the mean freshman family income, (c) the mean senior score in each school, (d) the standard deviation of the mean senior score, (e) the percentage of the top 10-percent performing candidates admitted to their most preferred schools, (f) the preference index for students in each income quartile and students with the bottom 10-percent family income, (g) the overall preference index, and (h) match rate.

I used the distribution of the mean freshman family income as the proxy for the distribution of educational opportunity. The less the standard deviation of this distribution, the less the inequality of educational opportunity. Using standard deviation to measure inequality has been supported by many scholars (Dorius, 2013). I also compared the standard deviations of the mean freshman family income under different policy settings to see which policy setting is more efficient in equalizing educational opportunity. I used the mean senior score as the proxy for school quality. If all of the mean senior scores across the schools are not significantly different, then it is possible to say that the qualities of all schools are similar. I also compared the standard deviations of the mean senior scores under different policy settings to see which policy setting is more efficient in equalizing school quality.

The reason to collect the admission information about the top 10-percent candidates was that the cases of justified envy suffered by elite students easily dominate the news, creating pressures on policymakers to change policies in favor of the top 10-percent. Therefore, how well elite students are admitted to their most preferred schools under a particular policy setting certainly concerns policymakers. When less than 100% of the top performing students are admitted to their most preferred schools, justified envy occurs. The magnitude of justified envy can be measured by the preference index (PI), which is calculated as follows:

PI = baseline preference rank – actual preference rank.

Baseline preference rank is the position of the baseline school in a student's preference list. Baseline school is the school assigned to a truth-telling student under deferred acceptance with no choice constraint. I used deferred acceptance without choice constraint as the baseline mechanism because it avoids strategy manipulation and justified envy (Pathak, 2011). Therefore, it is the most desired mechanism by elite students in competitive school districts, such as the Taipei School District. Actual preference rank is the position of the actual school in a student's preference list. Actual school is the school actually assigned to a student. For example, assuming Student *i*'s preference list, $P_i = \{2, 1, 4, 3, 5, 7, 6, 9, 10, 8\}$, if Student *i* would be assigned to School #4 under the baseline mechanism, then the student's baseline preference rank = 3 because School #4 is ranked third in the student's preference list. If the student is actually admitted to School #3, then the student's actual preference rank = 4 because School #3 is ranked fourth in the preference list. Therefore, Student *i*'s PI = 3 – 4 = -1.

A negative PI indicates the occurrence of justified envy or an admission loss experienced by a student. The larger the negative PI, the larger the scale of justified envy. On the other hand, a positive PI indicates an admission gain experienced by a student. While deferred acceptance without choice constraint avoids justified envy, it also sorts the students completely based on their scores. Under this policy setting, students with the bottom 10-percent family income probably will all stay in the lowest-quality schools when income and scores are highly and positively correlated. Some societies may not desire such result. The calculation of PI for students in each income quartiles and students with the bottom 10-percent family income provides useful information to evaluate the effects of an admission policy on various groups of students. The overall PI $= \frac{\sum |PI|}{N}$, where N is the total number of students. I used this index to examine whether the model passed the empirical output validation (see the section of verification and validation below.)

Match rate is the number of candidates who are assigned to schools divided by the number of total candidates. If there are a total of 1,000 seats and a total of 1,000 candidates in a school district, then the match rate can be as high as 100%. However, some students may not be admitted under certain admission policies. If the match rate of a matching mechanism is low, then the mechanism may not be efficient in assigning students. Match rate serves as an index to assess the allocation efficiency of a mechanism.

Data Analysis Plan

The purpose of this study was to answer the research question of whether the Taipei mechanism, in comparison to other commonly used mechanisms, and the free-tuition policy can help equalize educational opportunity and school quality in the Taipei School District. I simulated 300 combinations of the parameters to explore the possible results under different behavioral assumptions.

An advantage of running computational experiments is that computational experiments are controlled experiments, in which clear causal relationships can be defined (Chen, 2015; Epstein, 1999). The data collected from the simulations are clean. Little data cleaning and screening procedure are needed for computer-generated data, in comparison to human experiments. The software used to build the model and run the simulations was NetLogo 5.3, developed by Wilensky (1999). NetLogo 5.3 could automatically generate the needed data during simulations. Because the values of the computer-generated state variables, such as candidates' scores and preference lists, will be not the same in each simulation run, the result of each simulation even under the same scenario might not be the same. Multiple runs of simulations should be performed to make statistical analysis feasible. Therefore, I performed 30 simulation runs for each scenario listed in Table 5.

I exported the simulated data collected from the NetLogo 5.3 model to IBM SPSS v. 23 to perform descriptive statistics and paired samples *t*-tests with $\alpha = 0.01$ to compare the results of different matching mechanisms under the same scenario or the results under different scenarios with the same matching mechanism. I used Microsoft Excel to draw histogram graphs and tables to have a better visual comparison of policy outcomes. I also calculated Cohen's *d* to help determine the effect size.

Verification and Validation

A model must undergo verification and validation to be a rigorous simulation tool (Rand & Rust, 2011). For simulation models, model verification is to test whether a model performs as described. Model validation is to determine how well a model represents the reality that it is designed to represent (Rand & Rust, 2011). However, if the model is exploratory, model validation should focus on whether the modeler has a sound analytic strategy rather than whether the input and output of a model fit the empirical data (Bankes, 1993).

Verification

Rand and Rust (2011) suggested three steps to properly verify a model: documentation, programmatic testing, and test cases. I have fully documented the conceptual model in the section of model description and uploaded the model codes with clear descriptions to the website of OpenABM for other researchers to verify and replicate the model. For programmatic testing, I have performed the technique of unit testing by designing scenarios to test each submodel and the technique of code walkthrough by reading through the codes to examine whether they did as planned. I have also performed the technique of debugging walkthrough by running the program with a small number of student agents to ensure that it generated the correct result for each agent. For test cases, I have examined the specific scenarios to ensure that the model produced the three stylized facts as mentioned in the next section of validation. Therefore, the model in this study has been verified; i.e., the model performs as described.

Validation

In the field of ABM, how to validate and what to validate are constantly under debate. Simulation models are usually categorized into two types: consolidated models, or descriptive models, and exploratory models, or demonstration models (Bankes, 1993; Marks, 2013). Consolidated models are the models built on known facts or historical data and used as approximations of the real systems. Exploratory models are built with hypothetical details and mechanisms, often due to insufficient knowledge or inherent uncertainty, and used to explore the implications of the hypotheses and assumptions (Bankes, 1993). Schelling's (1971) segregation model is an example of exploratory models. For consolidated models, it may be possible to validate the model with historical data. However, if a model's input and output perfectly fit the historical data, it may be over-fitted by ignoring the inherent uncertainly in human behavior and produce little insight in prediction (Banke, 1993). For exploratory models, even though they cannot be validated with historical data, they help to explore the outcomes within the range of possible behaviors or help to hypothesize a plausible explanation and guide data collection to test the hypothesis (Bankes, 1993; Epstein, 2008). Schelling's segregation model serves as a good example. Although unable to be validated in a traditional sense, Schelling's model provides rich insight into segregation, showing us that macrobehavior can be very misleading in explaining the micromotives. Therefore, Bankes (1993) argued that the traditional concept of validation does not apply to exploratory models; instead, what should be focused is the validity of the analytic strategy. Since this study was an exploratory modeling, I adopted Bankes's argument to examine the validity of this study. In this study, the simulated data were collected by using the measures founded on the common concepts of educational opportunity and school quality. I ran each scenario 30 times to generate enough data for sound statistical analysis. I analyzed the data by using statistical techniques offered by SPSS. Therefore, this study had a valid analytic strategy.

Rand and Rust (2011) promoted the following four guidelines to validate a simulation model: (a) "micro-face" validation to make sure that the properties and mechanisms of a model correspond "on face" to the properties and mechanisms of the real-world system; (b) "macro-face" validation to show that the aggregate patterns generated by the model on face correspond to the aggregate patterns of the real-world system; (c) empirical input validation to ascertain a sufficient explanation about how the input of the model is derived, even if it cannot be calibrated to historical data; and (d) empirical output validation to show that the output of the implemented model corresponds to the stylized facts or empirical data of the real-world system. In the process of empirical output validation, an exploratory model is considered valid as long as its output corresponds to the stylized facts of the real world. If the model is for prediction purpose, then the model must demonstrate that one of the exhibited outputs of the model matches the empirical data of the real world. In the process of both micro-face and macro-face validation, no data are compared to the model (Rand & Rust, 2011).

In this model, the parameters and agent's behavioral rules were programmed based on the findings in the literature and the observations in the Taipei School District, which allowed the model to be micro-face valid. This model is macro-face valid because the matching processes in the model comply with those in the real world. This model met the guidelines of Rand and Rust's (2011) empirical input validation because candidates' scores and incomes generated from a distribution of which the values of the parameters were derived from the statistical data. This exploratory model also passed the empirical output validation defined by Rand and Rust (2011) because it reveals the following stylized facts of the matching mechanisms, which are also well proved in the literature: (a) serial dictatorship with full number of school choices is strategy-proof and avoids justified envy, (b) the Boston mechanism causes justified envy, and (c) the admission results under serial dictatorship and deferred acceptance are the same in one-sided matching markets (Abdulkadiroglu & Sonmez, 2003; Pathak, 2011; Chen et al., 2015). Since the Taipei mechanism is new, no related stylized fact was studied in the literature. I discuss the model's production of these stylized facts in Chapter 4.

Participant Protection

This study did not involve human experiment or any data containing private information. The values of the parameters in this model were assumed by reference to the published literature, and all inputs and outputs of this exploratory model were generated endogenously by the computer. This research has been carefully reviewed to ensure the compliance of Walden University's ethical standards and approved by the Walden Institutional Review Board (IRB approval No. 08-24-16-0304007) on August 24, 2016.

Summary

ABM is the right methodology to study the macrophenomena emerging from the interactions of the heterogeneous, autonomous, adaptive individual agents in complex adaptive systems (Borrill & Tesfatsion, 2011). It allows researchers to build models for complex educational systems with assumptions closer to the real world. The flexibility of ABM in performing scenario analysis makes it a proper approach to study a new education policy that involves anticipating human behaviors in its design. ABM also allows researchers to build causal hypotheses between microbehaviors and macrophenomena.

In this chapter, I describe the agent-based OLG model in detail according to the format of the ODD protocol (Grimm et al., 2010). All concepts, assumptions, and calculations underpinning this model have been clearly stated so that the model can be verified by others in the future. The values of the parameters, students' decision strategies, and the processes of the matching mechanisms were programmed by reference to the empirical literature or the real-world systems so that the model is valid in the on-face level and the empirical input level (Rand & Rust, 2011). The outputs of this model exhibit the stylized facts of the matching mechanisms; thus, this model is also valid in the empirical output level.

Agent-based simulations provide rich micro-, meso-, and macroinformation. What and how much data to collect depends on research questions. In this study, I collected the meso-and macroinformation related to the inequalities of educational opportunity and school quality. I analyzed these data by using descriptive statistics and paired samples *t*-tests and compared the outcomes of different matching mechanisms under the same behavioral scenario or the outcomes of different behavioral scenarios under the same matching mechanism. I discuss the simulation results in Chapter 4.

Chapter 4: Results

Introduction

The purpose of this study was to contribute to the development of ABM for educational policy analysis. I constructed an agent-based OLG model to represent qualitatively the environment of the Taipei School District. I have uploaded the model code (see Appendix A) to the website of OpenABM for anyone to download and replicate. I simulated a total of 300 combinations of the parameters (see Table 5) to explore the impacts of the Taipei mechanism (TM) and the free-tuition policy on equalities of educational opportunity and school quality, in comparison with the impacts of the following matching mechanisms: serial dictatorship (SD), the Boston mechanism (BM), deferred acceptance (DA), and the Chinese Parallel mechanism (CP). I analyzed the simulation results to answer the research questions listed below:

Question 1: Does the Taipei mechanism help equalize educational opportunities?

Question 2: Does the Taipei mechanism help school qualities converge upward?

Question 3: Does the Taipei mechanism with the free-tuition policy help equalize educational opportunities?

Question 4: Does the Taipei mechanism with the free-tuition policy help school qualities converge upward?

Additionally, I examined whether the model produced the following stylized facts to ensure that it passed the empirical output validation as defined by Rand and Rust (2011):

Stylized Fact #1: SD and DA with the full number of school choices are strategy-proof and avoid justified envy (Abdulkadiroglu & Sonmez, 2003; Pathak, 2011; Chen et al., 2015);

Stylized Fact #2: BM causes justified envy (Abdulkadiroglu & Sonmez, 2003; Pathak, 2011; Chen et al., 2015); and

Stylized Fact #3: The admission results under SD and DA are the same in one-sided matching markets (Abdulkadiroglu & Sonmez, 2003; Pathak, 2011; Chen et al., 2015).

The organization of this chapter is as follows. First, I review the data generation process and the appropriateness of the statistical analysis. I then examine the simulated stylized facts to ensure that the model passed the empirical output validation. I analyze the simulated standard deviation of the mean freshman family income (SD-Freshman) and the standard deviation of the mean senior scores (SD-Senior) to have an overall understanding of the effects of TM on educational opportunity and school quality in comparison with those of the other four student-assignment mechanisms. I also analyze the preference index (PI) defined in the section of model description in Chapter 3 to understand how TM, compared with the other four mechanisms, assigns the students in different groups to schools. Finally, a summary of the results concludes this chapter.

Data Collection and Analysis

Agent-based computational simulations are controlled experiments (Chen, 2015; Epstein, 1999). The inputs are preprogrammed and generated directly by the computer; the outputs are computer simulated through a predetermined process. The data is clean, and the causality is clear. There is no need to perform data cleaning or screening procedures. In this study, each simulation result is a sample observation because the input set in each simulation step is randomly generated from predefined distributions and thus different from each other. To have enough data points for statistical analysis, I simulated each scenario 30 runs, and there were 33 steps (years) in each run. As a result, each collected output variable had 990 data points, except that the variables related to senior's scores had only 930 data points. The reason is that in this model, a freshman agent takes two years to become a senior in the high schools (Grades 10 – 12), and thus there were no seniors in the first two steps (years) in each simulation run. Because the sample sizes were large enough, the central limit theorem applies to all data collected from the simulations.

Since this was an exploratory study, the purpose of the data analysis was to gain qualitative insights, rather than quantitative preciseness. Although I performed paired samples *t*-tests on SPSS v. 23 to facilitate the comparison of the results under TM and the other mechanisms, I mainly used graphing to find the relative patterns. The two essential assumptions under the paired samples *t*-test are the independence of the observations and the normality of the pair differences (van den Berg, 2014). Since each observation resulted from an independently generated input set and the same input set was processed independently under each matching mechanism, the independence assumption was met. Because of the central limit theorem, the normality assumption was also upheld. Therefore, it was appropriate to conduct the paired samples *t*-tests in this study. In the cases where a Cohen's *d* was reported to measure the effect size, I calculated the Cohen's *d* by using the calculator on the website of Dr. Lee A. Becker (Effect size Calculators), University of Colorado Colorado Springs.

Empirical Output Validation

Stylized Fact #1

Justified envy occurs when a higher ranked student loses the seat in his or her preferred school to a lower ranked student (Abdulkadiroglu & Sonmez, 2003). SD with the full number of choices avoids justified envy because a student with a higher rank can always choose a school before a student with a lower rank under this mechanism. PI serves as a measure of justified envy. It measures how much a student prefers the school actually assigned to him (the actual school) in comparison to the school the student would have been assigned to under the SD with the full choices (the baseline school). For example, PI = -2 means that the student's actual school is ranked 2 lower than his baseline school on his preference list. If a student does not suffer justified envy, his or her PI = 0. If a student is unassigned and his baseline school is ranked the second in his preference list, then the student's PI is 2 – 11 = -9.

Tables 8 and 9 show the average overall PI under each mechanism and strategic scenario with and without the free-tuition policy, respectively. The overall PI = $\frac{\sum |PI|}{N}$, where N is the total number of students. The average overall PI is the average of the overall PIs collected from each step and each run of the simulations under the same scenario. The only situation where the average overall PI = 0 is when the mechanism = SD or DA, the number of choices = 10, and *sort-extra-choice* = True. Since there are 10 schools in this model, the number of choices equal to 10 means that there is no limit to the number of schools that a student can choose. *Sort-extra-choice* = True indicates that students are truth tellers, who report their preference lists as their choice lists. Therefore, this model has correctly produced Stylized Fact #1: SD and DA with the full number of school choices are strategy-proof and avoid justified envy.

Table 8

The Average Overall Preference Index without the free-tuition policy

Number of choices	Strategy	Extra-in-order	SD	BM	DA	TM	CP
2	1	FALSE	1.97*	2.17*	1.97*	1.99	2.18*
		TRUE	1.97*	2.18*	1.97*	1.99	2.18*
	2	FALSE	2.35*	2.84*	2.41*	1.55	2.86*
		TRUE	2.45*	2.82*	2.38*	1.53	2.82*
	3	FALSE	2.17*	2.72*	2.14*	1.19	2.75*
		TRUE	2.16*	2.68*	2.18*	1.19	2.67*
4	1	FALSE	1.12*	1.84*	1.12*	1.16	1.43*
		TRUE	1.12*	1.84*	1.12*	1.16	1.43*
	2	FALSE	1.00*	2.28*	0.99*	0.60	1.71*
		TRUE	0.98*	2.53*	0.98*	0.56	1.67*
	3	FALSE	0.91*	2.06*	0.92*	0.39	1.50*
		TRUE	0.78*	2.37*	0.78*	0.31	1.50*
6	1	FALSE	0.65*	1.76*	0.65*	0.73	0.92*
		TRUE	0.65*	1.75*	0.65*	0.73	0.92*
	2	FALSE	0.67*	1.91*	0.66*	0.54	0.95*
		TRUE	0.38*	2.17*	0.40*	0.29	0.92*
	3	FALSE	0.56*	1.72*	0.57*	0.37	0.85*
		TRUE	0.19*	2.13*	0.21*	0.27	0.84*
8	1	FALSE	0.26*	1.78*	0.26*	0.41	0.47*
		TRUE	0.25*	1.80*	0.26*	0.41	0.47*
	2	FALSE	0.49	1.57*	0.49*	0.48	0.57*
		TRUE	0.08*	1.74*	0.07*	0.25	0.60*
	3	FALSE	0.47*	1.53*	0.50*	0.32	0.54*
		TRUE	0.05*	1.64*	0.05*	0.24	0.48*
10	1	FALSE	0.00*	1.76*	0.00*	0.24	0.25*
		TRUE	0.00*	1.76*	0.00*	0.24	0.24*
	2	FALSE	0.48	1.45*	0.48	0.45	0.53*
		TRUE	0.00*	1.76*	0.00*	0.24	0.49*
	3	FALSE	0.43*	1.34*	0.43*	0.34	0.47*
		TRUE	0.00*	1.76*	0.00*	0.24	0.40*

Note. * denotes that the value is statistically significantly different from that under the column of TM in the same scenario (the same row), $p < .01$.

Stylized Fact #2

Table 8 and Table 9 show that none of the overall PI under BM equals to 0, which means that BM always produces justified envy. Therefore, this model has successfully replicated Stylized Fact #2: BM causes justified envy.

Table 9

The Average Overall Preference Index with the free-tuition policy

Number of choices	Strategy	Extra-in-order	SD	BM	DA	TM	CP
2	1	FALSE	1.98	2.16	1.98	2.00	2.16
		TRUE	1.98	2.15	1.98	2.00	2.16
	2	FALSE	2.27	2.62	2.19	1.56	2.63
		TRUE	2.13	2.69	2.10	1.51	2.70
	3	FALSE	1.96	2.49	2.01	1.24	2.45
		TRUE	1.98	2.54	1.97	1.24	2.42
4	1	FALSE	1.14	1.95	1.14	1.19	1.49
		TRUE	1.14	1.95	1.14	1.19	1.49
	2	FALSE	0.95	2.27	0.99	0.74	1.73
		TRUE	0.98	2.53	0.96	0.64	1.75
	3	FALSE	0.81	2.08	0.87	0.45	1.58
		TRUE	0.79	2.47	0.79	0.43	1.61
6	1	FALSE	0.72	2.01	0.72	0.80	1.03
		TRUE	0.72	2.01	0.72	0.80	1.03
	2	FALSE	0.61	2.13	0.61	0.49	1.00
		TRUE	0.45	2.69	0.44	0.30	1.02
	3	FALSE	0.51	2.02	0.51	0.31	0.92
		TRUE	0.25^	2.59	0.24^	0.25	0.89
8	1	FALSE	0.35	1.99	0.35	0.48	0.65
		TRUE	0.35	2.00	0.35	0.48	0.65
	2	FALSE	0.48	1.90	0.48	0.44	0.59
		TRUE	0.09	2.28	0.10	0.26	0.57
	3	FALSE	0.40	1.90	0.41	0.34	0.53
		TRUE	0.05	2.26	0.05	0.26	0.53
10	1	FALSE	0.00	2.41	0.00	0.28	0.44
		TRUE	0.00	2.41	0.00	0.28	0.44
	2	FALSE	0.37	1.66	0.37	0.43	0.44^
		TRUE	0.00	2.41	0.00	0.28	0.46
	3	FALSE	0.35	1.72	0.35	0.29	0.41
		TRUE	0.00	2.40	0.00	0.28	0.36

Note. ^ denotes that the value is statistically insignificantly different from that under the column of TM in the same scenario (the same row), $p > .01$.

Stylized Fact #3

The literature has shown that other things being equal, the results of DA and SD in one-sided matching are equivalent (Zhu, 2014). Table 8, Table 9, and the rest of the tables in this chapter also show that the simulation results of SD and DA were insignificantly different in almost all scenarios, especially when the number of choices was large. The reason why there were discrepancies between the two is that stochasticity is applied in two procedures related to the student assignment process in this model (see the section of model description in Chapter 3 for details). The first is the tie-breaking procedure adopted from the common practice in many real-world school admission systems. The model randomly assigns ranks to the students involved in a tie and applies this random process in each simulation with a different matching mechanism. Therefore, the same student may be ranked and assigned differently between SD and DA. The second is the procedure to generate students' school-choice lists. When the number of choices is constrained and students need to make a selection among the schools, the model applies the principle of maximum entropy to make a choice for the students. Therefore, the same student may have a school-choice list under SD different from that under DA, resulting in assignment discrepancies between SD and DA.

The smaller the number of choices, the lower the probability of the same school to be selected twice. Therefore, the difference of the average overall PIs under SD and DA is larger when the number of choices is smaller, and vice versa. When the number of choices = 10, all schools are included in a student's choice list, which eliminates the lottery process of school selection, and thus the simulated results under SD and DA are identical or only less than 0.01 insignificantly different, $p < .01$. The insignificant difference results from the occasional occurrence of the random tie breaking. From the above analysis, it is safe to say that this model has successfully generated Stylized Fact #3: The admission results under SD and DA are the same in one-sided matching markets. In the rest of this chapter, when the results of SD and DA are equivalent or insignificantly different, I use "SD (DA)" to imply that the discussion applies to both SD and DA.

Effects of TM without the Free-tuition policy

Overall Effect on Educational Opportunity

SD-Freshman-N. As stated in Chapter 3, I defined equal educational opportunities as equal freshmen's average family income across the schools. I measured the inequality by using the standard deviation of the distribution of the mean freshman family incomes in the schools (SD-Freshman). The higher the standard deviation, the more the inequality. SD-Freshman-N denotes the SD-Freshman collected from simulations without the free-tuition policy. Table 10 summarizes the average SD-Freshman-N collected over the 33 simulated years in the 30 simulation runs for each scenario. The same data in Table 10 are also presented graphically in Figure 6.

I first examined the simulation results under TM in Table 10 and Figure 6 and summarize the findings below.

- When students used heterogeneous truth-telling strategies (Strategy #1), the greater the number of choices, the higher the SD-Freshman-N under TM.

- Strategy #1 produced lower SD-Freshman-Ns under TM than Strategy #2 or Strategy #3 as long as the number of choices was constrained (< 10 in our model).
- When students used Strategy #2 and the number of choices > 2, to sort the choices according to students' preference lists (*sort-extra-choice* = True) resulted in higher SD-Freshman-N than not to sort the choices (*sort-extra -choice* = False).
- When students used Strategy #3, *sort-extra-choice* = True resulted in higher SD-Freshman-Ns than *sort-extra -choice* = False in all cases.

Table 10

The average standard deviation of the mean freshman family income without the free-tuition policy (Average SD-Freshman-N)

Number of choices	Strategy	Extra-in-order	Scenario #	SD	BM	DA	TM	CP
2	1	FALSE	1	$373,600	$365,863*	$374,352	$373,065	$366,755*
		TRUE	2	$375,788*	$365,907*	$374,571	$372,486	$365,968*
	2	FALSE	3	†$813,078*	$830,445*	$833,570*	$715,455	$828,561*
		TRUE	4	$831,010*	$825,781*	$829,641*	$714,095	$821,971*
	3	FALSE	5	$788,836*	$794,645*	$781,408*	$677,684	$818,599*
		TRUE	6	$791,934*	$785,829*	$794,182*	$682,904	$800,938*
4	1	FALSE	7	$518,298*	$487,050*	$518,032*	$515,725	$510,853*
		TRUE	8	$517,200*	$487,098*	$517,498*	$515,675	$510,091*
	2	FALSE	9	†$634,528*	$764,334*	$629,649*	$613,066	$724,282*
		TRUE	10	$633,968*	$801,722*	$633,010*	$613,682	$722,457*
	3	FALSE	11	$624,110*	$715,601*	$625,876*	$611,737	$702,533*
		TRUE	12	$618,968*	$786,467*	$619,354*	$615,186	$706,921*

(Continued)

Number of choices	Strategy	Extra-in-order	Scenario #	SD	BM	DA	TM	CP
6	1	FALSE	13	†$566,061*	$527,617*	$564,510	$563,535	$557,478*
		TRUE	14	$564,378*	$525,074*	$563,715*	$561,825	$557,328*
	2	FALSE	15	$609,967	$669,068*	$609,843	$609,593	$615,142*
		TRUE	16	$606,513*	$657,893*	$606,502*	$613,795	$614,564
	3	FALSE	17	$608,858*	$640,310*	$608,253*	$610,715	$611,687
		TRUE	18	$613,901*	$643,539*	$613,750*	$617,233	$612,853*
8	1	FALSE	19	†$591,201	$532,099*	$592,844*	$590,036	$587,071*
		TRUE	20	$591,538	$531,457*	$591,311	$590,565	$587,327*
	2	FALSE	21	$600,505	$595,067*	$600,673	$600,031	$597,918*
		TRUE	22	$604,633*	$563,831*	$604,911*	$609,384	$597,681*
	3	FALSE	23	$600,505*	$594,896*	$600,263*	$603,785	$597,802*
		TRUE	24	$608,538*	$565,879*	$608,358*	$609,798	$598,461*
10	1	FALSE	25	$612,510*	$551,870*	$612,510*	$609,485	$606,689*
		TRUE	26	$611,901*	$551,620*	$611,901*	$609,836	$606,122*
	2	FALSE	27	$601,404	$573,069*	$601,403	$601,258	$600,187
		TRUE	28	$612,714*	$550,680*	$612,714*	$609,332	$597,945*
	3	FALSE	29	$601,336*	$576,949*	$601,335*	$603,423	$600,721*
		TRUE	30	$613,740*	$552,721*	$613,740*	$610,722	$601,751*

Note. * denotes that the value is statistically significantly different from that under the column of TM in the same scenario (the same row), $p < .01$. $ here represents Taiwan dollar. † denotes that the value is significantly different from that under the column of DA in the same scenario (the same row), $p < .01$.

I then compared the simulation results under TM with those under the other mechanisms on the scenario-by-scenario basis. The followings summarize the comparison findings.

- When Strategy = #1 or the number of choices < 6, the SD-Freshman-N under TM was smaller than that under SD (DA).
- When the number of choices ≥ 6 and Strategy = #2 or #3, there was no clear rule about the relationships of the SD-Freshman-Ns under TM and SD (DA).

- When the number of choices ≥ 8, the SD-Freshman-Ns under TM and SD (DA) were relatively stable, regardless of the school-choice strategy.

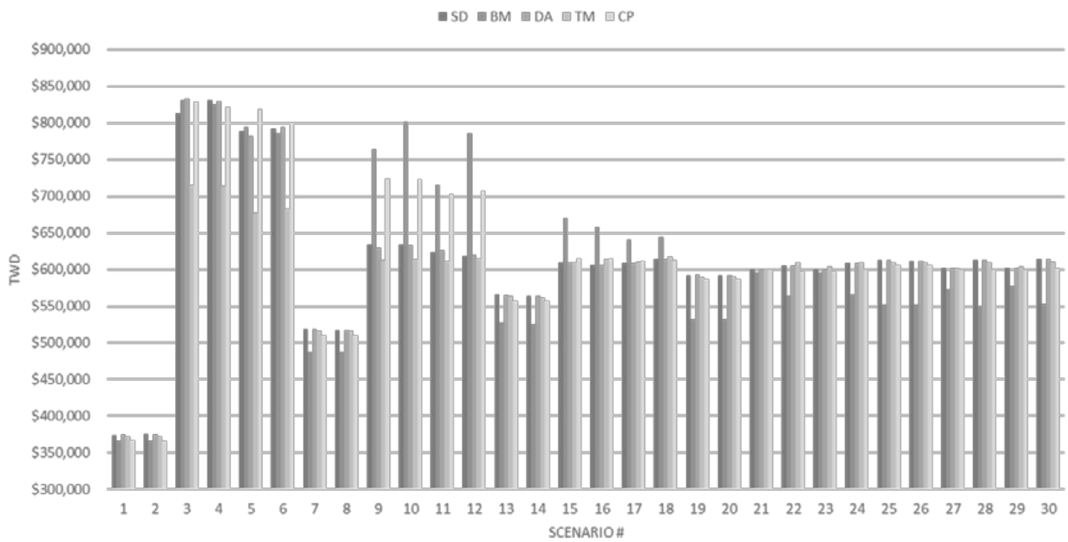

Figure 6. The average standard deviation of mean freshman family incomes without the free-tuition policy (SD-Freshman-N). The scenario # in this figure corresponds to that in Table 10.

When the numbers of choices were 6, 8, and 10, the SD-Freshman-Ns under TM was no more than 1.20%, 0.79%, and 0.55% different from the SD-Freshman-Ns under SD, respectively. Although some of the differences were statistically significant (see Table 10), a maximum of 1.20% change in the original SD-Freshman (SD-Freshman-N under SD) was deemed small, Cohen's d = 0.21 (Cohen, 1988). Therefore, I considered the values of the SD-Freshman-Ns under SD (DA) and TM in the above scenarios very close, TM ≈ SD (DA).

The simulation results under BM, compared to TM and the other mechanisms in each scenario, showed the followings.

- When Strategy = #1, BM produced the lowest SD-Freshman-N among the five mechanisms regardless of the number of choices.
- When Strategy = #2 or #3 and the number of choices > 6, BM still produced the lowest SD-Freshman-N among all mechanisms.
- When Strategy = #2 or #3 and the number of choices was 4 or 6, BM generated the highest SD-Freshman-N among the five mechanisms.
- When Strategy = #2 or #3 and the number of choices was 2, although SD-Freshman-N under BM was not the highest among the five mechanisms, it was still higher than the SD-Freshman-N under TM.

I then compared the SD-Freshman-Ns under TM and CP in each scenario and found the following relationships.

- When Strategy = #1, TM always generated higher SD-Freshman-N than CP, regardless of the number of choices.
- When Strategy = #2 or #3 and the number of choices > 6, TM still generated higher SD-Freshman-Ns than CP.
- When Strategy = #2 or #3 and the number of choices < 6, TM produced lower SD-Freshman-Ns than CP.
- When Strategy = #2 or #3 and the number of choices = 6, the relationships between TM and CP were mixed, depending on whether students sorted their school choices according to their preference lists.

Finally, I compared the five mechanisms together, scenario by scenario, and found the following properties.

- When Strategy = #1, the higher the number of choices, the higher the SD-Freshman-N under each mechanism.
- When Strategy = #1, the size of SD-Freshman-N produced by each matching mechanism was always in the following order: SD (DA) > TM > CP > BM.
- When Strategy = # 2 or #3, the SD-Freshman-N under TM was more robust to the change in the number of choices than BM, CP, and then SD (DA).
- When Strategy = #2 or #3 and the number of choices > 6, the SD-Freshman-N under each mechanism was in the following order: TM ≈ SD (DA) > CP > BM.
- When Strategy = #2 or #3 and the number of choices < 6, the SD-Freshman-N under each mechanism was in the following order: TM < CP, TM < BM, and TM < SD (DA).

When Strategy #2 or #3 was used, the number of choices 6 seemed to work as a bifurcation or turning point for the relationships of the SD-Freshman-N between TM and the other mechanisms. While TM produced the lowest SD-Freshman-Ns when the number of choices < 6, the SD-Freshman-Ns produced by TM were among the highest when the number of choices > 6.

Match rate. Match rate should be considered in the analysis of a student-assignment mechanism. If the system has enough seats for all candidates and the candidates do not leave any allowed choices blank, a student-assignment policy resulting in some students unassigned does not seem to be efficient. Match rate is the number of candidates assigned to schools divided by the number of total candidates. The simulated environment had a total of 1,000 seats (10 schools x 100 seats per school) for the 1,000 candidates per year. Table 11 summarizes the average match rate over the 33 simulation years in the 30 simulation runs for each scenario without the free-tuition policy. The same data in Table 11 are also presented graphically in Figure 7.

Table 11

The average match rate under each scenario without the free-tuition policy (Match-N)

Number of choices	Strategy	Extra-in-order	Scenario #	SD %	BM %	DA %	TM %	CP %
2	1	FALSE	1	90.47	87.14*	90.41	90.45	87.20*
		TRUE	2	90.39	87.13*	90.41	90.39	87.11*
	2	FALSE	3	59.86*	56.79*	58.76*	71.60	56.53*
		TRUE	4	58.72*	55.87*	59.59*	72.13	57.21*
	3	FALSE	5	61.78*	57.91*	62.47*	77.55	57.12*
		TRUE	6	61.81*	58.06*	61.78*	77.73	58.72*
4	1	FALSE	7	95.23	93.13*	95.25	95.16	94.19*
		TRUE	8	95.23	93.14*	95.24	95.25	94.15*
	2	FALSE	9	85.16*	72.83*	85.47*	92.43	75.06*
		TRUE	10	86.03*	67.49*	85.98*	93.85	75.27*
	3	FALSE	11	86.71*	74.72*	86.43*	95.84	77.44*
		TRUE	12	88.78*	68.85*	88.76*	97.63	77.22*
6	1	FALSE	13	97.67	95.33*	97.43*	97.64	97.53
		TRUE	14	97.61	95.37*	97.56	97.62	97.68
	2	FALSE	15	95.52	85.59*	95.52	95.68	92.53*
		TRUE	16	97.48*	82.35*	97.35*	98.55	92.86*
	3	FALSE	17	95.73*	87.33*	95.62*	96.97	93.08*
		TRUE	18	98.55	83.17*	98.48*	98.67	93.41*
8	1	FALSE	19	98.89*	98.29*	99.16	99.19	99.13
		TRUE	20	98.81*	98.27*	98.81*	99.13	99.07
	2	FALSE	21	98.03	94.83*	97.88	97.88	97.98
		TRUE	22	99.35*	94.86*	99.40*	99.71	97.69*
	3	FALSE	23	97.77*	95.21*	97.75*	98.50	97.69*
		TRUE	24	99.58	95.27*	99.55	99.68	97.85*

(Continued)

Number of choices	Strategy	Extra-in-order	Scenario #	SD %	BM %	DA %	TM %	CP %
10	1	FALSE	25	99.74	100.00*	99.74	99.80	99.78
		TRUE	26	99.67*	100.00*	99.67*	99.92	99.70*
	2	FALSE	27	98.41*	98.93	98.41*	98.77	98.27*
		TRUE	28	99.84	100.00*	99.84	99.85	98.76*
	3	FALSE	29	98.62	99.09*	98.62	98.85	98.48*
		TRUE	30	99.86*	100.00	99.86*	99.98	98.95*

Note. * denotes that the value is statistically significantly different from that under the column of TM in the same scenario (the same row), $p < .01$.

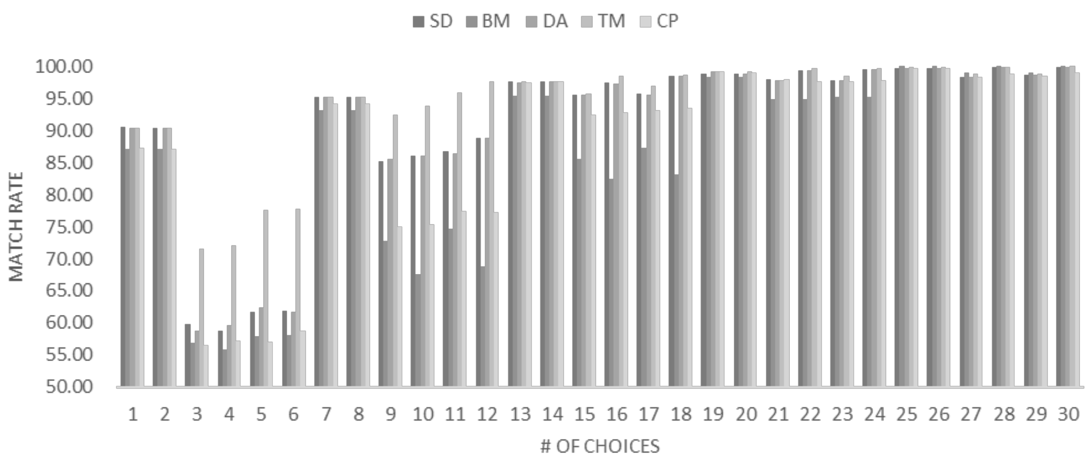

Figure 7. The average match rate under each scenario without the free-tuition policy (Match-N). The scenario # in this figure corresponds to that in Table 11.

Match-N denotes the match rate without the free-tuition policy. Figure 7 shows the trend that under the same mechanism and strategy, the higher the number of choices, the higher the Match-N. If a match rate of at least 95% is desired without regarding students' behaviors, TM, SD, and DA must be accompanied by an allowance of 6 or more school choices; CP, 8 or more; BM, the full number of choices.

Among the five mechanisms, only BM could reach a Match-N of 100%. All other mechanisms always had years in which some slots in the private school (School #3) were unfilled. The reasons for this phenomenon were as follows: (a) Without the free-tuition policy, poorer students (student's income < the average income of all candidates in that year) could not afford to choose School #3; and (b) students' school choices were not identical. In this study, the similarity of students' preferences was controlled by α, which was set to be 3 (see the subsection of details in Chapter 3 for the generation of students' preferences.) The higher the value of α, the more similar students' preferences are. When $\alpha = 3$, students' preferences are moderately-to-highly correlated. It was possible that some richer candidates who were eligible to attend School #3 preferred other schools to School #3. When this situation happened, those richer candidates would "crowd out" some poorer candidates with lower priorities and make them ended up with no school to attend under SD and DA because SD and DA emphasize students' priorities. The argument also applies to TM and CP because both TM and CP have the element of SD (DA).

This crowding out effect can be avoided in some situations under BM because BM emphasizes students' choices instead of priorities. That is, other things being equal, a student who ranks School *j* higher in the choice list has a better chance to be admitted to School *j* than a student who ranks School *j* lower in the choice list. Because of this property of BM, the more heterogeneous the students' choice lists are, the higher the chances they are assigned to their top choices. Under BM, the assignment in every step is final. That is, once a student is assigned to a school, no one can crowd the student out, regardless of the students' priorities. The simulated 100% Match-N under BM demonstrates that students' preferences generated under $\alpha = 3$ are heterogeneous enough for BM to avoid the crowding out effect in the scenarios where the number of choices = 10 and students are truth tellers.

I then compared the Match-Ns in each scenario and summarize the findings below.

- TM produced higher Match-N than CP and SD (DA) in almost all cases; in the cases where TM generated lower Match-N than CP and SD (DA), the difference was no more than 0.15%.
- When Strategy = #2 or #3 and the number of choices < 6, TM produced 7.27% to 15.95% higher Match-Ns than SD (DA).
- When Strategy = #1 or the number of choices ≥ 8, if the Match-Ns under TM were greater than those under SD (DA), the differences were no more than 0.73%, Cohen's $d = 0.31$.
- BM produced lower Match-Ns than the other mechanisms when the number of choices < 10 but slightly higher Match-Ns than the others when the number of choices = 10.

The above findings show that not only the matching mechanism but also the number of choices and student's strategy affect the assignment of students to schools. The interactions of these three factors also affect match rate. An efficient admission policy should produce a high match rate. Therefore, in the next paragraphs, I make further discussions about SD-Freshman-N by focusing on scenarios where the number of choices ≥ 8 (94% or higher of Match-Ns under all mechanisms).

The comparisons of the simulated SD-Freshman-Ns in the previous paragraphs were on the scenario-by-scenario basis, which implied that students use the same strategies under different admission policies. However, students may change their behaviors when the assignment policy changes (Chen et al., 2015; Lucas, 1996; Roth, 2002). Before or at the beginning of the implementation of a new policy, policymakers often do not have enough information to know whether and how students will change their behaviors. Even under the same mechanism, students may change their behaviors in response to a change in the number of choices. If how students behave is not explicitly known, it may be necessary to group the simulated results under various behavior assumptions together and make the group-to-group comparisons to estimate the best and worst consequences.

To calculate the best-worst estimates, I grouped the scenarios with different strategies under the same number of choices together for each mechanism. When the number of choices = 8, the highest and lowest SD-Freshman-Ns under SD were $608,538 and $591,201 (Taiwan Dollars), respectively (see Table 10). The highest and lowest SD-Freshman-Ns under TM were $609,798 and $590,036, respectively. If the mechanism is changed from SD to TM, the maximum increase in the original SD-Freshman can be $18,598 ($609,798 - $591,201) or 3.15% ($18,598/$591,201), Cohen's d = 0.92; the maximum decrease, $18,502 ($608,538 -$590,036) or 3.04% ($18,502/$608,538), Cohen's d = 0.91. I used the interval notation [] to present the estimates of the maximum increase and maximum decrease of a variable. Thus, the estimated maximum percentage increase and the estimated maximum percentage decrease in the original SD-Freshman caused by the change from SD to TM were reported as [3.15%, -3.04%]. Table 12 presents such best-worst estimates for a change from SD to BM, TM, and CP, respectively. Since I focused on high match rate, Table 12 only shows the best-worst estimates with the number of choices ≥ 8.

Table 12

The estimated maximum percentage increase and decrease in the original SD-Freshman resulting from the change from SD to the other mechanisms without the free tuition policy

Number of choices	To BM % Cohen's *d*	To TM % Cohen's *d*	To CP % Cohen's *d*
8	[0.65%, -12.67%] 0.11, -3.30	[3.15%, -3.04%] 0.92, -0.91	[1.23%, -3.53%] 0.32, -1.05
10	[0.00%*, -10.27%] -, -2.85	[2.06%, -2.02%] 0.42, -0.55	[0.89%, -2.57%] 0.23, -0.69

Note. The first percentage in the square brackets represents the estimated maximum percentage increase in the original SD-Freshman (SD-Freshman-N under SD) due to the change from SD to the new mechanism, and the second percentage in the square brackets denotes the estimated maximum percentage decrease. The corresponding Cohen's *d* is presented right below each percentage. * No Cohen's *d* was calculated for this percentage because BM did not increase the original SD-Freshman in any scenario within this category.

From Table 12, one can see that under the same number of choices, BM could offer the most decrease and least increase in the original SD-Freshman, followed by CP, and then TM. When the number of choices = 8, while TM had the potential to reduce 3% of the original SD-Freshman, it also had the potential to increase 3% of the original SD-

Freshman, depending on students' strategies before and after the change. If the number of choices increases to 10, the range of TM's effects on the original SD-Freshman could shrink about 1% on both ends. If we follow Cohen's (1988) suggestions on effect size, we should take TM's potential to increase the original SD-Freshman more seriously than CP's and BM's, because the effect size under TM was larger than the effect sizes under CP and BM as shown in Table 11.

Overall Effect on School Quality

SD-Senior-N. As stated in Chapter 3, I used the mean senior scores as the proxy for school quality and the standard deviation of the distribution of the mean senior scores (SD-Senior) to measure the magnitude of the inequality of school quality. The higher the SD-Senior, the greater the inequality of school quality. SD-Senior-N denotes the SD-Senior collected from simulations without the free-tuition policy. Since a freshman must study for two years to become a senior in this model, there was no SD-Senior-N in the first two steps (years) of each simulation run. Therefore, each SD-Senior-N listed in Table 13 was the average of the SD-Senior-Ns collected over the last 31 simulated steps in the 30 simulation runs. The same data in Table 13 are presented graphically in Figure 8.

Table 13

The average standard deviation of the mean senior scores without the free-tuition policy (average SD-Senior-N)

Number of choices	Strategy	Sort-extra-choice	Scenario #	SD	BM	DA	TM	CP
2	1	FALSE	1	13.92	13.48*	13.96	13.90	13.53*
		TRUE	2	13.94*	13.49*	13.97*	13.83	13.49*
	2	FALSE	3	30.06*	30.54*	30.78*	26.77	30.45*
		TRUE	4	30.49*	30.87*	30.53*	26.81	30.27*
	3	FALSE	5	29.63*	29.61*	29.31*	25.26	30.54*
		TRUE	6	29.75*	29.53*	29.76*	25.32	29.58*
4	1	FALSE	7	19.61*	17.07*	19.58*	19.46	18.82*
		TRUE	8	19.59*	17.07*	19.57*	19.50	18.78*
	2	FALSE	9	22.72*	26.79*	22.49*	21.73	26.40*
		TRUE	10	22.60*	29.76*	22.58*	21.68	26.47*
	3	FALSE	11	22.08*	26.00*	22.29*	21.59	25.80*
		TRUE	12	22.03*	29.37*	22.05*	21.83	25.97*
6	1	FALSE	13	20.93*	17.18*	20.81	20.74	19.93*
		TRUE	14	20.89*	17.15*	20.86*	20.72	20.00*
	2	FALSE	15	21.20*	22.72*	21.21*	21.49	21.38
		TRUE	16	21.42*	22.09	21.40*	21.87	21.27*
	3	FALSE	17	21.29*	22.05	21.29*	21.56	21.28*
		TRUE	18	21.82*	21.70*	21.80*	21.97	21.24*
8	1	FALSE	19	21.52*	16.59*	21.64*	21.43	21.01*
		TRUE	20	21.50*	16.51	21.48*	21.41	20.98*
	2	FALSE	21	20.94	19.89*	20.93	20.99	20.71*
		TRUE	22	21.61	17.96*	21.63	21.65	20.58*
	3	FALSE	23	20.83*	19.78*	20.80*	21.14	20.63*
		TRUE	24	21.76*	18.05*	21.75*	21.66	20.73*
10	1	FALSE	25	21.86*	16.83*	21.86*	21.64	21.29*
		TRUE	26	21.84*	16.83*	21.84*	21.68	21.27*
	2	FALSE	27	20.95	18.95*	20.95	21.09	20.81*
		TRUE	28	21.90*	16.80*	21.90*	21.66	20.97*
	3	FALSE	29	21.05*	19.13*	21.05*	21.23	20.97*
		TRUE	30	21.90*	16.84*	21.90*	21.70	21.14*

Note. * denotes that the value is significantly different from that under the column of TM in the same scenario (the same row), $p < .01$.

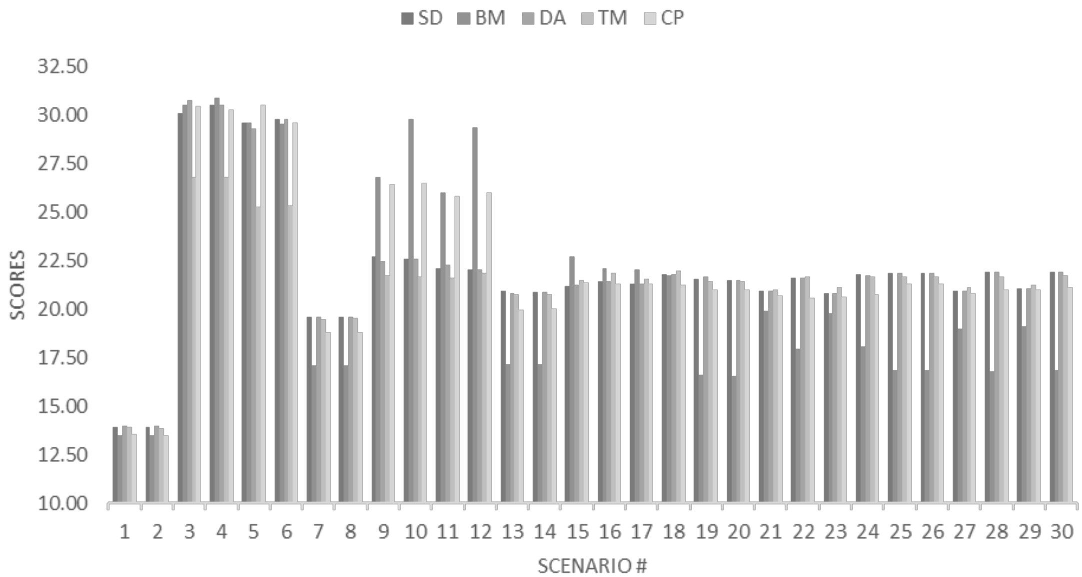

Figure 8. The average standard deviation of the mean senior scores without the free-tuition policy (average SD-Senior-N). The scenario # in this figure corresponds to that in Table 13.

An interesting phenomenon is that the graphic shape of the average SD-Senior-Ns (Figure 8) resembles that of the SD-Freshman-Ns (Figure 6). The reason is that in this study, students' family incomes were highly correlated with their scores (with a correlation coefficient of .8) and remained constant throughout the students' stay in the high schools. The algorithm to calculate senior scores was based on the consistent empirical findings since the 1966 Coleman's report. The findings have shown that after controlling students' socioeconomic status, schools contribute little to the difference in student performance and the most influential school factor is peer effect (Burke & Sass, 2013; Dearden, Ferri, & Meghir, 2002; Coleman et al., 1966; Hanushek, 1989; Jennings et al., 2015). Therefore, although some high school students' scores were adjusted and moved toward the average of their cohorts' scores in their schools (see the calculation of high school student's scores in the subsection of details in Chapter 3), the score change was not significant enough to transform the shape of the original distributions of scores.

Since Figure 8 resembles Figure 6, SD-Senior-N had the following properties similar to those observed in SD-Freshman-Ns based on the scenario-by-scenario comparisons.

- When Strategy = #1, the SD-Senior-N produced under each mechanism had the following order: SD (DA) > TM > CP ≥ BM; the higher the number of choices, the higher the SD-Senior-N under each mechanism.
- When Strategy = # 2 or #3 and the number of choices > 6, the relationships of the SD-Senior-Ns produced under SD (DA) and TM were mixed, although their differences were less than 0.32 points or 1.53% of the SD-Senior-N under SD, Cohen's *d* = .28.
- When Strategy = # 2 or #3 and the number of choices > 6, the SD-Senior-Ns under TM, CP, and BM were in the following order: TM > CP > BM.
- When Strategy = # 2 or #3 and the number of choices < 6, TM produced the lowest SD-Senior-Ns among all mechanisms.

Following the logic to calculate the values in Table 12, I estimated the maximum increase and maximum decrease in the original SD-Senior (SD-Senior-N produced by SD) caused by a mechanism change from SD to the other mechanisms and compiled the results in Table 14.

Table 14

The estimated maximum increase and decrease in the original SD-Senior resulting from the mechanism change from SD to the other mechanisms without the free tuition policy

Number of choices	To BM % Cohen's *d*	To TM % Cohen's *d*	To CP % Cohen's *d*
8	[0.00%, -24.13%] --, -7.52	[4.00%, -3.54%] 0.85, 0.80	[0.88%, -5.45%] 0.18, -1.09
10	[0.00%, -23.29%] --, -10.74	[3.62%, -3.72%] 0.66, 0.85	[1.65%, -4.98%] 0.29, -1.00

Note. The first percentage in the square brackets represents the maximum percentage increase in the original SD-Senior (SD-Senior-N under SD) due to the change from SD to the new mechanism, and the second percentage in the square brackets denotes the maximum percentage decrease. The corresponding Cohen's *d* is presented right below each percentage. * No Cohen's *d* was calculated for this percentage because BM did not increase the original SD-Freshman in any scenario within this category.

Like Table 12, Table 14 shows that under the same number of choices allowed, BM, followed by CP, and then TM, could offer the most decrease and least increase in the original SD-Senor. Table 14 also shows that while TM could reduce the original SD-Senior, TM could also increase the original SD-Senior, depending on students' behaviors before and after the policy change. Since the maximum increase in the original SD-Senior by TM could reach 3.62% or higher, Cohen's $d \geq 0.66$, this risk of increase should not be taken lightly.

Like SD-Freshman-N, SD-Senior-N emerged from the interactions among the matching mechanism, the number of choices, and student's strategies. SD-Senior-N measures the size of the inequality of school quality. However, it does not inform whether the average senior score moves upward after the change of the assignment mechanism. The comparison of the mean senior scores under different mechanisms in the following paragraphs could answer the question.

Mean Senior Score. I first calculated the mean senior score of each of the first 31 generations assigned to the high schools in each of the 30 simulation runs. I then calculated the average of these mean senior scores in each scenario. The results are presented in Table 15 and Figure 9.

Table 15

The mean scores of the high school seniors enrolled without the free-tuition policy

Number of choices	Strategy	Sort-extra-choice	Scenario #	SD	BM	DA	TM	CP
2	1	FALSE	1	59.38*	58.96*	59.40*	59.30	58.94*
		TRUE	2	59.40	58.96*	59.39	59.35	58.96*
	2	FALSE	3	61.17*	60.28*	61.78*	56.86	60.67*
		TRUE	4	61.35*	59.98*	61.68*	56.77	60.10*
	3	FALSE	5	59.94*	60.05*	59.75*	55.77	60.50*
		TRUE	6	60.40*	59.09*	60.13*	55.80	59.73*
4	1	FALSE	7	56.63	55.84*	56.64	56.59	56.32*
		TRUE	8	56.62*	55.83*	56.64*	56.49	56.38*
	2	FALSE	9	54.82*	56.37*	54.88	55.07	55.81*
		TRUE	10	54.91	55.60	54.87*	55.12	55.66*
	3	FALSE	11	55.15	56.27*	55.00	55.11	55.60*
		TRUE	12	54.89	55.77*	54.93	54.83	55.59*
6	1	FALSE	13	55.30	55.08*	55.45*	55.34	55.49*
		TRUE	14	55.34	55.09*	55.38	55.35	55.41
	2	FALSE	15	55.60*	54.97*	55.60*	55.39	54.93*
		TRUE	16	55.22*	54.03*	55.28*	54.86	55.16*
	3	FALSE	17	55.40	54.84*	55.43	55.39	55.00*
		TRUE	18	54.82	54.18*	54.86	54.84	55.04*

(Continued)

Number of choices	Strategy	Sort-extra-choice	Scenario #	SD	BM	DA	TM	CP
8	1	FALSE	19	54.77	55.50*	54.66	54.71	54.77*
		TRUE	20	54.78*	55.49*	54.79*	54.72	54.80*
	2	FALSE	21	55.03	54.42*	55.08	55.07	55.00
		TRUE	22	54.51*	54.44	54.49*	54.43	55.13*
	3	FALSE	23	55.17*	54.59*	55.17*	54.91	55.11*
		TRUE	24	54.42	54.46	54.43	54.44	55.04*
10	1	FALSE	25	54.40*	54.50	54.40*	54.47	54.45
		TRUE	26	54.42	54.46	54.42	54.43	54.46*
	2	FALSE	27	55.03	54.76*	55.03	54.95	55.10
		TRUE	28	54.36*	54.52*	54.36*	54.45	54.87*
	3	FALSE	29	54.98	54.67*	54.98	54.89	55.04
		TRUE	30	54.35*	54.49*	54.35*	54.40	54.72*

Note. * denotes that the value is significantly different from that under the column of TM in the same scenario (the same row), $p < .01$.

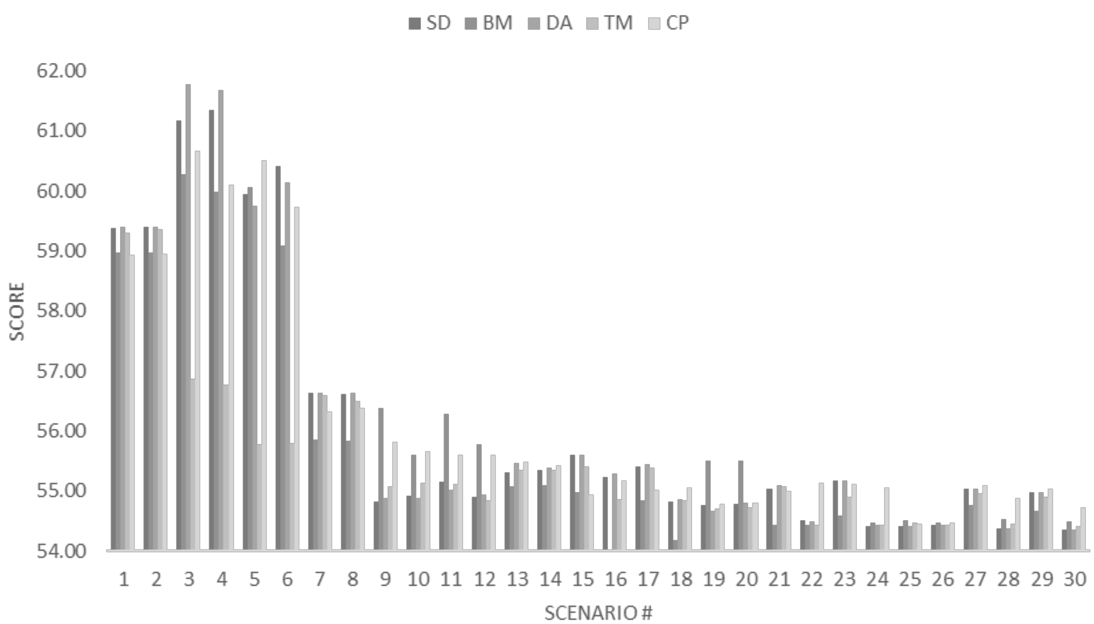

Figure 9. The average mean scores of the high school seniors enrolled without the free-tuition policy. The scenario # in this figure corresponds to that in Table 15.

In each school, only the scores of the students whose incomes were within a range of their cohort's mean income would be adjusted to move toward their cohort's mean scores. The income distribution and score distribution of a cohort determined how the scores were adjusted and thus the mean senior scores of that cohort. Since different mechanisms, behaviors, and the numbers of choices had different influences on freshmen's income and score distributions, the mean senior scores under different mechanisms were different in most scenarios as shown in Figure 9.

Like the graphs of SD-Freshman-N and SD-Senior-N, Figure 9 shows a spike of senior scores in the scenarios where the number of choices was small, and the scores became relatively stable when the number of choices ≥ 8. The relationships of the mean senior scores under different mechanisms were mixed. No single mechanism could outperform the other in all cases, although CP produced higher senior scores than the others in two third of the scenarios where the number of choices > 2. Nevertheless, when the number of choices > 2, the differences of the mean senior scores under CP and SD (DA) were no more than 1.85%, Cohen's *d* = 0.23; under TM and SD (DA), no more than 0.65%, Cohen's *d* = 0.17. When the number of choices > 6, the differences between CP and SD (DA), as well as TM and SD (DA), were even smaller. From this point of view, TM did not seem to be effective in improving senior scores; it could even reduce the original mean senior scores in some scenarios.

Following the same logic to calculate the best-worst estimates for SD-Freshman-N and SD-Senior-N, I calculated the best-worst estimates of the senior scores caused by a change from SD to the other mechanisms without the free-tuition policy as shown in Table 16. Here, we also see that with a mechanism change from SD to TM or BM, the range of the best-worst estimates shrank when the number of choices changed from 8 to 10. While the scenario-by-scenario comparison suggested that TM is likely to reduce senior scores, the best-worst estimates suggested that TM is also possible to increase the mean senior scores by more than 1%, Cohen's *d* = 0.62, depending on how students react to the policy change.

Table 16

The estimated maximum increase and decrease in the original senior score resulting from a mechanism change from SD to the other mechanisms without the free tuition policy

Number of choices	To BM % Cohen's *d*	To TM % Cohen's *d*	To CP % Cohen's *d*
8	[1.99%, -1.35%] 2.55, -0.45	[1.20%, -1.34%] 0.62, -0.59	[1.30%, -0.71%] 0.59, -0.31
10	[0.76%, -1.04%] 0.54, -0.45	[1.10%, -1.15%] 0.62, -0.51	[1.37%, -1.06] 0.59, -0.46

Note. The first percentage in the square brackets represents the maximum increase in the original senior score (senior score under SD) due to the change from SD to the new mechanism, and the second percentage in the square brackets represents the maximum percentage decrease. The corresponding Cohen's *d* is presented right below each percentage.

Impacts on Different Student Groups

OECD (2010) suggested that a policy aiming at reducing educational inequality should be able to help lower-performing and disadvantaged students. A competitive school admission system where students are prioritized mainly by their scores usually results in sorting if it employs a stable matching mechanism. Sorting means that the top-performing students go to the top-ranked schools, and the bottom-performing students go to the bottom-ranked schools. Many studies since the 1966 Coleman's report have shown that disadvantaged students benefit more in a mixed environment while advantaged students' performance is hardly affected by peers with lower socioeconomic statuses (Coleman et al., 1966; Carman & Zhang, 2012; Van de Werfhorst & Mijs, 2010). Therefore, to help disadvantaged students, a student-assignment mechanism may be desired to mix students and assign the disadvantaged students to higher ranked schools. SD-Freshman tells us the degree of dispersion of the schools' mean family incomes. It does not say much about how the students in different groups are assigned. The per-group information helps us understand the how and is presented in Tables 17 – 22 and Figures 10 – 12.

Table 17

The average percentage of the top 10% performing students being assigned to their top choices (top-choice match rate) without the free-tuition policy

Number of choices	Strategy	Extra-in-order	Scenario #	SD	BM	DA	TM	CP
2	1	FALSE	1	19.84	19.94	19.99	19.84	20.00
		TRUE	2	20.07	19.84	19.89	19.90	20.03
	2	FALSE	3	95.87	94.85	96.22	99.42	95.48
		TRUE	4	93.73	96.90	94.53	99.45	95.57
	3	FALSE	5	98.32	98.35	98.61	99.63	98.24
		TRUE	6	99.22	99.06	98.19	99.57	96.46
4	1	FALSE	7	40.11	39.97	39.85	40.23	40.36
		TRUE	8	40.07	40.11	39.93	40.15	40.09
	2	FALSE	9	99.53	89.57	99.68	99.53	96.48
		TRUE	10	99.85	96.09	99.75	99.56	97.30
	3	FALSE	11	99.87	99.18	99.69	99.60	98.74
		TRUE	12	99.99	98.29	99.98	99.60	99.00

(Continued)

Number of choices	Strategy	Extra-in-order	Scenario #	SD	BM	DA	TM	CP
6	1	FALSE	13	59.97	60.12	60.03	60.14	59.90
		TRUE	14	60.37	60.02	59.82	59.95	60.21
	2	FALSE	15	99.94	95.65	99.97	99.54	99.93
		TRUE	16	99.94	96.51	99.95	99.57	99.96
	3	FALSE	17	99.99	99.67	100.00	99.60	99.98
		TRUE	18	100.00	98.94	100.00	99.61	100.00
8	1	FALSE	19	80.34	80.03	80.20	80.00	80.12
		TRUE	20	80.00	80.10	80.24	80.37	80.11
	2	FALSE	21	99.95	99.80	99.96	99.55	99.97
		TRUE	22	99.85	99.82	99.85	99.58	99.95
	3	FALSE	23	100.00	99.84	100.00	99.64	100.00
		TRUE	24	100.00	99.98	100.00	99.64	100.00
10	1	FALSE	25	100.00	100.00	100.00	99.43	100.00
		TRUE	26	100.00	100.00	100.00	99.48	100.00
	2	FALSE	27	99.96	99.95	99.96	99.55	99.96
		TRUE	28	100.00	100.00	100.00	99.45	99.94
	3	FALSE	29	100.00	100.00	100.00	99.58	100.00
		TRUE	30	100.00	100.00	100.00	99.43	100.00

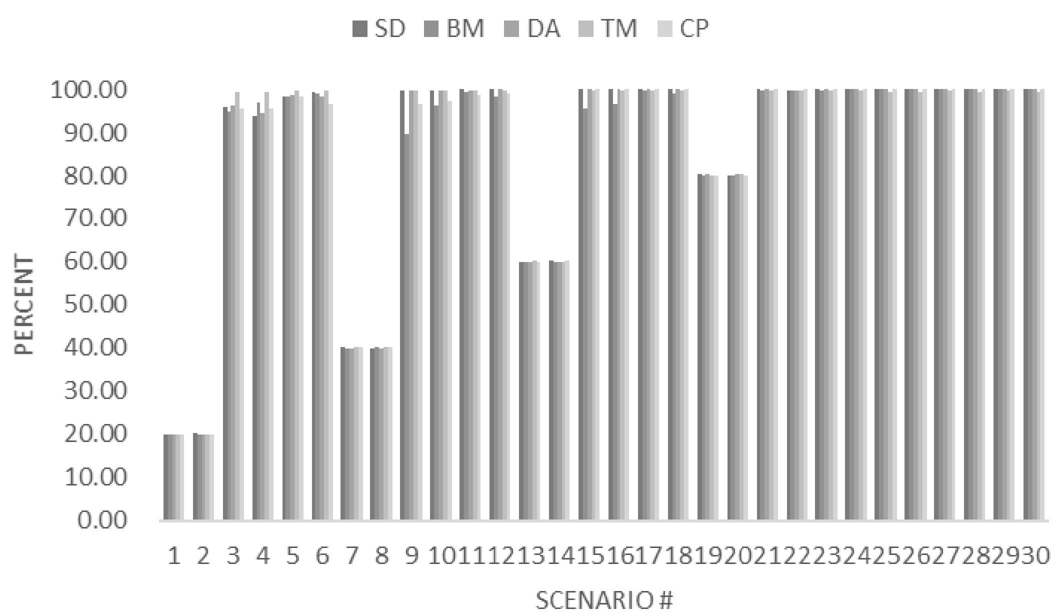

Figure 10. The percentage of the top 10% performing students being assigned to their top choices. The scenario # in this figure corresponds to that in Table 17.

Impacts on Top 10% Performing Students. In a competitive admission system like the Taipei School District, there are few objections against increasing disadvantaged students' educational opportunity, but there are serious objections against achieving that by creating justified envy for top performing students (Zhang, 2016). Policymakers seeking to change the system from sorting to mixing usually take this public opinion into account and will need the information about how the new policy affects the assignments of the top performing students. Table 17 and Figure 10 show the percentage of the top 10% performing students who were assigned to their top choice schools (top-choice match rate) under each mechanism and reveal the followings.

- When Strategy = #2 or #3, TM could reach a top-choice match rate of more than 99% regardless of the number of choices.
- When Strategy = #2 or #3, to reach a top-choice match rate of more than 99%, SD (DA) needed the number of choices to be higher than 2; CP, higher than 4; BM, higher than 6.
- If Strategy = 1, the greater the number of choices, the greater the top-choice match rate under all mechanisms.
- When the number of choices = 10, all mechanisms could produce a top-choice match rate of more than 99% regardless of the strategy used; if students were truth tellers (reporting their preference lists as their choice lists), the match rates could even reach 100% except for those under TM.

I defined the top 10% performing students as the students whose total raw scores are in the top decile. Since the model randomly breaks a tie, a top 10% performing student may be rejected by his top choice simply because the seats are full. Therefore, a top-choice match rate slightly less than 100% but more than 99.9% should not be interpreted as the occurrence of justified envy. However, the top-choice match rates under TM were no more than 99.64%, which seemed to be lower than the match rate resulting from mere random tie breaking. The reason might be TM's feature of coarse prioritization. Unlike the other mechanisms which prioritize students based on their raw scores, TM prioritizes students based on their total coarse-grained scores converted from their raw scores (see the subsection of the Taipei mechanism in Chapter 2 and the subsection of details in Chapter 3 for the conversion rules in the program.) In this prioritization process, Student *i* with higher total raw score may be prioritized lower than Student *j* with lower total raw score and thus lose the seat in his top-choice school to Student *j*. Therefore, the top 10% performing students under TM will always face the risk of justified envy, and the top-choice match rate under TM is hard to reach 100%.

Table 18

The average preference index (PI) of the students with the bottom 10% family income enrolled without the free-tuition policy

Number of choices	Strategy	Extra-in-order	Scenario #	SD	BM	DA	TM	CP
2	1	FALSE	1	(0.74)	(0.39)	(0.76)	(0.70)	(0.38)
		TRUE	2	(0.76)	(0.38)	(0.75)	(0.71)	(0.38)
	2	FALSE	3	(1.64)	(0.71)	(1.67)	(1.42)	(0.79)
		TRUE	4	(1.57)	(0.90)	(1.59)	(1.35)	(0.76)
	3	FALSE	5	(1.61)	(0.81)	(1.53)	(1.26)	(0.87)
		TRUE	6	(1.64)	(0.80)	(1.61)	(1.20)	(0.82)

(Continued)

Number of choices	Strategy	Extra-in-order	Scenario #	SD	BM	DA	TM	CP
4	1	FALSE	7	(0.51)	0.84	(0.50)	(0.44)	0.11
		TRUE	8	(0.50)	0.85	(0.51)	(0.42)	0.08
	2	FALSE	9	(0.70)	0.72	(0.67)	(0.51)	(0.38)
		TRUE	10	(0.65)	0.82	(0.63)	(0.41)	(0.43)
	3	FALSE	11	(0.72)	0.60	(0.68)	(0.37)	(0.46)
		TRUE	12	(0.63)	0.75	(0.63)	(0.16)	(0.47)
6	1	FALSE	13	(0.33)	1.57	(0.33)	(0.20)	0.12
		TRUE	14	(0.32)	1.57	(0.32)	(0.19)	0.12
	2	FALSE	15	(0.07)	1.56	(0.09)	(0.16)	0.38
		TRUE	16	(0.27)	1.97	(0.31)	(0.10)	0.35
	3	FALSE	17	(0.12)	1.54	(0.13)	(0.27)	0.27
		TRUE	18	(0.17)	1.98	(0.17)	(0.09)	0.29
8	1	FALSE	19	(0.13)	1.67	(0.12)	(0.03)	0.21
		TRUE	20	(0.13)	1.72	(0.12)	(0.04)	0.22
	2	FALSE	21	0.03	1.71	(0.00)	0.08	0.28
		TRUE	22	(0.05)	1.99	(0.05)	0.05	0.27
	3	FALSE	23	(0.04)	1.75	(0.02)	(0.06)	0.16
		TRUE	24	(0.04)	1.89	(0.04)	0.05	0.11
10	1	FALSE	25	0.00	2.88	0.00	0.05	0.50
		TRUE	26	0.00	2.89	0.00	0.05	0.50
	2	FALSE	27	0.11	1.72	0.11	0.23	0.26
		TRUE	28	0.00	2.88	0.00	0.05	0.25
	3	FALSE	29	0.11	1.69	0.11	0.08	0.22
		TRUE	30	0.00	2.89	0.00	0.05	0.20

Note. Values in parentheses are negative values.

Impacts on Students with Bottom 10% Family Income. Table 18 and Figure 11 show the average PI of the students with the bottom 10% family income in each scenario. As explained in the subsection of details in Chapter 3, a positive PI means that a student is assigned to a school that he prefers more to his baseline school and thus a gain to the student. In a complete sorting system, the bottom-performing students will be assigned to the bottom-ranked schools. Since students' income and scores are usually positively correlated, most bottom-performing students are disadvantaged students

(Pinkovskiy & Sala-i-Martin, 2009). Therefore, a mechanism designed to help disadvantaged students should generate positive PIs for the students with the bottom 10% family income.

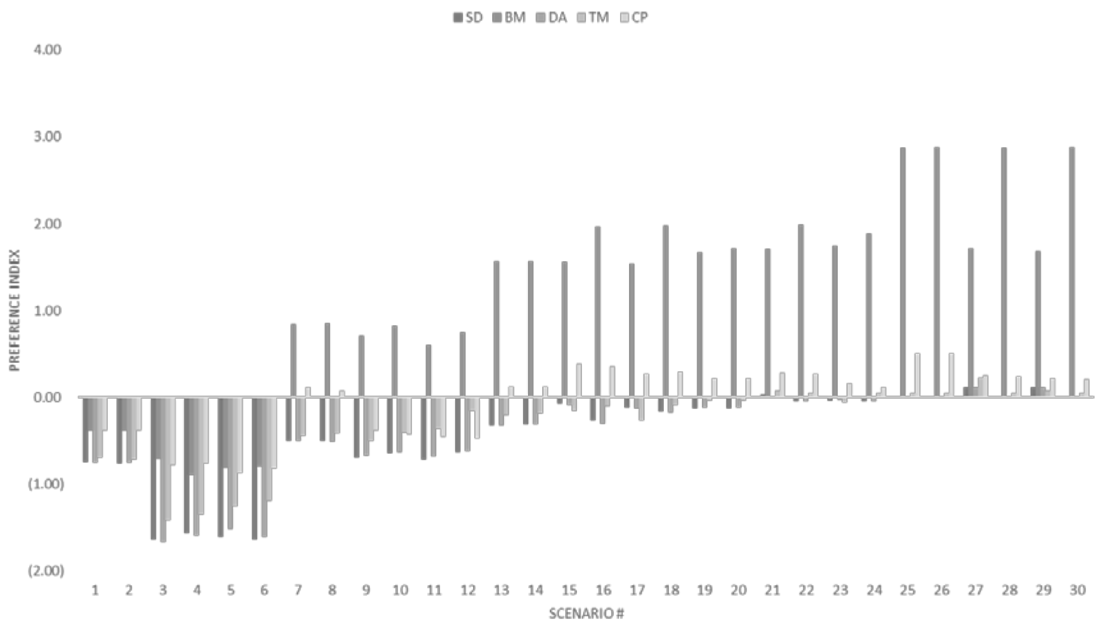

Figure 11. The average preference index (PI) of the students with the bottom 10% family income enrolled without the free-tuition policy. The scenario # in this figure corresponds to that in Table 18.

Table 18 and Figure 11 show the following simulated properties of TM compared with the other four mechanisms.

- TM could generate positive PIs for students with the bottom 10% income regardless of student's strategy used only when the number of choices = 10; BM, when the number of choices > 2; CP, when the number of choices > 4.
- Students with the bottom 10% income could hardly have positive PIs under SD (DA).

- When the number of choices > 6, BM could generate an average PI between 1.67 and 2.88 for students with the bottom 10% income; CP, between 0.11 and 0.50; TM, between -0.06 and 0.23; SD (DA), between -0.12 and 0.11.
- When the number of choices <6, TM generated less negative PIs for students with bottom 10% income than SD (DA).

From Figure 11, one can see that only BM could generate substantial positive PIs for the students with the bottom 10% family income; all other mechanisms generated little or no benefit for those students. However, the number of choices must be high for the benefits from BM to be realized.

Table 19

The average preference index (PI) of the students with the bottom quartile family income enrolled without the free-tuition policy

Number of choices	Strategy	Extra-in-order	Scenario #	SD25	BM25	DA25	TM25	CP25
2	1	FALSE	1	(0.65)	(0.54)	(0.66)	(0.62)	(0.52)
		TRUE	2	(0.66)	(0.53)	(0.66)	(0.62)	(0.52)
	2	FALSE	3	(2.26)	(1.58)	(2.30)	(1.76)	(1.66)
		TRUE	4	(2.22)	(1.78)	(2.23)	(1.69)	(1.64)
	3	FALSE	5	(2.20)	(1.67)	(2.12)	(1.45)	(1.72)
		TRUE	6	(2.21)	(1.67)	(2.18)	(1.41)	(1.66)
4	1	FALSE	7	(0.50)	0.47	(0.50)	(0.45)	(0.12)
		TRUE	8	(0.49)	0.49	(0.49)	(0.43)	(0.12)
	2	FALSE	9	(1.00)	(0.21)	(0.98)	(0.57)	(1.03)
		TRUE	10	(0.91)	(0.17)	(0.91)	(0.42)	(1.07)
	3	FALSE	11	(0.95)	(0.28)	(0.93)	(0.34)	(1.02)
		TRUE	12	(0.79)	(0.20)	(0.79)	(0.13)	(1.02)

(Continued)

Number of choices	Strategy	Extra-in-order	Scenario #	SD25	BM25	DA25	TM25	CP25
6	1	FALSE	13	(0.26)	1.12	(0.28)	(0.20)	0.03
		TRUE	14	(0.26)	1.12	(0.26)	(0.18)	0.04
	2	FALSE	15	(0.21)	0.87	(0.23)	(0.28)	(0.09)
		TRUE	16	(0.17)	1.11	(0.20)	(0.07)	(0.10)
	3	FALSE	17	(0.25)	0.84	(0.27)	(0.26)	(0.15)
		TRUE	18	(0.11)	1.14	(0.12)	(0.07)	(0.10)
8	1	FALSE	19	(0.08)	1.41	(0.08)	(0.01)	0.18
		TRUE	20	(0.09)	1.45	(0.09)	(0.01)	0.17
	2	FALSE	21	(0.03)	1.22	(0.04)	(0.06)	0.09
		TRUE	22	(0.03)	1.63	(0.03)	0.05	0.08
	3	FALSE	23	(0.09)	1.24	(0.08)	(0.09)	0.00
		TRUE	24	(0.02)	1.54	(0.02)	0.05	(0.01)
10	1	FALSE	25	0.00	2.31	0.00	0.06	0.35
		TRUE	26	0.00	2.31	0.00	0.06	0.35
	2	FALSE	27	0.04	1.25	0.04	0.04	0.11
		TRUE	28	0.00	2.31	0.00	0.06	0.11
	3	FALSE	29	0.03	1.21	0.03	(0.04)	0.08
		TRUE	30	0.00	2.31	0.00	0.06	0.06

Note. Values in parentheses are negative values.

Table 20

The average preference index (PI) of the students with the second quartile family income without the free-tuition policy

Number of choices	Strategy	Extra-in-order	Scenario #	SD50	BM50	DA50	TM50	CP50
2	1	FALSE	1	(0.37)	(0.67)	(0.38)	(0.38)	(0.66)
		TRUE	2	(0.39)	(0.67)	(0.38)	(0.39)	(0.67)
	2	FALSE	3	(3.01)	(3.13)	(3.08)	(1.93)	(3.21)
		TRUE	4	(3.09)	(3.22)	(3.05)	(1.90)	(3.15)
	3	FALSE	5	(2.79)	(3.11)	(2.75)	(1.44)	(3.16)
		TRUE	6	(2.79)	(3.06)	(2.79)	(1.42)	(3.05)

(Continued)

Number of choices	Strategy	Extra-in-order	Scenario #	SD50	BM50	DA50	TM50	CP50
4	1	FALSE	7	(0.42)	(0.66)	(0.42)	(0.43)	(0.62)
		TRUE	8	(0.42)	(0.65)	(0.42)	(0.41)	(0.62)
	2	FALSE	9	(1.07)	(1.90)	(1.06)	(0.49)	(1.76)
		TRUE	10	(1.03)	(1.94)	(1.03)	(0.40)	(1.72)
	3	FALSE	11	(0.96)	(1.77)	(0.99)	(0.26)	(1.51)
		TRUE	12	(0.78)	(1.87)	(0.77)	(0.13)	(1.52)
6	1	FALSE	13	(0.19)	(0.33)	(0.20)	(0.20)	(0.26)
		TRUE	14	(0.19)	(0.31)	(0.19)	(0.19)	(0.25)
	2	FALSE	15	(0.49)	(0.79)	(0.48)	(0.35)	(0.69)
		TRUE	16	(0.13)	(0.64)	(0.15)	(0.05)	(0.66)
	3	FALSE	17	(0.44)	(0.76)	(0.44)	(0.23)	(0.66)
		TRUE	18	(0.06)	(0.62)	(0.07)	(0.05)	(0.60)
8	1	FALSE	19	(0.04)	0.33	(0.04)	(0.01)	0.03
		TRUE	20	(0.05)	0.35	(0.04)	(0.01)	0.03
	2	FALSE	21	(0.30)	(0.03)	(0.31)	(0.27)	(0.33)
		TRUE	22	0.01	0.47	0.01	0.03	(0.35)
	3	FALSE	23	(0.32)	(0.08)	(0.33)	(0.16)	(0.35)
		TRUE	24	0.00	0.44	0.00	0.02	(0.31)
10	1	FALSE	25	0.00	0.79	0.00	0.05	0.05
		TRUE	26	0.00	0.79	0.00	0.05	0.05
	2	FALSE	27	(0.29)	0.11	(0.29)	(0.23)	(0.32)
		TRUE	28	0.00	0.80	0.00	0.05	(0.24)
	3	FALSE	29	(0.28)	0.08	(0.28)	(0.18)	(0.29)
		TRUE	30	0.00	0.78	0.00	0.05	(0.22)

Note. Values in parentheses are negative values.

Table 21

The average preference index (PI) of the students with the third quartile family income without the free-tuition policy

Number of choices	Strategy	Extra-in-order	Scenario #	SD75	BM75	DA75	TM75	CP75
2	1	FALSE	1	(0.49)	(0.65)	(0.50)	(0.53)	(0.65)
		TRUE	2	(0.51)	(0.65)	(0.51)	(0.53)	(0.67)
	2	FALSE	3	(2.21)	(2.85)	(2.30)	(1.27)	(2.93)
		TRUE	4	(2.36)	(2.81)	(2.24)	(1.25)	(2.83)
	3	FALSE	5	(1.94)	(2.71)	(1.92)	(0.85)	(2.75)
		TRUE	6	(1.92)	(2.65)	(1.98)	(0.85)	(2.64)
4	1	FALSE	7	(0.34)	(0.81)	(0.34)	(0.35)	(0.54)
		TRUE	8	(0.33)	(0.81)	(0.34)	(0.33)	(0.55)
	2	FALSE	9	(0.61)	(2.02)	(0.61)	(0.31)	(1.29)
		TRUE	10	(0.61)	(2.04)	(0.61)	(0.29)	(1.20)
	3	FALSE	11	(0.55)	(1.70)	(0.56)	(0.18)	(1.00)
		TRUE	12	(0.44)	(1.83)	(0.43)	(0.11)	(1.02)
6	1	FALSE	13	(0.17)	(0.77)	(0.18)	(0.19)	(0.30)
		TRUE	14	(0.17)	(0.77)	(0.17)	(0.19)	(0.29)
	2	FALSE	15	(0.38)	(1.45)	(0.37)	(0.26)	(0.61)
		TRUE	16	(0.20)	(1.38)	(0.21)	(0.07)	(0.60)
	3	FALSE	17	(0.31)	(1.21)	(0.31)	(0.18)	(0.50)
		TRUE	18	(0.08)	(1.30)	(0.09)	(0.05)	(0.51)
8	1	FALSE	19	(0.07)	(0.67)	(0.07)	(0.08)	(0.13)
		TRUE	20	(0.07)	(0.70)	(0.08)	(0.08)	(0.13)
	2	FALSE	21	(0.30)	(1.04)	(0.30)	(0.24)	(0.33)
		TRUE	22	(0.02)	(0.82)	(0.02)	(0.02)	(0.35)
	3	FALSE	23	(0.29)	(0.97)	(0.31)	(0.16)	(0.32)
		TRUE	24	(0.01)	(0.75)	(0.01)	(0.01)	(0.28)
10	1	FALSE	25	0.00	(0.88)	0.00	(0.01)	(0.05)
		TRUE	26	0.00	(0.88)	0.00	(0.02)	(0.05)
	2	FALSE	27	(0.31)	(0.72)	(0.31)	(0.22)	(0.33)
		TRUE	28	0.00	(0.88)	0.00	(0.02)	(0.27)
	3	FALSE	29	(0.27)	(0.67)	(0.27)	(0.17)	(0.28)
		TRUE	30	0.00	(0.88)	0.00	(0.02)	(0.20)

Note. Values in parentheses are negative values.

Table 22

The average preference index (PI) of the students with the top quartile family income without the free-tuition policy

Number of choices	Strategy	Extra-in-order	Scenario #	SD100	BM100	DA100	TM100	CP100
2	1	FALSE	1	(1.80)	(1.80)	(1.80)	(1.79)	(1.80)
		TRUE	2	(1.79)	(1.81)	(1.79)	(1.80)	(1.80)
	2	FALSE	3	(0.70)	(1.16)	(0.75)	(0.35)	(1.20)
		TRUE	4	(0.77)	(1.08)	(0.72)	(0.34)	(1.13)
	3	FALSE	5	(0.55)	(1.00)	(0.55)	(0.21)	(1.01)
		TRUE	6	(0.54)	(0.97)	(0.58)	(0.20)	(0.99)
4	1	FALSE	7	(0.69)	(0.83)	(0.69)	(0.70)	(0.71)
		TRUE	8	(0.69)	(0.84)	(0.69)	(0.69)	(0.72)
	2	FALSE	9	(0.18)	(0.89)	(0.17)	(0.10)	(0.41)
		TRUE	10	(0.18)	(0.82)	(0.18)	(0.10)	(0.37)
	3	FALSE	11	(0.17)	(0.63)	(0.16)	(0.06)	(0.28)
		TRUE	12	(0.12)	(0.65)	(0.12)	(0.03)	(0.29)
6	1	FALSE	13	(0.30)	(0.62)	(0.31)	(0.32)	(0.32)
		TRUE	14	(0.30)	(0.62)	(0.30)	(0.32)	(0.32)
	2	FALSE	15	(0.15)	(0.75)	(0.14)	(0.10)	(0.20)
		TRUE	16	(0.09)	(0.78)	(0.10)	(0.03)	(0.21)
	3	FALSE	17	(0.10)	(0.52)	(0.10)	(0.07)	(0.15)
		TRUE	18	(0.04)	(0.67)	(0.05)	(0.02)	(0.16)
8	1	FALSE	19	(0.12)	(0.65)	(0.12)	(0.13)	(0.14)
		TRUE	20	(0.12)	(0.66)	(0.12)	(0.14)	(0.14)
	2	FALSE	21	(0.11)	(0.77)	(0.12)	(0.09)	(0.12)
		TRUE	22	(0.04)	(0.89)	(0.04)	(0.02)	(0.14)
	3	FALSE	23	(0.11)	(0.62)	(0.11)	(0.06)	(0.11)
		TRUE	24	(0.02)	(0.76)	(0.02)	(0.01)	(0.10)
10	1	FALSE	25	0.00	(0.79)	0.00	(0.01)	(0.01)
		TRUE	26	0.00	(0.79)	0.00	(0.01)	(0.01)
	2	FALSE	27	(0.12)	(0.62)	(0.12)	(0.09)	(0.13)
		TRUE	28	0.00	(0.79)	0.00	(0.01)	(0.10)
	3	FALSE	29	(0.10)	(0.51)	(0.10)	(0.06)	(0.10)
		TRUE	30	0.00	(0.79)	0.00	(0.01)	(0.07)

Note. Values in parentheses are negative values.

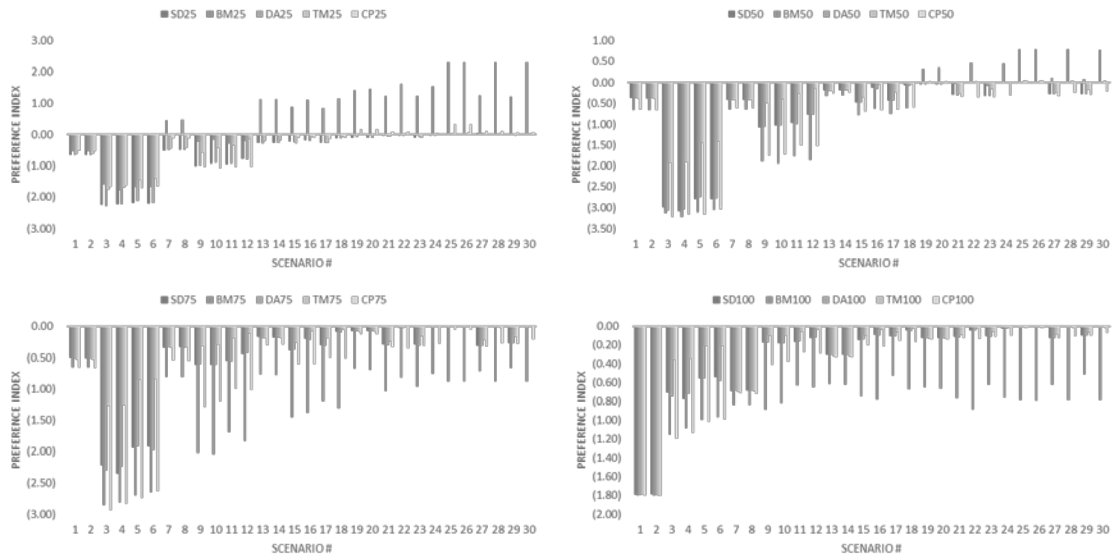

Figure 12. The average preference indexes (PIs) of the students in each income quartile enrolled without the free-tuition policy. The upper left is the PIs of the students in the first quartile; the upper right, the second quartile; the lower left, the third quartile; the lower right, the fourth quartile. The scenario # in this figure corresponds to that in Tables 19-22.

Impacts on Students in Each Income Quartile. Tables 19-22 and Figure 12 show the average PIs of the students in the four income quartiles and reveal the following properties of TM and the other mechanisms.

- Justified envy (negative PIs) always occurred to the students in the top and third income quartiles regardless of the mechanism or scenario except for the scenarios where mechanism = SD (DA), the number of choices = 10, and *sort-extra-choice* = True; in this exceptional case, the PI = 0.

- TM generated negative PIs for all students except that TM generated small positive PIs (< 0.06) for the students in the bottom and second income quartiles in some scenarios where the number of choices ≥ 8.
- Under SD (DA), students could only receive negative or zero PIs except that the students in the bottom and second income quartiles would have a small positive PI (< .04) in some scenarios where the number of choices ≥ 8.
- BM generated positive PIs for the students in the bottom income quartile in all scenarios where the number of choices ≥ 6 and for the students in the second income quartile in all scenarios where the number of choices = 10.
- CP generated positive PIs for the students in the bottom income quartiles in all scenarios where the number of choices = 10 and in most scenarios where the number of choices = 8.
- CP generated positive PIs for the students in the second income quartile only when the number of choices ≥ 8 and Strategy = #1.
- The positive PIs received by students were all less than 1 (less than one rank) except for the PIs received by the students in the bottom income quartile under BM in cases where the number of choices ≥ 8 and some cases where the number of choices = 6.
- When the number of choices = 10 and the mechanism = TM, BM, or CP, the students in the third income quartile had larger negative PIs than those in the top income quartile.

- When the number of choices > 6, the PIs under SD (DA) were between 0.04 and -0.32; under TM, between 0.06 and -0.27; under CP, between 0.18 and -0.35.
- When the number of choices > 6 and under BM, the PIs of the bottom-income-quartile students were between 1.20 and 2.31; the PIs of the second-income-quartile students, between 0.80 and -0.08; the PIs of the other students, between -0.51 and -0.89.

The above results showed us how the students were assigned. If a mechanism aiming at equality should benefit the more disadvantaged students, then BM is the best among all mechanisms because BM produced the highest positive PIs for those in the bottom income quartile. However, this benefit occurred only when the number of choices ≥ 6. When the number of choices was not high enough, many students, regardless of their groups, would end up having no school to attend because of their "bad" choices, which could be evidenced by the low match rates and substantially negative PIs in the cases where the number of choices was low as seen in Figure 7 and Figure 12, respectively. When the number of choices ≥ 8, BM could also generate the largest positive PIs or the least negative PIs for the second-income-quartile students and produced the largest negative PIs for the top- and third-income-quartile students. This finding suggests that BM mixes students the most, which might be the reason why BM had the lowest SD-Freshman-Ns when the number of choices ≥ 8. When the number of choices > 6, CP, by and large, produced more negative PIs for the top- and third-income quartile students than TM and SD (DA), which might also be the reason why CP had a slightly lower average SD-Freshman-Ns than TM and SD (DA). When the number of choices > 6, the relationships of the PIs in each quartile under TM and SD (DA) were mixed. However, their values were very close, which might be the reason why SD-Freshman-Ds under TM and SD (DA) were close.

Effects of TM with the Free-tuition policy

This section presents the data simulated with the free-tuition policy. I arranged this section in the same way as the previous section to have a better comparison of the outcomes with and without this policy.

Overall Effect on Educational Opportunity

SD-Freshman-Y. I defined SD-Freshman-Y as the SD-Freshman with the free-tuition policy. Table 23 and Figure 13 show the average SD-Freshman-Y across all the simulation steps in the 30 simulation runs in each scenario.

Table 23

The average standard deviation of the mean freshman family income under the free-tuition policy (SD-Freshman-Y)

Number of choices	Strategy	Extra-in-order	Scenario #	SD	BM	DA	TM	CP
2	1	FALSE	1	$327,710	$314,351	$327,890	$324,949	$314,063
		TRUE	2	$327,056†	$314,986	$326,390	$324,674	$313,398
	2	FALSE	3	$833,206	$827,860	$818,280	$735,152	$841,903
		TRUE	4	$817,636	$829,770	$809,773	$738,503	$835,469
	3	FALSE	5	$795,739	$804,619	$796,459	$697,264	$814,662
		TRUE	6	$806,719	$810,684	$792,117	$704,203	$799,248
4	1	FALSE	7	$502,868	$455,527	$502,773	$499,008	$489,158
		TRUE	8	$502,423	$455,205	$502,164	$499,536	$488,754
	2	FALSE	9	$648,400	$734,276	$656,049	$626,650	$743,329
		TRUE	10	$648,728	$791,605	$641,767	$618,485	$747,045
	3	FALSE	11	$631,826	$725,813	$639,992	$613,047*	$727,410
		TRUE	12	$631,019	$786,105	$632,518	$611,652*	$728,609

(Continued)

Number of choices	Strategy	Extra-in-order	Scenario #	SD	BM	DA	TM	CP
6	1	FALSE	13	$557,758	$475,594	$557,569	$554,080	$545,080
		TRUE	14	$558,334	$475,636	$558,049	$554,702	$545,812
	2	FALSE	15	$602,448	$652,995	$602,039	$605,337*	$609,972
		TRUE	16	$603,820	$714,579	$603,743	$609,219*	$609,150†
	3	FALSE	17	$602,984	$646,793	$603,149	$607,954*	$609,734†
		TRUE	18	$606,900	$705,442	$606,872	$611,182*	$608,849†
8	1	FALSE	19	$585,916	$491,894	$585,794	$582,426	$579,040
		TRUE	20	$587,554	$493,994	$587,332	$583,622	$579,919
	2	FALSE	21	$594,665	$582,670	$594,946	$597,339*	$591,458
		TRUE	22	$600,876	$545,027	$600,888	$601,875	$590,970
	3	FALSE	23	$595,813	$579,511	$595,783	$598,608	$593,155
		TRUE	24	$602,934†	$543,321	$602,915†	$602,872	$593,242
10	1	FALSE	25	$606,656	$490,788	$606,656	$603,347	$599,036
		TRUE	26	$605,986	$489,865	$605,986	$602,723	$598,154
	2	FALSE	27	$598,392†	$557,177	$598,362†	$598,522*	$597,267†
		TRUE	28	$606,084	$490,105	$606,084	$603,095	$595,657
	3	FALSE	29	$598,190	$554,051	$598,191	$599,712	$597,144
		TRUE	30	$607,571	$492,488	$607,571	$604,370	$599,414

Note. Each value under the columns of SD, BM, DA, and CP is significantly different from that under the column of TM in the same scenario (the same row), $p < .01$, except for the values with a †. * denotes that the SD-Freshman-Y under TM is insignificantly different from the corresponding SD-Freshman-N under TM, $p > .01$. $ here represents Taiwan dollar.

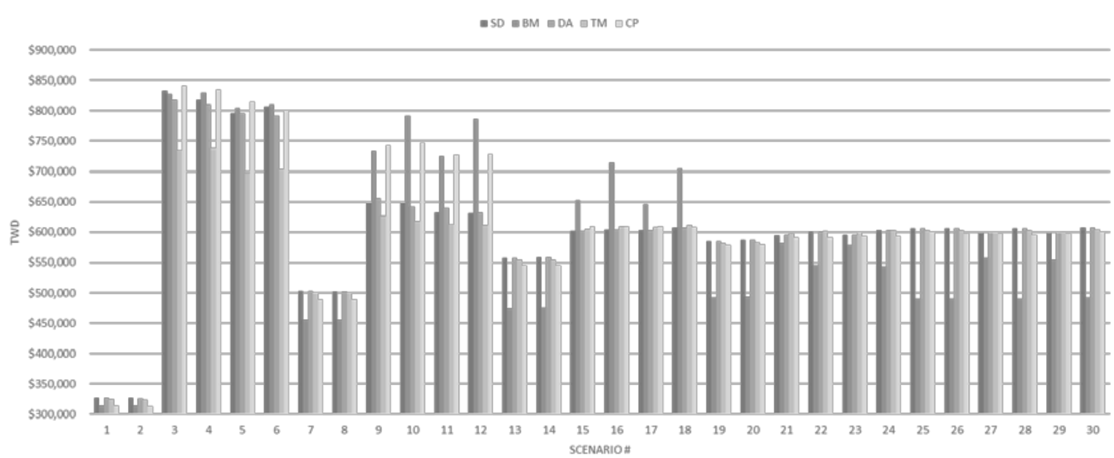

Figure 13. The average standard deviation of mean freshman family income under the free-tuition policy (SD-Freshman-Y). The scenario # in this figure corresponds to that in Table 23.

Figure 13 and its counterpart (Figure 6) share a similar pattern. The relationships among SD-Freshman-Ys based on the scenario-by-scenario comparisons were also similar to those among SD-Freshman-Ns and summarized below.

- When Strategy = #1, the size of SD-Freshman-Y produced by each of the mechanisms had the following orders: SD (DA) > TM > CP, and SD (DA) > TM > BM.
- When Strategy = #1 or the number of choices ≥ 6, the differences between the SD-Freshman-Ys under TM and SD (DA) were limited (less than 0.89% of the SD-Freshman-Y under SD, Cohen's *d* = 0.10).
- When Strategy = #2 or #3 and the number of choices > 6, the SD-Freshman-Y produced by each mechanism was in the following order: TM ≈ SD (DA) > CP > BM.
- When Strategy = #2 or #3 and the number of choices < 6, the SD-Freshman-Y produced by each mechanism was in the following orders: TM < SD (DA) < CP, and TM < BM; that is, TM generated the lowest SD-Freshman-Y in this condition.
- When Strategy = #2 or #3 and the number of choices = 6, the relationships of the SD-Freshman-Ys under the mechanisms depended on *sort-extra-order,* while the SD-Freshman-Y under BM was the highest among all mechanisms in this condition.

To see how well the free-tuition policy can reduce the SD-Freshman-N, I compared SD-Freshman-N and SD-Freshman-Y on the scenario-by-scenario basis as shown in Table 24 and summarized the findings below.

- Under SD, DA, TM, and CP, if Strategy = #1 or the number of choices ≥ 6, SD-Freshman-Y < SD-Freshman-N.
- Under BM, if Strategy = #1 or the number of choices > 6, SD-Freshman-Y < SD-Freshman-N.
- If Strategy = #2 or #3, SD-Freshman-Y > SD-Freshman-N in some scenarios other than the above.

Table 24

The comparisons of the standard deviations of the mean freshman family income (SD-Freshman) with and without the free-tuition policy

Number of choices	Strategy	Extra-in-order	Scenario #	A SD	B BM	C DA	D TM	E CP	F	G
2	1	FALSE	1	45,890	51,512	46,462	$48,116	52,692	535	48,651
		TRUE	2	48,731	50,922	48,182	$47,812	52,570	3,302	51,114
	2	FALSE	3	(20,128)	2,586	15,290	($19,697)	(13,341)	97,623	77,926
		TRUE	4	13,374	(3,989)	19,868	($24,407)	(13,498)	116,915	92,507
	3	FALSE	5	(6,903)	(9,974)	(15,051)	($19,580)	3,937	111,152	91,571
		TRUE	6	(14,784)	(24,855)	2,065	($21,299)	1,690	109,030	87,732
4	1	FALSE	7	15,430	31,523	15,259	$16,717	21,695	2,573	19,290
		TRUE	8	14,777	31,893	15,334	$16,139	21,337	1,525	17,664
	2	FALSE	9	(13,872)	30,059	(26,400)	($13,584)	(19,046)	21,462	7,878
		TRUE	10	(14,760)	10,117	(8,757)	($4,804)	(24,588)	20,287	15,483
	3	FALSE	11	(7,717)	(10,212)	(14,116)	($1,310)	(24,877)	12,373	11,063
		TRUE	12	(12,051)	362	(13,164)	$3,534	(21,688)	3,782	7,316

(Continued)

Number of choices	Strategy	Extra-in-order	Scenario #	A SD	B BM	C DA	D TM	E CP	F	G
6	1	FALSE	13	8,302	52,022	6,941	$9,455	12,398	2,525	11,980
		TRUE	14	6,044	49,439	5,666	$7,124	11,517	2,553	9,677
	2	FALSE	15	7,518	16,073	7,803	$4,255	5,170	374	4,629
		TRUE	16	2,693	(56,686)	2,759	$4,576	5,414	(7,283)	(2,707)
	3	FALSE	17	5,874	(6,482)	5,105	$2,761	1,953	(1,857)	904
		TRUE	18	7,001	(61,903)	6,878	$6,051	4,004	(3,333)	2,719
8	1	FALSE	19	5,285	40,205	7,049	$7,610	8,031	1,165	8,775
		TRUE	20	3,984	37,463	3,979	$6,943	7,408	973	7,916
	2	FALSE	21	5,841	12,397	5,727	$2,692	6,460	475	3,167
		TRUE	22	3,757	18,803	4,023	$7,508	6,711	(4,751)	2,758
	3	FALSE	23	4,693	15,386	4,480	$5,176	4,647	(3,279)	1,897
		TRUE	24	5,604	22,558	5,444	$6,926	5,219	(1,260)	5,666
10	1	FALSE	25	5,854	61,082	5,854	$6,138	7,653	3,024	9,163
		TRUE	26	5,916	61,755	5,916	$7,113	7,968	2,065	9,178
	2	FALSE	27	3,012	15,892	3,041	$2,736	2,920	145	2,882
		TRUE	28	6,630	60,575	6,630	$6,237	2,287	3,381	9,619
	3	FALSE	29	3,146	22,898	3,144	$3,711	3,577	(2,087)	1,625
		TRUE	30	6,170	60,233	6,170	$6,352	2,336	3,019	9,370

Note. Columns A to E = SD-Freshman-N minus SD-Freshman-Y under SD, BM, DA, TM, and CP, respectively. Column F = SD-Freshman-N under SD minus SD-Freshman-N under TM in the same scenario. Column G = SD-Freshman-N under SD minus SD-freshman-Y under TM in the same scenario. A value in parentheses is a negative value. $ here represents Taiwan dollar.

The intuition is that under the free-tuition policy, students with below-average family income can afford to choose and attend School #3 (the private school initially ranked the third best school in the model). With an increase in the population of lower-income freshmen, School #3's average Freshman family income could decrease, which in turn could reduce the overall SD-Freshman-Y. Table 24 shows a counterintuitive phenomenon that SD-Freshman-Y was larger than SD-Freshman-N in many cases where Strategy = #2 or #3 and the number of choices ≤ 6. A possible reason is that when the number of choices becomes less, school choice becomes more like a game. The free-tuition policy gives some players more options to bet on and increases the complexity of the game, which in turn may cause the policy to have a mixed effect on the equality of educational opportunity.

Columns F and G in Table 24 shows how well TM and the free tuition alone and combined could help reduce the SD-Freshman under Taipei's original mechanism, which was SD, when students' strategy was constant. As stated in the previous section, TM and SD (DA) should be accompanied by an allowance of six or more choices to generate a Match-N of at least 95% without regard to students' behaviors. This rule also applied to TM and SD (DA) with the free-tuition policy (see Table 25.) Assuming that high match rate is a requirement for efficiency, I focus the following discussion on the scenarios where the number of choices ≥ 6. The free-tuition policy always helped the original mechanism reduce SD-Freshman (Column A), while the effect of TM on the original mechanism depended on students' strategies (Column F). Combining TM and the free-tuition policy helped balance out TM's adverse effect and enhance TM's positive effect on the original SD-Freshman (see Column G.) If the number of choices > 6, TM with the free-tuition policy could decrease the original SD-Freshman in all scenarios (see Column G.)

The above findings were based on the scenario-by-scenario comparisons. I also calculated the best-worst estimates of the change in SD-Freshman-Y caused by the change from SD to the mechanisms with the free tuition policy when the number of choices ≥ 8. Table 25 shows the estimates.

Table 25

The estimated maximum percentage increase and decrease in the original SD-Freshman by a change from the SD without the free-tuition policy to the mechanisms with the free-tuition policy

Number of choices	To SD+Free % Cohen's *d*	To BM+Free % Cohen's *d*	To TM+Free % Cohen's *d*	To CP+Free % Cohen's *d*
8	[1.98%, -3.72%] 0.60, -1.14	[--*, -16.95%] --*, -5.32	[1.97%, -4.5%] 0.60, -1.32	[0.35%, -3.19%] 0.10, -1.48
10	[1.04%, -2.53%] 0.28, -0.76	[--*, -20.18%] --*, -5.49	[0.50%, -2.48%] 0.14, -0.62	[--*, -2.95%] --*, -0.76

Note. The first percentage in the square brackets represents the maximum percentage increase in the original SD-Freshman (SD-Freshman-N under SD) due to the change from SD to the new mechanism with the free-tuition policy (SD+Free, BM+Free, TM+Free, and CP+Free), and the second percentage in the square brackets is the maximum percentage decrease. The corresponding Cohen's *d* is presented right below each percentage. * denotes that the policy in this column did not increase the original SD-Freshman.

As in the scenarios without the free-tuition policy, BM with the free-tuition policy could provide the largest decrease and least increase in the original SD-Freshman among all mechanisms with the free-tuition policy. CP with the free-tuition policy also provided a minimal or no effect on increasing the original SD-Freshman. Additionally, with the free-tuition policy, CP, as well as TM and SD, could decrease the original SD-Freshman. However, their effect sizes were much less than that of BM with the free-tuition policy. The ranges and effect sizes of TM and SD (DA) with the free-tuition policy on the original SD-Freshman crossed zero, which means that a wrong prediction of students' behaviors might cause an opposite expectation of the effect of TM with the free-tuition policy on SD-Freshman.

In summary, when the number of choices > 6, the free-tuition policy helped TM reduce the original SD-Freshman. However, in this case, the SD-Freshman-Y under SD and the SD-Freshman-Y under TM were similar. The SD-Freshman-Y under SD measured the effect of the free-tuition policy alone on the original SD-Freshman (SD-Freshman-N under SD). SD-Freshman-Y under TM measured the combined effect of TM and the free-tuition policy on the original SD-Freshman. This finding implied that the free-tuition policy was the primary factor for TM with the free-tuition policy to reduce the original SD-Freshman. That is, a similar effect could be obtained by implementing the free-tuition policy alone without changing the mechanism from SD to TM. I further discuss the policy implication of this fining in Chapter 5.

Match Rate. Match-Y denotes the rate of the candidates assigned to the schools under a mechanism with the free-tuition policy. Table 26 and Figure 14 show the average Match-Y across all the simulation steps for each scenario.

Like their counterparts (Table 11 and Figure 7), Table 26 and Figure 14 show a trend that the higher the number of choices, the higher the Match-Y under all mechanisms. Also like their counterparts, the TM and SD (DA) with the free-tuition policy needed an allowance of six or more choices to generate a match rate of at least 95% without regard to students' strategies, while CP needed eight choices and BM needed ten choices. However, unlike Table 10 where only BM could reach the 100% match rate, all mechanisms with the free-tuition policy reached the 100% match rate when the number of choices = 10. That being said, the effect of the free-tuition policy on match rate under the same mechanism was nonlinear and crossed zero. When the number of choices < 10, the match rates in many scenarios with the free-tuition policy decreased instead. The reason might still be the increased complexity of the game caused by the free-tuition policy as explained in the previous paragraph.

Table 26

The average rate of students assigned to schools under a student-assignment mechanism with the free-tuition policy (Match-Y)

Number of choices	Strategy	Extra-in-order	Scenario #	SD %	BM %	DA %	TM %	CP %
2	1	FALSE	1	90.57	87.46*	90.52	90.50	87.46*
		TRUE	2	90.51	87.49*	90.50	90.51†	87.46*
	2	FALSE	3	55.87*	56.13*	57.52*	66.95†	55.90*
		TRUE	4	57.84*	54.39*	58.75*	67.47†	55.69*
	3	FALSE	5	60.42*	57.15*	59.94*	72.59†	57.73*
		TRUE	6	59.58*	56.24*	60.21*	72.42†	58.01*
4	1	FALSE	7	95.10*	91.52*	95.13*	94.99†	93.80*
		TRUE	8	95.13*	91.58*	95.13*	95.04†	93.81*
	2	FALSE	9	80.00*	71.55*	79.33*	86.62†	71.56*
		TRUE	10	80.34*	66.03*	80.89*	89.52†	70.86*
	3	FALSE	11	82.63*	72.48*	81.63*	92.88†	72.98*
		TRUE	12	83.62*	66.44*	83.29*	93.85†	72.79*
6	1	FALSE	13	97.60*	94.85*	97.59*	97.52†	97.53
		TRUE	14	97.59*	94.85*	97.60*	97.51†	97.56
	2	FALSE	15	95.29*	82.94*	95.33*	95.78	90.37*
		TRUE	16	97.47*	73.91*	97.50*	98.48	90.16*
	3	FALSE	17	95.42*	83.68*	95.37*	97.59	90.78*
		TRUE	18	98.44*	75.23*	98.50*	99.00	91.25*

(Continued)

Number of choices	Strategy	Extra-in-order	Scenario #	SD %	BM %	DA %	TM %	CP %
8	1	FALSE	19	98.91	97.81*	98.92	98.89†	98.95*
		TRUE	20	98.92	97.83*	98.90	98.91†	98.94
	2	FALSE	21	98.27*	93.50*	98.26*	98.01	97.96
		TRUE	22	99.61	92.29*	99.61	99.62	98.02*
	3	FALSE	23	98.31*	93.69*	98.29*	98.58	97.93*
		TRUE	24	99.61	92.55*	99.61	99.61	97.99*
10	1	FALSE	25	100.00	100.00	100.00	100.00†	100.00
		TRUE	26	100.00	100.00	100.00	100.00†	100.00
	2	FALSE	27	100.00	100.00	100.00	100.00†	100.00
		TRUE	28	100.00	100.00	100.00	100.00†	100.00
	3	FALSE	29	100.00	100.00	100.00	100.00†	100.00
		TRUE	30	100.00	100.00	100.00	100.00	100.00

Note. * denotes that the value is statistically significantly different from that under the column of TM in the same scenario (the same row), $p < .01$. † denotes that the Match-Y under TM was significantly different from the corresponding Match-N under TM, $p < .01$.

I then compared the match rates under TM and SD with and without the free-tuition policy on the scenario-by-scenario basis (Tables 11 and 25).

- When the number of choices = 10, the Match-Ys under TM and SD were both 100%.
- When the Strategy = #1, the match rates under TM alone, SD alone, SD with the free-tuition policy, and TM with the free-tuition policy were no more than 0.32% different.
- When Strategy = #2 or #3 and the number of choices < 6, TM alone produced the highest match rates, while SD with the free-tuition policy produced the lowest.

- When Strategy = #2 or #3 and the number of choices = 6, SD with the free-tuition policy produced the lowest match rates, while the relationships of the match rates under TM alone, SD alone, and TM with the free-tuition policy were mixed.
- When Strategy = #2 or #3 and the number of choices = 8, the relationships of the match rates under the four combinations were mixed but no more than 0.81% different, while SD with the free-tuition policy always produced higher match rates than SD alone (the original mechanism).

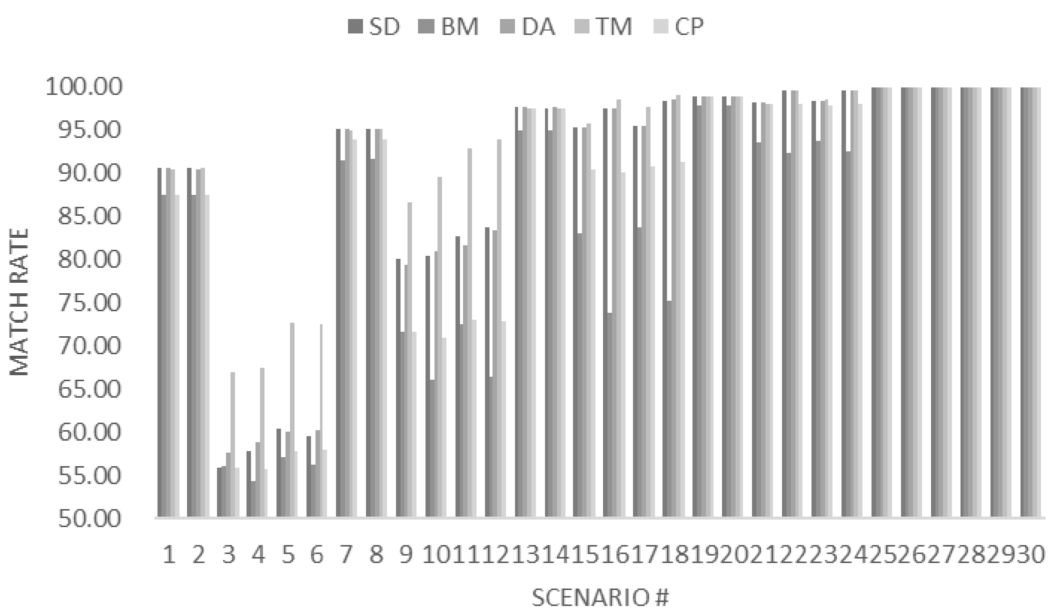

Figure 14. The average rate of students assigned to schools under a mechanism with the free-tuition policy (Match-Y). The scenario # in this figure corresponds to that in Table 26.

In summary, when the number of choices was high (more than 6 in the simulations), the free-tuition policy alone was enough to increase the match rate of the original mechanism. If the number of choices ≤ 6, then TM alone was effective enough to raise the match rate. TM with the free-tuition policy did not perform better than the free tuition alone when the number of choices > 6 or TM alone when the number of choices ≤ 6.

Overall Effect on School Quality

SD-Senior-Y. I defined SD-Senior-Y as the SD-Senior under a mechanism with the free-tuition policy. Table 27 and Figure 15 show the average SD-Senior-Y over the last 31 simulation steps in the 30 simulation runs in each scenario.

Table 27

The average standard deviation of the mean scores of the seniors enrolled under the free-tuition policy (average SD-Senior-Y)

Number of choices	Strategy	Extra-in-order	Scenario #	SD	BM	DA	TM	CP
2	1	FALSE	1	13.27	12.63	13.26	13.11	12.60
		TRUE	2	13.28	12.73	13.22†	13.17	12.67
	2	FALSE	3	31.10	30.90	30.55	27.95	31.18
		TRUE	4	30.50	31.31	30.09	28.21	30.61
	3	FALSE	5	30.09	30.46	29.90	26.46	30.38
		TRUE	6	30.45	30.56	29.97	26.93	30.03
4	1	FALSE	7	19.81	16.77	19.80	19.61	18.74
		TRUE	8	19.81	16.76	19.81	19.61	18.76
	2	FALSE	9	24.20	26.73	24.48	23.18	27.70
		TRUE	10	24.27	29.82	23.86	22.78	27.78
	3	FALSE	11	23.40	27.01	23.74	22.32	27.34
		TRUE	12	23.34	29.88	23.46	22.21	27.25

(Continued)

Number of choices	Strategy	Extra-in-order	Scenario #	SD	BM	DA	TM	CP
6	1	FALSE	13	21.14	15.98	21.16	21.00	19.96
		TRUE	14	21.18	15.99	21.16	20.99	19.96
	2	FALSE	15	21.44	23.04	21.40	21.78	21.62†
		TRUE	16	21.64	26.09	21.64	22.04	21.51
	3	FALSE	17	21.50	22.77	21.52	21.92	21.64
		TRUE	18	21.94	25.69	21.93	22.16	21.60
8	1	FALSE	19	21.68	15.68	21.69	21.46	20.82
		TRUE	20	21.70	15.66	21.70	21.48	20.85
	2	FALSE	21	21.07	19.48	21.09	21.35	20.65
		TRUE	22	21.80	17.65	21.80	21.71	20.68
	3	FALSE	23	21.17	19.28	21.17	21.49	20.84
		TRUE	24	21.86	17.50	21.86	21.74	20.82
10	1	FALSE	25	22.00	15.19	22.00	21.79	21.14
		TRUE	26	22.00	15.18	22.00	21.79	21.12
	2	FALSE	27	21.53†	18.61	21.53†	21.49	21.40†
		TRUE	28	22.01	15.24	22.01	21.80	21.39
	3	FALSE	29	21.51†	18.34	21.51†	21.61	21.40
		TRUE	30	21.99	15.22	21.99	21.78	21.51

Note. All values in the columns of SD, BM, DA, and CP are significantly different from the corresponding values in the column of TM except for those with a †, $p < .01$.

Table 27 and Figure 15 have a structure similar to those of their counterparts (Table 13 and Figure 8), although the gap between BM and the other mechanisms was wider in the former. Also largely similar to the relationships of SD-Senior-Ns, the relationships of SD-Senior-Ys were as follows: (a) When Strategy = #1, SD (DA) > TM > CP and SD (DA) > TM > BM; (b) when Strategy = #2 or #3 and the number of choices < 6, TM had the lowest SD-Senior-Ys among all mechanisms; and (c) when Strategy = #2 or #3 and the number of choices > 6, TM > CP > BM, while the relationships between TM and SD (DA) were mixed and depended on students' strategies.

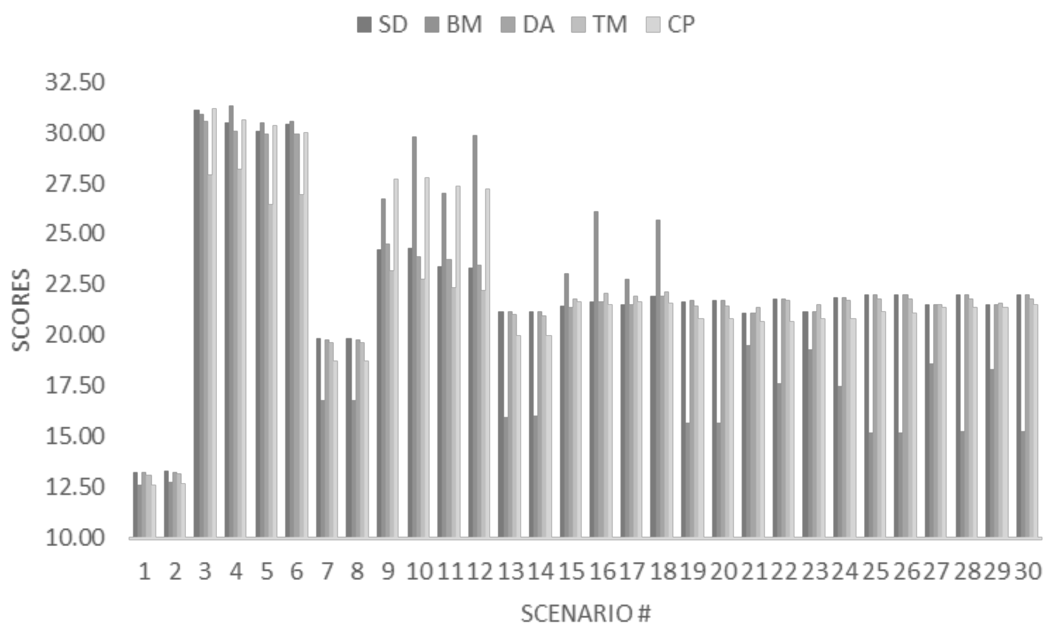

Figure 15. The average standard deviation of the mean scores of the seniors enrolled under the free-tuition policy (average SD-Senior-Y). The scenario # in this figure corresponds to that in Table 27.

Table 28

The differences of the standard deviations of the mean scores of the seniors enrolled with and without the free-tuition policy

Number of choices	Strategy	Extra-in-order	Scenario #	A SD	B BM	C DA	D TM	E CP	F	G
2	1	FALSE	1	0.65	0.85	0.69	0.79	0.92	0.02	0.81
		TRUE	2	0.66	0.76	0.74	0.66	0.82	0.11	0.77
	2	FALSE	3	(1.04)	(0.37)	0.23	(1.18)	(0.73)	3.30	2.12
		TRUE	4	(0.01)	(0.45)	0.45	(1.39)	(0.34)	3.68	2.29
	3	FALSE	5	(0.46)	(0.85)	(0.59)	(1.20)	0.16	4.37	3.17
		TRUE	6	(0.70)	(1.03)	(0.21)	(1.61)	(0.44)	4.43	2.82

(Continued)

Number of choices	Strategy	Extra-in-order	Scenario #	A SD	B BM	C DA	D TM	E CP	F	G
4	1	FALSE	7	(0.20)	0.30	(0.22)	(0.15)	0.07	0.16	0.01
		TRUE	8	(0.22)	0.31	(0.23)	(0.11)	0.03	0.09	(0.02)
	2	FALSE	9	(1.48)	0.06	(2.00)	(1.44)	(1.30)	0.99	(0.45)
		TRUE	10	(1.68)	(0.07)	(1.28)	(1.10)	(1.31)	0.92	(0.18)
	3	FALSE	11	(1.32)	(1.01)	(1.45)	(0.72)	(1.54)	0.48	(0.24)
		TRUE	12	(1.31)	(0.51)	(1.41)	(0.39)	(1.28)	0.20	(0.18)
6	1	FALSE	13	(0.22)	1.19	(0.36)	(0.26)	(0.02)	0.19	(0.07)
		TRUE	14	(0.28)	1.16	(0.30)	(0.26)	0.03	0.17	(0.09)
	2	FALSE	15	(0.24)	(0.32)	(0.19)	(0.28)	(0.25)	(0.30)	(0.58)
		TRUE	16	(0.23)	(3.99)	(0.24)	(0.17)	(0.24)	(0.46)	(0.63)
	3	FALSE	17	(0.20)	(0.72)	(0.23)	(0.36)	(0.36)	(0.27)	(0.63)
		TRUE	18	(0.12)	(3.99)	(0.13)	(0.19)	(0.36)	(0.15)	(0.34)
8	1	FALSE	19	(0.16)	0.91	(0.05)	(0.03)	0.18	0.09	0.06
		TRUE	20	(0.20)	0.86	(0.22)	(0.07)	0.13	0.09	0.02
	2	FALSE	21	(0.13)	0.42	(0.15)	(0.36)	0.06	(0.05)	(0.41)
		TRUE	22	(0.19)	0.30	(0.17)	(0.06)	(0.10)	(0.04)	(0.10)
	3	FALSE	23	(0.34)	0.50	(0.37)	(0.35)	(0.21)	(0.32)	(0.67)
		TRUE	24	(0.10)	0.55	(0.10)	(0.09)	(0.09)	0.10	0.02
10	1	FALSE	25	(0.14)	1.64	(0.14)	(0.16)	0.16	0.22	0.07
		TRUE	26	(0.16)	1.65	(0.16)	(0.11)	0.15	0.16	0.05
	2	FALSE	27	(0.58)	0.34	(0.58)	(0.40)	(0.59)	(0.14)	(0.54)
		TRUE	28	(0.11)	1.57	(0.11)	(0.15)	(0.42)	0.24	0.09
	3	FALSE	29	(0.46)	0.79	(0.46)	(0.38)	(0.43)	(0.19)	(0.56)
		TRUE	30	(0.09)	1.62	(0.09)	(0.08)	(0.37)	0.20	0.12

Note. Columns A to E = SD-Senior-N minus SD-Senior-Y under SD, BM, DA, TM, and CP, respectively. Column F = SD-Senior-N under SD minus SD-Senior-N under TM. Column G = SD-Senior-N under SD minus SD-Senior-Y under TM. A value in parentheses is a negative value.

To see how well the free-tuition policy could help each mechanism shape student compositions that promote lower SD-Senior-Ys, I compared the SD-Senior-Ns with the SD-Senior-Ys under the same mechanism on the scenario-by-scenario basis and show their differences in Table 28.

Columns A – E in Table 28 show that except for some scenarios where the number of choices = 2, the free-tuition policy increased the SD-Seniors under TM, SD, and DA (SD-Senior-Y > SD-Senior-N). With few exceptions, the free-tuition policy also increased SD-Seniors under CP when Strategy = #2 or #3. Contrarily, the free-tuition policy helped BM reduce SD-Seniors when Strategy = #1 or the number of choices > 6. Column F shows that a change of the mechanism from SD to TM reduced the original SD-Senior when students used Strategy #1 or the number of choices < 6, while TM's effects were mixed in other scenarios, depending on the number of choices and students' strategies. Since the free-tuition policy increased the original SD-Seniors in almost all cases (Column A), when combined with TM, the free-tuition policy weakened TM's positive effects and magnified TM's negative effects on the original SD-Seniors in all scenarios except for some scenarios where the number of choices = 2 (Column G). From this point of view, TM with the free-tuition policy was less effective than TM alone on reducing the original SD-Senior.

I also calculated the best-worst estimates for the original SD-Senior when there is a change from SD to the other mechanisms with the free-tuition policy. The estimates presented in Table 29 show that with the free-tuition policy, BM, followed by CP, still offered the most decrease and least increase in the original SD-Senior among all mechanisms. When the number of choices increased from 8 to 10, the effect range of TM with the free-tuition policy remained stable, while the effect of the free-tuition policy alone (the column of SD+Free) became more adverse by having an interval showing more increase and less decrease in the original SD-Senior. The comparison of the best-worst estimates with and without the free tuition policy showed that the free-tuition policy weakened TM's performance by enlarging the size of its maximum increase and reducing the size of its maximum decrease in the original SD-Seniors.

Table 29

The estimated maximum percentage increase and decrease in the original SD-Senior by a change from the SD without the free tuition policy to the other mechanisms with the free tuition policy

Number of choices	To SD+Free % Cohen's *d*	To BM+Free % Cohen's *d*	To TM+Free % Cohen's *d*	To CP+Free % Cohen's *d*
8	[4.97%, -3.19%] 1.09, -0.73	[0.00%*, -28.06%] --, -11.23	[4.41%, -1.89%] 0.97, -0.46	[0.11%, -5.12%] 0.02, -1.10
10	[5.06%, -1.81%] 0.94, -0.47	[0.00%*, -30.67%] --, -13.42	[4.09%, -1.90%] 0.75, -0.48	[2.71%, -3.56%] 0.44, -1.96

Note. The first percentage in the square brackets represents the maximum percentage increase in the original SD-Senior (SD-Senior-N under SD) by the change from SD to the new mechanism with the free-tuition policy (SD+Free, BM+Free, TM+Free, and CP+Free), and the second percentage in the square brackets is the maximum percentage decrease. The corresponding Cohen's *d* is presented right below each percentage. * denotes that the policy in this column did not increase the original SD-Freshman and thus no Cohen's *d* was calculated.

Mean Senior Score. To see whether the free-tuition policy helped increase the mean senior scores, I followed the same logic used to calculate the mean senior scores in Table 15 to calculate the mean senior scores in each scenario with the free-tuition policy and present the results in Table 30 and Figure 16.

Table 30

The average senior scores under each mechanism with the free-tuition policy

Number of choices	Strategy	Sort-extra-choice	Scenario #	SD	BM	DA	TM	CP
2	1	FALSE	1	58.60†	58.09	58.60†	58.57	58.13
		TRUE	2	58.62	58.08	58.65	58.55	58.08
	2	FALSE	3	61.67	59.64	61.21	57.72	59.71
		TRUE	4	60.61	58.82	60.75	57.41	59.18
	3	FALSE	5	60.16	58.57	60.12	56.36	59.30
		TRUE	6	60.58	58.23	59.99	56.21	59.06
4	1	FALSE	7	55.94†	55.41	55.94†	55.96	55.74
		TRUE	8	55.97†	55.41	55.97†	55.97	55.76
	2	FALSE	9	53.71†	54.91	53.89†	53.73	56.01
		TRUE	10	53.44	54.51	53.65†	53.81	56.14
	3	FALSE	11	53.81	55.19	53.76	54.39	55.73
		TRUE	12	53.69	54.09†	53.62	54.46	55.68
6	1	FALSE	13	55.06	55.43	55.04†	55.03	55.24
		TRUE	14	55.04†	55.44	55.04†	55.03	55.23
	2	FALSE	15	54.57†	53.89	54.57†	54.56	54.00
		TRUE	16	54.71	52.41	54.70	54.65	54.01
	3	FALSE	17	54.57	53.94	54.58†	54.62	54.16
		TRUE	18	54.62	52.93	54.61	54.66	54.18
8	1	FALSE	19	54.61	54.84	54.61	54.66	54.78
		TRUE	20	54.60	54.88	54.60	54.66	54.77
	2	FALSE	21	54.27†	54.48	54.28†	54.30	54.28†
		TRUE	22	54.35	54.11	54.35	54.39	54.31
	3	FALSE	23	54.28†	54.51	54.28†	54.31	54.29†
		TRUE	24	54.35	54.19	54.35	54.39	54.31
10	1	FALSE	25	54.32	54.67	54.32	54.35	54.31
		TRUE	26	54.33	54.61	54.33	54.37	54.33
	2	FALSE	27	54.12	54.63	54.13	54.22	54.13
		TRUE	28	54.32	54.65	54.32	54.35	54.15
	3	FALSE	29	54.14	54.68	54.14	54.25	54.15
		TRUE	30	54.32	54.60	54.32	54.36	54.17

Note. Each value in Columns SD, BM, DA, and CP are significantly different from the corresponding value in the column of TM (in the same row), except for the value with a †, $p < .01$.

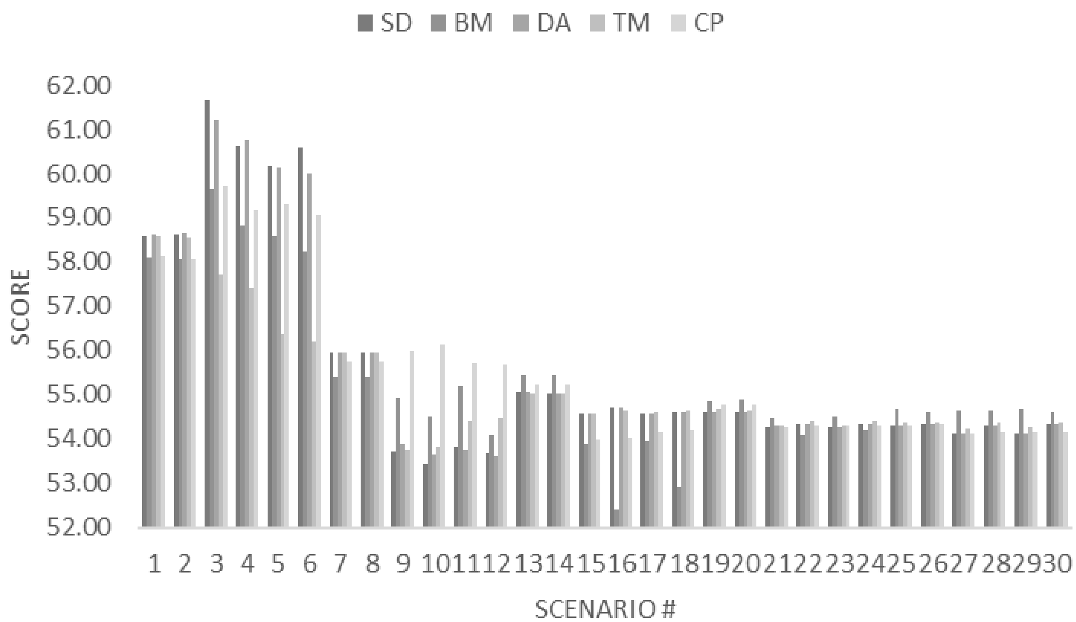

Figure 16. The average mean scores of the high school seniors enrolled with the free-tuition policy. The scenario # in this figure corresponds to that in Table 30.

When the number of choices ≤ 6, like their counterparts in Figure 9, the senior scores under each mechanism with the free-tuition policy fluctuated with students' strategies and the number of choices. When the number of choices > 6, unlike their counterparts, the senior scores under the free tuition policy became rather stable; the range (the highest score minus the lowest score) was less than 0.50 points under SD, DA, and TM and less than 0.80 points under BM and CP. The scenario-by-scenario comparison of the scores in the cases where the number of choices > 6 shows the followings.

- When the number of choices = 10, the average senior score under the mechanisms with the free-tuition policy had the following orders: BM > TM > SD (DA) and BM > TM > CP.
- When the number of choices = 8 and with the free tuition policy, TM resulted in higher senior scores than SD (DA), while the relationships between TM and the other mechanisms were mixed.
- The score differences between TM and SD (DA), with the free tuition policy, were small (no more than 0.11 points, Cohen's *d* = 0.27).

I also compared the effects of TM alone, the free-tuition policy alone, and the two interventions combined on the original senior scores (senior scores resulting from SD) on the scenario-by-scenario basis. Their differences are presented in Table 31.

Column (A) in Table 31 shows that the effects of TM alone on the original senior scores were mixed. However, if the number of choices > 6, the score differences between TM and SD were small and no more than 0.26 points different, Cohen's *d* = 0.18. On the other hand, the effect of the free-tuition policy alone on the original senior scores was quite clear. When the number of choices > 2, all values in Columns (B) are negative, which means that the free-tuition policy had an adverse effect on senior scores. Therefore, when these two interventions were combined, the free-tuition policy worsened TM's performance and made TM with the free-tuition policy produce a lower senior score than the original mechanism in almost all scenarios.

I also calculated the best-worse estimates of the senior score change caused by a change from SD to the mechanisms with the free-tuition policy as shown in Table 32.

Under the number of choices = 8 and with the free-tuition policy, the positive effect of the mechanisms on the original senior scores had the following order: BM > CP > TM > SD, while the sizes of their negative effects were about the same. Under the number of choices = 10, BM with the free-tuition policy could provide the most increase and least decrease in the original senior score. The rest of the mechanisms could hardly increase the original mean senior score, while their effect sizes on decreasing the original senior score were about medium.

Table 31

The differences of the mean senior scores resulting from SD alone, TM alone, SD with the free-tuition policy, and TM with the free-tuition policy

Number of choices	Strategy	Extra-in-order	Scenario #	(A) TM alone	(B) Free tuition alone	(C) TM with the free-tuition policy
2	1	FALSE	1	(0.08)	(0.78)	(0.81)
		TRUE	2	(0.04)	(0.77)	(0.84)
	2	FALSE	3	(4.31)	0.50	(3.45)
		TRUE	4	(4.58)	(0.74)	(3.94)
	3	FALSE	5	(4.17)	0.22	(3.58)
		TRUE	6	(4.60)	0.18	(4.19)
4	1	FALSE	7	(0.04)	(0.69)	(0.67)
		TRUE	8	(0.12)	(0.65)	(0.64)
	2	FALSE	9	0.25	(1.12)	(1.09)
		TRUE	10	0.21	(1.47)	(1.11)
	3	FALSE	11	(0.03)	(1.33)	(0.76)
		TRUE	12	(0.06)	(1.21)	(0.44)
6	1	FALSE	13	0.04	(0.24)	(0.27)
		TRUE	14	0.01	(0.31)	(0.32)
	2	FALSE	15	(0.20)	(1.03)	(1.04)
		TRUE	16	(0.36)	(0.51)	(0.57)
	3	FALSE	17	(0.01)	(0.84)	(0.78)
		TRUE	18	0.01	(0.21)	(0.17)

(Continued)

Number of choices	Strategy	Extra-in-order	Scenario #	(A) TM alone	(B) Free tuition alone	(C) TM with the free-tuition policy
8	1	FALSE	19	(0.06)	(0.16)	(0.10)
		TRUE	20	(0.06)	(0.19)	(0.13)
	2	FALSE	21	0.04	(0.75)	(0.73)
		TRUE	22	(0.08)	(0.16)	(0.12)
	3	FALSE	23	(0.26)	(0.89)	(0.86)
		TRUE	24	0.02	(0.07)	(0.03)
10	1	FALSE	25	0.07	(0.09)	(0.05)
		TRUE	26	0.01	(0.09)	(0.05)
	2	FALSE	27	(0.08)	(0.91)	(0.82)
		TRUE	28	0.09	(0.05)	(0.01)
	3	FALSE	29	(0.09)	(0.84)	(0.73)
		TRUE	30	0.05	(0.03)	0.00

Note. Column (A) is the mean senior score under TM without the free-tuition policy minus the mean senior score under SD without the free-tuition policy. Column (B) is the mean senior score under SD with the free-tuition policy minus the mean senior score under SD without the free-tuition policy. Column (C) is the mean senior score under TM with the free-tuition policy minus the mean senior score under SD without the free-tuition policy. A value in parentheses is a negative value.

In summary, when the number of choices > 6, whether TM alone could improve SD-Senior depended on students' behaviors before and after the policy change. Both the scenario-by-scenario analysis and the best-worst estimates showed that the free-tuition policy worsened TM's performance. Therefore, TM with the free-tuition policy could not increase the original senior score as much as TM alone, if any. When the number of choices = 10, TM with the free-tuition policy had little chance to increase the original senior score but probably could decrease it instead.

Table 32

The estimated maximum percentage increase and decrease in the original senior scores by a change from the SD without the free-tuition policy to the mechanisms with the free-tuition policy

Number of choices	To SD+Free % Cohen's *d*	To BM+Free % Cohen's *d*	To TM+Free % Cohen's *d*	To CP+Free % Cohen's *d*
8	[0.36%, -1.62%] 0.51, -0.69	[0.84%, -1.92%] 1.04, -0.58	[0.45%, -1.57%] 0.64, -0.68	[0.66%, -1.61%] 0.95, -0.68
10	[---*, -0.91%] --, -0.70	[0.32%, -0.44%] 0.64, -0.34	[0.01%, -0.81%] 0.07, -0.64	[--*, -0.91%] --, -0.70

Note. The first percentage in the square brackets represents the maximum percentage increase in the original senior score (senior score under SD without the free-tuition policy) by the change from SD to the new mechanism with the free-tuition policy (SD+Free, BM+Free, TM+Free, and CP+Free), and the second percentage in the square brackets is the maximum percentage decrease. The corresponding Cohen's *d* is presented right below each percentage. * denotes that the policy in this column did not increase the original SD-Freshman and thus no Cohen's *d* was calculated.

Impacts on Different Student Groups

Impacts on Top 10% Performing Students. Table 33 and Figure 17 show the top-choice match rate under the mechanisms with the free-tuition policy. The comparison between Figure 17 and its counterpart (Figure 10) on the scenario-by-scenario basis showed that the percentages under TM with and without the free tuition policy were less than 0.5% different. The same statement applied to SD (DA) when the number of choices > 2, CP when the number of choices > 4, and BM when the number of choices > 6. If the top-choice match rate was 100% in a scenario without the free-tuition policy, the percentage remained 100% after the implementation of the free-tuition policy. Therefore, the free-tuition policy had limited impact on the assignment of the top 10% performing students. However, when the number of choices < 10, Strategy #1 produced a significantly different top-choice match rate from Strategy #2 or #3, which implied that when the number of choices < 10 and students changed their behaviors, the implementation of the free-tuition policy could have a significant impact on the assignment of the top 10% performing students.

Table 33

The average percentage of the top 10% performing students assigned to their top choices under the mechanisms with the free-tuition policy

Number of choices	Strategy	Extra-in-order	Scenario #	SD	BM	DA	TM	CP
2	1	FALSE	1	19.87	20.13	19.99	19.98	19.94
		TRUE	2	20.07	19.93	20.15	20.01	19.85
	2	FALSE	3	96.53	96.19	97.01	99.08	93.92
		TRUE	4	96.36	96.76	96.80	99.62	94.28
	3	FALSE	5	99.15	98.56	98.69	99.68	97.01
		TRUE	6	98.65	96.84	98.92	99.72	97.78
4	1	FALSE	7	39.84	40.16	39.90	39.84	40.00
		TRUE	8	39.89	40.07	40.03	40.17	40.12
	2	FALSE	9	99.95	97.42	99.87	99.51	98.92
		TRUE	10	99.98	99.30	99.86	99.57	97.96
	3	FALSE	11	99.98	99.47	100.00	99.60	99.50
		TRUE	12	99.94	99.74	100.00	99.62	99.66
6	1	FALSE	13	60.28	59.93	59.80	60.01	59.71
		TRUE	14	60.22	59.84	59.57	60.04	60.04
	2	FALSE	15	99.95	99.30	99.95	99.55	99.93
		TRUE	16	99.93	99.83	99.93	99.64	99.95
	3	FALSE	17	100.00	99.75	100.00	99.59	100.00
		TRUE	18	100.00	99.98	100.00	99.61	100.00

(Continued)

Number of choices	Strategy	Extra-in-order	Scenario #	SD	BM	DA	TM	CP
8	1	FALSE	19	79.89	79.71	80.12	80.02	80.08
		TRUE	20	79.93	79.92	80.06	79.87	79.93
	2	FALSE	21	99.95	99.83	99.95	99.49	99.96
		TRUE	22	99.92	99.96	99.92	99.64	99.96
	3	FALSE	23	100.00	99.79	100.00	99.60	100.00
		TRUE	24	100.00	100.00	100.00	99.59	100.00
10	1	FALSE	25	100.00	100.00	100.00	99.42	100.00
		TRUE	26	100.00	100.00	100.00	99.45	100.00
	2	FALSE	27	99.96	99.97	99.96	99.54	99.95
		TRUE	28	100.00	100.00	100.00	99.44	99.93
	3	FALSE	29	100.00	100.00	100.00	99.60	100.00
		TRUE	30	100.00	100.00	100.00	99.38	100.00

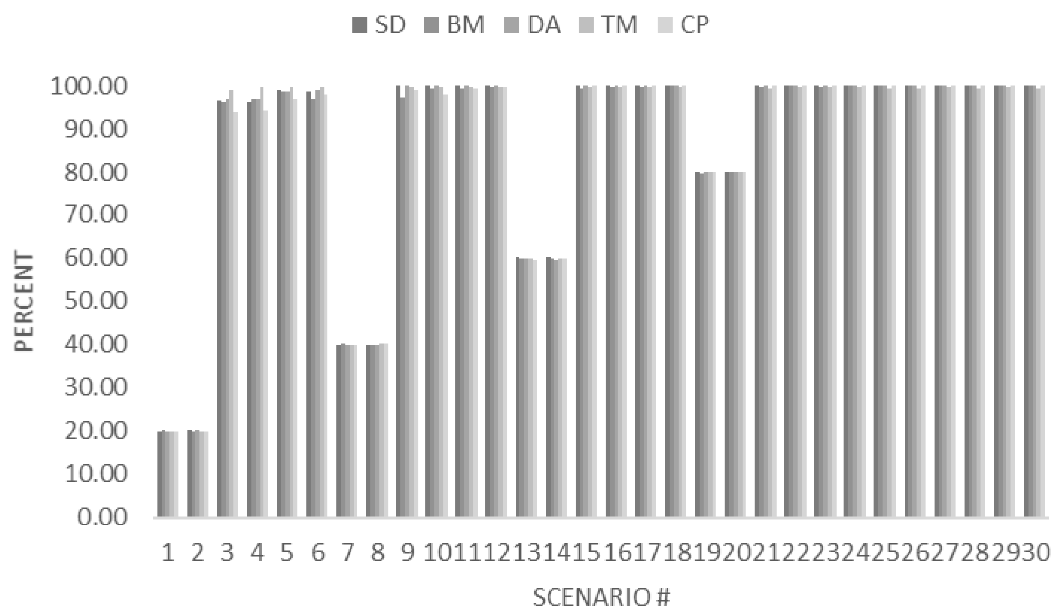

Figure 17. The percentage of the top 10% performing students assigned to their top choices under the mechanisms with the free-tuition policy. The scenario # in this figure corresponds to that in Table 33.

Impacts on Students with the Bottom 10% Family Income. Table 34 and

Figure 18 show the PIs of the students with the bottom 10% family income under the mechanisms with the free-tuition policy. Compared with its counterpart (Table 18), Table 34 shows that the free-tuition policy increased the PIs of the students in this group in almost all scenarios under SD, DA, and TM and all scenarios under BM and CP. Even if the PIs with the free-tuition policy were lower than the PIs without the free-tuition policy, their differences were no more than 0.06 if the number of choices ≤ 6 and no more than 0.02 if the number of choices > 6. Therefore, the free-tuition policy brought no harm, if no benefit, to the students with the bottom 10% income. When the free-tuition policy was implemented with an allowance of 6 choices, BM generated a PI between 2.07 and 3.10 for this group of students; CP, between 0.23 and 0.69; TM, between -0.05 and 0.58; SD (DA) between -0.13 and 0.47. Therefore, BM, followed by CP, still helped the students in this group the most.

Table 34

The average preference index (PI) of the students with the bottom 10% family income under the mechanisms with the free-tuition policy

Number of choices	Strategy	Extra-in-order	Scenario #	SD	BM	DA	TM	CP
2	1	FALSE	1	(0.20)	0.27	(0.21)	(0.19)	0.24
		TRUE	2	(0.20)	0.27	(0.22)	(0.16)	0.28
	2	FALSE	3	(0.75)	0.05	(0.76)	(0.80)	0.01
		TRUE	4	(0.74)	0.08	(0.73)	(0.74)	0.16
	3	FALSE	5	(0.71)	0.01	(0.69)	(0.77)	(0.06)
		TRUE	6	(0.77)	0.00	(0.74)	(0.72)	(0.05)

(Continued)

Number of choices	Strategy	Extra-in-order	Scenario #	SD	BM	DA	TM	CP
4	1	FALSE	7	(0.17)	1.30	(0.17)	(0.11)	0.54
		TRUE	8	(0.15)	1.32	(0.16)	(0.09)	0.56
	2	FALSE	9	(0.37)	1.35	(0.34)	(0.28)	0.35
		TRUE	10	(0.28)	1.63	(0.28)	(0.19)	0.27
	3	FALSE	11	(0.37)	1.28	(0.36)	(0.21)	0.19
		TRUE	12	(0.30)	1.62	(0.33)	(0.21)	0.27
6	1	FALSE	13	(0.16)	1.91	(0.15)	(0.01)	0.35
		TRUE	14	(0.15)	1.91	(0.15)	(0.01)	0.36
	2	FALSE	15	0.38	2.00	0.41	0.21	0.76
		TRUE	16	0.06	2.73	0.06	0.01	0.78
	3	FALSE	17	0.30	2.01	0.30	0.07	0.65
		TRUE	18	(0.01)	2.67	0.00	0.02	0.64
8	1	FALSE	19	(0.13)	2.53	(0.13)	(0.05)	0.23
		TRUE	20	(0.13)	2.53	(0.13)	(0.05)	0.23
	2	FALSE	21	0.39	2.09	0.36	0.36	0.69
		TRUE	22	(0.02)	2.94	(0.02)	0.06	0.66
	3	FALSE	23	0.33	2.20	0.33	0.25	0.58
		TRUE	24	(0.02)	2.87	(0.02)	0.05	0.58
10	1	FALSE	25	0.00	3.09	0.00	0.12	0.83
		TRUE	26	0.00	3.10	0.00	0.12	0.84
	2	FALSE	27	0.46	1.95	0.46	0.58	0.65
		TRUE	28	0.00	3.10	0.00	0.11	0.61
	3	FALSE	29	0.47	2.07	0.47	0.35	0.65
		TRUE	30	0.00	3.10	0.00	0.12	0.49

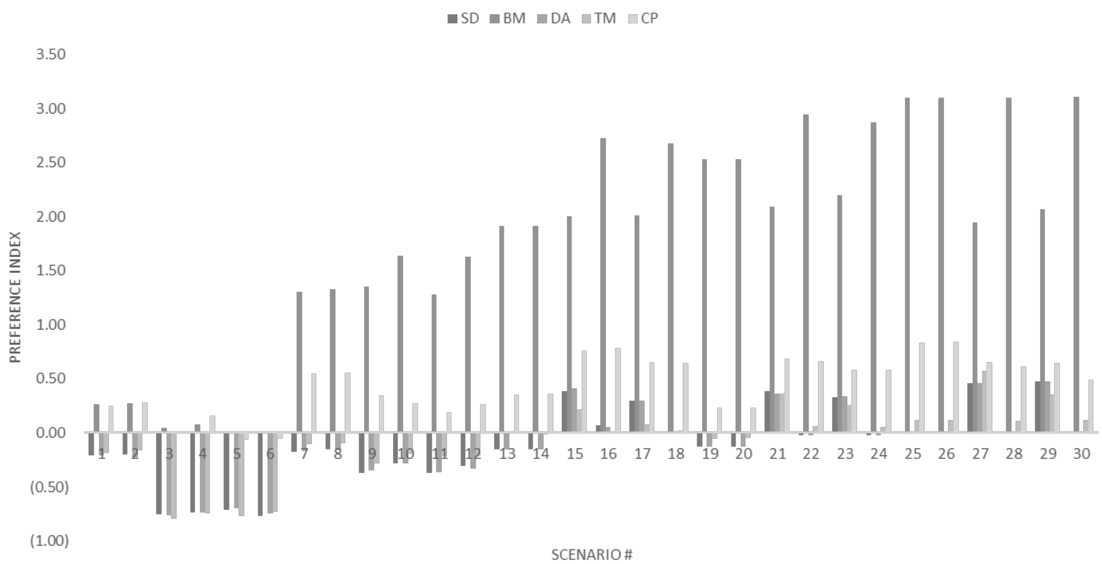

Figure 18. The average preference index (PI) of the students with the bottom 10% family income under the mechanisms with the free-tuition policy. The scenario # in this figure corresponds to that in Table 34.

Impacts on Students in Each Income Quartile. Tables 35-38 and Figure 19 show the PIs of the students in each quartile enrolled under the free-tuition policy. I compared the results under each mechanism with the free tuition policy (Figure 19) and without the free tuition policy (Figure 12) and summarize the findings below.

- The free-tuition policy helped BM and CP increase the PIs of the bottom-income-quartile students in all scenarios.
- The free-tuition policy helped SD (DA) remain or increase the PIs of the bottom-income-quartile students in all scenarios except for the scenarios where the number of choices = 8 and Strategy = #1.
- The free-tuition policy helped TM increase the PIs of the bottom-income-

quartile students in all scenarios except for the scenarios where the number of choices = 8 and Strategy = #1 and the scenario where the number of choices = 4, Strategy = #3, and *sort-extra-order* = True.

- The free-tuition policy helped all mechanisms increase the PIs of the second-income-quartile students in most scenarios, but there were more scenarios where the free-tuition policy harmed the second-income-quartile students than the scenarios where the free-tuition policy harmed the bottom-income-quartile students.
- The effects of the free-tuition policy on the third-income-quartile students were mixed; it helped SD, DA, TA, and CP improve these students' PIs in most scenarios where the number of choices > 6 and Strategy = 2 or 3 but helped BM improve these student's PI only in the scenarios where the number of choices = 10.
- The free-tuition policy helped remain or improve the PIs of the top-income-quartile students under SD (DA) and TM except for a few scenarios where the number of choices = 2 and under CP except for some scenarios where the number of choices ≤ 6.
- The free-tuition policy worsened the PIs of the top-income-quartile students under BM in all scenarios except for some scenarios where the number of choices ≥ 8 and Strategy = #2.
- When the number of choices ≥ 8, the PIs under SD (DA) were between ± 0.28; under TM, between 0.28 and -0.13; under CP, between 0.61 and -0.31.

- When the number of choices ≥ 8 and under BM, the PIs of the bottom-income-quartile students were between 2.69 and 1.57; the PIs of the second-income-quartile students, between 1.04 and 0.00; the PIs of the third- and top-income-quartile students, between -0.66 and -1.07.

The above findings show that with the free-tuition policy, BM was still the best mechanism to improve the assignments of the bottom-income-quartile students, followed by CP. However, the free-tuition policy made BM produce more negative PIs for the top-income-quartile students while helping all other mechanisms improve the PIs of the students in this group. Compared with Taipei's original mechanism (SD) on the scenario-by-scenario basis, TM alone or the free tuition alone improved the PIs of the bottom-income-quartile students in most scenarios but also decreased their PIs in a few scenarios. With the two interventions combined, the PIs of the students in the bottom income quartile were all higher than those under the original mechanism. From the viewpoint of OEDC (2010), since TM with the free-tuition policy benefited the disadvantaged students, it helped reduce educational inequality.

Table 35

The average preference index (PI) of the students with the bottom quartile family income enrolled under the free-tuition policy

Number of choices	Strategy	Extra-in-order	Scenario #	SD25	BM25	DA25	TM25	CP25
2	1	FALSE	1	(0.11)	0.10	(0.12)	(0.08)	0.10
		TRUE	2	(0.11)	0.12	(0.12)	(0.07)	0.12
	2	FALSE	3	(1.46)	(0.78)	(1.44)	(1.28)	(0.81)
		TRUE	4	(1.43)	(0.77)	(1.41)	(1.22)	(0.71)
	3	FALSE	5	(1.35)	(0.80)	(1.34)	(1.11)	(0.84)
		TRUE	6	(1.40)	(0.80)	(1.37)	(1.09)	(0.84)
4	1	FALSE	7	(0.20)	0.84	(0.20)	(0.15)	0.24
		TRUE	8	(0.19)	0.85	(0.20)	(0.15)	0.24
	2	FALSE	9	(0.80)	0.47	(0.79)	(0.52)	(0.38)
		TRUE	10	(0.71)	0.67	(0.71)	(0.36)	(0.46)
	3	FALSE	11	(0.73)	0.40	(0.74)	(0.30)	(0.47)
		TRUE	12	(0.64)	0.66	(0.67)	(0.23)	(0.42)
6	1	FALSE	13	(0.16)	1.42	(0.16)	(0.09)	0.24
		TRUE	14	(0.16)	1.42	(0.16)	(0.09)	0.24
	2	FALSE	15	0.07	1.33	0.09	(0.02)	0.20
		TRUE	16	0.05	1.75	0.05	0.01	0.20
	3	FALSE	17	0.01	1.29	0.01	(0.04)	0.14
		TRUE	18	0.01	1.73	0.01	0.01	0.14
8	1	FALSE	19	(0.11)	1.90	(0.11)	(0.04)	0.23
		TRUE	20	(0.11)	1.91	(0.11)	(0.03)	0.23
	2	FALSE	21	0.21	1.61	0.19	0.13	0.34
		TRUE	22	(0.01)	2.29	(0.01)	0.05	0.32
	3	FALSE	23	0.15	1.67	0.16	0.08	0.27
		TRUE	24	(0.02)	2.26	(0.02)	0.05	0.28
10	1	FALSE	25	0.00	2.69	0.00	0.10	0.60
		TRUE	26	0.00	2.69	0.00	0.10	0.60
	2	FALSE	27	0.28	1.57	0.28	0.28	0.39
		TRUE	28	0.00	2.68	0.00	0.10	0.37
	3	FALSE	29	0.27	1.69	0.27	0.15	0.35
		TRUE	30	0.00	2.68	0.00	0.10	0.26

Table 36

The average preference index (PI) of the students with the second quartile family income enrolled under the free-tuition policy

Number of choices	Strategy	Extra-in-order	Scenario #	SD50	BM50	DA50	TM50	CP50
2	1	FALSE	1	0.11	(0.12)	0.10	0.10	(0.11)
		TRUE	2	0.10	(0.12)	0.11	0.10	(0.11)
	2	FALSE	3	(2.72)	(2.49)	(2.63)	(1.89)	(2.51)
		TRUE	4	(2.56)	(2.55)	(2.53)	(1.80)	(2.53)
	3	FALSE	5	(2.35)	(2.41)	(2.38)	(1.46)	(2.39)
		TRUE	6	(2.40)	(2.44)	(2.37)	(1.44)	(2.39)
4	1	FALSE	7	(0.23)	(0.38)	(0.23)	(0.24)	(0.40)
		TRUE	8	(0.23)	(0.38)	(0.22)	(0.23)	(0.40)
	2	FALSE	9	(1.09)	(1.38)	(1.15)	(0.63)	(1.54)
		TRUE	10	(1.12)	(1.35)	(1.09)	(0.47)	(1.59)
	3	FALSE	11	(0.93)	(1.32)	(1.01)	(0.29)	(1.45)
		TRUE	12	(0.87)	(1.34)	(0.86)	(0.22)	(1.44)
6	1	FALSE	13	(0.17)	0.00	(0.17)	(0.18)	(0.21)
		TRUE	14	(0.18)	0.00	(0.17)	(0.17)	(0.20)
	2	FALSE	15	(0.39)	(0.54)	(0.39)	(0.24)	(0.61)
		TRUE	16	(0.12)	(0.50)	(0.12)	(0.03)	(0.63)
	3	FALSE	17	(0.35)	(0.53)	(0.34)	(0.12)	(0.59)
		TRUE	18	(0.05)	(0.44)	(0.05)	(0.02)	(0.56)
8	1	FALSE	19	(0.09)	0.22	(0.09)	(0.05)	(0.04)
		TRUE	20	(0.09)	0.23	(0.09)	(0.05)	(0.03)
	2	FALSE	21	(0.22)	(0.00)	(0.22)	(0.17)	(0.30)
		TRUE	22	0.01	0.45	0.01	0.03	(0.28)
	3	FALSE	23	(0.20)	0.01	(0.21)	(0.11)	(0.27)
		TRUE	24	0.00	0.46	0.00	0.03	(0.26)
10	1	FALSE	25	0.00	1.04	0.00	0.06	0.04
		TRUE	26	0.00	1.03	0.00	0.06	0.04
	2	FALSE	27	(0.14)	0.34	(0.14)	(0.14)	(0.17)
		TRUE	28	0.00	1.03	0.00	0.06	(0.16)
	3	FALSE	29	(0.16)	0.37	(0.16)	(0.07)	(0.19)
		TRUE	30	0.00	1.02	0.00	0.06	(0.15)

Table 37

The average preference index (PI) of the students with the third quartile family income enrolled under the free-tuition policy

Number of choices	Strategy	Extra-in-order	Scenario #	SD75	BM75	DA75	TM75	CP75
2	1	FALSE	1	(0.36)	(0.50)	(0.35)	(0.39)	(0.51)
		TRUE	2	(0.35)	(0.50)	(0.35)	(0.38)	(0.50)
	2	FALSE	3	(2.39)	(2.70)	(2.23)	(1.40)	(2.73)
		TRUE	4	(2.16)	(2.78)	(2.12)	(1.34)	(2.83)
	3	FALSE	5	(1.85)	(2.52)	(1.95)	(0.99)	(2.48)
		TRUE	6	(1.89)	(2.58)	(1.88)	(0.96)	(2.43)
4	1	FALSE	7	(0.27)	(0.94)	(0.27)	(0.29)	(0.52)
		TRUE	8	(0.27)	(0.94)	(0.27)	(0.30)	(0.52)
	2	FALSE	9	(0.60)	(1.88)	(0.66)	(0.35)	(1.33)
		TRUE	10	(0.65)	(1.94)	(0.63)	(0.28)	(1.36)
	3	FALSE	11	(0.47)	(1.61)	(0.52)	(0.15)	(1.15)
		TRUE	12	(0.46)	(1.87)	(0.44)	(0.13)	(1.16)
6	1	FALSE	13	(0.19)	(0.94)	(0.20)	(0.21)	(0.35)
		TRUE	14	(0.19)	(0.94)	(0.19)	(0.21)	(0.35)
	2	FALSE	15	(0.28)	(1.44)	(0.28)	(0.16)	(0.52)
		TRUE	16	(0.19)	(1.61)	(0.19)	(0.06)	(0.54)
	3	FALSE	17	(0.22)	(1.32)	(0.22)	(0.08)	(0.45)
		TRUE	18	(0.10)	(1.50)	(0.10)	(0.03)	(0.45)
8	1	FALSE	19	(0.10)	(0.73)	(0.10)	(0.10)	(0.17)
		TRUE	20	(0.10)	(0.72)	(0.10)	(0.10)	(0.17)
	2	FALSE	21	(0.24)	(1.07)	(0.23)	(0.13)	(0.28)
		TRUE	22	(0.01)	(0.90)	(0.01)	(0.01)	(0.28)
	3	FALSE	23	(0.18)	(1.05)	(0.19)	(0.09)	(0.24)
		TRUE	24	(0.00)	(0.87)	0.00	(0.01)	(0.24)
10	1	FALSE	25	0.00	(0.66)	0.00	0.01	(0.07)
		TRUE	26	0.00	(0.66)	0.00	0.01	(0.07)
	2	FALSE	27	(0.21)	(0.70)	(0.21)	(0.13)	(0.23)
		TRUE	28	0.00	(0.66)	0.00	0.01	(0.21)
	3	FALSE	29	(0.18)	(0.73)	(0.18)	(0.07)	(0.20)
		TRUE	30	0.00	(0.66)	0.00	0.01	(0.15)

Table 38

The average preference index (PI) of the students with the top quartile family income enrolled under the free-tuition policy

Number of choices	Strategy	Extra-in-order	Scenario #	SD100	BM100	DA100	TM100	CP100
2	1	FALSE	1	(1.79)	(1.80)	(1.79)	(1.80)	(1.80)
		TRUE	2	(1.79)	(1.81)	(1.79)	(1.80)	(1.82)
	2	FALSE	3	(0.78)	(1.17)	(0.72)	(0.38)	(1.21)
		TRUE	4	(0.69)	(1.20)	(0.67)	(0.37)	(1.27)
	3	FALSE	5	(0.52)	(1.03)	(0.57)	(0.24)	(0.99)
		TRUE	6	(0.55)	(1.05)	(0.54)	(0.22)	(0.95)
4	1	FALSE	7	(0.67)	(0.92)	(0.66)	(0.68)	(0.71)
		TRUE	8	(0.66)	(0.92)	(0.66)	(0.68)	(0.71)
	2	FALSE	9	(0.12)	(0.89)	(0.13)	(0.08)	(0.39)
		TRUE	10	(0.13)	(0.89)	(0.13)	(0.06)	(0.41)
	3	FALSE	11	(0.09)	(0.66)	(0.10)	(0.03)	(0.30)
		TRUE	12	(0.09)	(0.82)	(0.09)	(0.03)	(0.32)
6	1	FALSE	13	(0.30)	(0.83)	(0.30)	(0.32)	(0.34)
		TRUE	14	(0.30)	(0.84)	(0.30)	(0.31)	(0.33)
	2	FALSE	15	(0.08)	(0.81)	(0.07)	(0.05)	(0.14)
		TRUE	16	(0.07)	(1.01)	(0.07)	(0.03)	(0.14)
	3	FALSE	17	(0.06)	(0.69)	(0.06)	(0.02)	(0.11)
		TRUE	18	(0.04)	(0.95)	(0.04)	(0.01)	(0.11)
8	1	FALSE	19	(0.12)	(0.69)	(0.12)	(0.13)	(0.13)
		TRUE	20	(0.12)	(0.70)	(0.12)	(0.13)	(0.13)
	2	FALSE	21	(0.08)	(0.69)	(0.07)	(0.05)	(0.08)
		TRUE	22	(0.03)	(0.86)	(0.03)	(0.02)	(0.08)
	3	FALSE	23	(0.05)	(0.68)	(0.06)	(0.03)	(0.07)
		TRUE	24	(0.02)	(0.84)	(0.02)	(0.01)	(0.07)
10	1	FALSE	25	0.00	(0.98)	0.00	(0.01)	(0.01)
		TRUE	26	0.00	(0.97)	0.00	(0.01)	(0.01)
	2	FALSE	27	(0.07)	(0.59)	(0.07)	(0.05)	(0.07)
		TRUE	28	0.00	(0.97)	0.00	(0.01)	(0.08)
	3	FALSE	29	(0.06)	(0.56)	(0.06)	(0.03)	(0.06)
		TRUE	30	0.00	(0.97)	0.00	(0.01)	(0.04)

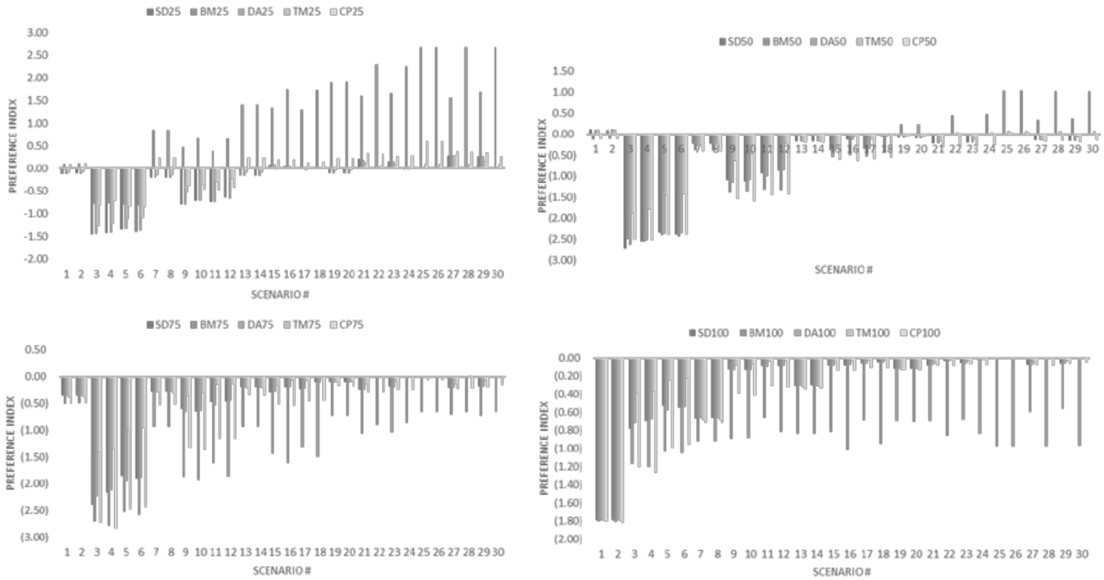

Figure 19. The average preference indexes (PIs) of the students in each income quartile enrolled under the free-tuition policy. The upper left is the PIs of the students in the first quartile; the upper right, the second quartile; the lower left, the third quartile; the lower right, the fourth quartile. The scenario # in this figure corresponds to that in Tables 35-38.

Summary

Educational equality and school quality are complex phenomena emerging from the interaction between students' behaviors and their environment. The simulation results demonstrated that whether TM alone or TM with the free-tuition policy helped equalize educational opportunities and converge school quality upward depended on the interactions of students' school-choice strategies, the admission policy, and the number of choices. Additionally, as shown in the best-worst analyses, a wrong expectation of students' behaviors might result in an opposite expectation of the effects of a policy on educational equality and school quality.

SD without the free tuition policy was Taipei's original admission policy. When the number of choices < 6 and students use Strategy #2 or #3, the replacement of SD with TM without the free-tuition policy helped equalize educational opportunity not only by reducing the SD-Freshman-Ns but also by increasing the match rates. When Strategy = #1, TM alone still produced less SD-Freshmans than SD, but the match rates produced under TM were similar to those under SD. When Strategy = #2 or #3 and the number of choices > 6, the results of TM and SD were mixed but similar regarding SD-Freshman and the match rate. However, the above analysis was based on the assumption that students' behaviors remain the same regardless of the mechanisms. If students changed their behavioral rules with the change of the mechanism, the effect of TM on SD-Freshman might change from positive to negative and vice versa.

The effects of TM alone on the equality of school quality were quite similar to its effects on educational opportunity. TM reduced the inequality in school quality by producing less SD-Senior-Ns than SD when (a) Strategy = #1 or (b) Strategy = #2 or #3 and the number of choices < 6. When Strategy = #2 or #3 and the number of choices > 6, the effects of TM and SD on the equality of school quality were close. However, the effect of TM alone on improving senior scores was limited. In some scenarios, TM even reduced the average senior scores.

The effects of the free-tuition policy also depended on the interaction between students' behaviors and their environment. When the number of choices < 6, the free-tuition policy worsened the performance of TM in reducing the original SD-Freshman in many scenarios where Strategy = #2 or #3. However, when the number of choices > 6, the free-tuition policy improved TM's performance and made TM outperform SD in reducing the original SD-Freshman without regard to students' behaviors. That being said, the benefit of the free-tuition policy might come at the cost of lower overall match rate, unless the number of choices = 10. When the number of choices = 10, the free-tuition policy made all mechanisms produce a 100% match rate, which was a phenomenon that only BM could achieve if there was no free-tuition policy. Again, if students changed their behaviors with the change of policy, the effect of TM with the free-tuition policy on the original SD-Freshman might be reversed from positive to negative, and vice versa.

The effects of the free-tuition policy on the equality of school quality were relatively consistent. As long as the number of choices > 2, the free-tuition policy worsened the inequality of school quality under SD, DA, and TM by increasing the original SD-Senior in each scenario. Senior students in the above scenarios also had lower average scores than those in the same scenarios without the free-tuition policy. However, if students changed their strategies with the change of policy, it was still possible that TM with the free-tuition policy could reduce the inequality of school quality and increase the average senior scores, depending on students' new strategies.

The simulations also showed the effects of TM and the free-tuition policy on the welfares of the students in each quartile. An ideal admission mechanism or policy is to increase the educational opportunity of the most disadvantaged students without hurting the top performing students. Both TM alone and TM combined with the free-tuition policy could produce a top-choice match rate of more than 99% for the top 10% performing students when Strategy # 2 or #3 was used or when the number of choices = 10. The above statement also applied to SD (DA) with and without the free-tuition policy. Therefore, the implementation of TM either with or without the free-tuition policy did not affect the welfare of the top 10% performing students. Additionally, TM and the free-tuition policy benefited the students with the bottom 10% income and in the bottom income quartile in most scenarios. However, there were still a few scenarios where TM alone or the free-tuition policy alone reduced the PIs of those students. When these two interventions were combined, the PIs of those students increased in all scenarios. Since TM with the free-tuition policy benefited the most disadvantaged students, it contributed to the improvement of educational equality in this sense.

Chapter 5 provides an in-depth interpretation of the simulation results, the limitation of this study, and the policy implication of this study. It concludes with recommendations for future research.

Chapter 5: Discussion, Conclusions, and Recommendations

Instruction

In this study, I proposed an agent-based OLG model to study the effects of school admission policies on educational equality and school quality. The traditional approach to studying macroscopic education policy is equation-based modeling, which assumes a fixed system structure and can hardly address the relationship between microbehaviors and macrophenomena (Chen, 2015). However, educational systems are CASs, where the macropatterns emerge from the interaction between the individual agents and their environment. Agents may change their behaviors in response to the change of policies, which in turn changes the system structures (Lucas, 1976). Therefore, it may not be proper to analyze policy implications simply based on a model assuming fixed system structures. ABM enables researchers to address agents' potential behaviors and let the macropatterns emerge from the interaction of the agents and the computational environment through the execution of agents' behavioral rules (Borrill & Tesfatsion, 2011; Macal & North, 2010). Since the macropatterns in an agent-based model are generated from the bottom-up, the causal relationship between individual behaviors and macrophenomena is clear. Therefore, ABM helps to answer not only what an educational policy can do but also how it works (Maroulis et al., 2010).

The Taipei government implemented TM and the free tuition policy in 2016, with the goal of enhancing educational opportunity and school quality. Since the government did not issue any research report to support their decisions, the goal seemed to be more ideology-based than research-based. Because the government had changed the student-assignment mechanism every year since 2014, parents criticized the government for treating their children as experimental subjects (Center for Educational Research and Evaluation, NTNU). It is understandable that to conduct human-subject experiments on new macroeducational policies is often impossible because of the constraints such as ethical issues, cost, time, and human fatigue (Chen, 2015; Roth, 2002). However, these policies could be tested by conducting computational experiments and scenario analysis, and ABM is the right complexity methodology to perform the above tasks.

I simulated the outcomes of TM and the other four real-world mechanisms, with and without the free tuition policy under 30 different scenarios in this study. The findings demonstrated that the effects of the matching mechanisms depend on how students react to the mechanisms and how many choices students are allowed to submit. The simulation results also reveal interesting counterintuitive outcomes that could not be obtained by using equation-based models. Therefore, this study contributes not only to the development of ABM in educational policy analysis but also to the literature on student-assignment mechanisms.

Interpretation of the Findings

This was an exploratory study. The quantities collected from the simulations cannot be interpreted as having a direct relationship with the quantities in the real world. For example, a 2% increase in SD-Freshman cannot be interpreted as a 2% increase in the real world. Instead, the quantitative amounts in the simulation results should be viewed as information regarding the qualitative magnitude of the system’s possible macroproperties.

Impacts on Educational Opportunities

In this study, I measured the equality of educational opportunity by using the standard deviation of the mean freshman family income across all schools (SD-Freshman). However, this measure only measured the educational inequality among the students who were admitted to schools. If a school system has enough seats for all candidates but a high percentage of candidates remain unassigned, then educational inequality is still high even with a low SD-Freshman. Therefore, match rate and SD-Freshman should be discussed together.

The simulated result reveals that like SD-Freshman, match rate depends on students' strategy and the number of choices. The number of choices needs to be large to have a robust high match rate, resistant to students' strategies. How large the number of choices is required depends on the mechanisms. The simulations showed that while TM needed the smallest number of choices, BM needed the most. However, when students were given the full number of choices, only BM could produce a 100% match rate; the other mechanisms could only produce a match rate between 98.2% and 99.9% unless they were accompanied by the free tuition policy. This result suggests that if a little less than 100% match rate is acceptable, there is no need to implement the costly free tuition policy to increase match rate, so long as the government does not constrain the number of choices. On the other hand, if 100% match rate is required, then the free tuition policy seems to be a must.

Compared with SD, which was Taipei's original mechanism, TM substantially reduced the inequality of educational opportunities, but only when the number of choices was low and all students used the same strategies. In other scenarios, the performance of TM was close to that of SD. As mentioned in the previous paragraph, a low number of choices may result in a low match rate. Therefore, if the government expects TM to reduce the inequality of educational opportunity, it should keep the number of choices low and expect a potential loss in the match rate.

In the simulations, the free tuition policy alone reduced the inequality of educational opportunity in all scenarios where students use heterogeneous strategies or the number of choices was high but increased the inequality in most scenarios where the number of choices was low and students use the same strategies. Supplemented by the free tuition policy, TM could reduce the inequality in almost all scenarios. However, when the number of choices was high, the SD-Freshmans between SD with the free tuition policy and TM with the free tuition policy were similar, which implies that a similar result may be obtained by simply implementing the free tuition policy without the complicated prioritization rules of TM, as long as students have enough number of choices. In fact, when the number of choices was high, the SD-Freshmans under SD and TM with or without the free tuition policy were all very close. On the other hand, when the number of choices was high, the SD-Freshmans under BM were significantly lower than those under the other mechanisms, and the free tuition policy further improved BM’s performance. Although BM seems to be a more efficient mechanism to reduce the inequality of educational opportunity in an environment with a high number of choices, BM may cause significant justified envy for students in the top and the third income quartiles, which may not be accepted by the parents of higher performing students in a competitive school district.

Under Taipei's old admission policy, students had the score and rank information to use the strategy similar to Strategies #2 or #3 in the model. At the beginning of the implementation of the new mechanism in 2014, the government released little score or rank information, hoping students would select schools based on their own preferences and considerations (Strategy #1). Without the experimental or empirical evidence, it is unclear whether students would behave as the government planned. If they did, then the simulated data support the government's strategy because the simulated SD-Freshman under Strategy #1 was lower than that under Strategy #2 or #3 regardless of the mechanisms, so long as there was a choice constraint. However, the government has constantly been under pressure and has compromised to release more rank and score information. My observation shows that students would coordinate themselves to use the same old strategy to make their choices. If students would not behave as the government expected, the best-worst estimates demonstrate that the actual result could be significantly different from or even opposite to the anticipated result. This finding stresses the importance of the Lucas critique and the importance of simulating students' potential behaviors in educational policy design.

Impacts on School Quality

In this study, the inequality of school quality was measured by the standard deviation of the mean senior scores in the schools (SD-Senior). The model assumes that a student's family income and scores are highly correlated. The model also assumes that the only factor that affects a student's high school scores is his or her peers. Peer effect will move the student's scores toward the peers' mean scores. However, only those students whose family incomes are within a tolerance range of the peers' average family income will be affected by their peers (see the subsection of details in Chapter 3 for the calculation of senior's scores.) Under these assumptions, it is not surprising to see that in the study, the graphic structure of SD-Senior resembles that of SD-Freshman. The inequality of school quality resulting from the distribution of students assigned by TM was substantially lower than that assigned by SD only when the number of choices was small and all students used the same strategy (Strategy #2 or #3 in this model). In other cases, the levels of the inequality of school quality under TM and SD were close.

When the number of choices was low, the simulated scores of the seniors enrolled under TM were lower than those of the seniors enrolled under SD in most behavior scenarios. When the number of choices was high, the relationships of the scores between TM and SD were mixed but close. This result suggests that TM does not help converge school quality upward. Neither does the free tuition policy. The simulations showed that except for some scenarios where the number of choices was low (4 or lower in the simulations) and some other scenarios under BM, students enrolled under the free tuition policy had an even lower average senior score than those enrolled without the policy.

The simulations showed that the free tuition policy, under TM, SD, and DA, worsens the inequality in school quality, except for some scenarios where the number of choices was extremely low (2 in the model). Under CP, whether the free tuition policy helped reduce the inequality of school quality depended on students' behaviors. If students used the same strategy, CP with the free tuition policy resulted in a wider disparity of school qualities than CP without the free tuition policy in almost all scenarios. If students used heterogeneous strategies, then the results were reversed in almost all scenarios. Under BM, the free tuition policy helped lower inequality of school quality when students used heterogeneous strategies or when the number of choices was high.

From the above discussion, it is interesting to see the complex effects of the free tuition policy on educational opportunity and school quality. When the number of choices was high, the free tuition policy did not work well with TM but worked quite well with BM in reducing the inequalities of educational opportunity and school quality. When students were truth tellers and allowed the full number of choices, the free tuition policy even helped BM to form a distribution of assignments that resulted in higher average senior scores. The simulated results suggest that BM, instead of TM, plus the free tuition policy has the potential to simultaneously reduce the inequalities of educational opportunity and school quality, maintain a high match rate, and achieve a better average senior score. Nevertheless, BM plus the free tuition policy may create more justified envy for the top-income-quartile students, which may not be accepted by parents in the Taipei School District.

Impacts on Students in Different Groups

Some researchers advocate SD and DA because these mechanisms avoid justified envy and are strategy free for all students. This argument is true only when there is no choice constraint (see the section of empirical output validation in this chapter). The simulated results demonstrate that under the same condition, CP and BM can also avoid justified envy for the elite candidates (top 10% performing students), even though they do create justified envy for other higher performing candidates. In the simulations, TM could not reach 100% top-choice match rate. However, when students used the same strategy or there is no choice constraint, TM could allow 99% of the elite students to attend their top-choice schools regardless of the number of choices allowed. In fact, when all students use the same strategies, all the five mechanisms could have at least 99% top-choice match rate for the elite students as long as the number of choices was high enough (at least 8 in this model). When the free tuition policy was implemented, the threshold could be lower (from 8 to 6). The above results demonstrate that for elite students, SD and DA are not superior to TM unless 100% top-choice match rate is required. Even if 100% top-choice match rate is required, SD and DA are not the only mechanisms that can reach that requirement. BM and CP can also reach the 100% requirement with the full number of choices.

If a policy is expected to benefit the most disadvantaged students (students with the bottom 10% income), SD and DA are the worst mechanisms in most cases. I used the assignment result of no justified envy as the baseline, which occurs when students report their preferences as their choices without any constraint under DA. In the simulations, under SD and DA, the bottom-10%-income students were worse off in all scenarios other than their baseline scenarios. TM and the free tuition policy alone or combined helped improve the assignments of these students but in a very limited way. By and large, the performances of CP and CP with the free tuition policy were better than those of TM and TM with the free tuition policy, respectively. However, the benefits CP, even with the free tuition policy, could generate for this group of students were small in comparison to those generated by BM, when the number of choices is high. With the free tuition policy, BM could assign these students to even better schools than BM without the free tuition policy. If we extend the definition of disadvantaged students to all students in the bottom income quartile, the above statements largely held true, except that the scenarios where CP could bring positive PIs to the students in this group became less and BM needed a higher number of choices to bring substantial benefits for these students.

In the simulations, when the number of choices was high, BM could produce the most positive PIs for the students in the bottom income quartile, the most positive PIs or the least negative PIs for the second income quartiles, and the most justified envy for the students in the third and top income quartiles among all mechanisms. This statement also applied to the scenarios under BM with the free tuition policy. This result implies that BM could mix students the most. CP produced the second largest justified envy for the students in the third and top income quartile, which implies that CP could mix students the second most. The above findings correspond to the simulation results that BM, followed by CP, produced the lowest inequality of educational opportunity when the number of choices was high, which might support the claim that mixing, compared with sorting, helps reduce educational inequality (Van de Werfhorst & Mijs, 2010).

Summary of the findings

Assuming students use the same strategies before and after the policy change, A simple answer to the research questions is as follows.

- TM helps equalize educational opportunities if the number of choices is low and students use the same school-choice strategy.
- TM helps school qualities converge upward in few behavioral and admission policy settings.
- TM with the free tuition policy helps equalize educational opportunities with few exceptions.
- TM with the free tuition policy performs worse than TM without the free tuition policy in helping school qualities converge upward.

However, the discussions in the previous subsections show that the simulation results tell more stories than the above simple answer and reveal the complex nature of the interventions. The effects of both TM and the free tuition policy were nonlinear, depending on and emerging from students' behaviors interacting with the interventions and the number of choices. If students change their behaviors with the change of the policy, the above answer may not apply. Therefore, if students' reaction to the new policy is unknown or uncertain, policymakers should perform the best-worst estimates to have a better understanding of the possible policy effects.

In addition to the answers to the research questions, the simulations provide information about the alternatives to the current policies. BM seems to be a better alternative to reduce the inequality of educational opportunities and converge school quality upward. I evaluate the simulated results of BM in the section of implications.

Limitations of the Study

ABM is a generative approach; the macroemergences are generated from the microspecifications (Epstein, 1999). Therefore, knowing the microspecifications of a system is essential to an agent-based modeler. However, unlike macropatterns, microspecifications are usually hard to obtain. In this model, students' preferences and school-choice strategies are assumptions based on my observations and the literature. Since I did not include all plausible preferences and strategies in the model due to the time and cost constraints, how robust the simulated findings are to students' preferences and strategies is unknown. Additionally, this study was exploratory. The parameter values were not calibrated to the real data. For example, there are more than 100 schools in the Taipei School District, while the model has only 10. It is uncertain whether scale has an effect on the macropatterns found in this study.

With the above limitations, the accuracy of the model to predict the outcomes of TM and the free tuition policy in the context of the Taipei School District may be challenged. An agent-based model for real-world prediction requires "close feedback between simulation, testing, data collection and development of theory" (Farmer & Foley, 2009, p. 686). This enormous task requires funding and interdisciplinary collaboration because ABM involves computer programming, human behaviors, and environment construction. If the above issues can be resolved, agent-based models have a potential to perform policy predictions better than their counterpart equation-based models for the following two reasons: (a) Agent-based models are not subject to the constraint of mathematical tractability and can build a virtual world with as many features as the modelers deem necessary; and (b) ABM relaxes the behavioral assumptions of rationality and optimization and allows human learning and adaptation (Epstein, 1999; Farmer & Foley, 2009).

Even though this exploratory model cannot be viewed as a quantitative prediction model for Taipei's new admission policies, the simulation results provide many insights into the nature of these interventions and the relative effectiveness of different matching mechanisms. The simulation results show a pattern change in the mechanisms' relationships at some point in the change of the number of choices, which is a phenomenon that has never been discussed in the literature. The simulation results also reveal the significant impacts of students' behaviors on the macroeffects of the mechanisms. The behavior of using a homogeneous strategy may lead to higher inequalities in educational opportunity and school quality than the behavior of using heterogeneous strategies if the number of choices is low, while the difference in their effects may become small but still exist if the number of choices is high. To the best of my knowledge, this phenomenon has never been discussed in the literature either. Whether the simulated findings are practical can be tested by empirical data collected in the future. In other words, the findings from agent-based simulations help researchers form hypotheses for empirical research and serve as a guide for data collection (Chen, 2015; Epstein, 2008).

Recommendations

In this study, I have tested the five student-assignment mechanisms with 30 different scenarios under a computer environment designed to qualitatively represent the

Taipei School District, which exhibits a moderate-to-high correlation among students' school preferences and an admission capacity adequate for all candidates. I did not test scenarios where students have higher or lower preference correlation than students in the Taipei School District or where the admission capacity is inadequate for all candidates. Therefore, it is not clear whether the findings in this study can apply to school districts with different preference correlation or different admission capacity. Future work can include a robustness test to examine how sensitive the distribution of student assignments is to preference correlation and admission capacity.

The simulated results have shown that students' behaviors play an important role in the formation of the distribution of students' assignments. The simulated scenarios only included a few assumptions of school-choice strategies, which can be divided into the maximum heterogeneous truth-telling strategies (Strategy #1) and the homogeneous strategies (Strategies #2 and #3). I designed Strategies #2 and #3 according to the commonly advised strategies in the Taipei School District (Sun, 2015; Zhang & Wang, 2015). Strategy #1 was based on the principle of maximum entropy. Since TM is new, there has not been any empirical evidence on students' strategies in response to this new mechanism. Further work may include the real-world strategic behaviors when the information becomes available.

The simulations show that the top-choice match rate of the top 10% performing students was seriously affected by the number of choices if Strategy #1 was used. Since students in this group have the top scores and the top priorities, it is possible that they do not need to play games but honestly report their top preferences as their top choices.

Future work can apply different strategies to different student groups. It will be interesting to see whether always truthfully reporting their preferences as choices by the top-performing students regardless of how other students make their decisions can make the assignments of the top-performing students resistant to the changes in the number of choices.

The computational environment in this study contains 10 schools and 1,000 candidates, which represents a system about one-tenth the scale of the Taipei School District. Future work can expand the computational environment by increasing the numbers of schools and students to test whether there is a scaling effect on the performance of the interventions.

Implications

Educational policies are powerful tools to make social changes. However, the history shows a myriad of cases where educational policies produced unexpected and undesired consequences (Groff, 2013). Lack of the right complexity model to study the complex educational problems may be one of the reasons. A prediction agent-based educational model may be difficult to construct. However, an exploratory model still can provide useful insights into the nature of a macroeducational policy. "Essentially, all models are wrong, but some are useful" (Box & Draper, 1987, p. 424). As argued by Epstein (2008), "by revealing tradeoffs, uncertainties, and sensitivities, models can *discipline the dialogue* about options and make unavoidable judgments more considered" (para. 1.7). The findings in this study indeed provide a disciplined basis for Taipei's policymakers and stakeholders to discuss the criticisms of the current admission policies and the solutions for the current problems.

One criticism of the free tuition policy is that this expensive policy is cost-inefficient not only because it would crowd out the programs that could directly benefit disadvantaged students but also because the high school attendance rate had been 93% before the implementation of this policy. The simulation results show that match rate was affected not only by the free tuition policy but also by the number of choices and the mechanism. The number of choices must be high to have a high match rate. In this case, the free tuition policy has a limited effect on reducing inequality of educational opportunity. Besides, when there was no choice constraint, even without the free tuition policy, all mechanisms could reach a match rate of at least 98%; BM could even reach 100%. The above findings seem to support the criticism. Although the free tuition policy is necessary to bring all mechanism to reach 100% match rate from 98%, policymakers should perform the cost-benefit analysis carefully to justify the implementation of the free tuition policy.

TM has been criticized for its design of assigning decreasing scores to student's choices. Many parents view this design as a punishment for student's bad choices (Zheng, 2015). The Taipei government's reason for adopting such design was that it could induce students to focus more on their true preferences than on school ranking and thus would result in more mixing than sorting. The simulation results show that when the number of choices was large, the distributions of assignments under TM and SD (Taipei's original mechanism) were similar. TM could substantially reduce educational inequality only when the number of choices was low. However, a low number of choices might cause the match rate to drop several to dozens of points. Whether this is acceptable to the society is doubtful. If the number of choices must be high, then the effect of TM is small. Whether it is worth implementing TM with a small effect but a heavy criticism is also debatable.

To ensure a high match rate and low inequalities in both the educational opportunity and school quality, BM seems to be the best solution if policymakers allow a high or a full number of choices. Additionally, BM could benefit the disadvantaged students the most even though BM could create the most justified envy for the students in the top income quartile. Nevertheless, BM with the full number of choices would not harm the top 10% performing students. BM emphasizes students' choices, which inspired the initial design of choice score in the algorithm of TM. However, because of the creation of severe justified envy, the role of BM was diminished in the revised algorithm of TM implemented in 2016. Therefore, despite all the benefits BM can provide, its acceptance by a society viewing the avoidance of justified envy as fairness is questionable.

As stated by Ostrom (2005), "if the individuals who are crafting and modifying rules do not understand how particular combinations of rules affect actions and outcomes in a particular ecological and cultural environment, rule changes may produce unexpected and, at times, disastrous outcomes" (p. 3). If the Taipei government intends to have a successful admission reform to reduce educational inequality, it may want to reconsider its priority and engage more in public opinion change. Unless the public agrees that fairness includes equal opportunity to attend each school, any implementation of a mechanism deviating from SD may be doomed to be heavily criticized.

An interesting phenomenon revealed by the simulation results is that a change in the relationships among the effects of the five mechanisms occurs at a certain point in the change in the number of choices. In the simulations, when students used the same strategy, the number of choices 6 acted like a bifurcation or turning point. The SD-Freshman under BM was among the highest and TM the lowest when the number of choices < 6; the relationship reversed when the number of choices > 6. Many school systems constrain the number of choices a student can submit. For example, the Taipei School District assigns choice scores only to the first 30 choices a student makes, while there are more than 100 schools in the district (New Taipei City Government, 2015). Similarly, Boston School District only allows up to 14 choices selected from their 125 schools (Boston Public Schools, 2017). A common reason for the constraints is the limited computational power. Other causes include stimulating students to contemplate their real preferences and making school-choice advice feasible (Liu, Liu, & Tu, 2012). This finding alerts the policymakers to the impact of choice constraints. If policymakers decide to constrain the number of choices, they must investigate carefully the impact of the number they choose, or they may experience an "unexpected and, at times, disastrous outcomes" (Ostrom, 2005, p. 3).

Conclusions

Educational systems are complex adaptive systems. Macroeducation policies cannot be evaluated easily by using linear regression models. Complexity tools are needed to understand what and how an educational policy's macroeffects emerge from the interactions between the students and the environment. This study demonstrates that ABM can provide counterintuitive insights into the impacts of admission policies that can hardly be found by the traditional equation-based models.

Intuitively, the benefits of the free tuition policy, which makes private schools affordable for poorer students, are twofold: (a) It gives poorer students more opportunities to attend higher quality private schools and thus can improve the match rate; and (b) with a higher proportion of poorer students, this policy helps reduce private schools' average family income and consequently result in a lower educational inequality. However, the agent-based simulation results counterintuitively show that the free tuition policy may instead reduce the match rate and increase the inequality if the number of choices is not high enough. The simulations also show a change in the relationships among the effects of the mechanisms at a certain point in the change of the number of choices. This nonlinear relationship can hardly be seen in an equation-based model either. Additionally, ABM is convenient to perform multilevel analysis. In this study, I analyzed not only macrolevel data but also mesolevel data. This model also allows me to collect microlevel data, including individual students' school-assignment results and their high-school scores. In this sense, ABM is a methodology that can generate big data, which help researchers to have a multifaceted understanding of the hierarchical properties of an educational system and the complex effects of educational policy.

As argued by Roth (2002), computational simulations "help us analyze games that are too complex to solve analytically" (p. 1374). This agent-based model helps us understand what and whom the admission policies work for, in what condition it works, and how it works. However, few agent-based educational models have been constructed to analyze macroeducational policies. More efforts are needed to produce full-fledged agent-based educational models and strengthen the development of ABM in educational research.

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F%97%E9%A1%98%E5%BA%8F%E6%89%A3%E5%88%86%E6%98%AF%E5%85%83%E5%87%B6

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Appendix A : NetLogo Code of the Model

```
extensions [r]

breed [students student]

breed [universities university]

globals [

  predict-score     ; the predict score of university (may be the min-score or max-rank)

  ave-score     ;;

  ave-score-sd     ; average score, serial dictator

  ave-score-bm     ; average score, boston mechanism

  ave-score-da     ; average score, deferred acceptance

  ave-score-tm     ; average score, taipei mechanism

  ave-score-cp     ; average score, chinese parallel

  min-score     ;;

  min-score-sd     ; minimum score, serial dictator

  min-score-bm     ; minimum score, boston mechanism

  min-score-da     ; minimum score, deferred acceptance

  min-score-tm     ; minimum score, taipei mechanism

  min-score-cp     ; minimum score, chinese parallel

  max-rank     ;; the ranking of the students who has the lowest scores and was
        admitted to the universities

  max-rank-sd     ; maximum rank, serial dictator

  max-rank-bm     ; maximum rank, boston mechanism
```

```
max-rank-da          ; maximum rank, deferred acceptance
max-rank-tm          ; maximum rank, taipei mechanism
max-rank-cp          ; maximum rank, chinese parallel
max-rank-score       ;; the score in this year according to the max-rank last year
high-school
num-univ             ;; number of universities
univ
u
mean-utility
average-mismatch-sd  ;; to measure the error, serial dictator
average-mismatch-bm  ;; to measure the error, boston mechanism
average-mismatch-da  ;; to measure the error, deferred acceptance
average-mismatch-tm  ;; to measure the error, Taipei mechanism
average-mismatch-cp  ;; to measure the error, Chinese parallel
sd-mismatch-sd       ;; standard deviation of the mismatch, serial dictator
sd-mismatch-bm       ;; standard deviation of the mismatch, boston mechanism
sd-mismatch-da       ;; standard deviation of the mismatch, deferred acceptance
sd-mismatch-tm       ;; standard deviation of the mismatch, taipei mechanism
sd-mismatch-cp       ;; standard deviation of the mismatch, chinese parallel
x                    ;; to produce random number
no-choices-1         ;; number of choices list 1
no-choices-2         ;; number of choices list 2
```

```
  score-mean              ;
  score-sdv               ;
  seven-letter-tiers
  mu
;;;;;  zipf-H                   ; array of H(n,n)
  income-average
  univ-rank               ; university rank
  univ-rank-sd            ; university rank, serial dictator
  univ-rank-bm            ; university rank, boston mechanism
  univ-rank-da            ; university rank, deferred acceptance
  univ-rank-tm            ; university rank, taipei mechanism
  univ-rank-cp            ; university rank, chinese parallel
  y1end-score             ; year-1 end score list, aligned to capacity, ordered according to
       university
  y1end-score-sd          ; year-1 end score list, serial dicator
  y1end-score-bm          ; year-1 end score list, boston mechanism
  y1end-score-da          ; year-1 end score list, deferred acceptance
  y1end-score-tm          ; year-1 end score list, taipei mechanism
  y1end-score-cp          ; year-1 end score list, chinese parallel
  y1end-income            ; year-1 end family income list, aligned to capacity, ordered
       according to university
  y1end-income-sd         ; year-1 end family income list, serial dicator
```

y1end-income-bm ; year-1 end family income list, boston mechanism

y1end-income-da ; year-1 end family income list, deferred acceptance

y1end-income-tm ; year-1 end family income list, taipei mechanism

y1end-income-cp ; year-1 end family income list, chinese parallel

y2end-score ; year-2 end score list, aligned to capacity, ordered according to university

y2end-score-sd ; year-2 end score list, serial dicator

y2end-score-bm ; year-2 end score list, boston mechanism

y2end-score-da ; year-2 end score list, deferred acceptance

y2end-score-tm ; year-2 end score list, taipei mechanism

y2end-score-cp ; year-2 end score list, chinese parallel

y2end-income ; year-2 end family income list, aligned to capacity, ordered according to university

y2end-income-sd ; year-2 end family income list, serial dicator

y2end-income-bm ; year-2 end family income list, boston mechanism

y2end-income-da ; year-2 end family income list, deferred acceptance

y2end-income-tm ; year-2 end family income list, taipei mechanism

y2end-income-cp ; year-2 end family income list, chinese parallel

y3end-score-ave

y3end-score-ave-sd

y3end-score-ave-bm

y3end-score-ave-da

y3end-score-ave-tm

y3end-score-ave-cp

y3end-score-stdev

y3end-score-stdev-sd

y3end-score-stdev-bm

y3end-score-stdev-da

y3end-score-stdev-tm

y3end-score-stdev-cp

income-quartile ;; the average misplacement of each quartile

income-quartile-sd

income-quartile-bm

income-quartile-da

income-quartile-tm

income-quartile-cp

mean-quartile ;; the average misplacement of each quartile

mean-quartile-sd

mean-quartile-bm

mean-quartile-da

mean-quartile-tm

mean-quartile-cp

mean-top10

mean-top10-sd

mean-top10-bm

mean-top10-da

mean-top10-tm

mean-top10-cp

mean-bottom10

mean-bottom10-sd

mean-bottom10-bm

mean-bottom10-da

mean-bottom10-tm

mean-bottom10-cp

max-top10

max-top10-sd

max-top10-bm

max-top10-da

max-top10-tm

max-top10-cp

percent-top10

percent-top10-sd

percent-top10-bm

percent-top10-da

percent-top10-tm

percent-top10-cp

income-mean

income-mean-sd

income-mean-bm

income-mean-da

income-mean-tm

income-mean-cp

income-stdev

income-stdev-sd

income-stdev-bm

income-stdev-da

income-stdev-tm

income-stdev-cp

y1-income-mean

y1-income-mean-sd

y1-income-mean-bm

y1-income-mean-da

y1-income-mean-tm

y1-income-mean-cp

y1-income-stdev

y1-income-stdev-sd

y1-income-stdev-bm

y1-income-stdev-da

y1-income-stdev-tm

y1-income-stdev-cp

private-list

univ-fill

mvrnorm-r-cmd

mvrnorm-matrix

mvrnorm-idx

zone-ln-income-mean

zone-ln-income-stdev

zone-score-mean

zone-score-stdev

univ-enrollment

univ-enrollment-sd

univ-enrollment-bm

univ-enrollment-da

univ-enrollment-tm

univ-enrollment-cp

end-score

end-income

average-score

average-income

```
]
universities-own [
  priority-students-list    ;;
  potential-students        ;;
  univ-zone                 ;; school location zone (0 or 1)
]
students-own [
  score                     ;; students' score, it is now the average of the score-subjects
  score-subjects            ;; students' score of each of the 5 subject
  preference-rank           ;; students' preference of school ranks
  preference-list           ;; student's preference of school ID
  choice                    ;; a list to store student's total choice (use n-value choices []
  seven-scale-score         ;; sum of seven scale for "Taipei mechanism"
  last-choice               ;; index for deferred acceptance
  next-choice               ;; index for deferred acceptance
  choice-1                  ;; a list of student's first choice under Chinese parallel
  choice-2                  ;; a list of student's second choice under Chinese parallel
  utility                   ;; utility the students have
  base-place                ;; where he could go
  actual-place              ;; where he goes actually
  misplacement-abs          ;; to measure the misplacement-abs (absolute distance of base-
place and what he really get)
```

```
  misplacement          ;; to measure misplacement gain or loss
  rank
  base-preference       ;; index of preference
  actual-preference     ;; index of enrollment
  income                ;; student's family income
  district              ;; student's living district (1 or 2)
]
to setup
  clear-all
  ;; initialize R interface
  ;; activate MASS library for mvrnorm function
  r:eval "library(MASS)"
  ;; initialize covariance matrix
  r:eval (word "Sig <- matrix(c("
    "1.0, 0.8, 0.8, 0.8, 0.8, 0.8, "
    "0.8, 1.0, 0.8, 0.8, 0.8, 0.8, "
    "0.8, 0.8, 1.0, 0.8, 0.8, 0.8, "
    "0.8, 0.8, 0.8, 1.0, 0.8, 0.8, "
    "0.8, 0.8, 0.8, 0.8, 1.0, 0.8, "
    "0.8, 0.8, 0.8, 0.8, 0.8, 1.0), nrow=6)")
  ;; initialize mvrnorm command string
  ;; This is needed because number of students may vary
```

```
  set mvrnorm-r-cmd (word "X <- mvrnorm(" num-students ", mu=rep(0, 6), Sigma = Sig,
empirical = TRUE)")
;;  set tolerance 0.5        ;;0.58
;;  set peer-effect 0.1       ;;0.58
  ;random-seed 0
  set-default-shape students "person"
  set-default-shape universities "house"
  set num-univ 10
  set zone-ln-income-mean [14.14 13.84]
  set zone-ln-income-stdev [0.52 0.47]
  set zone-score-mean [65.0 47.5]
  set zone-score-stdev [23.0 23.0]
  set univ-rank n-values num-univ [?]
  set univ-rank-sd univ-rank
  set univ-rank-bm univ-rank
  set univ-rank-da univ-rank
  set univ-rank-tm univ-rank
  set univ-rank-cp univ-rank
  set seven-letter-tiers [41 61 71 84 90 94 100]
  set mu 3
  set ave-score n-values num-univ [-1 - ?]
  set ave-score-sd ave-score
```

```
set ave-score-bm ave-score

set ave-score-da ave-score

set ave-score-tm ave-score

set ave-score-cp ave-score

set min-score n-values num-univ [0]

set min-score-sd min-score

set min-score-bm min-score

set min-score-da min-score

set min-score-tm min-score

set min-score-cp min-score

set y1end-score n-values (num-univ * capacity) [-1]

set y1end-score-sd y1end-score

set y1end-score-bm y1end-score

set y1end-score-da y1end-score

set y1end-score-tm y1end-score

set y1end-score-cp y1end-score

set y1end-income n-values (num-univ * capacity) [0]

set y1end-income-sd y1end-income

set y1end-income-bm y1end-income

set y1end-income-da y1end-income

set y1end-income-tm y1end-income

set y1end-income-cp y1end-income
```

set y2end-score n-values (num-univ * capacity) [-1]

set y2end-score-sd y2end-score

set y2end-score-bm y2end-score

set y2end-score-da y2end-score

set y2end-score-tm y2end-score

set y2end-score-cp y2end-score

set y2end-income n-values (num-univ * capacity) [0]

set y2end-income-sd y2end-income

set y2end-income-bm y2end-income

set y2end-income-da y2end-income

set y2end-income-tm y2end-income

set y2end-income-cp y2end-income

set y3end-score-ave n-values num-univ [-1 - ?]

set y3end-score-ave-sd y3end-score-ave

set y3end-score-ave-bm y3end-score-ave

set y3end-score-ave-da y3end-score-ave

set y3end-score-ave-tm y3end-score-ave

set y3end-score-ave-cp y3end-score-ave

set univ-enrollment n-values num-univ [0]

set univ-enrollment-sd univ-enrollment

set univ-enrollment-bm univ-enrollment

set univ-enrollment-da univ-enrollment

```
set univ-enrollment-tm univ-enrollment
set univ-enrollment-cp univ-enrollment
set max-rank n-values num-univ [num-students]
set max-rank-sd max-rank
set max-rank-bm max-rank
set max-rank-da max-rank
set max-rank-tm max-rank
set max-rank-cp max-rank
set income-quartile n-values 4 [0]
set income-quartile-sd income-quartile
set income-quartile-bm income-quartile
set income-quartile-da income-quartile
set income-quartile-tm income-quartile
set income-quartile-cp income-quartile
set mean-quartile n-values 4 [0]
set mean-quartile-sd mean-quartile
set mean-quartile-bm mean-quartile
set mean-quartile-da mean-quartile
set mean-quartile-tm mean-quartile
set mean-quartile-cp mean-quartile
set mean-top10-sd 0
set mean-top10-bm 0
```

```
set mean-top10-da 0
set mean-top10-tm 0
set mean-top10-cp 0
set mean-bottom10 0
set mean-bottom10-sd 0
set mean-bottom10-bm 0
set mean-bottom10-da 0
set mean-bottom10-tm 0
set mean-bottom10-cp 0
set max-top10-sd 0
set max-top10-bm 0
set max-top10-da 0
set max-top10-tm 0
set max-top10-cp 0
set percent-top10-sd 0
set percent-top10-bm 0
set percent-top10-da 0
set percent-top10-tm 0
set percent-top10-cp 0
set predict-score n-values num-univ [0]
set income-mean n-values num-univ [0]
set income-mean-sd income-mean
```

```
set income-mean-bm income-mean

set income-mean-da income-mean

set income-mean-tm income-mean

set income-mean-cp income-mean

set income-stdev 0

set income-stdev-sd income-stdev

set income-stdev-bm income-stdev

set income-stdev-da income-stdev

set income-stdev-tm income-stdev

set income-stdev-cp income-stdev

set y1-income-mean n-values num-univ [0]

set y1-income-mean-sd y1-income-mean

set y1-income-mean-bm y1-income-mean

set y1-income-mean-da y1-income-mean

set y1-income-mean-tm y1-income-mean

set y1-income-mean-cp y1-income-mean

set y1-income-stdev 0

set y1-income-stdev-sd y1-income-stdev

set y1-income-stdev-bm y1-income-stdev

set y1-income-stdev-da y1-income-stdev

set y1-income-stdev-tm y1-income-stdev

set y1-income-stdev-cp y1-income-stdev
```

```
  set end-score n-values num-students [-1]
  set end-income n-values num-students [0]
  set average-score n-values num-univ [-1 - ?]
  set average-income n-values num-univ [0]
;;  set top10-rank 0
  setup-schools                          ;; setup ten universities and high-school
  set univ-fill n-values num-univ [11 + ?]
  setup-students
;;;;;  setup-zipf num-univ alpha                ;; init zipf array index
  if behaviorspace-run-number < 2 [
    write-file-header
  ]

  reset-ticks
end
to setup-schools
  set univ n-values num-univ [?]
  let n 0
  let i 0
  let d (max-pxcor - min-pxcor) / num-univ
  repeat num-univ
```

```
 [
   set i n / d
    set univ replace-item i univ patches with [pycor > -10 and pxcor > (min-pxcor + n)
and pxcor < (min-pxcor + d + n)]
    ask patches with [pycor > -10 and pxcor > (min-pxcor + n) and pxcor < (min-pxcor +
d + n)] [
      ifelse i = 0 or i = 1 or i = 2 or i = 4 or i = 7 [
        set pcolor 105
      ] [
        set pcolor 102
      ]
    ]
    ask patch (min-pxcor + d / 2 + n) -10 [set plabel-color white set plabel (n / d  + 1) ]
    ask patch (min-pxcor + d / 2 + n) -11 [set plabel-color red set plabel item (n / d)
          predict-score]
    ask patch (min-pxcor + d / 2 + n) (max-pycor) [
          sprout-universities 1 [
        ;; hard code school zones
        ifelse i = 0 or i = 1 or i = 2 or i = 4 or i = 7 [
          set univ-zone 1
        ] [
          set univ-zone 2
```

```
      ]
      ;; hard code private schools without supplement
      ;; use color to code this flag
      ;; color white indicates public schools or private schools with supplement
      ;; color 27 indicates private schools without supplement
      ifelse i != 2 [
        set color white
      ] [
        set color 27
      ]
    ]
  ]
  set n (n + d)
]
set high-school patches with  [pycor < -11]
ask high-school [set pcolor lime - 4]
set private-list []
;; remove private school from list, hard coded for now
foreach sort universities with [color != white] [
  ask ? [
    set private-list lput (who + 1) private-list
  ]
```

```
  ]
end
to setup-students
  create-students num-students [
    set last-choice 0
    set next-choice 0
    setxy random-xcor (- 12 - random 5)
    set color magenta
    set size 1
    set district 2
  ]
  ask n-of round (num-students * 0.36) students [
    set district 1
  ]
end
to generate-rank-list
  set preference-rank []
  let z-value 0
  let j 1
  let school-list n-values zone-split [? + 1]
  repeat zone-split [
    set z-value zipf (zone-split + 1 - j) alpha
```

```
    set preference-rank lput item (z-value - 1) school-list preference-rank
    set school-list remove-item (z-value - 1) school-list
    set j (j + 1)
  ]
  set school-list n-values (num-univ - zone-split) [? + 1]
  set j 1
  repeat (num-univ - zone-split) [
    set z-value zipf (num-univ - zone-split + 1 - j) alpha
    set preference-rank lput (zone-split + item (z-value - 1) school-list) preference-rank
    set school-list remove-item (z-value - 1) school-list
    set j (j + 1)
  ]
end
to generate-preference
  set preference-list []
  let univ-zone-list []
  let t0 0
  let j 1
  repeat zone-split [
    set preference-list lput (1 + item (item (j - 1) preference-rank - 1) univ-rank)
preference-list
    set j (j + 1)
```

```
]
;; update school zone list here too
set j 1
repeat (num-univ - zone-split) [
  set t0 item (item (zone-split + j - 1) preference-rank - 1) univ-rank
  set preference-list lput (t0 + 1) preference-list
  set univ-zone-list lput get-univ-zone t0 univ-zone-list
  set j (j + 1)
]
; perform shuffle
; initialize starting list index
let temp-zone-idx zone-split
let temp-pref-list []
; check for list of schools in the same district
while [member? district univ-zone-list] [
  ; if the remaining schools are still in the list
  ifelse item 0 univ-zone-list != district [
    ; first school is not in the same zone, start shuffling
    ; get the first school in district from the list
    let univ-id item (temp-zone-idx + position district univ-zone-list) preference-list
    ; remove the first school from the list
    set preference-list remove univ-id preference-list
```

```
      ; insert the school to the front
      set preference-list se (se sublist preference-list 0 temp-zone-idx univ-id) sublist
preference-list temp-zone-idx length preference-list
      ; remove the school from zone list
      set univ-zone-list remove-item (position district univ-zone-list) univ-zone-list
    ]
    [
      ; current top school is in the same zone, remove it from list
      set univ-zone-list remove-item 0 univ-zone-list
    ]
    set temp-zone-idx (temp-zone-idx + 1)
  ]
  ; remove private school without grant
  if not grant and income <= income-average [
    foreach private-list [
      if member? ? preference-list [
        set preference-list remove ? preference-list
      ]
    ]
  ]
end
```

```
to-report get-univ-zone [id]
  let tval 0
  ask university id [
    set tval univ-zone
  ]
  report tval
end
to generate-students-priority
  set priority-students-list []
  let school-id who + 1
  ifelse mechanism = "Taipei mechanism" [
      set priority-students-list sort-on [
        (- (score / 1000000000 + seven-scale-score + (35 - floor (ifelse-value is-number?
(position school-id choice) [position school-id choice / group-size][35]))))
        ] students
    ] [
      set priority-students-list sort-on [(- score)] students
    ]
end
to go-to-high-school
  move-to one-of high-school
  set color magenta
```

```
  set last-choice 0
  set next-choice 0              ;; index for deferred acceptance
end

to go
  if ticks >= max-tick  [stop]
  set score-mean (random-normal population-mean (population-standard-deviation / 3.0))
  set score-sdv sqrt (random-gamma 30 0.5 * population-standard-deviation ^ 2 / 60)
  ;; generate normalized student income/score matrix
  r:eval mvrnorm-r-cmd
  ;; receive income/score matrix from R
  ;; matrix structure is as follows
  ;; items 0 * num-students to (1 * num-students - 1) => normalized delta of student's
          household income
  ;; items 1 * num-students to (2 * num-students - 1) => normalized delta of student's
          subject 1 score
  ;; items 2 * num-students to (3 * num-students - 1) => normalized delta of student's
          subject 2 score
  ;; items 3 * num-students to (4 * num-students - 1) => normalized delta of student's
subject 3 score
  ;; items 4 * num-students to (5 * num-students - 1) => normalized delta of student's
          subject 4 score
```

```
;; items 5 * num-students to (6 * num-students - 1) => normalized delta of student's
        subject 5 score
set mvrnorm-matrix r:get "X"
;; create index to easily map inome and subjects scores to mvrnorm-matrix
set mvrnorm-idx 0 ;;n-values 6 [? * num-students]
ask students [get-score]                ;; get score
set income-average mean [income] of students
ask students [generate-rank-list]
set no-choices-2 floor no-choices / 2
set no-choices-1 (no-choices - no-choices-2)
let mechanisms (list "Serial dictatorship" "Boston mechanism" "Deferred acceptance"
        "Taipei mechanism" "Chinese parallel mechanism")
if dbg-print [print (word "########## tick count: " ticks)]
ifelse All_mechanism? [
  foreach mechanisms [
    if dbg-print [print (word "<<<< mechanism: " ? " >>>>")]
    set mechanism ?
    update-begin-variables ?
    ask students [generate-preference]        ;; generate random preference list according
        to the probability matrix P
    ask students [get-seven-scale-score]
```

```
get-fair-place                 ;; to find out the base-place, i.e. the university that the
   student could go
ask students [go-to-high-school]
get-max-rank-score
ask students [
  set next-choice 0
  set last-choice 0
  choose
]
ask universities [generate-students-priority]
enroll
ask students [calculate-misplacement]
update-min-score-and-rank
update-label
class-mismatch
; update year 3 end score from year 2 end score
update-y3end-score
; update school ranking based on year 3 end score
update-univ-rank y3end-score-ave
; update year 2 end score from year 1 end score
update-y2end-score
; update year 1 end score from year 1 beginning score
```

```
    update-y1end-score
    update-dynamic-variables ?
  ]
  ifelse print-first-three [
    write-simulation-data
  ]
  [
    if ticks > 2 [write-simulation-data]
  ]
]
[
  update-begin-variables mechanism
  ask students [generate-preference]          ;; generate random preference list according
        to the probability matrix P
  ask students [get-seven-scale-score]
  get-fair-place                              ;; to find out the base-place, i.e. the university that the
        student could go
  ask students [go-to-high-school]
  get-max-rank-score
  ask students [
    set next-choice 0
    set last-choice 0
```

```
  choose
]
ask universities [generate-students-priority]
enroll
ask students [calculate-misplacement]
update-min-score-and-rank
update-label
class-mismatch
; update year 3 end score from year 2 end score
update-y3end-score
; update school ranking based on year 3 end score
update-univ-rank y3end-score-ave
; update year 2 end score from year 1 end score
update-y2end-score
; update year 1 end score from year 1 beginning score
update-y1end-score
update-dynamic-variables mechanism
ifelse print-first-three [
  write-simulation-data
]
[
  if ticks > 2 [write-simulation-data]
```

```
    ]
  ]
  tick
  ;;ask students [do-plots]
end
to-report cut-list [string]
  let temp (word map [(word ? ", ")] string)
  set temp remove-item 0 remove-item (length temp - 1) temp
  report temp
end
to get-score
  let i 0
  ;; get the current student map index
  let n mvrnorm-idx
  ;; update map index for next student
  set mvrnorm-idx (mvrnorm-idx + 1)
  ;; setup student's living district index
  let zone (district - 1)
  ;; calculate income by e ^ (log of income)
  set income exp (item zone zone-ln-income-stdev * item n mvrnorm-matrix + item zone
        zone-ln-income-mean)
```

```
  ;; calculate student's subject scores
  set score-subjects []
  while [i < 5] [
    ;; update mvrnorm index by adding num-students to n
    set n (n + num-students)
    set score-subjects se score-subjects (max (se 0.00001 min (se 99.99999 (item zone
        zone-score-stdev * item n mvrnorm-matrix + item zone zone-score-mean))))
    set i (i + 1)
  ]
  ;; calculate composite score
  set score mean score-subjects
end
to get-seven-scale-score
  let i 0
  let mult-scale 0.001
  set seven-scale-score 0
  repeat length score-subjects [
    let t-score item i score-subjects
    let j 1
    while [t-score >= item (j - 1) seven-letter-tiers] [set j (j + 1)]
    set seven-scale-score (seven-scale-score + (j * 1.01) + (j * mult-scale))
    set mult-scale (mult-scale * 0.1)
```

```
    set i (i + 1)
  ]
end
to get-max-rank-score
  set max-rank-score map [item (? - 1) sort-by > [score] of students ] max-rank          ;; get
          max-rank-score based on the max-rank last year and the scores this year
end
to get-fair-place
  let c-max 0
  set u n-values num-univ [0]
  foreach sort-on  [(- score)] students [
    ask ? [
      set c-max length preference-list
      let i 1                              ;; the i th univ that the students choose
      while [i <= c-max] [
        set base-place item (i - 1) preference-list  ;;of self)
        ifelse item (base-place - 1) u >= capacity [
          set i (i + 1)
        ] [
          set base-preference i
          set i c-max + 2                                                   ;; to end the circle
```

```
        set u replace-item (base-place - 1) u (item (base-place - 1) u + 1)        ;; count the
         students
      ]
      if i = c-max + 1 [
        set base-place num-univ + 1
        set base-preference num-univ + 1
      ]
    ]
  ]
 ]
end
to choose                              ;; to choose university
 ifelse scheme = "1" [
   produce-random-strategy
 ] [
   produce-c&c-strategy
 ]
 ;;  move-to one-of (item (first choice - 1) univ)
end
to produce-random-strategy
 let c-max length preference-list
 if c-max > no-choices [set c-max no-choices]
```

```
  set choice n-of c-max preference-list
  if mechanism = "Chinese parallel mechanism" [
    let t1 no-choices-1
    if t1 > c-max [set t1 c-max]
    set choice-1 sublist choice 0 t1
    set choice-2 sublist choice t1 c-max
  ]
end
to produce-c&c-strategy
  ifelse scheme = "2 or 3" [
    set predict-score  min-score
  ] [
    set predict-score  max-rank-score
  ]
  set choice []
  let school-left []
  let i 0
  let c-max length preference-list
  foreach preference-list [                              ;; get the F_i set
    ifelse score >= item (? - 1) predict-score [
      set choice lput ? choice
      set i (i + 1)
```

```
    ] [
      set school-left lput ? school-left
    ]
  ]
  if c-max > no-choices [set c-max no-choices]
  if i < c-max [
    let choice-left n-of (c-max - i) school-left
    set choice se choice choice-left
  ]
  set choice sublist choice 0 c-max
  ifelse (mechanism = "Chinese parallel mechanism") [
    chinese-choice
  ] [
    if extra-in-order = true [
      set choice sort-by-preference choice preference-list
    ]
  ]
end
to chinese-choice
  let c-max length choice
  let t1 no-choices-1
  if t1 > c-max [set t1 c-max]
```

```
  set choice-1 sublist choice 0 t1
  set choice-2 sublist choice t1 c-max
  if extra-in-order = true [
    set choice-1 sort-by-preference choice-1 preference-list
    set choice-2 sort-by-preference choice-2 preference-list
  ]
end
to enroll                              ;; the university give offer according to its enrollment plan
and the ranking of students
  set u n-values num-univ [0]                 ;; the number of students that enrolled
  if mechanism = "Serial dictatorship"
    [serial-dictatorship]
  if mechanism = "Boston mechanism"
    [boston-mechanism]
  if mechanism = "Deferred acceptance"
    [deferred-acceptance]
  if mechanism = "Taipei mechanism"
    ;;[taipei-mechanism]
    [deferred-acceptance]
  if mechanism = "Chinese parallel mechanism"
    [chinese-parallel]
end
```

```
to serial-dictatorship
  foreach sort-on  [(- score)] students [
    ask ? [
      let c-max length choice
      let i 1                                ;; the i th univ that the students choose
      while [i <= c-max] [
        set actual-place (item (i - 1) choice) ;;[choice] of self)
         ifelse item (actual-place - 1) u >= capacity [
          set i (i + 1)                                        ;; over capacity, not placed, test for
next choice
        ] [
          set actual-preference position actual-place preference-list + 1
          set i (c-max + 2)                                      ;; to end the circle
                                                         ;; set utility (12 / 11 - actual-place / 11 -
score / 1089)          ;; set the utility
                                                         ;; set misplacement abs (base-place -
actual-place)                  ;; set the misplacement
          move-to one-of (item (actual-place - 1) univ)                    ;; show placement
          ifelse district = 1 [set color yellow] [set color lime]                        ;; change display
color
```

```
        set u replace-item (actual-place - 1) u (item (actual-place - 1) u + 1)        ;; count
the students that enrolled
      ]
      if i = c-max + 1 [
        ;; move-to one-of high-school set color white        ;;the student go back to high
school if got no offer]
        set actual-place num-univ + 1
        set actual-preference num-univ + 1
        ;; set misplacement abs (base-place - 11)]
      ]
    ]
  ]
 ]
end
to boston-mechanism
 let n 0
 while [n < no-choices and (any? students with [color = magenta])] [
   foreach n-values num-univ [?] [
     foreach sort-on [(- score)] students with [item n (se choice univ-fill) = ? + 1 and color
= magenta ] [  ; ? represents 0, 1, 2,..., 9
       ask ? [                                                            ;; students from highest
score to lowest score
```

```
      set actual-place item n choice ;;[choice] of self
      if item (actual-place - 1) u <  capacity
      [
       move-to one-of (item (actual-place - 1) univ)
       ifelse district = 1 [set color yellow][set color lime]                    ;; change display
color
       set u replace-item (actual-place - 1) u (item (actual-place - 1) u + 1)   ;; count the
students that enrolled
       set actual-preference position actual-place preference-list + 1
      ]
     ]
    ]
   ]
   set n n + 1
  ]
  if any? students with [color = magenta] [
   ask students with [color = magenta] [
    set actual-place num-univ + 1
    set actual-preference num-univ + 1
   ]
  ]
end
```

```
to deferred-acceptance
 let excluded-students []
 let choice-sum 1
 let temp []
 let c-max 0
 while [choice-sum != 0] [
   ;; create a sublist of unassigned students
   ;; initialize the list
   set excluded-students students
   foreach sort universities [
     ask ? [
       set temp students with [item next-choice choice = ([who] of ? + 1) and color =
magenta ]
       let univ-priority [priority-students-list] of ?
       ;show univ-priority
       set potential-students (sort-by-preference sort temp univ-priority)
       ;show potential-students
       if length potential-students >= capacity [
         set potential-students sublist potential-students 0 capacity
       ]
       ;show potential-students
```

```
    set excluded-students (sort-by-not-listed sort excluded-students potential-students)
    foreach (sort temp) [
      if not member? ? potential-students [
        ask ? [
          set c-max length choice
          set last-choice next-choice
          set next-choice next-choice + 1
          if next-choice >= (c-max - 1) [
            set next-choice (c-max - 1)
          ]
        ]
      ]
    ]
  ]
]
;; detect end of loop
;show excluded-students
set choice-sum 0
foreach (sort excluded-students) [
  ask ? [
    set c-max length choice
```

```
      if last-choice != (c-max - 1) [
        set choice-sum (choice-sum + 1)
      ]
    ]
  ]
  ;show choice-sum
]
; enroll
foreach sort universities [
  ask ? [
    foreach potential-students [
      ask ? [
        ifelse district = 1 [set color yellow][set color lime]                ;; change display color
        set actual-place ([who] of myself + 1)
        set actual-preference position actual-place preference-list + 1
        set u replace-item (actual-place - 1) u (item (actual-place - 1) u + 1)
        move-to one-of (item (actual-place - 1) univ)
      ]
    ]
  ]
]
if any? students with  [color = magenta]
```

```
  [ask students with  [color = magenta]
   [
     set actual-place num-univ + 1
     set actual-preference num-univ + 1
   ]
  ]
end
to chinese-parallel
  let c-max 0
  foreach sort-on [(- score)] students [
    ask ? [
      set c-max length choice-1
      let i 1                       ;; the i th univ that the students choose
      while [i <= c-max] [
        set actual-place (item (i - 1) choice-1) ;;[choice-1] of self)
          ifelse item (actual-place - 1) u >= capacity [
            set i (i + 1)                   ;; over capacity, not placed, test for next choice
          ] [
          ifelse district = 1 [set color yellow][set color lime]          ;; change display color
          set u replace-item (actual-place - 1) u (item (actual-place - 1) u + 1)       ;; count
the students that enrolled
          move-to one-of (item (actual-place - 1) univ)              ;; show placement
```

```
        set i (c-max + 2)                                  ;; to end the circle
        set actual-preference position actual-place preference-list + 1
        ]
;       ]
       if (i = c-max + 1) and (no-choices-2 = 0) [
        ;; move-to one-of high-school set color white        ;;the student go back to high
school if got no offer]
        set actual-place num-univ + 1
        set actual-preference num-univ + 1
        ;; set misplacement abs (base-place - 11)]
       ]
      ]
     ]
   ]
  if no-choices-2 != 0 [
    foreach sort-on  [(- score)] students with [color = magenta ] [
     ask ? [
      set c-max length choice-2
      let i 1                              ;; the i th univ that the students choose
      while [i <= c-max ] [
        set actual-place (item (i - 1) choice-2) ;;[choice-2] of self)
         ifelse item (actual-place - 1) u >= capacity [
```

```
    set i (i + 1)
   ] [
   set i (c-max + 2)                                ;; to end the circle
                                                   ;; set misplacement abs (base-
place - actual-place)                 ;; set the misplacement
   move-to one-of (item (actual-place - 1) univ)
   ifelse district = 1 [set color yellow][set color lime]                  ;; change display
color
   set u replace-item (actual-place - 1) u (item (actual-place - 1) u + 1)  ;; count the
students that enrolled
   set actual-preference position actual-place preference-list + 1
   ]
  if i = c-max + 1 [
   ;; move-to one-of high-school set color white        ;;the student go back to high
school if got no offer]
   set actual-place num-univ + 1
   set actual-preference num-univ + 1
   ;;set misplacement abs (base-place - 11)]
  ]
 ]
]
]
```

```
  ]
end
to calculate-misplacement
  set misplacement-abs abs (base-preference - actual-preference)        ;; set the
misplacement-abs
  set misplacement (base-preference - actual-preference)               ;; set misplacement
gain or loss
end
to class-mismatch
  set income-quartile [0 0 0 0]
  let quartile-turtles [0 0 0 0]
  (foreach [0 1 2 3] [0 0.25 0.5 0.75] [0.25 0.5 0.75 1]
    [set quartile-turtles replace-item ?1 quartile-turtles turtle-set sublist sort-on [income]
students int (?2 * num-students) int (?3 * num-students)])
  set income-quartile replace-item 0 income-quartile (mean [misplacement] of item 0
quartile-turtles)
  set income-quartile replace-item 1 income-quartile (mean [misplacement] of item 1
quartile-turtles)
  set income-quartile replace-item 2 income-quartile (mean [misplacement] of item 2
quartile-turtles)
  set income-quartile replace-item 3 income-quartile (mean [misplacement] of item 3
quartile-turtles)
```

```
  set mean-quartile [0 0 0 0]
  ; get the 25% 50% 75% quartile using the "TI-83" method
  ; divide to students into four group according to their score
  set quartile-turtles [0 0 0 0]
  (foreach [0 1 2 3] [0 0.25 0.5 0.75] [0.25 0.5 0.75 1]
    [set quartile-turtles replace-item ?1 quartile-turtles turtle-set sublist sort-on [score]
students int (?2 * num-students) int (?3 * num-students)])
  ; calculate the mean of misplacement of each quartile
  set mean-quartile replace-item 0 mean-quartile (mean [misplacement] of item 0 quartile-
turtles)
  set mean-quartile replace-item 1 mean-quartile (mean [misplacement] of item 1 quartile-
turtles)
  set mean-quartile replace-item 2 mean-quartile (mean [misplacement] of item 2 quartile-
turtles)
  set mean-quartile replace-item 3 mean-quartile (mean [misplacement] of item 3 quartile-
turtles)
  ; get the top 10% of students
  let top10 turtle-set sublist sort-on [score] students int (0.9 * num-students) num-students
  set mean-top10 mean [misplacement] of top10
  set max-top10 max [misplacement-abs] of top10
  set percent-top10 count top10 with [actual-preference = base-preference] / count top10
```

```
  ; get the bottom 10% of students
  let bottom10 turtle-set sublist sort-on [income] students 0 int (0.1 * num-students)
  set mean-bottom10 mean [misplacement] of bottom10
end
to update-min-score-and-rank
  set ave-score n-values num-univ [0]
  set min-score n-values num-univ [0]
  set max-rank n-values num-univ [num-students]
  let i 0
  repeat num-univ [
    if any? students with [actual-place = (i + 1)] [
      set ave-score replace-item i ave-score (mean [score] of students with [actual-place =
(i + 1)])
      set min-score replace-item i min-score ([score] of min-one-of students with [actual-
place = (i + 1)] [score])
      set max-rank replace-item i max-rank (position (item i min-score) (sort-by > [score]
of students) + 1)
    ]
    set i i + 1
  ]
end
to update-label
```

```
  let n 0
  let d (max-pxcor - min-pxcor) / num-univ
  repeat num-univ  [
    ask patch (- (max-pxcor - d / 2) + n) -11  [set plabel-color red set plabel  round item (n
/ d)  min-score]
    set n (n + d)
  ]
end
to-report sort-by-preference [list1 list2]        ;;define a function to sort list1 according to
list2
  let newlist []
  foreach sort map [position ? list2] list1 [
    set newlist lput item ? list2 newlist
  ]
  report newlist
end
to-report sort-by-not-listed [list1 list2]        ;;define a function to sort list1 according to
NOT in list2
  let newlist []
  foreach sort list1 [
    if position ? list2 = false [
      set newlist lput ? newlist
```

```
    ]
  ]
  report newlist
end
to do-plots
  set-current-plot "E1"
  set-current-plot-pen "E1"
  plotxy ticks mean [misplacement-abs] of students
  ;; set-current-plot "E2"
  ;; set-current-plot-pen "E2"
  ;; plotxy ticks mean [misplacement-abs] of students with [base-place <= 10]
  ;; set-current-plot "E3"
  ;;set-current-plot-pen "E3"
  ;;plotxy ticks mean [misplacement-abs] of students with  [color = yellow ]
end
; draw zipf (zeta) distribution
;;---------------------- generate random number in Zeta distribution (or Zipf distribution)
;; -----  1. Generate a uniform random number z in [0,1]
;; ------ 2. The cumulative distribution function F(x) = \frac{H_{x,a}}{H_{n,a}}, where
H_{n,a} = \sum_{n=1}^{\infty}  \frac{1}{n^a}
;;------- 3. The random number we get is F^{-1}(z)
;; ---------------------------------------------------------------------------------------------------
```

```
to-report zipf [n a]
 let  H 0
 let zipf-value 0
 ;draw a uniform random number z in [0,1]
 let z random-float 1
 ; calculate  H_{n,a} = \sum_{n=1}^{\infty}  \frac{1}{n^a}
 let i 1
 repeat n [
   set H (H + (1 / i) ^ a)
   set i i + 1
 ]
 ;map z to the to F^{-1}(z)
 let sum-pro 0
 set i 1
 repeat n [
   set sum-pro (sum-pro + 1 / (H * i  ^ a))
   if (sum-pro >= z) [
     set zipf-value i
     report zipf-value
     stop
   ]
   set i i + 1
```

```
  ]
end
to update-univ-rank [rank-list]
  let sorted-rank-score sort-by > rank-list
  let i 0
  repeat num-univ [
    let temp-position position (item i sorted-rank-score) rank-list
    set univ-rank replace-item i univ-rank temp-position
    set rank-list replace-item temp-position rank-list (min rank-list - 1)
    set i (i + 1)
  ]
end
to update-endyear-score [begin-score begin-income]  ;; this is to calculate the avergae
score of a cohort in each school
  set end-score begin-score
  set end-income begin-income
  set average-score n-values num-univ [-1 - ?]
  set average-income n-values num-univ [0]
  let n 0
  repeat num-univ [
    let idx-a n * capacity
    let temp sublist begin-score idx-a (idx-a + capacity)
```

```
let enrollment-count capacity
if min temp = -1 [
  set enrollment-count position -1 temp
]
let idx-b (idx-a + enrollment-count)
if idx-b != idx-a [
  let income-bot mean sublist end-income idx-a idx-b
  set average-income replace-item n average-income income-bot
  let income-top income-bot * (1 + tolerance)
  set income-bot income-bot * (1 - tolerance)
  let score-effect mean sublist temp 0 enrollment-count
  set score-effect score-effect * peer-effect
  let idx-c idx-a
  ;; the following is to calculate the new scores of the students in a school
  repeat enrollment-count [
    let temp-income item idx-c end-income
    if (temp-income < income-top and temp-income > income-bot) [
      let temp-score item idx-c end-score
      set temp-score temp-score * (1 - peer-effect) + score-effect
      set end-score replace-item idx-c end-score temp-score
    ]
    set idx-c idx-c + 1
```

```
    ]
    set average-score replace-item n average-score mean sublist end-score idx-a idx-b
   ]
   set n (n + 1)
  ]
end
to update-y1end-score
  let y1start-score n-values (num-univ * capacity) [-1]
  let y1start-income n-values (num-univ * capacity) [0]
  set univ-enrollment n-values num-univ [0]
  let n 0
  let idx-a 0
  let idx-b 0
  repeat num-univ [
   set idx-a (n * capacity)
   set idx-b idx-a
   let temp students with [actual-place = (n + 1)]
   set univ-enrollment replace-item n univ-enrollment count temp ;;To count how many
students are in each university
   foreach sort-on  [(- score)] temp [
    ask ? [
```

```
        set y1start-score replace-item idx-b y1start-score (score)  ;; make a list of freshmen's start scores in each school. This list contains all students.
        set y1start-income replace-item idx-b y1start-income (income) ;; make a list of freshmen's incomes in each school. This list contains all students.
      ]
      set idx-b idx-b + 1
    ]
    set n n + 1
  ]
  update-endyear-score y1start-score y1start-income
  set y1-income-mean average-income
  set y1-income-stdev standard-deviation average-income
  set y1end-score end-score
  set y1end-income end-income
  if dbg-print [
    print (word "y1-end enrollment: " univ-enrollment)
    print (word "y1-end score average: " average-score)
  ]
end
to update-y2end-score
  update-endyear-score y1end-score y1end-income
  set y2end-score end-score
```

```
  set y2end-income end-income
  if dbg-print [
    print (word "y2-end score average: " average-score)
  ]
end
to update-y3end-score
  update-endyear-score y2end-score y2end-income
  set y3end-score-ave average-score
  set income-mean average-income
  set income-stdev standard-deviation average-income
  let temp-ave y3end-score-ave
  let n 0
  repeat num-univ [
    if item n temp-ave < 0 [
      set temp-ave replace-item n temp-ave 0
    ]
    set n n + 1
  ]
  set y3end-score-stdev standard-deviation temp-ave
  if dbg-print [
    print (word "y3-end score average: " average-score)
  ]
```

```
end
to update-dynamic-variables [m]
  if m = "Serial dictatorship" [
    set average-mismatch-sd mean [misplacement-abs] of students ;show  average-
mismatch-sd
    set sd-mismatch-sd standard-deviation [misplacement-abs] of students
    set ave-score-sd ave-score
    set min-score-sd min-score
    set max-rank-sd max-rank
    set income-quartile-sd income-quartile
    set mean-quartile-sd mean-quartile
    set mean-top10-sd mean-top10
    set mean-bottom10-sd mean-bottom10
    set max-top10-sd max-top10
    set percent-top10-sd percent-top10                    ;; to report misplacement-abs
    set univ-rank-sd univ-rank
    set y1end-score-sd y1end-score
    set y1end-income-sd y1end-income
    set y2end-score-sd y2end-score
    set y2end-income-sd y2end-income
    set income-mean-sd income-mean
    set income-stdev-sd income-stdev
```

```
    set y1-income-mean-sd y1-income-mean
    set y1-income-stdev-sd y1-income-stdev
    set y3end-score-ave-sd y3end-score-ave
    set y3end-score-stdev-sd y3end-score-stdev
    set univ-enrollment-sd univ-enrollment
;;    set top10-rank-sd top10-rank
  ]
  if m = "Boston mechanism" [
    set average-mismatch-bm mean [misplacement-abs] of students ;show average-
mismatch-bm
    set sd-mismatch-bm standard-deviation [misplacement-abs] of students
    set ave-score-bm ave-score
    set min-score-bm min-score
    set max-rank-bm max-rank
    set income-quartile-bm income-quartile
    set mean-quartile-bm mean-quartile
    set mean-top10-bm mean-top10
    set mean-bottom10-bm mean-bottom10
    set max-top10-bm max-top10
    set percent-top10-bm percent-top10                    ;; to report misplacement-abs
    set univ-rank-bm univ-rank
    set y1end-score-bm y1end-score
```

```
    set y1end-income-bm y1end-income
    set y2end-score-bm y2end-score
    set y2end-income-bm y2end-income
    set income-mean-bm income-mean
    set income-stdev-bm income-stdev
    set y1-income-mean-bm y1-income-mean
    set y1-income-stdev-bm y1-income-stdev
    set y3end-score-ave-bm y3end-score-ave
    set y3end-score-stdev-bm y3end-score-stdev
    set univ-enrollment-bm univ-enrollment
;;    set top10-rank-bm top10-rank
  ]
  if m = "Deferred acceptance" [
    set average-mismatch-da mean [misplacement-abs] of students ;show  average-
mismatch-da
    set sd-mismatch-da standard-deviation [misplacement-abs] of students
    set ave-score-da ave-score
    set min-score-da min-score
    set max-rank-da max-rank
    set income-quartile-da income-quartile
    set mean-quartile-da mean-quartile
    set mean-top10-da mean-top10
```

```
    set mean-bottom10-da mean-bottom10
    set max-top10-da max-top10
    set percent-top10-da percent-top10                    ;; to report misplacement-abs
    set univ-rank-da univ-rank
    set y1end-score-da y1end-score
    set y1end-income-da y1end-income
    set y2end-score-da y2end-score
    set y2end-income-da y2end-income
    set income-mean-da income-mean
    set income-stdev-da income-stdev
    set y1-income-mean-da y1-income-mean
    set y1-income-stdev-da y1-income-stdev
    set y3end-score-ave-da y3end-score-ave
    set y3end-score-stdev-da y3end-score-stdev
    set univ-enrollment-da univ-enrollment
;;    set top10-rank-da top10-rank
  ]
  if m = "Taipei mechanism" [
    set average-mismatch-tm mean [misplacement-abs] of students ;show  average-
mismatch-tm
    set sd-mismatch-tm standard-deviation [misplacement-abs] of students
    set ave-score-tm ave-score
```

```
    set min-score-tm min-score
    set max-rank-tm max-rank
    set income-quartile-tm income-quartile
    set mean-quartile-tm mean-quartile
    set mean-top10-tm mean-top10
    set mean-bottom10-tm mean-bottom10
    set max-top10-tm max-top10
    set percent-top10-tm percent-top10              ;; to report misplacement-abs
    set univ-rank-tm univ-rank
    set y1end-score-tm y1end-score
    set y1end-income-tm y1end-income
    set y2end-score-tm y2end-score
    set y2end-income-tm y2end-income
    set income-mean-tm income-mean
    set income-stdev-tm income-stdev
    set y1-income-mean-tm y1-income-mean
    set y1-income-stdev-tm y1-income-stdev
    set y3end-score-ave-tm y3end-score-ave
    set y3end-score-stdev-tm y3end-score-stdev
    set univ-enrollment-tm univ-enrollment
;;    set top10-rank-tm top10-rank
  ]
```

```
  if m = "Chinese parallel mechanism" [
    set average-mismatch-cp mean [misplacement-abs] of students ;show  average-
mismatch-cp
    set sd-mismatch-cp standard-deviation [misplacement-abs] of students
    set ave-score-cp ave-score
    set min-score-cp min-score
    set max-rank-cp max-rank
    set income-quartile-cp income-quartile
    set mean-quartile-cp mean-quartile
    set mean-top10-cp mean-top10
    set mean-bottom10-cp mean-bottom10
    set max-top10-cp max-top10
    set percent-top10-cp percent-top10                    ;; to report misplacement-abs
    set univ-rank-cp univ-rank
    set y1end-score-cp y1end-score
    set y1end-income-cp y1end-income
    set y2end-score-cp y2end-score
    set y2end-income-cp y2end-income
    set income-mean-cp income-mean
    set income-stdev-cp income-stdev
    set y1-income-mean-cp y1-income-mean
    set y1-income-stdev-cp y1-income-stdev
```

```
    set y3end-score-ave-cp y3end-score-ave
    set y3end-score-stdev-cp y3end-score-stdev
    set univ-enrollment-cp univ-enrollment
;;    set top10-rank-cp top10-rank
  ]
end
to update-begin-variables [m]
  if m = "Serial dictatorship" [
    ;set ave-score ave-score-sd
    set univ-rank univ-rank-sd
    set min-score min-score-sd
    set max-rank max-rank-sd
    set y1end-score y1end-score-sd
    set y1end-income y1end-income-sd
    set y2end-score y2end-score-sd
    set y2end-income y2end-income-sd
    set y3end-score-ave y3end-score-ave-sd
  ]
  if m = "Boston mechanism" [
    ;set ave-score ave-score-bm
    set univ-rank univ-rank-bm
    set min-score min-score-bm
```

```
  set max-rank max-rank-bm
  set y1end-score y1end-score-bm
  set y1end-income y1end-income-bm
  set y2end-score y2end-score-bm
  set y2end-income y2end-income-bm
  set y3end-score-ave y3end-score-ave-bm
]
if m = "Deferred acceptance" [
  ;set ave-score ave-score-da
  set univ-rank univ-rank-da
  set min-score min-score-da
  set max-rank max-rank-da
  set y1end-score y1end-score-da
  set y1end-income y1end-income-da
  set y2end-score y2end-score-da
  set y2end-income y2end-income-da
  set y3end-score-ave y3end-score-ave-da
]
if m = "Taipei mechanism" [
  ;set ave-score ave-score-tm
  set univ-rank univ-rank-tm
  set min-score min-score-tm
```

```
    set max-rank max-rank-tm
    set y1end-score y1end-score-tm
    set y1end-income y1end-income-tm
    set y2end-score y2end-score-tm
    set y2end-income y2end-income-tm
    set y3end-score-ave y3end-score-ave-tm
  ]
  if m = "Chinese parallel mechanism" [
    ;set ave-score ave-score-cp
    set univ-rank univ-rank-cp
    set min-score min-score-cp
    set max-rank max-rank-cp
    set y1end-score y1end-score-cp
    set y1end-income y1end-income-cp
    set y2end-score y2end-score-cp
    set y2end-income y2end-income-cp
    set y3end-score-ave y3end-score-ave-cp
  ]
end
to write-file-header
  if write-mismatch [
    write-header-type2 "mismatch.csv" "sd," "bm," "da," "tm," "cp"
```

```
]
if write-sd-mismatch [
  write-header-type2 "sd_mismatch.csv" "sd," "bm," "da," "tm," "cp"
]
if write-ave-score [
  write-header-type2 "ave_score.csv"
      "sd 1,sd 2,sd 3,sd 4,sd 5,sd 6,sd 7,sd 8,sd 9,sd 10,"
      "bm 1,bm 2,bm 3,bm 4,bm 5,bm 6,bm 7,bm 8,bm 9,bm 10,"
      "da 1,da 2,da 3,da 4,da 5,da 6,da 7,da 8,da 9,da 10,"
      "tm 1,tm 2,tm 3,tm 4,tm 5,tm 6,tm 7,tm 8,tm 9,tm 10,"
      "cp 1,cp 2,cp 3,cp 4,cp 5,cp 6,cp 7,cp 8,cp 9,cp 10"
]
if write-min-score [
  write-header-type2 "min_score.csv"
      "sd 1,sd 2,sd 3,sd 4,sd 5,sd 6,sd 7,sd 8,sd 9,sd 10,"
      "bm 1,bm 2,bm 3,bm 4,bm 5,bm 6,bm 7,bm 8,bm 9,bm 10,"
      "da 1,da 2,da 3,da 4,da 5,da 6,da 7,da 8,da 9,da 10,"
      "tm 1,tm 2,tm 3,tm 4,tm 5,tm 6,tm 7,tm 8,tm 9,tm 10,"
      "cp 1,cp 2,cp 3,cp 4,cp 5,cp 6,cp 7,cp 8,cp 9,cp 10"
]
if write-mean-quartile-score [
  write-header-type2 "mean_quartile_score.csv"
```

```
      "sd 25%,sd 50%,sd 75%,sd 100%,"
      "bm 25%,bm 50%,bm 75%,bm 100%,"
      "da 25%,da 50%,da 75%,da 100%,"
      "tm 25%,tm 50%,tp 75%,tp 100%,"
      "cp 25%,cp 50%,cp 75%,cp 100%"
    write-header-type2 "income_quartile_score.csv"
      "sd 25%,sd 50%,sd 75%,sd 100%,"
      "bm 25%,bm 50%,bm 75%,bm 100%,"
      "da 25%,da 50%,da 75%,da 100%,"
      "tm 25%,tm 50%,tp 75%,tp 100%,"
      "cp 25%,cp 50%,cp 75%,cp 100%"
  ]
  if write-mean-top10 [
    write-header-type2 "mean_top10.csv" "sd," "bm," "da," "tm," "cp"
    write-header-type2 "mean_bottom10.csv" "sd," "bm," "da," "tm," "cp"
  ]
  if write-max-top10 [
    write-header-type2 "max_top10.csv" "sd," "bm," "da," "tm," "cp"
  ]
  if write-percent-top10 [
    write-header-type2 "percent_top10.csv" "sd," "bm," "da," "tm," "cp"
;;    write-header-type2 "top10_rank.csv" "sd," "bm," "da," "tm," "cp"
```

```
]
if write-univ-rank [
  write-header-type2 "univ-rank.csv"
     "sd 1,sd 2,sd 3,sd 4,sd 5,sd 6,sd 7,sd 8,sd 9,sd 10,"
     "bm 1,bm 2,bm 3,bm 4,bm 5,bm 6,bm 7,bm 8,bm 9,bm 10,"
     "da 1,da 2,da 3,da 4,da 5,da 6,da 7,da 8,da 9,da 10,"
     "tm 1,tm 2,tm 3,tm 4,tm 5,tm 6,tm 7,tm 8,tm 9,tm 10,"
     "cp 1,cp 2,cp 3,cp 4,cp 5,cp 6,cp 7,cp 8,cp 9,cp 10"
  write-header-type2 "univ-enrollment.csv"
     "sd 1,sd 2,sd 3,sd 4,sd 5,sd 6,sd 7,sd 8,sd 9,sd 10,"
     "bm 1,bm 2,bm 3,bm 4,bm 5,bm 6,bm 7,bm 8,bm 9,bm 10,"
     "da 1,da 2,da 3,da 4,da 5,da 6,da 7,da 8,da 9,da 10,"
     "tm 1,tm 2,tm 3,tm 4,tm 5,tm 6,tm 7,tm 8,tm 9,tm 10,"
     "cp 1,cp 2,cp 3,cp 4,cp 5,cp 6,cp 7,cp 8,cp 9,cp 10"
]
if write-senior-ave [
  write-header-type2 "senior-ave.csv"
     "sd 1,sd 2,sd 3,sd 4,sd 5,sd 6,sd 7,sd 8,sd 9,sd 10,"
     "bm 1,bm 2,bm 3,bm 4,bm 5,bm 6,bm 7,bm 8,bm 9,bm 10,"
     "da 1,da 2,da 3,da 4,da 5,da 6,da 7,da 8,da 9,da 10,"
     "tm 1,tm 2,tm 3,tm 4,tm 5,tm 6,tm 7,tm 8,tm 9,tm 10,"
     "cp 1,cp 2,cp 3,cp 4,cp 5,cp 6,cp 7,cp 8,cp 9,cp 10"
```

```
    write-header-type2 "senior-stdev.csv"  "sd," "bm," "da," "tm," "cp"
    write-header-type2 "mean-income.csv"
      "sd 1,sd 2,sd 3,sd 4,sd 5,sd 6,sd 7,sd 8,sd 9,sd 10,"
      "bm 1,bm 2,bm 3,bm 4,bm 5,bm 6,bm 7,bm 8,bm 9,bm 10,"
      "da 1,da 2,da 3,da 4,da 5,da 6,da 7,da 8,da 9,da 10,"
      "tm 1,tm 2,tm 3,tm 4,tm 5,tm 6,tm 7,tm 8,tm 9,tm 10,"
      "cp 1,cp 2,cp 3,cp 4,cp 5,cp 6,cp 7,cp 8,cp 9,cp 10"
    write-header-type2 "stdev-income.csv" "sd," "bm," "da," "tm," "cp"
    write-header-type2 "y1-mean-income.csv"
      "sd 1,sd 2,sd 3,sd 4,sd 5,sd 6,sd 7,sd 8,sd 9,sd 10,"
      "bm 1,bm 2,bm 3,bm 4,bm 5,bm 6,bm 7,bm 8,bm 9,bm 10,"
      "da 1,da 2,da 3,da 4,da 5,da 6,da 7,da 8,da 9,da 10,"
      "tm 1,tm 2,tm 3,tm 4,tm 5,tm 6,tm 7,tm 8,tm 9,tm 10,"
      "cp 1,cp 2,cp 3,cp 4,cp 5,cp 6,cp 7,cp 8,cp 9,cp 10"
    write-header-type2 "y1-stdev-income.csv" "sd," "bm," "da," "tm," "cp"
  ]
end
to write-simulation-data
  if write-mismatch [
    write-data-type1 "mismatch.csv" average-mismatch-sd average-mismatch-bm average-
mismatch-da average-mismatch-tm average-mismatch-cp
   ]
```

```
  if write-sd-mismatch [
    write-data-type1 "sd_mismatch.csv" sd-mismatch-sd sd-mismatch-bm sd-mismatch-da
sd-mismatch-tm sd-mismatch-cp
  ]
  if write-ave-score [
    write-data-type2 "ave_score.csv" ave-score-sd ave-score-bm ave-score-da ave-score-
tm ave-score-cp
  ]
  if write-min-score [
    write-data-type2 "min_score.csv" min-score-sd min-score-bm min-score-da min-score-
tm min-score-cp
  ]
  if write-mean-quartile-score [
    write-data-type2 "mean_quartile_score.csv" mean-quartile-sd mean-quartile-bm mean-
quartile-da mean-quartile-tm mean-quartile-cp
    write-data-type2 "income_quartile_score.csv" income-quartile-sd income-quartile-bm
income-quartile-da income-quartile-tm income-quartile-cp
  ]
  if write-mean-top10 [
    write-data-type1 "mean_top10.csv" mean-top10-sd mean-top10-bm mean-top10-da
mean-top10-tm mean-top10-cp
```

```
    write-data-type1 "mean_bottom10.csv" mean-bottom10-sd mean-bottom10-bm mean-
bottom10-da mean-bottom10-tm mean-bottom10-cp
  ]
  if write-max-top10 [
    write-data-type1 "max_top10.csv" max-top10-sd max-top10-bm max-top10-da max-
top10-tm max-top10-cp
  ]
  if write-percent-top10 [
    write-data-type1 "percent_top10.csv" percent-top10-sd percent-top10-bm percent-
top10-da percent-top10-tm percent-top10-cp
;;    write-data-type1 "top10_rank.csv" top10-rank-sd top10-rank-bm top10-rank-da
top10-rank-tm top10-rank-cp
  ]
  if write-univ-rank [
    write-data-type2 "univ-rank.csv" univ-rank-sd univ-rank-bm univ-rank-da univ-rank-
tm univ-rank-cp
    write-data-type2 "univ-enrollment.csv" univ-enrollment-sd univ-enrollment-bm univ-
enrollment-da univ-enrollment-tm univ-enrollment-cp
  ]
  if write-senior-ave [
    write-data-type2 "senior-ave.csv" y3end-score-ave-sd y3end-score-ave-bm y3end-
score-ave-da y3end-score-ave-tm y3end-score-ave-cp
```

```
   write-data-type1 "senior-stdev.csv" y3end-score-stdev-sd y3end-score-stdev-bm y3end-score-stdev-da y3end-score-stdev-tm y3end-score-stdev-cp
   write-data-type2 "mean-income.csv" income-mean-sd income-mean-bm income-mean-da income-mean-tm income-mean-cp
   write-data-type1 "stdev-income.csv" income-stdev-sd income-stdev-bm income-stdev-da income-stdev-tm income-stdev-cp
   write-data-type2 "y1-mean-income.csv" y1-income-mean-sd y1-income-mean-bm y1-income-mean-da y1-income-mean-tm y1-income-mean-cp
   write-data-type1 "y1-stdev-income.csv" y1-income-stdev-sd y1-income-stdev-bm y1-income-stdev-da y1-income-stdev-tm y1-income-stdev-cp
  ]
end

to write-header-type2 [fn c1 c2 c3 c4 c5]
  if file-exists? fn and keep_files = FALSE [file-delete fn]
  if NOT file-exists? fn [
    file-open fn
    file-print (word
      "run,"
      "ticks,"
      "scheme,"
      "alpha,"
```

```
    "capacity,"
    "order,"
    "choices,"
    "choices-2,"
    "grant,"
    c1
    c2
    c3
    c4
    c5
    )
   file-close
  ]
end
to write-data-type1 [fn c1 c2 c3 c4 c5]
  file-open fn
  file-print (word
   behaviorspace-run-number
   " ," ticks
   " ," scheme
   " ," alpha
   " ," capacity
```

```
    " ," extra-in-order
    " ," no-choices
    " ," no-choices-2
    " ," grant
    " ," c1
    " ," c2
    " ," c3
    " ," c4
    " ," c5
    )
  file-close
end
to write-data-type2 [fn c1 c2 c3 c4 c5]
  file-open fn
  file-print (word
    behaviorspace-run-number
    " ," ticks
    " ," scheme
    " ," alpha
    " ," capacity
    " ," extra-in-order
    " ," no-choices
```

```
    " ," no-choices-2
    " ," grant " ,"
    cut-list c1
    cut-list c2
    cut-list c3
    cut-list c4
    cut-list c5
    )
  file-close
end
```