2016

Algebraic Content and Pedagogical Knowledge of Sixth Grade Mathematics Teachers

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COLLEGE OF EDUCATION

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2016

Abstract

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Master of Education in Educational Management (International Programme),

Mahidol University, 2010

Bachelor of Science Education (Honours) in Mathematics and Accounting,

Universiti Brunei Darussalam, 2006

Doctoral Study Submitted in Partial Fulfillment

of the Requirements for the Degree of

Doctor of Education in

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Abstract

Algebra test scores of the Maldivian students from grade 6 through 12 are the lowest compared to any other area of mathematics. Algebra is a fundamental topic in mathematics and lays the foundation for mathematical reasoning and complex problem-solving. Research shows that strengthening algebra instruction could improve student achievement. This concurrent mixed methods study examined the algebraic content and pedagogical knowledge of 5 sixth grade mathematics teachers who teach in 5 different schools across the Maldives. Shulman’s major categories of teacher knowledge and Ball, Thames, and Phelps’ domains of mathematical knowledge for teaching guided this study. The research questions examined the relationship between teachers’ perceptions of their mastery of algebraic content and pedagogical knowledge, and what teachers actually know about algebraic content and pedagogy. Purposive sampling was used to select the 5 participants. Quantitative data were collected using the Diagnostic Teacher Assessments of Mathematics and Science – Middle Mathematics Teacher Assessments and qualitative data were gathered through lesson observations, interviews, and analysis of teachers’ lesson plans and notes. All participants believed that they were proficient in both algebraic content and pedagogical knowledge. However, the results of this study showed that all participants lacked both algebraic content and pedagogical knowledge. Findings of this study were used to inform and design mathematics professional development to meet the needs of the participants. This mathematics professional development is expected to improve the instructional delivery of algebra through enhanced algebraic content and pedagogical knowledge. This could positively contribute to the improvement of student achievement in algebra.

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Dedication

This dissertation is dedicated to my son Muhammad Khalaf Shakeeb who has grown into a wonderful eight year old in spite of his mother spending so much time away from him working on this dissertation and having a busy full-time job in academia.

Acknowledgments

First and foremost, I am grateful to Allah for showers of blessings and the good health and wellbeing that were necessary to complete my doctoral studies successfully. *Al-ḥamdu li-llāhi* (الحمد لله). The following document summarizes four years' worth of effort, frustration, and achievement. There are several people to whom I am indebted for their contribution to the research represented by this dissertation.

At Walden, the first person that deserves my gratitude is my dissertation chair Dr. Samson Lyman. Sam has given me enormous freedom to wander intellectually, with only an occasional tug on my sleeve to bring me back to topic. There have been numerous occasions where I remember feeling disheartened and stumped about the direction of my research, but inevitably, a meeting with Sam would reinvigorate my enthusiasm and raise my spirits immeasurably. Sam's steadfast support and intellectual guidance have been invaluable over the years and I feel incredibly privileged to have him as my dissertation chair. I would like to express my utmost thanks and sincere gratitude to Sam for his mentorship throughout this exciting journey. Sam, know that your words of wisdom and advice have always been received with the highest degree of respect. Your confidence in me and loyalty to this research were instrumental to its successful completion.

I have also benefited greatly from my dissertation committee members. I would like to extend my thanks and gratitude to Dr. Irene McAfee and Dr. Kathleen Montgomery, for their invaluable advice that not only helped this project but also was a great influence in shaping me as a researcher. I am also grateful to Walden University Writing Center editor Dr. Steve Lehman, who reviewed this dissertation for form and style. Steve, many thanks for catching those spelling mistakes, suggesting those additions and deletions that seemed so obvious but that never occurred to me, and correcting the APA Style errors. Last but not least, I would like to offer a special thank you to Dr. Matthew Jones, Walden University CAO Designee, for his clear and constructive feedback, direction, and assistance.

I am very grateful for the participants who gave their time, stories and insights; and without whom this research would not exist.

I am also grateful to several mentors from my pre-Walden days. From Mahidol University, I would like to thank Dr. Patreeya Kitcharoen and Dr. Arisara Leksansern. And from Universiti Brunei Darussalam, I would like to thank Professor Keith Wood and Professor M. A. (Ken) Clements. Each one of them has been an inspiration to me and has helped me in many ways. In particular, I would like to express my deep appreciation to Professor Keith Wood for his thoughtful suggestions, encouraging comments and advice during the initial stages of the dissertation proposal development.

I would like to express my profound gratitude to my former employer, Dr. Ahmed Anwar and my former boss, Aishath Mohamed Didi whose commitment to the success of their employees has truly been an inspiration. I would like to acknowledge their kindness, consideration, and continued support during my first three years as a doctoral candidate. I am also grateful to my former colleagues who provided positive words of encouragement during overwhelming times.

My heartfelt thanks goes to those who gave me their indispensable generous assistance in the form of loans and grants. Without their support and financial help, it would not have been possible for me to complete this Doctoral project successfully. Thank you for you constant support through the ups and downs of my academic career.

Yes, it has been bumpy at times, but your confidence in me and your support for this project has enhanced my ability to get through it all and succeed in the end.

I have been blessed with a very loving and a supportive family. I believe I would never have completed this work without the encouragement and unwavering support of my parents and sister. I am grateful to my mother Fathmath Rasheeda, who emphasized the importance of education, and gave her unconditional love and support throughout my life. I am also indebted to my father, Mohamed Naseer, who has been my role-model for hard work, persistence and personal sacrifice, and who inspired me to set high goals and to have the confidence to achieve them. I deeply appreciate the kindness of my bonus mom, Shakeela Hamid, who taught me the art of patience. Her belief in me was strong and she constantly reminded me that tough times never lasted, something that seemed so impossible during the final year of my studies. This acknowledgement would be incomplete if not for my sister, Aminath Sheeza Naseer, who has been my emotional anchor through not only the vagaries of graduate school, but my entire life. I love you all very much.

I would like to thank my partner who has been proud and supportive of my work and who has shared the many uncertainties, challenges and sacrifices for completing this dissertation. Thank you for putting up with me; thank you for letting me be mean to you constantly every second of the day; thank you for listening to me complain about my life; thank you for calling me back when I hang up on you; thank you for making me laugh and most importantly, thank you for always being there for me. I love you.

As for my son, Muhammad Khalaf Shakeeb, I find it difficult to express my appreciation because it is so boundless. Considering that he is an eight year old it is hard to imagine how he could contribute to a doctoral dissertation, but in fact, it is his bright smile that puts it all into perspective for me. He reminds me daily of all that is yet to be. My son, a source of unending joy and love, has been wonderfully understanding throughout the dissertation process and has been awaiting the day when I would be finished. Khalaf, now we will be able to spend more time together! I need to thank my little boy not only for being such a bundle of joy and laughter but also for allowing me time away from family stuff to get this dissertation completed, and to you this dissertation is dedicated. You are my world. I love you very much!

Finally, I would like to thank my cousins, relatives, and close friends for their love, support and belief in me to achieve my goals over many years of higher education, even if they do not always understand what it is that I am doing and why. This project has been, without a doubt, the single largest test of my own commitment, patience, and perseverance, spanning four years. But I have not achieved this alone. Along the way, I have received support from too many people to count. Though you may not see your names here, know that your various contributions have not gone unnoticed or unappreciated.

Mariyam Shahuneeza Naseer

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Section 1: The Problem

Introduction

A teacher's ability to teach mathematics content is influenced by the mathematical content and pedagogical knowledge of the teacher (Piccolo, 2008; Strand & Mills, 2014). Shulman (1987) discussed categories of teacher knowledge that included content knowledge and pedagogical knowledge. In 2008, Ball, Thames, and Phelps (2008) refined Shulman's major categories of teacher knowledge and developed a model called domains of mathematical knowledge for teaching, which included mathematical content knowledge and pedagogical knowledge. Content knowledge was defined as the subject matter knowledge whereas pedagogical knowledge referred to the unique knowledge required to teach the subject-specific content (Ball, Thames, & Phelps, 2008; Shulman, 1987).

Mathematical content and pedagogical knowledge of teachers had been associated with students' ability and performance in subjects including algebra (El Mouhayar & Jurdak, 2013; Strand & Mills, 2014; Tajudin, 2014; Tennant & Colloff, 2014). Algebra is a fundamental topic in mathematics that serves as a gateway to student skills necessary for mathematical reasoning and complex problem-solving (Cheng-Yao, Yi-Yin, & Yu-Chun, 2014; Strand & Mills, 2014). Research indicated that algebra is a topic students find difficult. A baseline study conducted in the Maldives between 2012 and 2013 indicated students scored the lowest in algebra compared to any other topics in mathematics (United Nations Children's Fund & National Institute of Education [UNICEF & NIE], 2014). The study reported that students lacked conceptual understanding of algebra. An interview with a local researcher who served as the local project manager of this baseline study indicated the importance of studying the algebraic content and pedagogical knowledge of the in-service teachers because it was thought to be linked to student performance (A. Shareef, personal communication, April 22, 2015).

The purpose of this study was to examine the algebraic content and pedagogical knowledge of sixth grade mathematics teachers in the Maldives to determine their relative strengths and weaknesses in order to provide the foundation for the development of a teacher professional development curriculum. This study measured the algebraic content knowledge and pedagogical knowledge of sixth grade mathematics teachers using Diagnostic Teacher Assessments in Mathematics and Science – Middle Mathematics Teacher Assessments (DTAMS). Analysis of algebra lesson plans and lesson notes, observations of algebra lessons, and interviews with sixth grade mathematics teachers were used to gain an in-depth understanding of the algebraic content and pedagogical knowledge of the sixth grade mathematics teachers in addition to the crystallization of the findings from the DTAMS. The preliminary findings of this study were presented at the 11th Annual Education and Development Conference held in Bangkok March 5-7, 2016. Moreover, the findings of this study were used to tailor professional development curricula for in-service teachers in the Maldives and were shared with schools in the Maldives. In addition, the strengths and weaknesses of teacher content and pedagogical knowledge were communicated to teacher education institutions to strengthen the teacher education programs, thereby contributing positively towards the betterment and enhancement of mathematics education.

Definition of the Problem

The problem examined in this study is the algebraic content and pedagogical knowledge of sixth grade mathematics teachers. It is commonly assumed that teachers who have a deep understanding of the content and pedagogy foster better student performance as they will employ better instructional practices than teachers who lack an in-depth understanding of the content and pedagogy (Ball, Lubienski, & Mewborn, 2001; Brown, Davis, & Kulum, 2011; Darling-Hammond & Sykes, 2003; Ojose, 2014; Shirvani, 2015). Brown et al. (2011) reported that students taught by teachers equipped with the ability and knowledge to teach mathematics effectively produced six times better results compared to the students taught by teachers who lacked the ability and knowledge to teach mathematics effectively. Though it is unclear whether it is content knowledge or pedagogical knowledge or both content and pedagogical knowledge that lead to better student performance, it is apparent from the literature that the ability to teach mathematics depends on the mathematical content and pedagogical knowledge of the teachers (Begle, 1979; Piccolo, 2008; Strand & Mills, 2014).

Research on teacher knowledge and student performance has produced mixed results (Ball et al., 2001; Buschang, Chung, Delacruz, & Baker, 2012; Ferrini-Mundy, Floden, McCrory, Burrill, & Sandow, 2005; Shirvani, 2015). Some studies showed teacher knowledge had a positive effect on student performance while other studies showed that teacher knowledge had a negative effect on student performance (Begle, 1979; Shirvani, 2015; Strand & Mills, 2014; Thames, 2006). According to Thames (2006) these inconsistent results led researchers to ponder whether there was a problem in the way teacher knowledge was measured given that the number of mathematics courses taken at university was used as a measure of teacher knowledge in many studies. Use of variables such as number of mathematics courses taken at university or grade point average to measure teacher knowledge was seen as inappropriate and a poor approximation of teacher knowledge due to the complexity in measuring such variables (Begle, 1979; Buschang et al., 2012 Ferrini-Mundy et al., 2005; Thames, 2006). These mixed results led researchers to explore what types of knowledge teachers should really have in order to teach mathematics (Begle, 1979; Thames, 2006).

Shulman (1987) explained the complexity and diversity of teachers' knowledge by establishing the categories of knowledge required for teaching. The list included content knowledge and pedagogical knowledge. Mathematical content knowledge, which is also referred to as subject matter knowledge, is a critical aspect of teacher knowledge since teaching requires helping others to learn and therefore understanding what is to be taught is fundamental to teaching (Ball & McDiarmid, 1990; Ball et al., 2008). Pedagogical knowledge covers the knowledge required to teach the subject specific content, which includes understanding students' misconceptions and knowing how to remedy those misconceptions (Ball et al., 2008; Chick, Pham, & Baker, 2006; Welder, 2012).

Many students fail to achieve basic algebraic literacy, and that can prove to be a barrier to entry into careers in the sciences, engineering, technology, and business (Brown et al., 2011; Massey & Riley, 2013; Strand & Mills, 2014; Welder, 2012). Research indicated that students face numerous difficulties in understanding algebra due to lack of understanding of symbols and letters and of manipulation of algebraic expressions and equations (Booth; Kieran; Kuchemann; MacGregor & Stacey, as cited in Banerjee, & Subramaniam, 2012; Strand & Mills, 2014). Welder (2012) pointed out that these difficulties could be due to the existing knowledge students have that may be incomplete or misunderstood.

Teachers who introduce algebra to students are responsible for helping students build a solid foundation on which they can later construct a more sophisticated algebraic understanding (Strand & Mills, 2014). Numerous studies have been conducted to identify the difficulties and misconceptions students have in learning algebra (Baroudi, 2006; Behr, Erlwanger, & Nichols, 1976; Carpenter, Levi, & Farnsworth, 2000; MacGregor & Stacey, 1997; Steinle, Gvozdenko, Price, Stacey, & Pierce, 2009; Welder, 2012). Research indicated that at times students incorrectly interpreted letters as objects (MacGregor & Stacey, 1997; Steinle et al., 2009; Welder, 2012). Tennant and Colloff (2014) linked this incorrect interpretation of letters to the approach used by teachers in introducing early simplification of algebra that is influenced by the algebraic content and pedagogical knowledge of the teacher (Ball & McDiarmid, 1990; Chick et al., 2006). Understanding the algebraic content and pedagogical knowledge strengths and weaknesses of mathematics teachers would contribute positively towards improving mathematics education in general and strengthening algebra instruction in particular.

Rationale

Evidence of the Problem at the Local Level

A teacher's ability to deliver mathematics content depends on the mathematical content and pedagogical knowledge of the teacher (Piccolo, 2008). Research has linked content and pedagogical knowledge of teachers to student performance (El Mouhayar & Jurdak, 2013; Tennant & Colloff, 2014). According to a baseline study conducted by UNICEF and NIE during 2012 and 2013 in the Maldives, algebra had the lowest pass percentage compared to any other area of mathematics. The study reported that only 27.1% of the students who were studying in Grade 7 at the time of study were able to obtain the correct answers for algebra questions (UNICEF & NIE, 2014). In the baseline study, it was reported that students lacked conceptual understanding. In other studies, lack of conceptual understanding has been associated with the way teachers introduced early algebraic concepts, which depends on the algebraic content and pedagogical knowledge of the teacher (Ball & McDiarmid, 1990; Chick et al., 2006; Tennant & Colloff, 2014). An interview with the local project manager of the UNICEF and NIE study indicated the importance of researching the algebraic content and pedagogical knowledge of the teachers in order to identify the teachers' needs so that necessary support could be provided to improve algebra instruction (A. Shareef, personal communication, April, 22, 2015).

The project manager explained that the project team wanted the teachers to take the same tests; however, the idea was postponed as teachers might feel insulted by the request to administer the same tests to the teachers simultaneously. The manager further explained that the project team intended to check the mathematical content knowledge of the teachers at some point of this project and highlighted that it would be best if they could assess both mathematical content and pedagogical knowledge of the teachers (A. Shareef, personal communication, April, 22, 2015).

A retired teacher educator who has been conducting professional development for mathematics teachers across the country stated that some of the teachers had difficulty in explaining concepts relating to fractions and algebra (M. Qasim, personal communication, April 13, 2015). Qasim stressed the importance of studying teacher content and pedagogical knowledge to find out "whether the teachers are equipped with the content and pedagogical knowledge required to teach" (M. Qasim, personal communication, April 13, 2015). Moreover, Qasim highlighted the importance of this study in understanding algebraic content and pedagogical knowledge of the in-service teachers that could inform teacher education institutions about the needs of the teachers (M. Qasim, personal communication, April 13, 2015).

One of the primary school heads also stated "the teachers are very weak. We have had complaints from students and parents that teachers don't know how to explain in a way that these students understand. We have had cases in which teachers teach concepts, particularly algebraic concepts incorrectly" (A. Waheed, personal communication, April 7, 2015). The primary school head also emphasized the importance of studying the algebraic content and pedagogical knowledge of the in-service teachers. The head of one of the teacher training institutions in the Maldives acknowledged that there are problems in the system and that checking the algebraic content (and mathematical content in general) and pedagogical knowledge of in-service mathematics teachers was needed. However, this individual lacked the resources required to conduct such a study (A. Gasim, personal communication, June 5, 2015) but was keen to know the results of this study, believing it would be valuable to the institution in designing teacher training courses (A. Gasim, personal communication, July 4, 2015).

Through these interviews one thing became clear: the need to examine the mathematical content and pedagogical knowledge of teachers in the Maldives. All the interviewees' agreed upon the need for such a study. They all believed that to improve algebra instruction it is important to examine the algebraic content and pedagogical knowledge of mathematics teachers in the Maldives. Although the interviewees believed that teachers' mathematical content and pedagogical knowledge is connected to student achievement, research on teacher knowledge and student performance had produced inconclusive and inconsistent results (Buschang et al., 2012; Ferrini-Mundy et al., 2005; Shirvani, 2015).

Evidence of the Problem from the Professional Literature

The ability to teach mathematics content is influenced by the mathematical content and pedagogical knowledge of the teachers (Piccolo, 2008). Teachers who lack a deep understanding of mathematics fail to teach students to develop conceptual understanding (Ma, 1999; Stoddart, Connell, Stofflett, & Peck, 1993). Conceptual understanding is important as students who lack conceptual understanding tend to forget how to apply the concepts. Moreover, attaining new knowledge becomes challenging to those students (Welder, 2012). Conceptual understanding has been linked with the ability of the teacher to explain certain concepts. For example, Tajudin (2014) stated that lack of conceptual understanding in algebra could be the result of the way students were taught algebra. Often teachers who lack algebraic content knowledge emphasized computational procedures, which led students to make errors and develop misconceptions with regard to the concept of variables, algebraic expressions, algebraic equations, and word problems (Ball et al., 2001; Koency & Swanson, 2000; Tajudin, 2014). Tennant and Colloff (2014) linked students' misconceptions to the approach teachers used in introducing and explaining algebraic concepts and simplifying algebraic processes. El Mouhayar and Jurdak (2013) reported that middle school students had difficulties in the areas in which teachers had difficulties. This is parallel to the findings of Shirvani (2015), who reported that teachers' knowledge affected students' performance on mathematics assessments. Shulman (1987) stated that the "teacher has special responsibilities in relation to content knowledge, serving as the primary source of student understanding of subject matter" (p. 9). This is parallel to the findings of Ojose (2014) that indicated teachers' knowledge of mathematical content significantly affected the way concepts were taught and specifically pointed out that teachers who lacked mathematical content knowledge tended to assume that students knew and understood the concepts. This limited the mathematical content to which students were exposed (Ojose, 2014; Strand & Mills, 2014).

It is evident from professional literature that mathematical content and pedagogical knowledge of teachers play a crucial role in teaching mathematical concepts in a way that helps students develop conceptual understanding of the subject matter. Therefore, it is significant to find out the algebraic content and pedagogical knowledge strengths and weaknesses of mathematics teachers in the Maldives in order to determine the focus of professional development curriculum of in-service teachers. Hence, the purpose of this study is to examine the algebraic content and pedagogical knowledge of the sixth grade mathematics teachers in the Maldives.

Definitions

Content Knowledge

Content knowledge is subject matter knowledge. This covers not only the facts and concepts of the content the teachers teach, but also understanding the structures of the subject matter (Shulman, 1986). Although the phrase “content knowledge” gives the impression that it is just the knowledge of content, Shulman (1986) stated that it extends beyond the knowledge of content to include understanding why something is so. This definition is equivalent to the explanation proposed by Ball et al. (2008), which specifically covered knowledge and skills of mathematics used in settings other than teaching, knowledge and skills uniquely needed by mathematics teachers to teach, and the knowledge of how the mathematics curriculum is spread cross the grades. Hiebert and Carpenter (1992) used the term procedural knowledge whereas Skemp (1987) used the term instrumental understanding to refer to what Ball et al. (2008) categorized as common content knowledge, which included the knowledge and skills of mathematics used in settings other than teaching (Ball et al., 2008; Hiebert & Carpenter, 1992; Saderholm, Ronau, Brown, & Collins, 2010; Skemp, 1987). Ball et al.’s explanation of specialized content knowledge is parallel to what Hiebert and Carpenter referred to as conceptual understanding, while Skemp used the term relational understanding (Ball et al., 2008; Hiebert & Carpenter, 1992; Saderholm et al, 2010; Skemp, 1987).

Pedagogical Knowledge

Pedagogical knowledge is the understanding of what makes the learning of specific topics easy or difficult (Shulman, 1986). Shulman described this knowledge as the knowledge that linked content knowledge and teaching (Ball et al, 2008; Shulman, 1986, 1987). This is the unique knowledge required to teach the subject specific content (Ball et al., 2008). This type of knowledge covers knowing to identify the teaching materials that make the subject accessible to students, being able to identify places where students may make common errors, being able to prevent the formation of misconceptions, and being able to identify and remedy the misconceptions students have (Ball et al., 2008; Shulman, 1986).

DTAMS

DTAMS was developed by teams of mathematicians, mathematics educators, and middle-school teachers to measure the content knowledge and pedagogical content knowledge of middle-school mathematics teachers. (DTAMS, n.d.; Saderholm et al., 2010). The breadth of the mathematics content was determined by reviewing extensive literature regarding what mathematics middle-school students and teachers should know while the depth of the mathematics knowledge was based on the types of mathematics knowledge defined in research on models and frameworks for knowledge (Saderholm et al., 2010). In this study, DTAMS for Algebra was used to measure the algebraic content and pedagogical knowledge of sixth grade mathematics teachers.

Significance

A baseline survey conducted in the Maldives during 2012 and 2013 showed that student performance in algebra is the lowest compared to any other area of mathematics (UNICEF & NIE, 2014). It is believed that highly qualified teachers of mathematics lead to better student performance although it is unclear exactly what types of knowledge contribute to these performance gains (Ball et al., 2001; Begle, 1979; Brown et al., 2011; Darling-Hammond & Sykes, 2003; Ojose, 2014; Shirvani, 2015). Hence, this study was designed to examine the algebraic content and pedagogical knowledge of sixth grade mathematics teachers in the Maldives.

Research has been conducted to identify specific types of teacher knowledge that promote conceptual understanding that leads to better student outcomes. Ball et al. (2008) have categorized teacher knowledge into content knowledge and pedagogical knowledge. Content knowledge is the subject specific knowledge required to teach the subject, whereas pedagogical knowledge refers to the knowledge required to teach subject specific content in a way that helps the students to learn and understand those concepts (Ball et al., 2008; Chick et al., 2006; Shulman, 1986).

Piccolo (2008) argued that mathematical content and pedagogical knowledge of a teacher influence the teacher's ability to teach mathematics. Therefore, this study is significant as it is expected to determine the algebraic content and pedagogical knowledge of sixth grade mathematics teachers in the Maldives. In the Maldives, algebra is first introduced to students in sixth grade. All sixth graders study algebra, and this is the very first time they are exposed to algebraic concepts. The transition from arithmetic to algebra is considered a difficult process for students due to its abstract nature (Gurbuz & Toprak, 2014). It is the teacher’s ability to deliver the content, which is influenced by the algebraic content and pedagogical knowledge that could make this transition as smooth as possible. In fact Gurbuz and Toprak (2014) reported that when students are taught in a “proper way” during the transition from arithmetic to algebra, students were found to be more successful in learning concepts of algebra. Misconceptions that students have with regard to algebraic concepts have been linked to how they were first exposed to algebra (Tennant & Colloff, 2014).

Professional literature has linked algebraic content and pedagogical knowledge of teachers to students’ success in algebra courses. As this is the first study of its kind in the Maldives, and students are first exposed to algebra at sixth grade, it is essential that algebraic content and pedagogical knowledge of sixth grade mathematics teachers are examined. This would help to identify the algebraic content and pedagogical knowledge strengths and weaknesses of sixth grade mathematics teachers. Sixth grade mathematics teachers are the ones who lay the foundation upon which more complex algebraic concepts are built. Identifying the algebraic content and pedagogical knowledge strengths and weaknesses of sixth grade mathematics teachers might in turn bring a positive social change by informing the direction of needed teacher professional development. Moreover, understanding the algebraic content and pedagogical knowledge of the mathematics teachers would help the field of mathematics education in preparing teachers to teach the subject. Mathematics education researchers have been calling for the better preparation of mathematics teachers (Gurbuz & Toprak, 2014; Ojose, 2014;

Tajudin, 2014). Therefore, this study also could also contribute towards improving mathematics education in the country by informing teacher training institutions about teachers' algebraic content and pedagogical knowledge strengths and weaknesses, which could prove invaluable in designing their courses.

Guiding Research Questions and Hypotheses

The ability to effectively teach algebra depends on the algebraic content and pedagogical knowledge of mathematics teachers (Piccolo, 2008). Therefore, this study aimed to examine the algebraic content and pedagogical knowledge of sixth grade mathematics teachers in the schools of the Maldives. Specifically, this research aimed to answer the following questions:

RQ1: Qualitative: What is the relationship between teachers' perceptions of their mastery of algebraic content and pedagogical knowledge, and what teachers know about algebraic content and pedagogy as measured by DTAMS?

$H_{0:}$ There is no significant relationship between teachers' perceptions about their algebraic content and pedagogical knowledge and what they know about algebra content and pedagogy as measured by DTAMS.

$H_{a:}$ There is a significant relationship between teachers' perceptions about their algebraic content and pedagogical knowledge and what they know about algebra content and pedagogy as measured by DTAMS.

RQ1.1: Quantitative: Based on responses on the DTAMS, what is the algebraic content knowledge of sixth grade mathematics teachers?

RQ1.2: Quantitative: Based on responses on the DTAMS, what are the strengths and weaknesses of the algebraic pedagogical knowledge of sixth grade mathematics teachers?

RQ1.3: Qualitative: Based on responses to the interviews, what are the perceptions of sixth grade mathematics teachers about their own algebraic content and pedagogical knowledge strengths and weaknesses?

RQ2: Qualitative: As measured by DTAMS, what are the specific algebraic content and pedagogical knowledge strengths and weaknesses of sixth grade mathematics teachers?

Review of the Literature

Search Strategy

Saturation for the literature review was reached after researching peer-reviewed journals in education databases. The databases searched included ERIC, Educational Research Complete, SAGE Premier, ProQuest Central, Science Direct, and Academic Search Complete. Boolean search terms included, but not limited to: *algebra teacher knowledge*, *algebra instruction*, *content knowledge*, *content and pedagogical knowledge*, *knowledge base for teaching*, *knowledge required to teach*, *knowledge required to teach algebra*, *mathematical knowledge*, *pedagogical knowledge*, *teacher knowledge*, *teacher content knowledge*, and *teacher pedagogical knowledge*.

Review of the Broader Problem

The capability to teach mathematics content is greatly influenced by mathematical content and pedagogical knowledge of the teacher (Piccolo, 2008; Strand & Mills, 2014).

Several studies have been conducted to understand teachers' mathematical content and/or pedagogical knowledge since Shulman (1986, 1987) put forward the idea of categories of required knowledge for teaching, upon which Ball et al. (2008) developed the domains of mathematical knowledge required for teaching (Browning et al., 2014; Buschang et al., 2012; Hauk, Toney, Jackson, Nair, & Tsay, 2014; Hill, Ball, & Schilling, 2008; Jing-Jing, 2014; Kleickmann et al., 2015; Lange, Kleickmann, & Moller, 2012; Liu, 2010; Shirvani, 2015; Thanheiser et al., 2014). In this section, evidence of the problem in the broader context as well as in the local context will be discussed briefly. This will be followed by a detailed description of the teacher training model used to train sixth grade mathematics teachers in the Maldives. A brief description of the models used to study the knowledge base required for teaching is provided before moving on to the theoretical frameworks guiding this study.

Evidence of the problem from professional literature. It is widely believed that students taught by teachers who have a strong mathematical content and pedagogical knowledge perform better in mathematics compared to those who are taught by teachers who lack an in-depth understanding of mathematical content and pedagogical knowledge (Ball et al., 2001; Begle, 1979; Brown et al., 2011; Darling-Hammond & Sykes, 2003; Ojose, 2014; Shirvani, 2015; Strand & Mills, 2014). Teachers who lack content knowledge found it difficult to explain concepts to the students and made mathematical errors in the classroom while teaching (Shirvani, 2015). Ojose (2014) reported that teachers who had the content knowledge but lacked pedagogy offered "skeletal explanations loaded with routines" (p. 41), while Tajudin (2014) reported that teachers who lacked content knowledge focused on procedures. This resulted in students being offered mathematically incorrect answers when they sought understanding by asking questions (Ojose, 2014). These clearly indicate that in order to teach mathematics well teachers are not only expected to have a commanding knowledge of the content they teach but they also need to know the right approach that could be used to teach the specific content for the target audience (Ojose, 2014; Shirvani, 2015; Strand & Mills, 2014).

Evidence of the problem at the local level. During 2012 and 2013 a baseline survey was conducted in the Maldives, and the results indicated that students performed poorest in algebra compared to any other area in mathematics (UNICEF & NIE, 2014). Interviews with researchers, teacher educators, and heads of schools revealed that they all believed the poor performance of students in algebra was the result of deficient algebraic content and pedagogical knowledge of the mathematics teachers (A. Gasim, personal communication, April 23, 2015; A. Shareef, personal communication, April 22, 2015; A. Waheed, personal communication, March 7, 2015; M. Qasim, personal communication, April 13, 2015). The algebraic content and pedagogical knowledge of mathematics teachers in the Maldives have never been studied. Therefore, this study is important, as this study would confirm or contradict the belief that the poor performance of students in algebra was the result of a deficiency in the algebraic content and pedagogical knowledge of mathematics teachers.

Sixth grade mathematics teacher training in the Maldives. This section sheds light on the background of the training undertaken by teachers who teach mathematics at sixth grade in the Maldives. The Maldives National University, the only university in the country, was established in the year 2011. However, teacher training has been in effect since 1984 with the establishment of the Institute for Teacher Education that later became the Faculty of Education of the Maldives National University (Maldives National University, n.d.). The Maldives National University was the only institution that had been training teachers to teach grades six and seven until recently. According to the head of one of the departments in the Faculty of Education, the trained in-service teachers who are teaching sixth grade at present have completed one of the following programs offered at the Maldives National University (W. Fikuree, personal communication, July 24, 2015):

1. Bachelor of Education (Primary), which is a four-year course preparing teachers to teach grades one through six.
2. Diploma in Teaching (Primary), awarded upon completion of the first year of the Bachelor of Education (Primary) degree.
3. Advanced Diploma in Teaching (Primary), awarded upon completion of the first two years of Bachelor of Education (Primary) degree.
4. Bachelor of Teaching (Primary), awarded upon completion of the first three years of Bachelor of Education (Primary) degree.

It is noteworthy that there are no sixth grade mathematics teachers who have a certification with a specialization in mathematics as the only teacher qualification offered is a four-year degree major in education, which prepares teachers to teach all the subjects taught in primary grades with the exception of Dhivehi (the native language), Islam, Arabic, and Quran. Bachelor of Education (Primary) prepares teachers to teach Mathematics, English, Science, Social Studies, ICT, Creative Arts, and Health and Physical Education (L. Mohamed, personal communication, August 10, 2015; W. Fikuree, personal communication, July 26, 2015).

Table 1 shows the mathematics content and pedagogy courses all certified sixth grade mathematics teachers are required to complete in order to obtain a teaching qualification to teach sixth grade mathematics (S. Abdullah, personal communication, August 10, 2015; W. Fikuree, personal communication, August, 10, 2015).

Table 1

Mathematics Modules and Qualifications Obtained

Year	Name of Module	Name of Qualification
1	Essential Mathematics	Diploma in Teaching Primary
2	Teaching and Learning Mathematics 1 for Primary	Advanced Diploma in Teaching Primary
3	Teaching and Learning Mathematics 2 for Primary	Bachelor of Teaching Primary
4	Mathematics Through Exploration – Primary	Bachelor of Education (Primary)

Essential mathematics. Essential mathematics is a mathematics content course that covers pre-University level mathematics and is equivalent to Pearson International

Advanced Level Mathematics. This module is worth 150 learning hours (R. Fathimath, personal communication, August 11, 2015; S. Abdullah, personal communication, August 10, 2015; W. Fikuree, personal communication, August, 10, 2015).

Content covered in essential mathematics. Content covered in this module includes algebra, coordinate geometry, sequences and series, functions, trigonometry, differentiation, and integration (R. Fathimath, personal communication, August 11, 2015; S. Abdullah, personal communication, August 10, 2015; W. Fikuree, personal communication, August, 10, 2015).

Learning outcomes of essential mathematics. By the end of the successful completion of the module students are expected to be able to find the nth term of a series; find the sum of arithmetic series and geometric series; and the sum to infinity of geometric series; understand the basic concepts of algebra; simplify algebraic fractions; add, subtract, multiply and divide one polynomial by another; factorize polynomials and trinomials; use factor theorem and the remainder theorem along with long division and partial fractions; factorize quadratic functions using completing the square method; solve quadratic equations by factoring, by completing the square, and by using the quadratic formula; solve simultaneous equation when two equations are linear and when one equation is linear and the other is quadratic; use sine, cosine and tangent ratios to find angles and sides of right angle triangles; extend sine and cosine function to angles between 90 and 360; use trigonometric identities; find the equation of a straight line when two points are given; parallel and perpendicular lines; find the mid-point of a line; find the equation of a circle; sketch and transform graphs of functions; use laws of logarithm to simplify expressions involving exponential and logarithms; solve exponential and logarithmic equations; differentiate functions of the form $y = x^n$; classify increasing and decreasing functions; find turning points, tangents and normals to curves; integrate functions of the form $y = x^n$; find definite and indefinite integrals; and find the area of region bounded by curves and lines (R. Fathimath, personal communication, August 11, 2015; S. Abdullah, personal communication, August 10, 2015; W. Fikuree, personal communication, August, 10, 2015).

Teaching and learning mathematics 1 for primary. This is a mathematics pedagogy course that focuses on problem-solving and investigational work as a basis for developing student teachers' understanding of mathematical knowledge. Furthermore, this module also aims to develop deeper understanding of selected topics from the National Primary School Mathematics Curriculum. This module is worth 150 learning hours (R. Fathimath, personal communication, August 11, 2015; S. Abdullah, personal communication, August 10, 2015; W. Fikuree, personal communication, August, 10, 2015).

Content covered in teaching and learning mathematics 1 for primary. Content covered in this module include attitude towards mathematics learning, learning theories and mathematics instruction in the primary grades, the Maldives National Mathematics Curriculum, scheme of work and concept mapping, lesson planning, teaching mathematics skills and procedures, problem-solving in mathematics, number sense and estimation, introducing the concept of place value, early number operations (addition, subtraction, multiplication, and division), number sequences leading to algebra, measurement in the lower grades, introduction to measurement and length (perimeter), early concept of algebra, introduction for formulas, early concept of volume, early concepts of shapes and space, assessment in mathematics, types and purpose of assessment instruments, designing a table of specifications, and designing test instrument and marking schemes (R. Fathimath, personal communication, August 11, 2015; S. Abdullah, personal communication, August 10, 2015; W. Fikuree, personal communication, August, 10, 2015).

Learning outcomes of teaching and learning mathematics 1 for primary. By the end of the successful completion of the module student teachers are expected to be able to examine their own experiences of learning mathematics and discuss how it has affected their own attitude towards mathematics; examine the use of problem-solving and investigational work in developing children's mathematical thinking and application skills; have a broader understanding of different number systems and number bases; have a better understanding of the Primary Mathematics curriculum; and have a broader understanding of some topics from the school mathematics curricula, namely, early number, place value, early number operations shapes and space and measurement (R. Fathimath, personal communication, August 11, 2015; S. Abdullah, personal communication, August 10, 2015; W. Fikuree, personal communication, August, 10, 2015).

Teaching and learning mathematics 2 for primary. This is a mathematics pedagogy course that focuses on problem-solving and investigational work as a basis for developing student teachers' understanding of mathematical knowledge. Furthermore, this module also aims to develop deeper understanding of selected topics from the National Primary School Mathematics Curriculum. This module is worth 150 learning hours (R. Fathimath, personal communication, August 11, 2015; S. Abdullah, personal communication, August 10, 2015; W. Fikuree, personal communication, August, 10, 2015).

Content covered in teaching and learning mathematics 2 for primary. Content covered in this module includes mathematics and language, initial concept formation of fractions, equivalent fractions, operations on fractions, problem-solving and misconceptions of students in learning about fractions, initial concept formation and comparison of decimals, operations and problem-solving on decimals, concept formation and problem-solving on ratio and proportions, percentage teaching ideas and concept formation, volume and capacity teaching ideas, directed numbers, geometry, practical graphs on spread sheet and straight line graphs, data handling in lower grades, how to teach algebra, a framework for developing effective questioning, and developing and using resources to teach mathematics (R. Fathimath, personal communication, August 11, 2015; S. Abdullah, personal communication, August 10, 2015; W. Fikuree, personal communication, August, 10, 2015).

Learning outcomes of teaching and learning mathematics 2 for primary. By the end of the successful completion of the module student teachers are expected to have a broader understanding of some topics from the school mathematics curricula, namely; fractions, decimals, ratio & proportions, percentages, directed numbers, geometry, straight line graphs, data handling, volume and capacity and algebra; be able to examine the use of problem-solving and investigational work in developing children's mathematical thinking and application skills; have a broader understanding of different number systems and number bases; have a better understanding of the Primary Mathematics curriculum; and have a broader understanding of some topics from the school mathematics curricula, namely, early number, place value, early number operations shapes and space and measurement (R. Fathimath, personal communication, August 11, 2015; S. Abdullah, personal communication, August 10, 2015; W. Fikuree, personal communication, August, 10, 2015).

***Mathematics through exploration for primary*.** This is a mathematics pedagogy course that focuses on providing student teachers an opportunity to develop a broader appreciation of mathematics by exploring ways in which the artistic, aesthetic, intellectual and humanistic aspects of mathematics are as important as its utility. This module is worth 150 learning hours (R. Fathimath, personal communication, August 11, 2015; S. Abdullah, personal communication, August 10, 2015; W. Fikuree, personal communication, August, 10, 2015).

Content covered in mathematics through exploration for primary. Content covered in this module include history of mathematics, learning theories in mathematics, problem-solving and investigational work in mathematics, early number and number sense, the concept of place value, operations on numbers and fractions, sequences and investigations in numbers, algebraic representation and algebraic manipulation, equations and inequalities, geometric properties, shapes and space, concept formation of perimeter, area, and volume, misconceptions and misinterpretations in measurement, handling data in lower grades, concept formation and operations in decimals, teaching methods used in teaching mathematics, assessment in mathematics teaching, analyze different types of assessment in mathematics, and develop assessment instruments (R. Fathimath, personal communication, August 11, 2015; S. Abdullah, personal communication, August 10, 2015; W. Fikuree, personal communication, August, 10, 2015).

Learning outcomes of mathematics through exploration for primary. By the end of the successful completion of the module student teachers will improve communicating mathematical ideas clearly and succinctly; work with concepts in Mathematics Scheme of Work for Primary School; think logically; construct logical arguments and be aware of the values of using real life situations in teaching Mathematics (R. Fathimath, personal communication, August 11, 2015; S. Abdullah, personal communication, August 10, 2015; W. Fikuree, personal communication, August, 10, 2015).

Passing grade for the modules is 50% or above. Students who scored below 45% must repeat the module. If a student scores between 45% and 49% (inclusive) the student is eligible for a re-test. Students are given two weeks to prepare for the re-test. If a student failed to score 50% in the re-test, then the student is required to repeat the module. This indicates that there will be a lot of variability of mastery among the teachers completing these courses (R. Fathimath, personal communication, August 11, 2015; S. Abdullah, personal communication, August 10, 2015; W. Fikuree, personal communication, August, 10, 2015).

In a nutshell, a teacher who graduated with a Diploma in Teaching Primary would take only one mathematics content course whereas a teacher who has an Advanced

Diploma in Teaching Primary would take one mathematics content course and one mathematics pedagogy course. A teacher who has completed Bachelor of Teaching Primary would take one mathematics content course and two mathematics pedagogy courses whereas, Bachelor of Education (Primary) graduates would take one mathematics content course and three mathematics pedagogy courses. Each of these modules are worth 150 learning hours, and a student must score at least 50% in order to obtain a pass in the module.

Models used to study the knowledge base required for teaching. Algebra is a fundamental topic in mathematics which majority of the students find difficult due to its abstract nature (Cheng-Yao et al., 2014; Strand & Mills, 2014). According to a baseline survey conducted during 2012 and 2013 in the Maldives, algebra was found to have a lowest pass percentage compared to any other topic in mathematics (UNICEF & NIE, 2014). Through literature review and the interviews, the importance of studying algebraic content and pedagogical knowledge of mathematics teachers became apparent (A. Gasim, personal communication, April 23, 2015; A. Shareef, personal communication, April 22, 2015; A. Waheed, personal communication, March 7, 2015; M. Qasim, personal communication, April 13, 2015). Hence, this study was designed to examine the algebraic content and pedagogical knowledge of the sixth grade mathematics teachers.

The literature review was conducted to learn about various models that were used in studying teacher knowledge and to identify the most suitable model to study algebraic content and pedagogical knowledge of the sixth grade mathematics teachers. After an exhaustive literature review, the following models became apparent as models used for knowledge base required for teaching:

1. Shulman's major categories of teacher knowledge (Fernandez, 2014; Shulman, 1987)
2. Grossman's model of teacher knowledge (Grossman, as cited in Fernandez, 2014)
3. Carlsen's domains of teacher knowledge (Carlsen, 1999; Fernandez, 2014)
4. Magnusson, Krajcik, and Borko's components of pedagogical content knowledge for science teaching (Fernandez, 2014; Magnusson, Krajcik, & Borko, 1999)
5. Magnusson, Krajcik, and Borko's model of the relationships among the domains of teacher knowledge (Magnusson, Krajcik, & Borko, 1999)
6. Park and Oliver's hexagonal model of pedagogical content knowledge for science teaching (Fernandez, 2014; Park & Oliver, 2008)
7. Rollnick and colleagues' tailored model for PCK (Fernandez, 2014; Rollnick et al., 2008)
8. Morine-Dershimer and Kent's categories contributing to pedagogical content knowledge (Fernandez, 2014; Morine-Dershimer & Kent, 1999)
9. Morine-Dershimer and Kent's facets of pedagogical knowledge (Fernandez, 2014; Morine-Dershimer & Kent, 1999)
10. Abell's model of science teacher knowledge (Abell, 2008; Fernandez, 2014)

11. Consensus model of PCK from PCK Summit (Fernandez, 2014; Helms & Stokes, 2013)
12. Ball and colleagues' domains of mathematical knowledge for teaching (Ball et al., 2008)

After meticulous analysis of the models, Shulman's model and Ball's model were chosen to guide the study for the following reasons:

1. Shulman introduced the idea of pedagogical knowledge and all the models were based on Shulman's major categories of teacher knowledge (Abell, 2008; Ball 2008; Fernandez, 2014; Carlsen, 1999; Helms & Stokes, 2013; Magnusson, Krajcik, & Borko, 1999; Morine-Dershimer & Kent, 1999; Park & Oliver, 2008; Rollnick et al., 2008).
2. Shulman's model discussed the knowledge base for teaching while Ball et al. built upon Shulman's model to identify the domains of mathematical knowledge for teaching (Ball et al., 2008) whereas the other models mentioned above either focused on a domain or a few domains from Shulman's model or they focused on teaching of subjects other than mathematics such as science teaching (Abell, 2008; Ball 2008; Fernandez, 2014; Carlsen, 1999; Helms & Stokes, 2013; Magnusson, Krajcik, & Borko, 1999; Morine-Dershimer & Kent, 1999; Park & Oliver, 2008; Rollnick et al., 2008).
3. Since 1986 when Shulman first put forward the idea of pedagogical knowledge, it has been cited in more than 1,200 refereed journal articles in

125 different journals in "professions ranging from law to nursing to business, and regarding knowledge for teaching students preschool through doctoral studies" (Ball et al., 2008, p. 392).

4. Ball et al.'s domains of mathematical knowledge for teaching gained popularity since it was first proposed in 2008 and has been cited 2,316 times to date to study the knowledge required to teach mathematics (For example, Cheng-Yao et al., 2014; Hauk et al., 2014; Kleickmann et al., 2015; Shirvani, 2015; Tajudin, 2014).

Shulman focused on the categories of knowledge required to teach in general which included content knowledge and pedagogical knowledge, whereas Ball et al. built on, expanded and refined Shulman's idea which linked content knowledge and teaching to focus on mathematics teaching (Ball et al., 2008). Therefore, the two theoretical frameworks, namely, Shulman's major categories of teacher knowledge (1987) and Ball et al.'s domains of mathematical knowledge for teaching (2008) were found to be the most appropriate for this study as this study aimed to examine the algebraic content and pedagogical knowledge of sixth grade mathematics teachers.

Theoretical Frameworks Guiding the Study

This study is guided by two theoretical frameworks, namely, Shulman's major categories of teacher knowledge (1987) and the domains of mathematical knowledge for teaching proposed by Ball, Thames, and Phelps (2008). Lee Shulman outlined the categories of knowledge required by a teacher to teach, and for the first time pedagogical content knowledge was mentioned in education. Ball et al. developed Shulman's idea of pedagogical content knowledge and linked that to content knowledge. In addition, Ball et al. developed a model that focused specifically on the knowledge required to teach mathematics.

Shulman's major categories of teacher knowledge. The first is the Shulman's major categories of teacher knowledge. In 1987 Shulman outlined seven categories of teacher knowledge required for a teacher to teach. According to Shulman (1987, p. 8) they are:

1. Content knowledge;
2. General pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter;
3. Curriculum knowledge, with particular grasp of the materials and programs that serve as "tools of the trade" for teachers;
4. Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding;
5. Knowledge of learners and their characteristics;
6. Knowledge of educational contexts, ranging from the workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures; and
7. Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds.

Shulman's definition of content knowledge was not just limited to the knowledge of concepts, theories, ideas, and proofs but also covered the approaches to develop this knowledge (Fernandez, 2014). Shulman (1986) argued that in order to teach a subject it is crucial that teachers knew more than just the facts and concepts. He believed that teachers knowing that something is *so* is not enough. In addition teachers should understand the *why* something is so. Shirvani (2015) reported that teachers who had a strong content knowledge were capable of implementing more flexible teaching strategies that helped students better understand more complex mathematical concepts. This is parallel to the findings of Strand and Mills (2014) who reported that in order to tailor instruction in a way that develops students' understanding, it is important that teachers have a strong understanding of the algebraic content related pedagogy.

Shulman's second category, general pedagogical knowledge, included "educational purposes and values and, in addition requires a cognitive, social and developmental theory of learning and how they apply within the classroom" (Fernandez, 2014, p. 83). Shulman (1986) defined curricular knowledge as "the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances" (p. 10).

Shulman further subdivided curricular knowledge into lateral curriculum knowledge and vertical curriculum knowledge (Ball et al., 2008). Lateral curriculum knowledge is the knowledge of how the curriculum relates to the curriculum that is taught to students in other classes while vertical curriculum knowledge is used to represent the knowledge of curriculum that is taught to students across the grades in the same subject area (Ball et al., 2008).

In 1986 Shulman introduced and in 1987 developed the idea of pedagogical content knowledge for the first time (Ball et al., 2008; Fernandez, 2014). This was a departure from what was focused in education research those days (Ball et al, 2008). Shulman referred to this as the missing paradigm in research on teaching and teacher knowledge (Fernandez, 2014). Shulman (1986) described pedagogical content knowledge as the knowledge of subject matter knowledge required for teaching. He went on to explain that pedagogical content knowledge also includes understanding of what makes learning of certain concepts easy or difficult. In addition, he explained that pedagogical content knowledge covers the preconceptions and misconceptions associated with learning of specific concepts according to age and the background of the students. In particular, knowing the strategies that could be used to address those shortcomings and reorganize the understanding of the students come under pedagogical content knowledge. The last three categories addressed the general dimensions of teacher knowledge that were the backbone of teacher education programs of that time, hence, were not the main focus of Shulman's work (Ball et al., 2008).

Domains of mathematical knowledge for teaching. The second theoretical framework that guided this study is the domains of mathematical knowledge for teaching proposed by Ball, Thames, and Phelps (2008). The domains of mathematical knowledge for teaching was built on Shulman's theoretical framework connecting content knowledge to practice of teaching (Ball et al., 2008). According to Ball et al. (2008), in 1986 Lee Shulman and colleagues put forward a domain of teacher knowledge which linked content knowledge and teaching, and they called it pedagogical content knowledge. Since then, this domain has gained the popularity and Shulman's idea has been cited in more than 1,200 refereed journal articles in 125 different journals in "professions ranging from law to nursing to business, and regarding knowledge for teaching students preschool through doctoral studies" (Ball et al., 2008, p. 392).

Ball et al. (2008) refined Shulman's categories and proposed the model in Figure 1. Figure 1 shows the domains of mathematical knowledge for teaching. This has been cited 2,316 times since then.

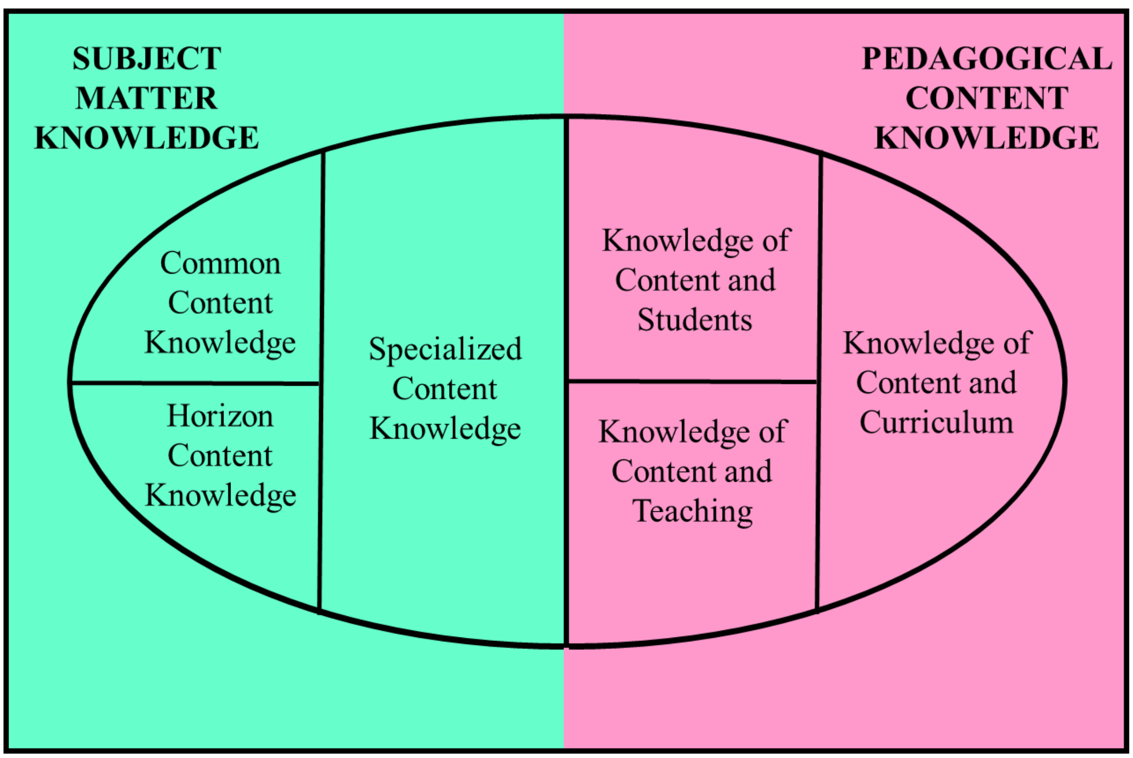


Figure 1. Domains of mathematical knowledge for teaching (Ball et al., 2008).

Ball et al. (2008) proposed six domains, namely, common content knowledge; horizon content knowledge; specialized content knowledge; knowledge of content and students; knowledge of content and teaching; and knowledge of content and curriculum (as shown in Figure 1). These six domains come under two main categories – subject matter knowledge and pedagogical content knowledge.

Subject matter knowledge. Subject matter knowledge is the subject specific knowledge of content. This category is subdivided into common content knowledge, horizon content knowledge, and specialized content knowledge.

Common content knowledge. Common content knowledge is defined as "the mathematical knowledge and skill used in settings other than teaching" (Ball et al., 2008, p. 399). This includes knowing the materials they teach; being able to identify incorrect answers given by students; recognizing inaccurate definitions presented in textbooks; and being able to use terms and notations correctly. In short, common content knowledge is defined as "the mathematical knowledge known in common with others who know and use mathematics" (Ball et al., 2008, p. 403). Some research indicated that teachers' high school mathematics knowledge is positively correlated with students' learning gains (Shirvani, 2015).

Horizon content knowledge. Horizon content knowledge is defined as the awareness of how mathematical topics are related over the span of mathematics included in the curriculum (Ball et al., 2008, p. 403). For example, primary teachers may need to know how categorization of shapes and pattern generalizations can set the mathematical foundation for formal algebra, which is taught in sixth grade in the Maldives.

Specialized content knowledge. Specialized content knowledge is defined as "the mathematical knowledge and skill unique to teaching" (Ball et al., 2008, p. 400). An example of this could be the analysis of a student's incorrect answer to find out the nature of the error because, this would require dexterity in thinking about numbers, being aware of the patterns, and being able to critically examine the meaning in ways that are unique to teaching (Ball et al., 2008).

Pedagogical content knowledge. Ball et al. (2008) cited Shulman's definition of pedagogical content knowledge as follows:

> The most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the most useful ways of representing and formulating the subject that make it comprehensible to others. Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (p. 391 – 392)

Magnusson, Krajcik, and Borko (as cited in Ball et al., 2008) defined pedagogical content knowledge as a teacher's understanding of how to help students understand specific subject matter. It includes knowledge of how particular subject matter topics, problems, and issues can be organized, represented and adapted to the diverse interests and abilities of learners, and then presented for instruction. The defining feature of pedagogical content knowledge is its conceptualization as the result of a *transformation* of knowledge from other domains (p. 394).

In summary it can be said that pedagogical content knowledge is the unique knowledge required to teach subject-specific content. Ball et al. (2008) subdivided the pedagogical content knowledge into three domains. These domains are knowledge of content and students; knowledge of content and teaching; and knowledge of content and curriculum. Pedagogical knowledge of the teacher is important as teachers who have content knowledge but lack pedagogical knowledge not only find it difficult to explain mathematical concepts to the students in a way that they would understand but also are likely to make errors in classroom teaching (Ojose, 2014).

Knowledge of content and students. Knowledge of content and students is defined as the “knowledge that combines knowing about students and knowing about mathematics” (Ball et al., 2008, p. 401). This means teachers should be able to recognize how the students would think when presented with a certain problem and also teachers should be able to judge what the students would find confusing with the presented problem. In addition, teachers not only should be able to predict what kind of examples would make it easier for the students to grasp the concept but also what type of examples would maintain or build their interest in the lesson presented.

Knowledge of content and teaching. Ball et al. (2008) defined knowledge of content and teaching as “knowing about teaching and knowing about mathematics” (p. 401). In other words, teachers should be able to identify which instructional strategies would suit a certain lesson and also the sequence of the lesson, for instance, teachers being able to recognize which example would work best in the beginning or the introduction phase of the lesson.

Knowledge of content and curriculum. Ball et al. (2008) cited Shulman's definition of curricular knowledge instead of proposing a definition. Shulman defined curricula knowledge as

> [knowledge] represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances (as cited in Ball et al., 2008, p. 391).

Ball et al. (2008) developed the framework domains of mathematical knowledge for teaching after analyzing existing literature on knowledge base for teaching. This specifically focused on mathematics teaching and had identified the necessary aspects of mathematics knowledge for teaching. Moreover, this framework had been cited 2,316 times since then. No obvious blind spots to use of this framework in examining mathematical content knowledge and pedagogical knowledge have been identified. Remarkably, during the course of this study, no blind spots to use of this framework were uncovered.

How Theory Relates to the Research Questions

There were two main research questions and three sub questions under the main research question one. The main research questions were:

RQ1: What is the relationship between teachers' perceptions of their mastery of algebraic content and pedagogical knowledge and what do teachers know relative to algebraic content and pedagogy as measured by DTAMS?

RQ2: As measured by DTAMS, what are the specific algebraic content and pedagogical knowledge strengths and weaknesses of sixth grade mathematics teachers with respect to their algebraic content and pedagogical knowledge?

RQ1 aimed to identify whether there was any relationship between what teachers perceive to know and what they actually know about algebraic content and pedagogical knowledge, and the main research question two sought to identify the specific algebraic content and pedagogical knowledge strengths and weaknesses of sixth grade mathematics teachers with respect to their algebraic content and pedagogical knowledge. This also related to the first four categories of Shulman's major categories of teacher knowledge (1987) and the domains of mathematical knowledge for teaching (Ball et al., 2008). In general these research questions helped in gaining an in-depth understanding of the algebraic content and pedagogical knowledge of the sixth grade mathematics teachers in addition to helping crystallize the findings from DTAMS.

RQ1.1: Based on responses on the DTAMS, what is the algebraic content knowledge of sixth grade mathematics teachers?

This subquestion sought to identify the algebraic content knowledge of sixth grade mathematics teachers using DTAMS. This related to the first and third category identified by Shulman (1987) which are content knowledge and curriculum knowledge respectively. Ball et al. (2008) extended the idea of content knowledge to include general mathematical knowledge and skills, knowledge of how the mathematics curriculum is spread across the grades, and mathematical knowledge and skills required for teaching. Ball et al. referred to this as subject matter knowledge.

> RQ1.2: Based on responses on the DTAMS, what are the strengths and weaknesses of the algebraic pedagogical knowledge of sixth grade mathematics teachers?

This subquestion aimed to investigate the pedagogical knowledge of sixth grade mathematics teachers relative to algebra using DTAMS. This related to the second and fourth categories identified by Shulman (1987), which covered general pedagogical knowledge and pedagogical content knowledge. This also related to the second of the two broader categories identified by Ball et al. (2008). Ball et al. (2008) defined pedagogical knowledge to include knowledge of content, students, teaching, and curriculum that played a key role in delivering the content in a way that is easier for the students to grasp. This definition is aligned with the aspects covered in the models proposed for teacher knowledge (Abell, 2008; Ball 2008; Fernandez, 2014; Carlsen, 1999; Helms & Stokes, 2013; Magnusson, Krajcik, & Borko, 1999; Morine-Dershimer & Kent, 1999; Park & Oliver, 2008; Rollnick et al., 2008).

> RQ1.3: Based on responses to the interview, what are the perceptions of sixth grade mathematics teachers about their own algebraic content and pedagogical knowledge strengths and weaknesses?

This subquestion explored the teacher’s perception with regard to their algebraic content and pedagogical knowledge. This related to the first four categories identified by

Shulman (1987), namely, content knowledge, general pedagogical knowledge, curriculum knowledge, and pedagogical content knowledge. In addition, this question also related to the two broader domains of mathematical knowledge required for teaching proposed by Ball et al. (2008), which are subject matter knowledge and pedagogical content knowledge.

Summary

In the literature review mathematical content and pedagogical knowledge have been addressed. Moreover, the subcategories that come under each of the two broader categories have been discussed. Through the review of the literature, it became evident that mathematical content and pedagogical knowledge of teachers played a key role in delivering the content in a way that could be easily grasped by the students. Results of the baseline survey conducted in the Maldives as well as international studies showed that algebra is a topic students find difficult to understand. Additionally, some studies indicated that students found algebra concepts easier to grasp when taught by teachers with a strong background of algebraic content and pedagogical knowledge.

Implications

Educational leaders are concerned about the growing problem of teachers not possessing the content knowledge required to teach mathematics (Ojose, 2014). Mathematics education researchers have been calling for the better preparation of mathematics teachers (Gurbuz & Toprak, 2014; Ojose, 2014; Tajudin, 2014). Education researchers advocate that professional development activities should be planned to address these issues and in-service mathematics teachers should be encouraged to take part in these professional development activities (Ojose, 2014).

As this is the first study of its kind in the Republic of Maldives, this study could contribute to the improvement in algebra instruction as a result of what was learned about the algebraic content and pedagogical knowledge strengths and weaknesses of sixth grade mathematics teachers. The results of this study could contribute in to the improvement in algebra instruction by suggesting ways teachers could modify their instruction of algebraic concepts to ensure students build conceptual understanding of the subject. In addition, the results of this study could be used to inform teacher-training needs and specific needs of professional development programs. Furthermore, the results from this this study could be used to inform the pre-service teacher training curriculum.

Summary

Algebra is fundamental to any study of science, technology, engineering and mathematics. Algebra plays an important role in advanced mathematics. Yet, algebra is considered as a subject students find difficult due to its abstract nature. Many students consider algebra as the obstacle that prevented them from pursuing careers in mathematics, science and engineering. Teachers are considered the primary source of content knowledge students learn, especially at the early stages of their student life as they entirely depend on teachers rather than textbooks and other resources.

A teacher’s ability to teach is directly affected by the content and pedagogical knowledge of the teacher. Knowing either the content or pedagogy alone is not enough for teaching that content. Hence, in order to continuously improve algebra instruction and identify the specific needs of mathematics professional development it is vital to assess the algebraic content and the pedagogical knowledge of teachers. This study aimed to examine the algebraic content and pedagogical knowledge of sixth grade mathematics teachers in the schools of the Maldives. Shulman’s major categories of teacher knowledge and Ball et al.’s domains of mathematical knowledge for teaching are the two frameworks that guided this study. The nature of this study is a mixed methods collective case study design. In the next section the research design and approach; research instruments and their validity and reliability; setting and sample; strategy used for data collection and the sequence; data analysis and the validity and the trustworthiness of the findings; and limitations of this study are described. Data collection started during the fall of 2015 after Walden University Institutional Review Board (IRB) approval (approval number 10-28-15-0398995).

Section 2: The Methodology

Introduction

The purpose of this study was to examine the algebraic content and pedagogical knowledge of sixth grade mathematics teachers in the Maldives to determine their relative strengths and weaknesses in order to provide a foundation for the development of a teacher professional development curriculum. This study is a mixed methods collective case study design. According to Merriam (2009), a case study is “an in-depth description and analysis of a bounded system” (p. 40) in which the “bounded system” refers to a unit around which there are boundaries that could be a single person, a program, a group, an institution, a community, or a specific policy. Yin (2008) defined case study as “an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when boundaries between phenomenon and context are not clearly evident” (p. 18). Wolcott (1992) described case study as “an end-product of field-oriented research” (p. 36). Creswell (2012) defined case study as “an in-depth exploration of a bounded system … based on extensive data collection” (p. 465) and explained bounded as being “separated out for research in terms of time, place, or some physical boundaries” (p.465). When case studies include more than one case, it is called a collective case study (Creswell, 2012; Stake, 1995) whereas Merriam (2009) referred to this as multisite case studies.

Mixed Method Design and Approach

As the purpose of this study was to examine the algebraic content and pedagogical knowledge of sixth grade mathematics teachers in the Maldives, a mixed-methods collective case study design was the most appropriate in which the "bounded system" refers to the sixth grade algebra teachers. Creswell (2012) stated "in-depth understanding requires that only a few cases be studied, because for each case examined, the researcher has less time to devote to exploring the depths of any one case" (p. 465), which is in agreement with Miles and Huberman (1994). Creswell described the research conducted by Kos (as cited in Creswell, 2012) of four middle school students who had reading disabilities and the research conducted by Padula and Miller (as cited in Creswell, 2012) of four women who had joined university as full-time students as examples of a case study (Creswell, 2012).

This study is considered a mixed methods study as both qualitative and quantitative data were collected from the cases. According to Ross and Onwuegbuzie (2012), mixing of both qualitative and quantitative research improved the findings in mathematics education research. In a mixed methods study, the strengths of both quantitative and qualitative study are merged, enhancing the understanding of the phenomena under study (Lopez-Fernandez & Molina-Azorin, 2011). Mixed methods research is gaining acceptance among researchers as the method that gives answers to questions which cannot be answered by using either quantitative or qualitative research methods alone (Caruth, 2013; Creswell, 2012; Lopez-Fernandez & Molina-Azorin, 2011). In fact, in mathematics education research, qualitative and quantitative data have been used to complement one another and to more thoroughly understand the relationships between observation and assessment data (Ross & Onwuegbuzie, 2012).

According to Lopez-Fernandez and Molina-Azorin (2011), triangulation and complementarity are the most widely stated reasons for use of mixed methods research. Triangulation of data improves the validity and reliability of the results, whereas complementarity clarifies the data collected by one method by applying the other (Jick, as cited in Lopez-Fernandez & Molina-Azorin, 2011; Venkatesh, Brown, & Bala, 2013). As this study sought to gain an in-depth understanding of algebraic content and pedagogical knowledge of sixth grade mathematics teachers in the Maldives, a mixed methods design was believed to be the most appropriate design for this study.

Quantitative Data

Quantitative data were collected using DTAMS. The instrument was used to establish the baseline algebraic content and the pedagogical knowledge of the sixth grade mathematics teachers. It was developed by teams of mathematicians, mathematics educators, and middle-school teachers to measure the content knowledge and pedagogical knowledge of middle-school teachers (DTAMS, n.d.; Saderholm et al., 2010). The coverage of the algebraic content was determined by reviewing a wide range of literature regarding what algebraic concepts middle-school students and teachers should know. According to DTAMS (n.d.), some of the reviewed literature on algebraic ideas included the following:

1. MacGregor, M., & Stacey, K. (1997). Students' understanding of algebraic notation: 11–15. *Educational Studies in Mathematics, 33*(1), 1-19.

2. Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992). Development of the process conception of function. *Educational Studies in Mathematics*, *23*(3), 247-285.
3. Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research*, *60*(1), 1-64.
4. MacGregor, M., & Stacey, K. (1993). Cognitive models underlying students' formulation of simple linear equations. *Journal for Research in Mathematics Education*, *24*(3), 217-232.
5. Even, R. (1993). Subject matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education*, *24*(2), 94-116.
6. Bishop, J. (2000). Linear geometric number patterns: Middle school students' strategies. *Mathematics Education Research Journal*, *12*(2), 107-126.
7. Ferrandez-Reinisch, A. M. (1985). The acquisition of inverse proportionality: A training experiment. *Journal for Research in Mathematics Education*, *16*(2), 132-140.
8. Rosnick, P. (1981). Some misconceptions concerning the concept of variable. *The Mathematics Teacher*, *74*(6), 418-450.

The depth of the mathematics knowledge was based on the types of knowledge defined in research on models and frameworks for knowledge (Saderholm et al., 2010). DTAMS for algebra includes questions from three subdomains:

relations/functions/patterns, equations/inequalities, and expressions/polynomials. The assessment consists of 20 questions, 10 multiple choice questions and 10 open response questions. Open-response questions had two parts. The first part assessed the algebraic content knowledge required to solve the problem while the second part assessed the pedagogical knowledge and skills required to teach the concept and address the student misconceptions (Saderholm et al., 2010).

A sample multiple choice question is as follows:

Solve the equation for N: $\frac{200lbs}{270lbs} = \frac{E}{N}$.

$a)N = \frac{270}{200}E$ $\quad$ $b)N = \frac{270}{200E}$ $\quad$ $c)N = \frac{200E}{270}$ $\quad$ $d)N = \frac{200}{270E}$

A sample open response question is:

One of your sixth-grade students is having difficulty graphing lines. After asking a few questions, you realize that this student seems to be unable to graph lines without plotting three, four, five, or more points on the line. Further, the student does not seem to grasp the concept of slope as 'rate of change.' Describe how you would help this student understand the relationship between the slope of a line and the coordinates on the line.

Reliability of DTAMS. Three types of reliability, internal reliability, equivalency reliability, and inter-scorer reliability, were established (DTAMS, n.d.; Saderholm et al., 2010). Between May 1, 2005 and March 1, 2006, nearly 3,500 assessments were administered to 2,300 middle mathematics teachers in 38 projects across 17 states of the United States (DTAMS, n.d.). Some of the teachers completed a single assessment whereas others completed two or more in the form of pretest and posttest. Results of the assessments were used to determine reliability, and it was reported that the reliability score for each of these forms was greater than 0.8, which is considered acceptable for research purposes (DTAMS, n.d.; Saderholm et al., 2010).

Internal reliability was established computing Cronbach's alpha, which gave the value 0.87 (DTAMS, n.d.). Equivalency reliability was established by calculating Pearson product moment correlation coefficient for pairs of parallel assessments completed by the same teachers, whereas interscorer reliability was determined using intraclass correlation coefficients and percentage of agreements among three graduate students who developed and used the scoring guides for scoring open response questions to evaluate the teacher responses (DTAMS, n.d.; Saderholm et al., 2010). Correlation coefficient calculated to establish equivalency reliability and interscorer reliability was found significant at 0.01 level (DTAMS, n.d.).

Validity of DTAMS. Content validity of the assessment was established by aligning the assessment design to United States recommendations, objectives of standardized assessments, and research on misconceptions of students and teachers of middle school (DTAMS, n.d.; Saderholm et al., 2010). According to Saderholm et al. (2010) and the list published on DTAMS (n.d.), some of the documents that were reviewed included the following:

1. *Curriculum and Evaluation Standards for School Mathematics* published by National Council of Teachers of Mathematics in 1989.

2. *Principles and Standards for School Mathematics* published by National Council of Teachers of Mathematics in 2001.
3. *Benchmarks for Science Literacy* published by American Association for the Advancement of Science in 1993.
4. Objectives and items from the National Assessment of Educational Progress 1996 and 2000.
5. Research findings on middle school students' misconceptions and learning deficiencies about mathematics and research findings on middle school teachers' misconceptions about mathematics.
6. *Professional Standards for Teaching Mathematics* published by National Council of Teachers of Mathematics in 1991.
7. *Foundations for Success* published by Achieve, Inc. Mathematics Achievement Partnership in 2001.
8. *Trends in International Mathematics and Science Study* 1995, 2000 and 2003.
9. *Mathematical Education of Teachers* published by Conference Board of Mathematical Sciences in 2001.
10. *Model Standards in Mathematics for Beginning Teacher Licensing and Development: Middle School* published by Council of Chief State School Officers - The Interstate New Teacher Assessment and Support Consortium in 1995.

11. Objectives and the released items of the mathematics portions of Professional Assessments for Beginning Teachers/Educational Testing Service - Middle School Mathematics.

United States reviewers including mathematicians, mathematics educators, and middle school teachers assessed the appropriateness of items to establish the construct validity of the assessment (DTAMS, n.d.; Saderholm et al., 2010). It was reported that each item was reviewed by at least 36 different reviewers for construct validity of the assessment items (DTAMS, n.d.).

Content validity of DTAMS to use in the Maldives was established by checking alignment of the assessment tasks against the learning outcomes of the National Primary Mathematics Curriculum and the learning outcomes of the mathematics content and pedagogy courses offered to primary teachers at teacher training institutions in the Maldives. Construct validity of DTAMS was established by eight national reviewers including mathematics educators and mathematics lecturers. Five of the eight reviewers were employed at teacher training institutions in the Maldives at the time of this study, while the other three reviewers had worked in various positions at teacher training institutions, particularly in the mathematics department, before moving on to higher positions in academia. Two of the reviewers were initially trained as primary teachers and had taught sixth grade mathematics. These eight reviewers checked the appropriateness of assessment tasks for the teachers teaching sixth grade mathematics in the Maldives.

Qualitative Data

Qualitative data were collected through observations of sixth grade algebra lessons, interviews with sixth grade mathematics teachers, and analyses of teacher's algebra lesson plans and lesson notes. In order to develop an in-depth understanding of the algebraic content and pedagogical knowledge of the sixth grade mathematics teachers multiple forms of data need to be collected (Caruth, 2013; Creswell, 2012). For instance, Ball et al. conducted an extensive qualitative analysis of teaching practice to study the mathematical knowledge for teaching (Ball et al., 2008). Central to the qualitative approach used by Ball et al. was the analysis of videotaped and audiotaped classroom lessons, transcripts, copies of students' written classwork, homework, and quizzes, along with teacher's lesson plans, notes, and reflections (Ball et al., 2008). An observation checklist and related interview questions are in Appendix B and Appendix C, respectively.

The geographical nature of the Maldives required the researcher to travel to different islands for data collection. As a result, data were collected from one research site at a time. Data were collected in the following order:

1. Researcher observed an algebra lesson;
2. After the first observation of an algebra lesson an interview was conducted;
3. DTAMS was administered right after the interview;
4. More algebra lessons were observed and follow up interviews were conducted to clarify what was observed; and

5. Researcher made arrangements to conduct telephone interviews (if needed) before traveling to another island. Some of the participants were contacted over the telephone for clarification while analyzing the lesson plans and notes.

The main reason for using the above mentioned order in data collection was to avoid any bias that could arise if there was a lapse between interview and the administration of DTAMS. For example, if there was a lapse between interview and administering of DTAMS, a participant could actually study before taking the DTAMS assessment. Therefore the order was used in data collection to minimize such occurrences.

Setting and Sample

The Maldives

Maldives is one of the lowest lying nations in the world (United Nations Development Program [UNDP] & Ministry of Finance and Treasury, 2014). This small island nation well-known for its underwater beauty has a population of approximately 341,000 people spread over 188 inhabited islands (National Bureau of Statistics, 2015). According to the National Bureau of Statistics (2015), the majority of the islands have a population between 84-1,000 people. Only six islands have a population exceeding 4,000 people (National Bureau of Statistics, 2015).

According to the Maldives Human Development Report 2014, the disparity and unequal distribution of wealth and resources between the capital island and the rest of the islands is extremely high and ensuring equitable distribution of developmental gains among the islands has proved to be a challenge (UNDP & Ministry of Finance and

Treasury, 2014). It was reported that the gap between the highly populated islands and the rest of the islands is higher mainly in terms of education. For example, a person living in the capital island is likely to complete three years more of schooling compared to a person living in any other island (UNDP & Ministry of Finance and Treasury, 2014).

The education system of the Maldives has three stages: primary education (grades 1-7, age between 6 and 13), lower secondary education (grades 8-10, age between 13 and 15), and higher secondary education (grades 11-12, age between 15 and 17). There are 408 schools in the Maldives providing education for approximately a quarter of the entire population (UNDP & Ministry of Finance and Treasury, 2014). Twenty-seven of the schools in the Maldives are located in the capital island and cater to 30% of the nation's student population. Most of the teachers are trained teachers. According to the Maldives Human Development Report 2014 only 2% of the untrained teachers are employed in the capital island while the rest are employed in schools in the other islands and most of the untrained teachers are employed in primary schools (UNDP & Ministry of Finance and Treasury, 2014). Trained teachers are employed mostly in schools located in the capital island and in schools located in the very few highly populated islands.

The majority of the trained local teachers refuse to work in small islands due to the lack of resources and facilities (UNDP & Ministry of Finance and Treasury, 2014). The majority of the trained teachers working in small islands are expatriate teachers. For the purpose of this research all the 14 public schools with sixth grade classes were selected from the highly populated islands including the capital island in order to maintain a high level of homogeneity in terms of the teacher certification, and the facilities and resources available to the teachers.

Population and Sampling

As this study aimed to examine the algebraic content and pedagogical knowledge of sixth grade mathematics teachers in the Maldives, the sampling strategy used for this study was homogeneous, purposive sampling. In homogenous sampling the researcher purposefully selects individuals based on common characteristics (Creswell, 2012). Marshall et al. stated that the adequate sample size is directly related to the concept of saturation which was defined as researcher gathering data to the point that nothing new is being added (Marshall, Cardon, Poddar, & Fontenot, 2013). Guest, Bunce, and Johnson (2006) conducted a study to find out the degree of data saturation needed and made evidence-based suggestions for purposive sample sizes for interviews. They found that a sample size of six was sufficient for studies with a high level of homogeneity among the population (Guest et al., 2006). Creswell argued that a sample size of four was sufficient for a collective case study (Creswell, 2012). Baker and Edwards (2015) and Mason (2010) stated that the guiding principle should be the concept of saturation while Charmaz (2006) highlighted that some studies might achieve saturation faster than other studies depending on the aims of the study. Charmaz (2006) stated that “a small study with modest claims might allow proclaiming saturation early” (p.114). Guest et al. (2006) stated “saturation has, in fact, become the gold standard by which purposive sample sizes are determined” (p. 60). After reviewing 24 books on research methods and seven data bases Guest et al. (2006) concluded that although there are no published guidelines or tests of adequacy for estimating the sample size required to reach saturation, saturation is the key to excellent qualitative work. Therefore, it is evident that the adequate sample sizes in qualitative research is justified in terms of saturation (Baker & Edwards, 2015; Charmaz, 2006; Creswell, 2012; Guest et al., 2006; Marshall et al., 2013; Mason, 2010). Due to high level of homogeneity among the participants, saturation was achieved after three interviews, however, data were collected from all five subjects who consented to take part in the study.

As the purpose of this study was to understand the algebraic content and pedagogical knowledge of the sixth grade mathematics teachers in the Maldives, all sixth grade mathematics teachers are eligible to take part in the study. In order to select the participants, initially all sixth grade mathematics teachers employed in the selected 14 schools were contacted. The teachers were given the consent form that included a detailed description of the study. Of the 14 teachers who were approached only seven consented to participate in the study. The seven teachers who consented to take part in the study were provided with the data collection coordination request (Appendix D) and confidentiality agreement (Appendix E). During the interview, one of the participants withdrew while a second participant decided to drop out after the second observation. Therefore, five participants continued till the end of the study. As all Maldivian sixth grade mathematics teachers were trained from the same institution, and all the participants were recruited from public schools, a high level of homogeneity among the population was observed.

Eligibility criteria that were used in selecting and eliminating potential participants are summarized below:

1. Participant must be a sixth grade mathematics teacher trained from the Maldives National University.
2. Participant must be employed in one of the 14 public schools selected based on the population of the islands.

All five participants of this study are from five different schools. Four of the five participants are females. The number of years of experience teaching sixth grade algebra of these five participants were two years, seven years, 13 years, 18 years, and 20 years. Of the five participants who continued until the end of this study, two participants had Bachelor of Teaching Secondary (Major Mathematics); two participants had Advanced Diploma in Teaching Mathematics; and one participant had a Diploma in Teaching Secondary Dhivehi (native language of the Maldivians) as their highest educational qualification at the time of this study. Prior to obtaining their above-mentioned highest educational qualifications, all five participants completed a Certificate in Primary Teaching.

All five participants completed 15 hours of professional development each year, which was mandatory for public school teachers. It was noteworthy that none of the five participants attended any professional development sessions on mathematics. The reason participants cited was that they did not get any opportunity to attend a professional development session that was focused on mathematics as their schools organized no such sessions. Furthermore, they stated that the professional development sessions organized by their schools always focused on general areas such as classroom management, time management, behavior management, and use of PowerPoint presentations in teaching. All five participants stated that the professional development sessions did not contribute towards the enhancement of their algebraic content or pedagogical knowledge. During the discussions that took place on the margins of the interviews and lesson observations it became apparent that participants never thought of their own professional development other than attending the mandatory 15 hours that the schools ensured that they attended. Three of the five participants with highest number of years of experience in teaching sixth grade algebra never even considered the possibility that professional development could contribute towards enhancement of their algebraic content and pedagogical knowledge as they had been teaching the 'same stuff' throughout their 'whole life.'

Ethical Considerations

According to Demirdirek (2011) researchers have an ethical responsibility toward research participants in their research in order to ensure that there are no disruptions to the participants' life and also to ensure no harm may come to the participants due to taking part in the research subsequent to dissemination of research findings. Participants should be treated with respect and their feedback must be considered as part of the research process itself. Researchers must refrain from exploitation of the participants. During the data collection process an objective but friendly relationship was maintained with the participants. Informed consent was obtained prior to data collection and all participants were provided with a confidentiality agreement (Appendix E) stating that the data collected would be kept confidential and results that may identify any of the participants would not be reported. Upon completion of the research the findings were shared with the participants and as a thank you, two complimentary professional development sessions were conducted for them in the areas they identified.

In order to protect the participants, all data collected were confidential. Data were de-identified within 24 hours to minimize the risk of inappropriate disclosure of personal information. Data will be stored in electronic format for five years, which is the minimum required duration of Walden University (Walden University, 2014). After the minimum required duration the researcher will dispose of the data by shredding the documents and removing the electronically stored files from the database. The following precautions were taken not to disclose to anyone else any part of the data that was linkable to a participant's identity:

1. Pseudonyms were used instead of the real names of the participants.
2. Interviews were conducted at places the participants identified in order to ensure the participants felt comfortable.
3. During the interviews only the researcher and the participant were present.
4. Audio recording of the interview, interview transcripts and the field notes of observations of algebra lessons were stored in a locked cabinet where only the researcher had access.

Data Collection Strategies

Data were collected concurrently. According to Creswell (2012) a concurrent mixed methods design allows both quantitative and qualitative data to be collected simultaneously. Data collection strategies included observations of algebra lessons, interviews with sixth grade mathematics teachers, analysis of teacher’s algebra lesson plans and lesson notes, and administration of DTAMS.

Qualitative Sequence

Qualitative data were collected through analysis of algebra lesson plans and lesson notes, observations of algebra lessons, and sixth grade mathematics teacher interviews. Observations of algebra lessons provided the researcher with a better understanding as to how algebraic concepts were explained to the students at sixth grade which is influenced by the teachers’ algebraic content and pedagogical knowledge (Piccolo, 2008). As a result, this provided the researcher with information with regard to the algebraic content and pedagogical knowledge of sixth grade mathematics teachers in the Maldives.

Chick (2006) reported that pedagogical knowledge could be observed at many levels such as through teacher’s lesson planning and lesson notes, teacher’s explanations and interactions in the classroom, and through mathematics competency. In-depth, open-ended, semi-structured interviews with teachers were conducted after the observations of lessons. Sixth grade mathematics teacher interviews provided details on why a certain approach was used to explain algebraic concepts, thereby helping the researcher to gain an in-depth understanding of algebraic content and pedagogical knowledge of the five sixth grade mathematics teachers who took part in the study.

Creswell (2012) stated that open-ended questions not only allowed the participants to best voice their experiences without any perspectives of the researcher but also gave the opportunity for participants to share their perceptions without being forced into response possibilities. Interviews are fundamental to understanding the way people think about their world and how those definitions are formed (Bogdan & Biklen, 2007). Conducting in-depth, open-ended, semi-structured interviews with the sixth grade mathematics teachers helped to gain an insight into the teachers' perceptions of their own algebraic content and pedagogical knowledge.

Observations. Algebra lessons of all the sixth grade mathematics teachers who consented to take part in the study were observed. A lesson observation checklist was developed after a thorough analysis of teaching practicum observation forms used in teacher certification programs (L. Mohamed, personal communication, February 13, 2015; W. Aishath, personal communication, April 14, 2015). The observation checklist is in Appendix B. Content validity of the observation checklist was established by checking alignment of the items against the teaching practicum evaluation form used in the teacher training institutions in the Maldives. Construct validity of the checklist was established by five national reviewers who served as supervisors for primary teaching practicum students and were employed at teacher training institutions in the Maldives at the time of this study.

Document analysis. According to Patton (2002), documents and written materials provide rich information and evidence in qualitative studies. Algebra lesson plans and lesson notes of the sixth grade mathematics teachers who consented to take part in this study were collected and analyzed to gain an in-depth understanding of the algebraic content and pedagogical knowledge of the sixth grade mathematics teachers. According to Chick (2006), pedagogical knowledge of teachers could be assessed through teachers' lesson plans and lesson notes. The algebra lesson plans and notes were collected before the lesson observation so that the researcher could go through the lesson plans and notes before observing the lesson. The reason for collecting the algebra lesson plans and notes ahead of the lesson observation was to gain an insight into what the teacher expected the students to know before the lesson (students' prior knowledge) in addition to gaining insight into the algebraic content to be covered during the lesson.

Interviews. Patton (2002) stated that interviewing allowed the researcher to gain an understanding of people's perceptions as what people perceive about themselves or others is not observable. The purpose of interview was stated as "to find out what is in and on someone else's mind" (Patton, 2002, p. 341). The purpose of interviews in this study was to gain an in-depth understanding of the algebraic content and pedagogical knowledge of the teachers, their perceptions regarding their own algebraic content and pedagogical knowledge, and to clarify what was unclear from the lesson observations. All sixth grade mathematics teachers who consented to take part in the study were interviewed.

Interview questions were developed to gain an in-depth understanding of the sixth grade mathematics teachers' algebraic content and pedagogical knowledge. An advantage of having an interview guide is that it provides the interviewer with a clear set of instructions and hence can provide reliable and comparable qualitative data (Newby, 2010). Hancock and Algozzine (2006) emphasized the importance of having such an interview guide for case study research. Interview questions were developed keeping in mind the purpose of this study. Interview questions are in Appendix C.

Content validity of the interview questions was established by checking alignment of the questions against the learning outcomes of the mathematics content and pedagogy courses these teachers take during their teacher certification programs. Six national reviewers established construct validity of the interview questions. Five of these six reviewers not only supervised primary teaching practicum students but also lectured primary teaching students on mathematics content and pedagogy. They were employed at teacher training institutions in the Maldives at the time of this study. The remaining reviewer was a retired mathematics teacher educator who specialized in teaching middle school mathematics. This teacher educator supervised primary teaching practicum students for more than 15 years. Moreover, this teacher educator was a mathematics consultant during the initial stages of the development of the present primary teacher training curriculum in the Maldives.

Quantitative Sequence

The quantitative data collection instrument used in this study was DTAMS. This was used to establish the baseline algebraic content and pedagogical knowledge of the sixth grade mathematics teachers in the Maldives. At the end of the initial interview, each interviewee was asked to complete the DTAMS. The reason for administering DTAMS right after the interview was to avoid any bias that could arise if there was a lapse between interview and administering of DTAMS. For example, after the interview, participants could study algebraic content and learn about the pedagogy before they completed DTAMS.

Data Analysis

Qualitative data were collected through observations, interviews, and analysis of documents while quantitative data were collected using DTAMS. Qualitative data were used to gain an in-depth understanding of the algebraic content and pedagogical knowledge while quantitative data were used to establish the baseline algebraic content and the pedagogical knowledge of the sixth grade mathematics teachers. Figure 2 depicts how both the qualitative and quantitative data were simultaneously collected and analyzed, which resulted in collecting strengths of one form of data to offset the weakness of relying upon the other form of data (Creswell, 2012).

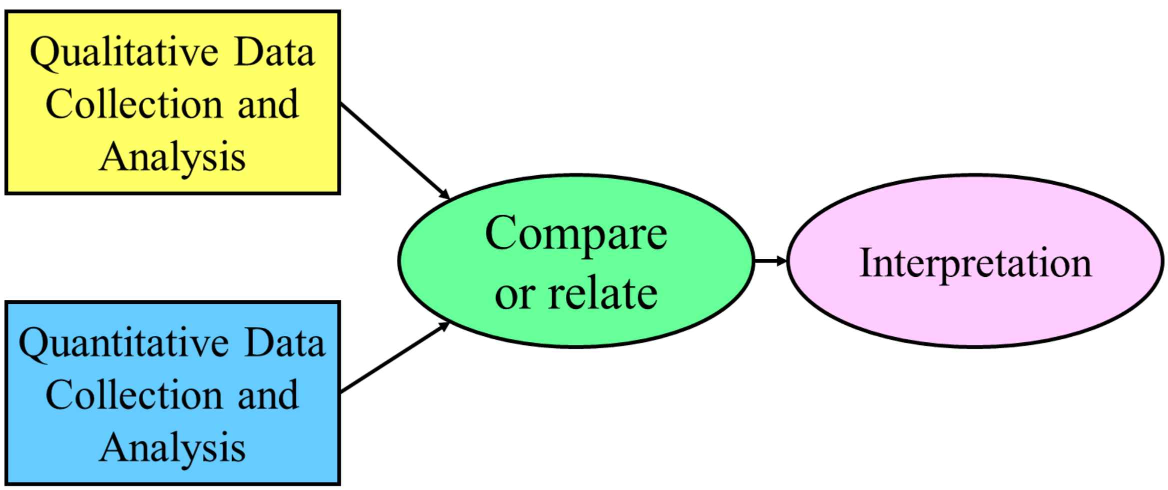


Figure 2. Data collection and analysis in concurrent mixed methods design (Creswell, 2012, p. 541).

Qualitative Data Analysis

Data collected through observations of sixth grade algebra lessons, interviews with sixth grade mathematics teachers, and teachers' algebra lesson plans and lesson notes were reviewed and analyzed for themes and patterns such as content knowledge, pedagogical knowledge, and any other sub-categories which might emerge from the data. According to Merriam (2009) all qualitative data analysis involves content analysis. Krippendorff (2003) stressed that content analysis is a scientific tool and defined content analysis as a "research technique for making replicable and valid inferences from texts (or other meaningful matter) to the contexts of their use" (p. 18). The process of content analysis involved coding of raw data and categorizing them according to characteristics of the content gathered.

According to Lockyer (2004) coding is an organized way to reduce broad data sets into smaller analyzable units through the creation of categories and concepts derived from the data. Creswell (2012) defined coding as the process of breaking down data and labelling text to form descriptions and broad themes in the data. The objective of coding is to make sense out of text data by identifying patterns and categorizing them which helps to narrow down the data to a few broader themes (Creswell, 2012; Lockyer, 2004; Merriam 2009). Data could be categorized according to variables, and numbers or even colors could be used to categorize these variables (Bourque, 2004; Creswell, 2012). Qualitative data were coded as they were collected and this facilitated in organizing, retrieving, and in interpreting the data which speeded up the process of analyzing qualitative data and in arriving at conclusions. Various colors were assigned to pre-defined codes and texts were analyzed for those codes. In addition open coding was used in case any other themes emerged. Pre-defined codes included, common content knowledge, horizon content knowledge, specialized content knowledge, knowledge of content and students, knowledge of content and teaching, knowledge of content and curriculum, teachers' perception of their own teaching and teachers' perception of their content and pedagogical knowledge. New codes that emerged from the data included fruit-salad algebra approach, assessment of learning, response to students' quarries, and areas for professional development. All these codes were categorized to three broader themes, namely, algebraic content knowledge, pedagogical knowledge, and the teachers' own perception with regard to their algebraic content and pedagogical knowledge, and their teaching.

Content analysis of data gathered to identify themes and patterns was the most appropriate method for this study as this study sought to examine the algebraic content and pedagogical knowledge of sixth grade mathematics teachers in the Maldives. In order to ensure strength of the data and the quality of the analysis, data collected were coded before the next trip of data collection as per the advice of Miles and Huberman (1994). This ongoing coding helped in surfacing incomplete data that could be clarified next time out. As Miles and Huberman stated "the ultimate power of field research lies in researcher's emerging map of what is happening and why" (p. 65). This early and continuing analysis not only helped to accomplish the goal of gaining an insight into what was happening and staying focused, but also accelerated the data analysis process.

Observations. A total of 80 lessons were observed at five different sites over a 10 week period. The lessons were categorized into nine subcategories. These subcategories are named after what the teachers told their students at the beginning of the lesson. The subcategories are:

1. Add and subtract algebraic terms

2. Multiply two algebraic terms
3. Remove brackets and solve algebraic expressions
4. Substitute given values and get numerical values for expressions
5. Solve simple linear equations of addition and subtraction
6. Solve simple linear equation of multiplication and division
7. Practice sessions
8. Revision sessions
9. Class test sessions

Two periods were allocated for each of the six subtopics, that is, one period for teaching, and the other period for practice. Upon completion of subtopic three, that is after period six, the seventh period was spent to revise the first three subtopics of algebra and a topic that was completed before algebra. This was followed by a class test that was conducted during the eighth period that was based on the topics revised the previous day. After that, the remaining three subtopics were covered and the fifteenth period was allocated to revise Algebra (that is, all the six subtopics) and a class test covering all six subtopics were conducted during the sixteenth period.

The six 'teaching periods' consisted of more teacher activity in terms of explanations and examples. Remaining periods were utilized for practice where students were provided a worksheet and were expected to do the problems. Towards the end of those lessons or last quarter of those lessons, the teacher gave either a printed sheet with answers or wrote the answers on the board followed by instructions for students to check their answers. At times students were called to the board and asked to work out the problems on the board.

As this study aimed to examine the algebraic content and pedagogical knowledge of sixth grade mathematics teachers in the Maldives, the focus of the observations were on the teacher activities such as the explanation of the lesson, materials that were used in the lesson, how teachers' dealt with the students who provided incorrect answers, and how the teachers differentiated instruction to meet the diverse needs of the students.

Add and subtract algebraic terms. Add and subtract algebraic terms was the introductory lesson where students were first exposed to the word 'algebra'. During the lesson students were taught how to simplify algebraic expressions that involved addition and subtraction only. After the introduction, the teacher demonstrated how to add and subtract algebraic terms and students were given a few questions to complete during the lesson. The answers to these questions were shared with the students at the end of the lesson. To conclude the lesson, some teachers asked selected students to work out those questions on the board, whereas, other teachers simply recalled the facts and stated that more practice will be done in the following lesson. It was noteworthy that one of the teachers gave the students the worksheet that they would be doing on the following day and instructed the students to paste the worksheet on their notebook and come to the class the following day (during the post-lesson discussion which took place before the interview, it became clear that teachers were instructed to complete all the work within the class time and encouraged not to give homework to the students at that level).

Algebraic content knowledge. All the five teachers introduced algebra using what Tennant and Colloff (2014) referred to as 'fruit salad' approach to algebra. Two of the five teachers actually used fruits while one used fruits and vegetables, one used stationery (that is, books and pens), and the remaining teacher used front and the back of the teacher's hands to demonstrate like and unlike terms. This clearly indicated that the teachers lacked the common content knowledge as they were unaware that the "letters" or "variables" used in algebra represented numbers, not objects. Ball et al. (2008) placed common content knowledge as a subdivision of subject matter knowledge (content knowledge) and defined it as knowledge of mathematics that is common to everyone who knew and used mathematics. In summary, the introduction of the lesson was vague and that led students to raise serious concerns such as "why can't we write $3a+2b=5ab$" to which the teacher simply said that "you cannot write because they are unlike terms" [in this particular case, the teacher first showed three apples and asked the students how many apples they see. This was followed by a picture of two bananas. Then, the teacher asked how many did the teacher have altogether, to which students responded five. Then teacher said that "*a*" means *apples* and "*b*" means *bananas* and wrote *3a* and *2b* to represent three apples and two bananas, respectively. Then the teacher put the two pictures together and wrote $3a+2b$ and said that since apples and bananas are different that is how it should be written.]. Four of the five teachers clearly communicated the objective of the lesson. The teachers seemed confident in teaching the lesson although the materials used were inappropriate and they lacked algebraic content knowledge.

Pedagogical knowledge. None of the teachers related the lesson to real-life (however, during the interviews it became clear that teachers believed by using stationery or using fruits and vegetables they had related the lesson to real-life). Also, it was observed that none of the teachers identified or even mentioned common errors or potential misconceptions students might have (in fact, teachers ignored the incorrect answers given by students and hurried to write the correct answers given by students on the board). It was also observed that none of the teachers used a different approach to explain to the students who did not understand the lesson. Either the same example was repeatedly explained or (as in most cases) another student was asked to help the student who did not understand the lesson. It was observed that teachers were unable to differentiate instruction to cater the needs of the students. Differentiated instruction had shown improvements in students' performance (Rittle-Johnson, Matthews, Taylor, & McEldoon, 2011). Teachers gave questions to try as to keep the students engaged and it was observed that teachers encouraged individual students to answer by calling out their names (later on it became clear that teachers invited the same crowd to answer questions and students who had questions were always referred to these students). In short, it became clear that teachers lacked pedagogical knowledge required to teach algebra. It is noteworthy that teachers' lack of algebraic content and pedagogical knowledge could lead students to develop misconceptions with regard to the concept of variables, making it difficult for students to understand problems involving algebraic expressions or equations (Ball et al., 2001; Koency & Swanson, 2000; Tajudin, 2014).

Multiplying two algebraic terms. All participants introduced the lesson by communicating to the students that they were going to learn how to multiply two algebraic terms. Noticeably, in all the questions and the worksheets, the two terms consisted of numbers and letters. Each of the two terms had a number and one or more letters each of which were different. For example, $3r \times 5s$ or $-5xy \times 3z$. During the post lesson discussion it became clear that the teachers had to use different letters as "rules of indices are taught in seventh grade." Also, participants were asked why they used only integers and not fractions, the typical answer was "it is not in the syllabus" while one of the participants said that there were no such examples in the textbook and hence the participant did not think of it.

Algebraic content knowledge. It was unclear from the teachers' explanations that teachers had sufficient knowledge to teach algebra at sixth grade. However, it became apparent that teachers were procedurally fluent, that is, the teachers were able to show how to arrive at the right answer without even knowing why the students were doing what they were doing. The explanations provided by the teachers (all five teachers) were "multiply the numbers and write the letters next to the product in alphabetical order". One of the students called the teacher and asked whether it would be incorrect if the letters were not in alphabetical order, the teacher then went to the student and guided the student. During the post-lesson discussion I inquired about it and the teacher said that student was told to "order the letters and write the final answer." I then asked if the teacher would accept the students' answer if the letters are not in order, to which the teacher hesitated and said, in the examples [textbook] the letters were always in order.

Two of the teachers used a PowerPoint presentation whereas the other three teachers used a whiteboard to explain the lesson to the class. Simple examples where both of the numbers were natural numbers, and complex examples [in teachers' terms] where directed numbers (integers) were used were discussed.

Pedagogical knowledge. Three of the five teachers recalled the multiplication rule of directed numbers right before demonstrating how to multiply two algebraic terms. The other two teachers straightaway gave examples of how to multiply two algebraic terms. It was observed that none of the teachers promoted conceptual understanding, critical-thinking, and creativity. Students were passive recipients of knowledge. The students were taught how to arrive at the answer. They were not offered an explanation as to why they were doing what they were doing (that is, multiplying the numbers and writing the letters next to the numbers). Interestingly, all the students were able to do the problems and except for one student who asked whether the student could write the letters in any order, no other questions were raised. The only mistake was that some students obtained incorrect answers when multiplying the numbers, to which the teacher recommended to revisit the multiplication table. It was obvious from the observation that teachers focused on explaining the examples presented in the textbook and they were using the textbook as the main resource (and their lesson plans listed textbook, workbook, and the teachers' guide as resources). Through the lesson observations it became clear that the teachers not only lacked pedagogical knowledge required to teach mathematics at sixth grade, but also lacked deep understanding of mathematical concepts. Hence, they were unable to teach to develop students' conceptual understanding (Ma, 1999; Stoddart, Connell, Stofflett, & Peck, 1993).

Remove bracket and solve algebraic expressions. Students were told that they were going to learn how to remove bracket and solve [the correct term should be simplifying because in mathematics, when you solve an expression, you find the value of the variable] algebraic expressions. Questions that required removing brackets involved one term outside the bracket and two terms inside the bracket. Moreover, the expressions were formed in a way that all the variables that needed multiplication were different (that is, the highest power of the variables is one). It was also observed that too much time was allocated for one question. For example, students were given three minutes to "solve the bracket" [expand] $5(a+b)$, and four minutes to expand $-p(-2q-3r)$. Students rarely asked questions and they were able to obtain the right answer most of the time. It was also noticed that teachers addressed individual questions to the same selected group of students – who always gave the correct answer. Teachers concluded this class by informing the students about the upcoming class test. One of the observations was that students were very quiet during the lesson. Two of the teachers said that it could be because they have a 'stranger' in the class. The other three teachers cited having no repeaters (that is, students who are studying sixth grade for a second time) in the class as the reason why the students were always quiet. It was also observed that the participants were having less and less time to have post-lesson discussions. At this point, the only time for post-lesson discussion was three to five minutes that I got while they walked to their work station. One of the participants called the night before my visit and asked how the participant could deliver the lesson. However, as this study aimed to examine algebraic content and pedagogical knowledge of the sixth grade mathematics teachers, I had to decline as it could bias the findings of this study.

Algebraic content knowledge. As all the participants directly instructed the students to multiply each of the terms inside the bracket by the term outside the bracket, which was followed by five to eight examples, it was difficult to conclude whether the teachers had the knowledge required to teach algebra at sixth grade. It was obvious that they were able to solve the problems without making any mistakes [procedural fluency]. Two of the teachers used a PowerPoint presentation and the other participants used a whiteboard to show how to expand algebraic expressions.

Pedagogical knowledge. None of the teachers related the topic to real-life. Moreover, it became apparent through observations that the teachers lacked pedagogical knowledge required to teach algebra at sixth grade. Teachers did not use various teaching strategies, nor did they promote conceptual understanding, critical thinking or creativity. Teachers gave various questions that helped students to master the procedure rather than understand what they were doing and why they were doing it that way. In one of the classes, it was also observed that when students gave incorrect answers, instead of checking where the student went wrong, the teacher asked one of the students who obtained the correct answer to help the other students. The student who obtained the correct answer simply passed the notebook to the other student who copied the correct answer. It became clear that teacher avoided incorrect answers instead of identifying and addressing the potential error or misconception students might have. In another class, it was observed that teacher ignored to acknowledge incorrect answers given by students and when a student shouts the correct answer this particular teacher hurried to the whiteboard and wrote the correct answer on the board.

Substitute given values and get numerical value for expressions. Students were informed that they were going to learn how to find the numerical value for a given expression. Values for variables were given and students were asked to find the value of 15 algebraic terms. Typical questions were as follows:

1. Given $a = 3$ and $b = 5$, find the values of the following: $7ab$, $-9ab$
2. Given $x = 2$ and $y = -7$, find the values of the following: $12xy$, $-xy$

First question above is a ‘simple’ question while the second question above was considered a ‘complex’ question. None of the examples that were discussed nor the exercise questions involved any fractions, only integers were used. As always, the teacher explained how to do the problems and the role of students included listening and doing the exercises. Two of the teachers who used PowerPoint presentations gave the answers to the questions at the end of the lesson, whereas the remaining teachers called individual students to come to the whiteboard and solve the questions on the board. These teachers instructed students to check their answers and make corrections if needed.

Algebraic content knowledge. Students were told what they were going to learn, followed by teachers demonstrations of how to substitute given values into an algebraic expression and obtain a numerical value of the expression. This clearly indicated that the teachers lacked conceptual understanding of the concepts which is essential to teaching for understanding (Ma, 1999; Ojose, 2014; Strand & Mills, 2014; Stoddart et al., 1993).

Two of the teachers used PowerPoint presentations while the other three used a whiteboard. Teachers had sufficient knowledge to demonstrate how to do the calculations and they were confident in showing examples. Through the observation it was clear that teachers were able to solve the problems correctly, however, it was difficult to conclude anything about their algebraic content knowledge. Tajudin (2014) reported that teachers who lacked content knowledge focused on procedures. Therefore, it could be possible that these teachers lacked algebraic content knowledge.

Pedagogical knowledge. It was observed that teachers did not mention anything about real-life applications of the concept, and it became clear that teachers were using one teaching strategy, that is, explain through the use of examples how to get the correct answer. Students' roles were to follow the steps in solving problems. It was clear that teachers promoted procedural fluency over conceptual understanding, critical-thinking, and creativity. From the teaching approach, it became clear that teachers lacked pedagogical knowledge. Teachers offered "skeletal explanations loaded with routines" (Ojose, 2014, p. 41). Moreover, it became apparent that examples discussed and exercise questions were the type of questions that will be asked in the examinations. Hence, making it clear that teachers were preparing students to obtain a good score [or at least make them all pass] in the examinations.

Solve simple linear equations of addition and subtraction. Teachers told the students that they were going to learn how to solve equations. All the teachers used PowerPoint presentations for the lesson. Interestingly, all the teachers presented the same example to begin the lesson. That is, students were told that equations were made up of two quantities with an equal sign and went on giving examples of equations. The two examples that all the teachers used were $2+3=5$ is an arithmetic equation and $x+5=9$ is an algebraic equation. Later on it became clear from the example given that all the teachers used the examples that were in the textbook, hence, the same examples. Teachers used the second example to demonstrate how to solve for x. Instructions of the teachers were as follows:

1. First, bring all the x terms of one side and the numbers to the other side.
2. When you take a number from one side to the other, its sign changes to opposite. Teachers emphasized the point by saying "here, when you move five to the other side it will be $x=9-5$ and then wrote the final answer $x=4$.

After the class, teachers were asked why students were told "when you take a number from one side to the other" to which four of the teachers said that, that was how they had been teaching and also how they were taught when they were studying. The remaining teacher told that "it is not moving the numbers, but if I tell them that five is subtracted then the students will get confused. Now they will be able to do the questions [students will be able to obtain the correct answer]." Four teachers out of five explained questions involving four terms at most. An example of such questions would be $7x+10=2x+75$. Only one of the teachers used the following example to summarize the lesson: $5x+4-3x-2=x+5$. The teacher concluded the lesson by asking what the answer was and some of the students told the teacher $x=0$ whereas the others told it was $x=4$. The teacher asked if any of the students obtained a three, and it was observed that none of the students obtained a three as the value of x. It was time for the next period so the teacher concluded the lesson saying that the teacher would explain the question on the following day. The following day, the teacher showed how to solve the problem as follows:

1. Teacher wrote the question on the board
2. Instructed the class to bring all x terms to one side and all the numbers to the other side giving $5x-3x-x=5-4+2$. At this point, a lot of students asked why $-3x$ did not become $+3x$. The teacher said that the number [term] was not moved and said “okay, I will explain in another way” and erased the board.
3. Alternative explanation offered was to “rearrange the terms and simplify the sides before moving the terms” and wrote: $5x-3x+4-2=x+5$. Again, students asked why $-3x$ did not become $+3x$ to which the teacher responded that “we did not move it [the term] to the other side of the equal [sign].”
4. Teacher wrote the answer and asked the students to copy and not to worry about the question as it would not be asked in the exam. Teacher also mentioned that the students needed to complete the worksheet and since the teacher had limited time teacher would solve all the questions on the board and students could copy the answers.

The teacher gave the students some time to copy and stood at one side of the class. I signaled to the teacher and requested from him if I could check the answers students wrote the previous day at the end of the class. At the end of the class the teacher collected the notebooks of four students and handed them over to me. There were two different answers [probably the students worked in pairs as they were sitting together]. The two solutions are presented below:

Solution 1:	Solution 2:
$5x+4-3x-2=x+5$	$5x+4-3x-2=x+5$
$5x+3x-2-4=x+5$	$8x-2=x+5$
$8x-6=x+5$	$8x-x=5+2$
$8x-x=5+6$	$7x=7$
$7x=11$	$x=0$
$x=11-7$	
$x=4$	

The teacher was asked what the teacher thought about the students' answers and what the opinion of the teacher was regarding the mistake, to which the teacher responded that the students had forgotten the rule ["when you take a number from one side to the other, its sign changes to opposite"] and with more practice they would be able to remember. The teacher also stated that "anyway, this is not in the syllabus. I got the students confused right? I will stick to what is in the textbook from now on." It was also observed that none of the students volunteered to do the questions on the board. As a result, the teachers solved the problems on the board and asked the students to copy the answers as they did not have enough time to go through the questions. However, teachers mentioned that the teachers would clarify students' doubts the following day.

Algebraic content knowledge. The teachers started by stating the lesson objective in terms of what the students were going to learn and proceeded to give examples. The introduction was limited to showing an example of an arithmetic equation and an algebraic equation. Teachers then proceeded to show examples of how equations involving at most four terms (two terms on each side) were solved. Observations revealed that all five teachers lacked algebraic content knowledge required to teach successfully at sixth grade. This was further proven by the explanation that was given to the students regarding what happened to the sign of a term when "moving terms from one side to the other."

Pedagogical knowledge. It became clear from the teachers' explanations that they did not possess the pedagogical knowledge required to teach algebra at the sixth grade level. Teachers not being able to offer a sensible explanation to students' questions emphasized the fact that the teachers lacked pedagogical knowledge. Teachers did not use various teaching strategies, in fact, when students asked questions, teachers repeated the example saying "this is how it is done", which clearly indicated that teachers lacked pedagogical knowledge. Teachers did not relate the topic to real-life, nor did the teachers identify and address the potential errors and misconceptions students must have. It was also observed that this lesson was a lesson where teachers could have identified students' errors and misconceptions and addressed these. However, instead of giving students the opportunity to make mistakes, teachers hurried to give the correct answers to the students. This practice limited the opportunity for students to learn the concepts. Teachers did not promote conceptual understanding, critical thinking and creativity. For instance, the teacher who gave students a question with more than four terms (discussed above) could have used the students' incorrect answers to teach about the misconceptions and take corrective steps to remedy those. Students were merely passive recipients of knowledge. Numerous research studies have indicated that teachers who had a deep understanding of the content and the pedagogy of the concepts they taught had better instructional practices (Ball et al., 2001; Brown et al., 2011; Darling-Hammond & Sykes, 2003; Ojose, 2014; Shirvani, 2015). Therefore, it could be concluded from the observations that the teachers not only lacked algebraic content knowledge but also pedagogical knowledge.

Solve simple linear equations of multiplication and division. Solve simple linear equations of multiplication and division was the last of the six subtopics under algebra that was covered in grade six. It was observed that all five teachers used the same examples which were $2a = 12$, $9y = -72$, and $\frac{x}{5} = 10.$ Teachers demonstrated how to solve the three equations and instructed students to follow those examples in doing the worksheet. Teachers emphasized that if there was ‘no sign between a number and the letter then it is multiplication’ and also advised the students to ‘remember that division is the opposite of multiplication.’ After the demonstration students were asked to do the worksheet. Teachers checked how individual students were doing the problems and assisted the students. Four of the five teachers explained the questions on the board and students were instructed to check their solutions and make corrections. Once the lesson was over I was able to talk to two of the teachers and they said that students were making the same mistakes and that was the reason they explained all the questions on the board. One of these teachers reported that the students ‘subtracted when they had to divide’ whereas the other teacher explained that the students were making 'silly mistakes because they did not practice the previous day's work.' This teacher did not see the need to explain the concept again to ensure students understood it, rather placed the blame on the students. Interestingly, teachers' beliefs are prominent in shaping mathematics teachers' decisions around instructions (Nathan & Koedinger, 2010). Furthermore, teachers' with limited content knowledge tended to assume students understood the concepts, which limited what students were exposed to in terms of mathematical content they were taught (Ojose, 2014; Strand & Mills, 2014). The remaining teacher called individual students and asked to do the worksheet questions on the board. Typical misconceptions observed included students writing $3x = -15 \Rightarrow x\frac{-15}{-3}$, and $7x = 21 \Rightarrow x = 21 - 7$. It was also observed that students were able to do the "cross multiplication" correctly when fractions were involved. However, they made mistakes in obtaining solutions. From two classes it was observed that the mistakes or misconceptions students had were almost the same. This could be the result of distorted explanations such as "sign changes when you move a number from one side to the other". Naseer (2015) and Schnepper and McCoy (2013) stated that the root cause of misconceptions were the result of incomplete or distorted definitions.

Algebraic content knowledge. Teachers started the lesson by communicating the objective of the lesson. The only material used was the textbook. It was clear from the explanations that teachers did not have sufficient knowledge to teach algebra at sixth grade. For instance, all five teachers said that 'when you move the number from one side to the other its sign change to opposite.’ Teachers who lacked content knowledge focused on computational procedures (Ball et al., 2001; Koency & Swanson, 2000; Tajudin, 2014). A person with sufficient knowledge would have stated that they are ‘not moving’ numbers but treating both sides equally either by dividing both sides by the same number or multiplying both sides by the same number to ensure the numbers statement (equation) is true.

Pedagogical knowledge. From the observation as well as the post-lesson discussion it became clear that teachers lacked pedagogical knowledge. First of all, instead of identifying what caused the mistake, teachers worked out all the questions on the board. Second, some teachers believed that the reason for the students’ mistakes was lack of practice. None of the teachers used different teaching strategies or promoted conceptual understating or creativity. Instead, teachers focused on computational procedures. Teachers did not identify or address potential errors and misconceptions. In fact, they failed to detect students’ misconceptions raising serious questions about teachers having sufficient algebraic content and pedagogical knowledge to teach sixth grade algebra. Identifying students’ misconceptions and correcting those is essential to advancing students’ conceptual understandings (Russell, O’Dwyer, & Miranda, 2009).

From the observations of algebra lessons, it became clear that teachers not only lacked algebraic content knowledge but also pedagogical knowledge. For example, all five teachers were unable to distinguish abbreviations from variables. They all used the “fruit-salad” approach to introduce algebra, indicating that all five teachers treated the variables as objects. Moreover, when a student asked questions, teachers were unable to differentiate instruction to cater the individual needs of students. Teachers repeated the same explanation and each time the tone became harsher and the voice became louder. Another significant observation was that teachers' main focus was on the computational procedures and their explanations and examples were limited to the prescribed textbook and workbook. In addition, it was also observed that teachers did not ask any questions that required students to use their problem-solving skills, critical-thinking, or creativity. Teachers were unable to identify students' misconceptions. Table 2 presents the summary analysis of lesson observations.

Table 2

Summary Analysis of Lesson Observations

Algebraic Content Knowledge	Pedagogical Knowledge
✓ Used objects to represent variables. ✓ Offered incomplete or distorted explanations. ✓ Ignored incorrect answers given by students. ✓ Failed to detect errors and misconceptions. ✓ Focused on procedures and exam type questions. ✓ Examples and exercise questions were limited to the prescribed textbook and workbook.	✓ No real-life applications. ✓ Unable to differentiate instruction. ✓ Students were blamed for incorrect answers. ✓ No questions required critical thinking or problem-solving skills. ✓ If majority of the students were found making mistakes, teachers gave solutions to the problems on the board. ✓ Questions were addressed to those students who were likely to give the correct answers.
Conclusion: All five teachers lacked algebraic content knowledge	*Conclusion: All five teachers lacked pedagogical knowledge*

Document analysis. Documents and written materials such as lesson plans and notes can provide rich information and evidence in qualitative studies. Pedagogical knowledge of teachers could be observed through teachers' lesson plans and lesson notes (Chick, 2006; Patton, 2002). Algebra lesson plans and lesson notes of the sixth grade mathematics teachers who consented to take part in this study were analyzed to gain an in-depth understanding of the algebraic content and pedagogical knowledge of the sixth grade mathematics teachers.

***Lesson plans*.** Only one of the five teachers had a lesson plan. Interestingly, it was the teacher who had two years teaching experience and that lesson plan was prepared when she taught mathematics for the first time which was two years ago. The lesson plan had three sections, namely, lesson starter, main activity, and plenary. Each of these sections had two parts, that is, teacher activity and student activity. The lesson plan was a one page document and it lacked details.

Lesson starter. The teacher activity included greeting the students and introducing the lesson. Student activity was ‘greet back’ [teachers enter the class and say “good morning/ afternoon students” and the students are expected to say “good morning/ afternoon teacher”]. That was the only role students were expected to play in the “lesson starter” activity.

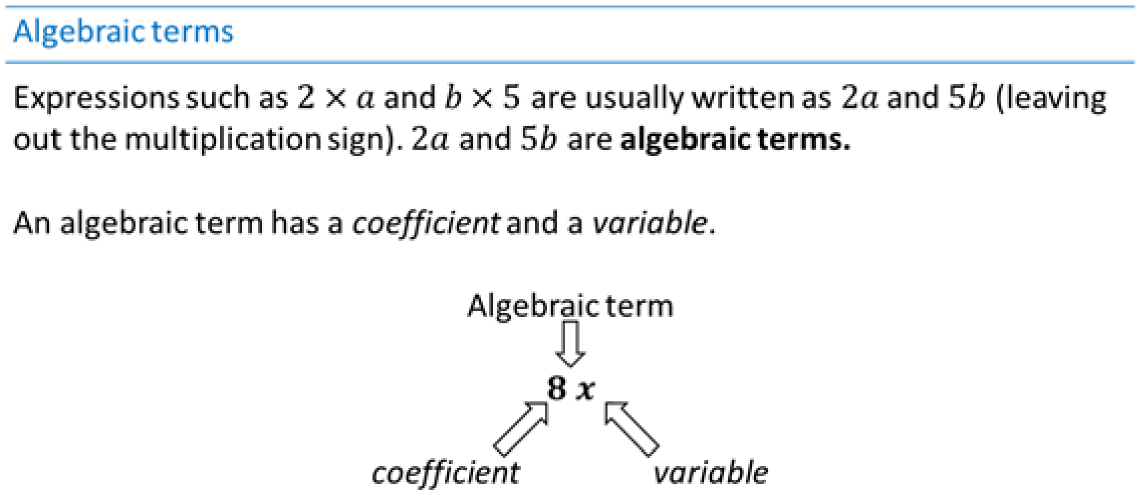
Main activity. The teacher activity was to explain examples on a certain page in the textbook. The student activity was to listen, clarify the doubts, doing the exercises on a given page, and give the students chance to present their answers. No other information was included.

Plenary. The teacher activity in this section read “summarize the lesson and check students’ work” whereas student activity read “let them response to the teacher.”

Four of the five teachers did not have a lesson plan. Some of them cited length of their teaching experience as the reason for not having a lesson plan while others said that they used the textbook so there was no need to have one.

Lesson notes**.** Teachers used Mathematics for Maldivian Schools Six (Adam & Naseer, 2002), Mathematics for Maldivian Schools Six Workbook (Adam & Naseer, 2002), Mathematics for Maldivian Schools Six Teacher's Resource Book (Adam & Naseer, 2003), and Mathematics in the National Curriculum Key Stage 2 (National Institute of Education (NIE), 2014) as lesson notes.

Mathematics for Maldivian Schools Six. The main textbook teachers used in their lessons was Mathematics for Maldivian Schools Six. This was the prescribed textbook for grade six mathematics. All five teachers used this book as their main resource in classroom instruction. This book offered little explanation of the topics and focused more on worked examples promoting procedural fluency over conceptual understanding. The explanation provided in the textbook did not include what the topic was about or why a certain approach was used. Instead, the book focused on explaining the steps that would guide students to obtain the correct answer to problems of similar kind. Moreover, the explanation offered with each step of the example showed how the steps were carried out instead of explaining or justifying the approach used in solving the problem. Furthermore, real-life applications of the topic were not included in the text. This could be the reason why teachers did not relate the topics to real-life or the misconception teachers had with regard to relating the topics to their real-life applications. For example, teachers thought using fruits or stationery to objectify variables was a real-life application of the topic. Figure 3 shows an extract from the book showing how the topic "algebraic terms" is explained in the book and Figure 4 shows how examples are presented in the book (Adam & Naseer, 2002).

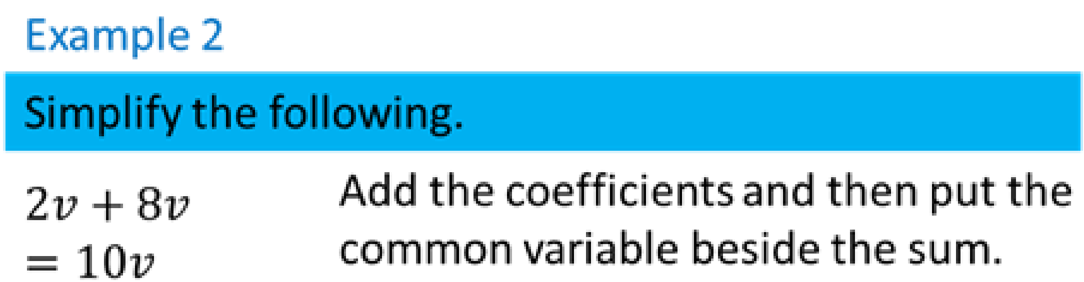


Algebraic terms

Expressions such as $2 \times a$ and $b \times 5$ are usually written as $2a$ and $5b$ (leaving out the multiplication sign). $2a$ and $5b$ are **algebraic terms.**

An algebraic term has a *coefficient* and a *variable*.

Figure 3. Algebraic terms (Adam & Naseer, 2002, p. 55).



Example 2

Simplify the following.

$2v + 8v$ $= 10v$	Add the coefficients and then put the common variable beside the sum.

Figure 4. Example of addition of like terms (Adam & Naseer, 2002, p. 56).

It was also observed that the explanations provided for similar kind of questions were not consistent. Figure 5 shows an extract from the book showing how the explanations for subtraction of like terms differed in two examples provided in the text. It is not only important to maintain consistency in explanations to ensure students develop a firm understating of the procedures but also offer complete explanations and justify why a certain approach was used and why it works all the time. For example, explanation would have been clear-cut, complete and consistent if it were mentioned that subtracting the coefficients is the same as adding the coefficients using the directed numbers addition rule. If the students are not aware that subtracting the coefficients is the same as adding the coefficients using the directed numbers addition rule, students might wonder why they were "subtracting" in one question and applying "the directed numbers addition rule" in another question. Students had raised these kinds of concerns time and again and teachers had dismissed those kinds of questions as "silly" questions. Incomplete or inconsistent explanations followed by dismissal of students concerns could lead to formation of misconceptions. As Naseer (2015) and Schnepper and McCoy (2013) stated, incomplete explanations and distorted definitions were the root cause of misconceptions.

Example 3

Simplify the following.

$13w - 9w$
$= 4w$

Subtract the coefficients and then put the common variable beside the difference.

Example 6

Simplify the following.

$10c - 2c$
$= 8v$

- Add the coefficients using the directed numbers addition rule.
- Put the common variable beside the coefficient.

Figure 5. Different explanations for subtraction of like terms (Adam & Naseer, 2002, p. 56-57).

Analysis of lesson notes revealed that teachers followed the book in explaining the concepts to students. For example, teachers would state what the students were going to learn. After that the teacher would briefly explain the topic in two or three statements. This would be followed by various worked examples. Students rarely asked questions, and when they asked questions (if at all) they were told to ‘listen carefully’ or ‘look at the example and follow the steps’ and the teacher would repeat the same explanation all over again. Figure 6 shows how the topic ‘removing brackets’ was explained in the text (which is exactly how all five teachers explained the topic).

Removing brackets

Brackets are used to group terms together. An expression with brackets in it can be replaced by an equivalent one without brackets. This is called **removing brackets**.

To remove brackets:
Multiply the term outside the brackets by each of the terms inside the brackets.

Example 1

Write the following without brackets.

$2(a+3)$
$= 2a+6$

Multiply the term outside the brackets by each of the terms inside the brackets.

Figure 6. Removing brackets explained in the book (Adam & Naseer, 2002, p. 60).

In summary, this textbook focused on explaining the procedures that would lead students to obtain the correct solutions to the problems given (which were of similar kind or pattern). The examples, explanations, and given questions did not promote conceptual understanding, critical thinking or even creativity. Lack of higher-order questions that required analytical skills and critical thinking made it difficult to identify whether students fully understood what was taught. Moreover, it hindered the opportunity to bring out misconceptions students might have.

Mathematics for Maldivian Schools Six Workbook. Mathematics for Maldivian Schools Six Workbook contained questions for students to do which were very similar to the examples given in the textbook. For example, a question given in this book asking students to write without brackets is $2(x+4)$ which is very similar to the given example (Adam & Naseer, 2002, p. 57). In short, questions given in this book were very similar to the examples given in the book Mathematics for Maldivian Schools Six. Precisely, the difference between the examples in the Mathematics for Maldivian Schools Six and the exercise questions in the Mathematics for Maldivian Schools Six Workbook could be the use of a different a number or use of a different variable. For example, if how to expand $2(a+3)$ is explained in the Mathematics for Maldivian Schools Six, then in the corresponding exercise in the Mathematics for Maldivian Schools Six Workbook would include questions such as $4(a+3)$ and $2(w+3)$ for students to practice. The difference between the example $2(a+3)$ and the problem $4(a+3)$ is that the number 2 had been replaced with a 4 while, in the difference between $2(a+3)$ and $2(w+3)$ is that the letter a had been replaced with the letter w. There were no questions that required students to put to use their critical thinking ability, analytical skills, or problem-solving strategies. This book was used by all five teachers in all the lessons to assign classwork for students.

Moreover, the worksheets students were given contained questions from the exercises given in the book.

Mathematics for Maldivian Schools Six Teacher's Resource Book. Only three of the five teachers used this book. Interestingly those who used this book were the teachers who had the longest teaching experience. The remaining two teachers were not even aware of this textbook. Interestingly, the schools to which these two teachers belonged did not even own a copy of this book. This book contained the objectives of the topic (which appeared only in this book); answers (only answers) to the questions given in the book Mathematics for Maldivian Schools Six Workbook; some additional questions of similar kind to the questions given in the books Mathematics for Maldivian Schools Six and Mathematics for Maldivian Schools Six Workbook; and a unit test at the end of each topic. Although this book is called 'Teacher's Resource Book' it did not include anything on mathematics teaching pedagogy or various teaching strategies that could be used to teach the topics.

Mathematics in the National Curriculum Key Stage 2. Only one of the five teachers used Mathematics in the National Curriculum Key Stage 2. The one who used this book as a resource cited "to learn different ways of explaining since I do not have that much experience." This book was mentioned to the other four just to check whether they were aware of the existence of the book. All four of them said that they were aware as they attended a workshop on new curriculum and cited "that book is for new curriculum [which was to be implemented in 2016]" as the reason for not using this book as a resource.

Analysis of Mathematics in the National Curriculum Key Stage 2 revealed that this book was very different from the above mentioned books. Unlike the above mentioned three books, this book was developed by a team of three curriculum developers from two institutions and eight mathematics teachers from eight different schools in the Maldives. This book included learning outcomes, indicators that showed the learning outcomes had been achieved in addition to teaching strategies.

In-depth analysis of the topic Algebra showed that some of the explanations, suggested teaching strategies, and indicators were incorrect. This raised questions as to whether the team members were equipped with algebraic content and pedagogical knowledge required to develop such a book or teach algebra at sixth grade (or any grade for that matter). Figure 7 shows such an example taken from this book (NIE, 2014).

Topic: Algebraic Techniques

Outcome PA2.1:	**Indicators:**
Model algebraic expressions and carryout addition and subtraction.	***This is evident when the student:***
	a. Models algebraic expressions using objects. Eg: %%%%= 4% !!!!!!!!= 8! b. Uses objects to add and subtract simple algebraic expressions. Eg: ff + fff = 2f + 3f = 5f

Figure 7. An incorrect indicator of learning (NIE, 2014, p. 78).

The example given in part (a) under indicators is what Tennant and Colloff (2014) referred to as “fruit salad” algebra approach. This approach is one of the main factors contributing to the formation of misconceptions. Kuchemann (as cited in Welder, 2012) reported that only a few students between ages 11-13 were able to identify letters as a generalized number. The majority of the students identified variables as objects which is incorrect (Tennant & Colloff, 2014; Welder, 2012). For example, in part (a) it was also stated $!!!!!!!! = 8!$ which is incorrect as $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40{,}320$ in mathematics. The example in the first part of (b) read $ff + fff = 2f + 3f = 5f$ which is mathematically incorrect as $ff = f^2$ and $fff = f^3$ in algebra (in the Maldives students will come across multiplication of like terms in grade seven). It is worth mentioning that during the lesson observation it was noticed that when it came to multiplication of algebraic terms all five teachers said "multiply the numbers and write the letters next to the number" which is how it was explained in the textbooks used.

Unfortunately, these misconceptions go undetected at this level due the way mathematics curriculum is spread across the grades. It is also noteworthy that this book would be used in 2016 with the new curriculum, and this is a book written by a team of three curriculum developers and eight mathematics teachers. This not only raises serious questions regarding the state of the state of mathematics education in the country but also urgency in addressing the issue. As the focus of this study was to examine algebraic content and pedagogical knowledge of sixth grade mathematics teachers in the Maldives, the issues detected in this curriculum guide would not be further explored in this study.

In summary, document analysis showed that the prescribed textbooks and other resource books prepared for the Maldivian Schools had focused on computational procedures rather than developing conceptual understanding, problem-solving skills, or analytical skills. It also became clear that teachers were following the textbook and resource materials sent to them word for word. However, in order to promote knowledge growth and conceptual understanding it is important that explicit attention is given to differentiated instruction and open discussions rather than simple textbook exposure (Rittle-Johnson et al., 2011). Moreover, in-depth analysis of the textbooks and resource materials used by these teachers revealed that algebraic concepts were explained incorrectly in the prescribed textbooks, and resource materials. Teachers not being able to detect these incorrect explanations provide evidence that these teachers lack common content knowledge as well as knowledge of content and teaching (Ball et al., 2008). Ball et al. placed common content knowledge as a sub-category of subject matter knowledge (which in this study is algebraic content knowledge) whereas knowledge of content and teaching was placed under pedagogical knowledge. In short, this confirmed that teachers lack algebraic content and pedagogical knowledge required to teach at sixth grade. Table 3 presents the summary analysis of documents.

Table 3

Summary Analysis of Documents

Algebraic Content Knowledge	Pedagogical Knowledge
✓ Relied heavily on the prescribed textbook and workbook. ✓ Unable to detect mistakes in the textbook and followed the textbook word for word. For example, questions discussed in the class were the examples presented in the textbook. *Conclusion: All five teachers lacked common content knowledge, which is a sub-category of algebraic content knowledge.*	✓ Only one teacher used lesson plans. Lesson plans lacked important details such as activities, teaching strategies for different levels of students, assessment methods. ✓ All five teachers used lesson notes which were taken from the prescribed textbook and workbook. They all followed textbook word for word. *Conclusion: All five teachers lacked pedagogical knowledge.*

Interviews. Interviews were broken into three sections: the first section collected personal information related to the profession whereas the second and third sections of the interview collected information on algebraic content knowledge and pedagogical knowledge, respectively. Before starting the interviews the participants were reminded about the research purpose and the confidentiality of their comments made during the interview. The researcher also reconfirmed the participants' permission to audio record the interviews.

Personal information**.** Personal information collected included the length of experience teaching sixth grade algebra, highest degree achieved and major, mathematics content courses studied in college, pedagogy courses studied in college, number of professional development sessions on mathematics attended, and whether the professional development enhanced their algebraic content and pedagogical knowledge. Table 4 summarizes the personal information of the participants with regard to years of experience in teaching sixth grade mathematics, highest qualification achieved and major, number of mathematics content courses taken in college, number of mathematics pedagogy courses taken in college, and the number of professional development sessions on mathematics.

Table 4

Personal Information

ID	MSN001	MSN002	MSN003	MSN005	MSN006
Years of Experience	13	7	20	2	18
Highest Qualification	B. Ed.[1]	Dip.[2]	Adv. Dip.[3]	B. Ed.	B. Ed.
Major	Secondary Math	Dhivehi[4]	Math	Secondary Math	Primary[5]
Number of Content Courses	7	None	2	6	2
Number of Pedagogy Courses	2	None	None	None	2
Number of Professional Development Sessions on Mathematics	None	None	None	None	None

[1]Bachelor of Education, [2]Diploma in Teaching, [3]Advanced Diploma in Teaching, [4]Dhivehi is the native language of the Maldivians, [5]All the subjects taught in Primary with the exception of Dhivehi, Islam, and Quran.

Only one of the participants who taught mathematics at the sixth grade level had a language qualification whereas all others were qualified teachers either in teaching primary or teaching mathematics. It was discovered during the interview that the participant who had a language qualification was teaching mathematics at sixth grade because when the participant joined the school, there were no posts related to the participant's area of study. The participant was further asked what made the participant to agree to teach mathematics, the response was that "I can do all the questions in the grade six textbook without difficulty so I agreed to teach mathematics. Since then I have been teaching mathematics and so far I have had no complaints from students or their parents."

The variations in number of mathematics content and pedagogy courses for the same qualification from the same institution was due to the variations in time of graduation. It is also noticeable that three of the participants reported that they had not studied any mathematics pedagogy modules, however, they mentioned that they have studied general pedagogy modules in which they learned Bloom's taxonomy, questioning skills, and different approaches to teaching. During the interview it became clear that four of the participants had studied at least one algebra module in college. The participant who did not study any mathematics content or pedagogy modules was the one who qualified as a language teacher.

Interestingly, only one of the participants among the five was qualified to teach at primary level while three of the five participants were qualified to teach mathematics at secondary level. This clearly indicated that there was a mismatch in teachers' qualifications and their jobs, which should be further explored in order to identify the reasons for these mismatches and possible solutions to address this issue. Also, another important observation was that each of the teachers had been undertaking 15 hours of professional development per year (since 2009) as it was mandatory for public school teachers. However, none of them attended a mathematics professional development. The reason they cited was that schools focused on general topics such as using PowerPoint presentations, classroom management, and behavior management. One of the participants mentioned that the professional development day was a “waste of time” as it was more seen as a day to have fun at school without students around. All five participants agreed that the professional development did not contribute towards enhancement of their algebraic content knowledge or pedagogical knowledge as they never got the opportunity to attend a professional development session tailored for mathematics teachers. Taton (2015) argued for the importance of having subject-specific professional development and stated that most of the professional development available for teachers were not useful as they were more generic. All five participants stated that the current professional development practice was useless and expressed their interest to attend a mathematics professional development.

Algebraic content knowledge. All five teachers stated that they were very confident in teaching sixth grade algebra and cited that they were able to do all the questions in the textbook as the reason for their confidence. One of the teachers went on to state that the teacher was very confident to teach sixth grade algebra but if the teacher had to teach seventh grade algebra the teacher “would be dead.” One of the five teachers said that when a student gave an incorrect answer, the teacher asked a “good student” to explain the question to the student who obtained the incorrect answer. In this teacher’s opinion, when a good student explained it became easier for the weak students to understand instead of the teacher explaining “the same thing repeatedly”. The remaining four of the five teachers said that they explained the problem on the board or at times individually to the student. The lack of analysis of student mistakes indicated that the teachers lack what Ball et al. (2008) referred to as Specialized Content Knowledge that is a form of subject matter knowledge (which in this study is algebraic content knowledge). The teachers repeatedly offering the same explanation when a student asked a question indicated that they lack pedagogical content knowledge and specifically, Knowledge of Content and Students (Ball et al., 2008).

Four of the five teachers believed that students are first exposed to algebraic concepts when they are in grade six indicating that they lack Horizon Content Knowledge, which is the knowledge of how the curriculum is spread across the grades (Ball et al., 2008). The remaining teacher believed that although students are taught formal algebra in grade six, they encounter algebraic concepts in earlier grades. This teacher stated:

> From primary onwards they are exposed to algebra. Because they are actually doing perimeter and area. There they have length into breadth. Which is l into b. that is algebra. But formally as a topic we introduce in grade 6. They are exposed to the concept before grade 6.

All five teachers stated that they did not use any additional materials other than the prescribed textbooks and resources prepared for sixth grade. They all mentioned that the new thing they have started in the year [2015] was use of PowerPoint presentations to keep students interested. It was also noted all five teachers believed that teacher explanations for how to do problems was the best and the most effective strategy when it came to algebra.

All five teachers identified removing brackets and factorizing [factoring] as the areas they needed help most. They also mentioned that since factorizing is not in the grade six syllabus, learning factorizing was not a priority as they “didn’t really need that [factorization].” One of the five teachers mentioned that the teacher needed help in explaining like terms. On the contrary, all the five teachers believed that they have sufficient knowledge to teach sixth grade algebra very well. In fact, one of the teachers stated that “I can even teach [with] my eyes closed.” The most common reasons they gave included being able to do all the questions in the sixth grade textbook, and the experience they have in teaching “the same thing” [that is, sixth grade mathematics] repeatedly for a long period of time.

Pedagogical knowledge. From the interview it became clear that all teachers lacked pedagogical knowledge. For example, all the teachers cited use of fruits and vegetables or use of stationery to represent variables (which is incorrect) as relating the concept to real-life. Remarkably, all five teachers believed that the lesson they delivered was perfect and there was nothing they would change, if they were to re-teach the lesson again. They also believed that all the students understood the lesson because the students did not ask any questions. However, one of the teachers mentioned that students ask “silly questions” because they were not paying attention. The “silly question” student asked was “why not $5ab$?” when the teacher explained that 3 apples and 2 bananas can be written as $3a+2b$. Categorizing this question as “silly” and considering students not asking questions as an indicator of learning suggested that this teacher lacked Specialized Content Knowledge, Knowledge of Content and Students, and Knowledge of Content and Teaching. These three types of knowledge are covered under subject matter knowledge or content knowledge, and pedagogical knowledge (Ball et al., 2008; Shulman, 1987).

Interestingly, teachers introduced algebra using the “fruit salad” approach because the teachers thought by using that approach they were relating algebra to real-life. All five teachers mentioned that the only teaching materials they used in teaching were PowerPoint presentations and the reason was that students liked the use of PowerPoints. They believed the best teaching strategy for their students is PowerPoint presentations. Later on it became clear that the use of PowerPoints was fairly recent [beginning in January 2015], hence the excitement. Teachers believed that students found the lesson easy. From the observations it became clear that teachers focused on computational procedures and the majority of the students were able to follow that. Teachers identified sign mistakes as the only misconception students had. However, observations revealed that students had misconceptions due to incomplete or distorted explanations offered. For example, teachers said that “when you take a number to the other side, its sign changes to opposite” and this resulted in students incorrectly writing $7x = 21 \Rightarrow x = 21 - 7$. It was also discovered that teachers were unable to detect misconceptions that indicated that they lacked not only algebraic content knowledge but also pedagogical knowledge (Ball et al., 2008; Shulman, 1987). Table 5 summarizes the findings from the interviews.

Table 5

Summary Analysis of Interviews

Case	MSN001	MSN002	MSN003	MSN005	MSN006
Years of Experience	13 years	7 years	20 years	2 years	18 years
Mathematics Professional Development	None of the participants attended mathematics professional development				
Content Knowledge	All five participants lacked algebraic content knowledge				
Pedagogical Knowledge	All five participants lacked pedagogical knowledge				
Areas they need help	All five participants named 'factoring' and 'expansion of algebraic expressions'				
Perception with regard to teacher's own teaching	All five participants were very satisfied with their teaching. Interestingly, and all five participants cited "students did not ask any questions" as an indication of learning.				
Perception with regard to teacher's own algebraic content and pedagogical knowledge	All five participants believed they did not need any help and the reasons included their teaching experience, and being able to do all the questions given in the textbook.				

Summary of Qualitative Data Analysis

The combination of lesson observations, document analysis and interviews not only maximized the depth of the examination of algebraic content and pedagogical knowledge but also helped in exploring teachers' perceptions, viewpoints of the instructional strategies, pedagogical decisions made, and specific actions observed during lesson observations. Through observations, interview and document analysis it became clear that the teachers lacked algebraic content knowledge as well as pedagogical knowledge. For example, teachers were unable to recognize incorrect explanations given in the textbook, none of the teachers were able to recognize that categorization of shapes and pattern generalizations can set the mathematical foundation for formal algebra, none of the teachers analyzed the incorrect answers given by students. In fact they avoided students who gave incorrect answers and questions were addressed to those students who were likely to give correct answers. Moreover, teachers were unable to differentiate instruction that would suit the diverse needs of students. However, teachers believed that they had sufficient knowledge of algebra to teach at sixth grade. Teachers based this decision either on their teaching experience or their ability to compute problems given in the textbook without difficulty. Table 6 presents the summary of qualitative analysis.

Table 6

Summary Analysis of Qualitative Data

	Algebraic Content Knowledge	Pedagogical Knowledge
Observations	From the observations it became clear that all five teachers lacked algebraic content knowledge. For example, the teachers treated variables as objects.	It was observed that all five teachers lacked pedagogical knowledge. For instance, teachers were unable to detect students' misconceptions.
Document Analysis	From the documents it became apparent that teachers lacked algebraic knowledge. An example would be, all five teachers followed the textbooks word for word. Interestingly, teachers were also unable to detect mistakes in the textbook. Moreover, teachers did not include any questions that required students to use their critical thinking or analytical skills, or their creativity.	All five teachers focused on computational procedures. Exercise questions and even exam questions were taken from the prescribed textbooks. Teachers used only one teaching strategy. Teacher would demonstrate how to do problems and students were merely passive recipients of knowledge. Students' activities were limited to listening.
Interviews	It became apparent from the interviews that all five teachers lacked algebraic content knowledge. However, they believed that they had sufficient knowledge to teach algebra at sixth grade.	Interviews confirmed that teachers lacked pedagogical knowledge, yet they believed that their teaching was "perfect" and that they would not bring any changes to the lessons they delivered.

Quantitative Data Analysis

Data collected using DTAMS were sent to the Center for Research in Mathematics and Science Teacher Development (CRMSTD) staff for a detailed analysis of the algebraic content and pedagogical knowledge of the five sixth grade teachers that participated in the study. The analysis was comprised of a summary of each teacher's performance that included scores on individual items, on each mathematics subdomain in algebra, and on algebraic content knowledge and pedagogical knowledge. Content knowledge was further analyzed for memorized knowledge, conceptual understanding, and higher-order thinking and problem-solving, all of which comes under common content knowledge in Ball's framework (Ball et al., 2008). Pedagogical knowledge covered the most useful form of representation of algebraic ideas; the most powerful analogies, illustrations, examples, explanations and demonstrations; and identification of student misconceptions and providing strategies to correct them that promote understanding, reasoning and proficiency. All of these come under knowledge of content and students, and knowledge of content and teaching in Ball's framework (Ball et al., 2008). This analysis of performance on specific items, subdomain topics, and knowledge levels allowed for an in-depth understanding of teacher's algebraic content and pedagogical knowledge of sixth grade mathematics teachers in the Maldives.

Figure 8 shows a detailed analysis of algebraic content knowledge and pedagogical knowledge of the participants.

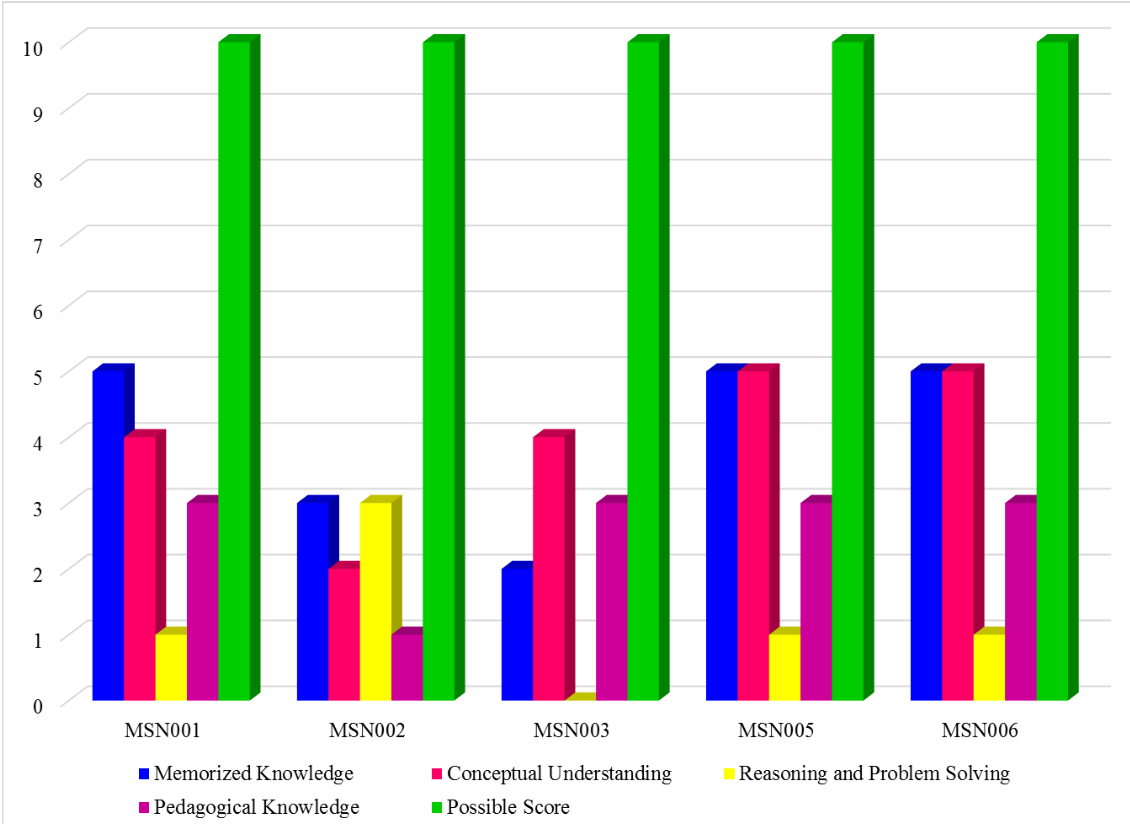


Figure 8. In-depth analysis of algebraic content and pedagogical knowledge.

In-depth analysis of DTAMS showed that the highest score obtained was for memorized knowledge that was five out of a possible 10 points. This score was obtained by three of the five teachers, whereas the other two scored below five out of a possible 10 points. The highest score for conceptual understanding was also five out of a possible 10 points that was scored by only two out of five participants. Reasoning and problem-solving was the lowest scored domain among the five participants. The highest score obtained in the domain was three, while the lowest was a zero out of a possible 10 points. These scores indicated that teachers lacked sufficient algebraic content knowledge required to teach sixth grade algebra. As DTAMS questions were from middle school mathematics syllabus and aligned with the sixth grade mathematics curriculum, teachers teaching sixth grade mathematics were expected to score 10/10. No other data were available for comparison. DTAMS results also showed that teachers lacked the pedagogical knowledge required to teach algebra. The highest score that was three out of a possible 10 points was obtained by four of the five participants while the remaining participant scored only one point out of 10.

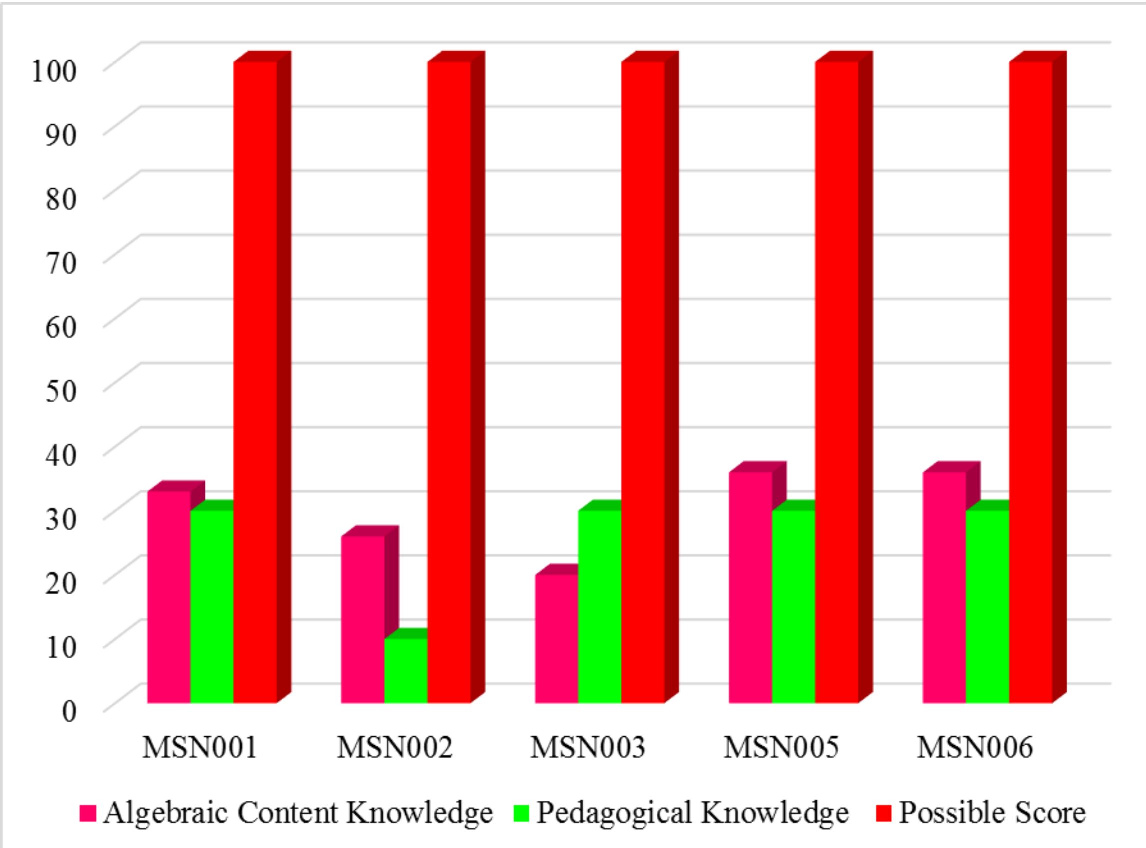


Figure 9. Overall algebraic content and pedagogical knowledge.

Figure 9 illustrates the overall algebraic content and pedagogical knowledge of the participants. The overall content knowledge of the individual participants was below 35% while the pedagogical knowledge of the individual participants was below 30% indicating that participants lacked algebraic content and pedagogical knowledge required to teach algebra at sixth grade. Moreover, it was observed that the experience of the participants did not have any impact on these results. The score expectation for a well-qualified teacher to teach sixth grade algebra is 100% as DTAMS is based on the content of middle school algebra. In other words, DTAMS covered the algebra content sixth grade students are expected to learn from these teachers. Therefore, these teachers are expected to score 100% from the DTAMS.

Summary of Quantitative Data Analysis

Analysis of DTAMS showed that teachers lacked algebraic content knowledge and pedagogical knowledge required to teach algebra. The highest performing domain was memorized knowledge with a score of five out of a possible 10 points and the lowest was reasoning and problem-solving. It was not surprising that memorized knowledge was the highest performing domain considering that it was observed the teachers taught straight out of the book with no variation. Notably, one of the five participants scored a zero in reasoning and problem-solving which indicated that the participant lacked reasoning and problem-solving skills. Furthermore, pedagogical knowledge of the participants was either three or below out of a possible 10 points indicating the teachers lacked the understanding of what made the learning of specific topics easy or difficult (Shulman, 1986). Pedagogical knowledge is what actually linked content knowledge to teaching (Ball et al., 2008). Algebraic content knowledge and pedagogical knowledge is highly essential as they affected a teacher's ability to teach algebra (Begle, 1979; Piccolo, 2008; Strand & Mills, 2014). Table 7 summarizes the DTAMS results.

Table 7

Summary Analysis of DTAMS out of a Possible 100%

Case	MSN001	MSN002	MSN003	MSN005	MSN006
Content knowledge	33%	26%	20%	36%	36%
Pedagogical knowledge	30%	10%	30%	30%	30%

Mixed Methods Results

This concurrent mixed methods study combined the strengths of qualitative and quantitative data to explore algebraic content and pedagogical knowledge of five Maldivian sixth grade mathematics teachers. Qualitative data were collected through lesson observations, analysis of algebra lesson plans and lesson notes, and interviews whereas quantitative data were collected using DTAMS. The results of qualitative data and quantitative data were consistent. From the lesson observations, analysis of lesson plans and lesson notes, interviews, and the results from DTAMS indicated that teachers lacked algebraic content and pedagogical knowledge required to teach algebra.

For example, from the observations it became clear that teachers only asked questions of the type given in the textbook, which only required memorized knowledge. None of the teachers asked any questions that required critical-thinking, creativity, or problem-solving skills. DTAMS results showed that teachers scored the lowest in questions that required reasoning and problem-solving. The highest score that was three was obtained by only one of the teachers while three of the teachers scored one and one of the teachers scored a zero out of a possible 10 points.

From the observations and the interviews it became clear that teachers lacked pedagogical knowledge and this was confirmed by DTAMS results. Four of the five teachers scored three while the remaining teacher scored a one out of a possible 10 points allocated for pedagogical knowledge. It was also observed that teachers focused on computational procedures rather than conceptual understanding. Moreover, teachers were unable to detect students' misconceptions. All these indicated the teachers not only lacked pedagogical knowledge but also conceptual understanding that was later shown by the results from DTAMS. DTAMS showed that teachers lacked conceptual understanding. Only two of the five teachers scored five points, two scored four points, and the remaining teacher scored a two out of a possible 10 points. Conceptual understanding is an important aspect of content knowledge. Lack of content knowledge limited what students were exposed to in terms of the mathematical content they were taught (Ojose, 2014; Strand & Mills, 2014).

In summary, results obtained from qualitative data and quantitative data collected proved that teachers lacked algebraic content and pedagogical knowledge, although they believed that they had sufficient knowledge to teach algebra at sixth grade. The teachers' perception of their algebraic content and pedagogical knowledge was based on their ability to work out all the problems in the sixth grade mathematics textbook. Researchers and mathematics educators agreed that this was faulty thinking as being able to solve problems given in a textbook did not attest to having sufficient knowledge and skills required to teach the content (E. Ashworth, J. K. Corkett, R. R. Perez, E. V. Chua, personal communication, March 7, 2016; I. Hassan, personal communication, March 9, 2016; A. Shareef, personal communication, March 10, 2016). All five teachers believed that their lessons were very good and that they did not need any help. Moreover, they stated that they did not intend bring any changes to their lessons. Example of responses received from the teachers when they were asked whether they would bring any changes to the lessons if they were to reteach the lesson include:

- "No. I will teach it this way. I have been teaching this lesson this way and students understand. So no point in changing it."
- "No changes. Because this is the best way to teach for my students."
- "No changes. Because students understood the lesson. [How do you know that?] They did not ask any questions."
- "No change. Because more than 90% understood the lesson. I think."
- "No changes. I have been teaching it this way and students always understand."

Specifically to answer the research question one, there was no relationship between teachers' perceptions of their mastery of algebraic content and pedagogical knowledge and what teachers knew relative to algebraic content and pedagogy as measured by DTAMS. Based on responses on the DTAMS, algebraic content knolwedge of the teachers was on average 30.2% while pedagogical knowledge was on average 26%. Based on responses on the DTAMS, there were no strengths of algebraic content and pedagogical knowledge whereas the weaknesses included lack of cocneptual undertanding, reasoning and problem-solving and lack of pedagogical knowledge. Based on the responses in interviews, teachers' perceptions about their own algebraic content and pedagogical knolwdge was that they had sufficient knowledge to teach algebra at sixth grade and there were no weaknesses relating to their algebraic content and pedagogical knolwedge.

To specifically answer the research question two, as meaured by DTAMS, there were no specific algebraic content and pedagogical knowledge strengths of the sixth grade mathematics teachers with respect to their algebraic content and pedagogical knowledge. However, the weaknesses relating to the sixth grade mathematics teachers' algebraic content and pedagogical knowledge included emphasis on memorized knowledge, lack of conceptual understanding, lack of reasoning and problem-solving, and lack of pedagogical knowledge required to teach algebra. Table 8 presents the summary of mixed methods findings.

Table 8

Summary of Mixed Methods Results

Case	MSN001	MSN002	MSN003	MSN005	MSN006
Years of experience	13 years	7 years	20 years	2 years	18 years
Content knowledge	All five participants lacked algebraic content knowledge				
Pedagogical knowledge	All five participants lacked pedagogical knowledge				
Perception	All five participants believed that they have sufficient knowledge to teach algebra at the sixth grade level.				

Validity and Trustworthiness of the Findings

Data were collected from four different sources (DTAMS, interviews with sixth grade mathematics teachers, observations of sixth grade algebra lessons, and analysis of teacher's algebra lesson plans and lesson notes) to ensure the validity of the research. It is commonly believed that qualitative researchers could never capture an objective "truth" or "reality" (Merriam, 2009). Therefore, three different strategies were employed to increase the credibility of the qualitative findings. First, triangulation by using multiple sources of data was employed by using three methods of data collection – observation, documents, and interviews. Second, investigator triangulation, which is two or more persons independently analyzing the same qualitative data and comparing their findings, was employed to ensure the internal validity of the qualitative findings (Bogdan & Biklen, 2007; Creswell, 2012; Merriam, 2009). A former mathematics teacher educator who obtained a PhD in Mathematics Education by conducting a doctoral level qualitative study served as a second investigator during the data analysis stage. The second investigator independently analyzed the qualitative data collected for themes and patterns. The findings of the second investigator were then compared with the findings of the researcher, and it was found out that nothing new emerged from the data that was not identified by the researcher. This investigator triangulation ensured the internal validity of the qualitative findings. Third, respondent validation or member check was employed to ensure internal validity or credibility of the qualitative findings. According to Maxwell (2005),

> this [member checks] is the single most important way of ruling out the possibility of misinterpreting the meaning of what participants say and do and the perspective they have on what is going on, as well as being an important way of identifying your own biases and misunderstanding of what you observed. (p. 111)

Findings from interviews were shared with the participants to ensure what was said during the interview was exactly what was understood. Also, field notes on observed algebra lessons were shared with respective teachers to ensure what was observed was what actually took place. The goal of member checking was to ensure that the results were authentic and original (Bogdan & Biklen, 2007; Creswell, 2012; Merriam, 2009).

According to Richardson (2000), in postmodern research, “we do not triangulate; we *crystallize*. We recognize that there are far more than three sides from which to approach the world" (p. 934). Crystallization of data collected from four different sources – observations, documents, interviews, and DTAMS – helped to ensure the research findings are valid while the triangulation of qualitative data helped to improve the accuracy, credibility, validity, and transferability of this study (Bogdan & Biklen, 2007; Creswell, 2012; Merriam, 2009).

Assumptions, Limitations, and Delimitations

All sixth grade mathematics teachers must be qualified primary teachers, so the assumption is that these teachers are uniformly informed about mathematics content and pedagogical knowledge. Over the years, the names of mathematics content and mathematics pedagogy courses have been changed. However, the number of mathematics or mathematics education modules, or learning hours allocated to mathematics. or mathematics education modules in primary teacher certification program is consistent. In light of this, it was assumed that these teachers would have the same experience and exposure to mathematics content and pedagogy.

The findings of this study cannot be generalized to the general population due to the small sample size and use of purposive sampling. The participants were selected from highly populated islands. These teachers have an advantage over teachers teaching in rest of the islands in the Maldives in term of their accessibility to resources and materials available for them to teach. They have undergone teacher training. Some of the teachers teaching in rest of islands are without any teaching qualification or training. Moreover, they do not have access to the resources and facilities available to the teachers teaching in highly populated islands due to geographical nature of the islands which result in unequal distribution of resources. The findings of this study might apply to the others but the data does not assure that the findings could be applied to others.

Delimitations of the study include that the participants are trained from the same institution and belong to the same nationality. It is noteworthy that there were teachers from other countries or teachers trained from other countries teaching sixth grade mathematics, however, they were not eligible to take part in this study as they did not meet the inclusion criteria which were used to assure homogeneity among the sample.

Conclusion

This study aimed to examine the algebraic content and pedagogical knowledge of five sixth grade mathematics teachers in the Maldives using a concurrent mixed methods multi case study approach. The main reason for selecting a mixed methods approach was to gain an in-depth understanding of the problem under study. Mixed methods research is found to complement the results obtained through either quantitative or qualitative approach only, making the results more meaningful in terms of what could be done in future to address the issue studied.

Fourteen sixth grade algebra teachers employed in the selected schools were eligible to take part in the study. These 14 teachers were approached and the seven teachers who consented to take part in the study were selected. However, two of the teachers withdrew after the first observation. Therefore, only five continued through the end of the study. As this is an in-depth study, a study of a few cases would suffice. All data collected were kept confidential to prevent the participants from any harm or negative impact that may come due to the findings of the study. Qualitative data were collected through observations of algebra lessons, analysis of algebra lesson plans and lesson notes, and interviews with the sixth grade algebra teachers while quantitative data were collected using DTAMS.

Content of the qualitative data collected was analyzed for themes and patterns while quantitative data were sent to Center for Research in Mathematics and Science Teacher Development (CRMSTD) staff for a detailed analysis of the algebraic content and pedagogical knolwege of the sixth grade mathematics teachers in the Maldives. Both qualitatve findings and quantitative findings were in agreement. Analysis of both qualitative and quantitative data showed that teachers lacked algebraic content and pedagogical knowledge necessary to teach algebra at sixth grade although the teacher-participants believed that they had sufficient knowledge to teach sixth grade algebra.

In short, there was no relationship between teachers' perceptions of their mastery of algebraic content and pedagogical knowledge and what teachers knew relative to algebraic content and pedagogy as measured by DTAMS. Also, as meaured by DTAMS, there were no specific algebraic content and pedagogical knowledge strengths of the sixth grade mathematics teachers with respect to their algebraic content and pedagogical knowledge. However, identified weaknesses included emphasis on memorized knowledge, lack of conceptual understanding, lack of reasoning and problem-solving, and lack of pedagogical knolwedge required to teach agebra.

The preliminary findings of this study were presented at the 11th Annual Education and Development Conference 2016 held in Bangkok from 5-7 March 2016. After taking into consideration the feedback and recommendations received from the participants of the conference and discussions with my dissertation chair and second committee member it was decided that the most suitable artifact would be a professional development curriculum for the teachers. One of the professors from Nipissing University, Canada said to me “Mariyam, if you want to change the system you have to change the teachers. The best thing you could do [to address the issue] is to conduct professional development for in-service teachers” (J. K. Corkett, personal communication, March 7, 2016). The feedback and recommendations I received from the other education researchers from Australia, Canada, New Zealand, Philippines, and South Africa were along the same lines (J. Barnett, D. Ford, M. H. Aranga, S. L. Ferguson, R. R. Perez, E. V. Chua, M. K. Mhiolo, personal communication, March 7, 2016).

In addition, the researcher who served as the local project manager of the baseline study and two school heads also suggested that the best I could do to contribute to improve the current practice [teaching sixth grade algebra] would be to come up with a mathematics professional development curriculum for the in-service teachers (A. Shareef, personal communication, March 9, 2016; A. Waheed, personal communication, March 4, 2016; M. Majeed, personal communication, March 10, 2016). Therefore, the project genre chosen based on the findings from this research, and recommendations from the experts in the field is Professional Development/Training Curriculum and Materials. The goals of this project and its rationale, related literature, implementation and evaluation of the project, implications including social change are discussed in Section 3.

Section 3: The Project

Introduction

A baseline survey conducted by UNICEF and NIE to assess student performance during 2012 and 2013 in the Maldives showed that algebra is the lowest performing area of mathematics, with only 27.1% students obtaining the correct answers to algebra questions. Algebra is a fundamental topic in mathematics that lays foundation for further studies in mathematics and quantitative sciences. Interviews with the local project manager of the baseline study, mathematics teacher educators, heads of two 1-10 schools, and heads of teacher training institutions gave rise to serious questions regarding the algebraic content and pedagogical knowledge of the teachers, and, therefore, the teachers' ability to teach the subject. In the Maldives, algebra is first introduced in sixth grade, and teachers who introduce algebra are responsible for laying a solid foundation on which students can later construct their algebraic understanding. Therefore, the research represented by this dissertation aimed to examine the algebraic content and pedagogical knowledge of sixth grade mathematics teachers to identify their strengths and weaknesses in these areas.

This mixed methods concurrent multicase study design research was guided by Shulman's major categories of teacher knowledge and the domains of mathematical knowledge for teaching proposed by Ball et al. (2008). Data collection techniques used in this study included observations of algebra lessons, analysis of algebra lesson plans and lesson notes, interviews, and administration of DTAMS. The results of this study revealed that the sixth grade mathematics teachers who participated in this study lacked algebraic content as well as pedagogical knowledge. However, they believed that they had sufficient knowledge to teach algebra at sixth grade, which was based on their teaching experience and their ability to solve problems given in the sixth grade textbook. The results of this study were shared with the local project manager of the baseline study, mathematics teacher educators, heads of K-12 schools, heads of teacher training institutions, and the participants of the study. Moreover, the results were also presented at the 11th Annual Education and Development Conference 2016 held March 5-7, 2016 in Bangkok, Thailand. Academics in the country as well as the researchers who attended the conference and my dissertation committee recommended planning and implementing mathematics professional development for the participants to address the issues identified. It is believed that improving algebra instruction through enhancing algebraic content and pedagogical knowledge of the participants would lead to better student performance in algebra.

Four basic genres of projects were listed in the mixed method final study checklist provided by Walden University as project options. The four genres listed were evaluation report, curriculum plan, professional development/training curriculum and materials, and policy recommendation with detail. The evaluation report option was for evaluation studies. As this study was not an evaluation study, this was not an option. The remaining three options were available for this project. The curriculum plan option required the researcher to prepare lesson plans and teaching materials for students. Preparing lesson plans and teaching materials and presenting those materials to the teachers to deliver those lessons was not seen as appropriate for two reasons. First, the teachers heavily depend on the textbook and follow the textbook word-for-word even if the concept presented in the textbook is incorrect. Second, the results showed that teachers lacked algebraic content and pedagogical knowledge. So to bring a lasting change, it was important to educate these teachers. As Professor Corkett said (J. K. Corkett, personal communication, March 7, 2016), if one wanted to change the system (algebra teaching), one must change the teachers (educate the teachers).

The policy recommendation with detail option was also not seen as appropriate to address the issue identified from this study as the policy recommendation required the researcher to present the problems with the existing policies, which was not the focus of the study. The professional development/training curriculum and materials option was seen as the most appropriate option for the project based on the findings of this study and the recommendations from the academics. Findings of this study showed that the participants lacked algebraic content and pedagogical knowledge. The recommendations received from the academics were to conduct mathematics professional development as educating the teachers would bring a lasting change and improve classroom instruction. In this section, the purpose, goals, learning outcomes, and target audience of this project are discussed.

Purpose of This Project

The problem examined in this study is algebraic content and pedagogical knowledge of sixth grade mathematics teachers. The purpose of this study was to identify the algebraic content and pedagogical knowledge strengths and weaknesses of the five teachers from the selected schools of the Maldives who gave consent to take part in this study. Identifying algebraic content and pedagogical knowledge strengths and weaknesses is essential to enhancing algebra instruction by building on the strengths and improving areas of weaknesses. Moreover, it was believed that identification of algebraic content and pedagogical knowledge of sixth grade mathematics teachers' relative strengths and weaknesses could serve as the foundation for the development of mathematics teacher professional development curriculum. The purpose of this project was to prepare a mathematics teacher professional development curriculum to build on the strengths and improve the weaknesses identified in this study in order to enhance algebra instruction.

Goals of This Project

Professional development curriculum serves to improve instruction in order to improve student learning. According to the National Council of Teachers of Mathematics (2010), research on professional development advocated that mathematics professional development was effective when it promoted mathematics teachers' growth in four major areas. The areas highlighted in National Council of Teachers of Mathematics (2010, p. 1) included:

1. Builds teachers' mathematical knowledge and their capacity to use it in practice.
2. Builds teachers' capacity to notice, analyze, and respond to students' thinking.
3. Builds teachers' productive habits of mind.
4. Builds collegial relationships and structures that support continued learning.

In the Maldives it is mandatory for public school teachers to undergo 15 hours of professional development each academic year. It is noteworthy that all five participants completed 15 hours of professional development each year since 2009. However, none of them attended a mathematics professional development. Hence, the participants reported that professional development did not contribute towards the enhancement of mathematics content or pedagogical knowledge. Due to limited resources and trained teachers, schools release teachers on three full days to complete 15 hours of mandatory professional development each academic year, two days during the first semester and one day during the second semester as per the calendar set by the Ministry of Education. On each day teachers complete five hours of professional development. In order to make this project realistic and as practical as possible, the mathematics professional development is designed for three full days accounting for a total of 15 hours, which will be completed within a year.

In light of this and the findings of this study, the goals of this project to address the problem identified were:

1. Advance algebraic content and pedagogical knowledge of the teachers through the development of their conceptual understanding, critical-thinking, and problem-solving ability.
2. Expand the capacity of the teachers to detect errors and misconceptions students have in algebra through students’ responses and answer scripts and identify ways to remedy and prevent the formation of these errors and misconceptions.

3. Promote collegial relationships and structures that support continuous mathematics professional development.

Learning Outcomes of This Project

Mathematics professional development curriculum is designed in a way that each full day of professional development addresses one of the goals identified in the above section. Specific learning outcomes of this project are as follows. Upon successful completion of the Professional Development Day 1, the participants will be able to:

1. Develop a comprehensive conceptual understanding of basic to advanced algebraic concepts required to teach algebra at sixth grade.
2. Apply the knowledge gained from this module in solving problems and forming problems that promote critical thinking and analytical abilities among sixth grade mathematics students.
3. Understand how algebraic concepts relate to everyday life and explain algebraic concepts using developmentally-appropriate strategies to students of varying abilities and levels.
4. Critically analyze textbooks and other resources to identify inaccurate definitions and explanations.

Upon successful completion of the Professional Development Day 2, the participants will be able to:

1. Identify the common errors students make and misconceptions they have by analyzing students' answers.
2. Categorize the common errors students make and misconceptions they have.

3. Develop strategies to remedy the common errors students make and misconceptions they have.
4. Prepare and deliver lessons that would prevent making errors and the formation of misconceptions.

Upon successful completion of the Professional Development Day 3, the participants will be able to:

1. Identify various strategies that could be used to enhance one's own professional learning.
2. Effectively plan professional learning.
3. Collaborate to research, plan and design effective teaching strategies and programs.
4. Self-evaluate the impact of the mathematics professional development curriculum on one's own algebraic content and pedagogical knowledge using DTAMS post-test.

Detailed outline of the components, timeline, activities with trainer notes, module formats, implementation plan, evolution plan, and specific hour-by-hour detail of this training is in Appendix A.

Rationale

Professional development was chosen as the project genre to address the issues identified in this research. Desimone (2009) and Zehetmeier (2014) highlighted the fact that numerous education reforms count on teacher learning and improved instruction to enhance student achievement. Education reform is in fact equal to teacher professional development (Sykes, 1996; Zehetmeier, 2014). The main reason for choosing professional development was that research has indicated the positive impact of professional development on teaching quality and student achievement (Cohen & Hill, 2001; Garet, Birman, Porter, Desimone, & Herman, 1999; Hill & Ball, 2004; Killion & Roy, 2009; Lane et al., 2015; Polly, 2015; Taton, 2015; Yoon, Duncan, Lee, Scarloss, & Shapley, 2007). Specifically, mathematics professional development has been identified as a critical component of mathematics education reform as there is evidence that mathematics professional development enhanced instruction which in turn improved student achievement (Clarke, 2003; Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009; Higgins & Parsons, 2009; Lane et al., 2015; Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010; Polly, 2015; U.S. Department of Education, 2008).

Data analysis of this study which was the focus of Section 2 showed that teachers lacked algebraic content and pedagogical knowledge. However, participants perceived that their lessons were perfect and that they did not need any help. Their perception was mainly based on the fact that they have been teaching the “same stuff” for a long time and students not having (or asking) any questions. All five participants believed that they had sufficient knowledge to teach algebra at sixth grade. The reasons given by the participants to justify their belief included:

- “I have a degree.”
- “I can do all the questions given in the textbook.”
- “I have been teaching the same stuff for 20 years.”
- “I have already learned these topics.”

- "I can even teach [with] my eyes closed [pauses and laughs] but if you ask anything from grade seven I am dead."

Algebraic content and pedagogical knowledge weaknesses identified through observations of algebra lessons, analysis of algebra lesson plans and lesson notes, interviews, and administration of DTAMS included:

1. Participants lacked conceptual understanding of algebraic concepts.
2. Participants' were deficient with regards to reasoning and problem-solving skills.
3. Participants lacked pedagogical knowledge required to teach algebraic ideas and concepts.
4. Participants followed the textbook word for word.
5. Participants were unable to detect incorrect explanations given in the textbooks.
6. Participants ignored the incorrect answers given by the students.
7. Participants taught for the exams instead of understanding.
8. Participants never attended a mathematics professional development.
9. Participants identified factorizing (factoring) and removing brackets (expansion of algebraic expressions) as areas where they needed help.

The findings of this study were shared with academics, mathematics educators, principals, and researchers. The unanimous recommendation was to conduct professional development for these teachers as a significant number of researches indicated teacher learning led to improvement in student performance (Clarke, 2003; Cohen & Hill, 2001;

Darling-Hammond et al., 2009; Garet et al., 1999; Higgins & Parsons, 2009; Hill & Ball, 2004; Killion & Roy, 2009; Lane et al., 2015; Loucks-Horsley et al., 2010; Polly, 2015; U.S. Department of Education, 2008; Yoon et al., 2007). Moreover, in order to improve the current practice, algebraic content and pedagogical knowledge weaknesses of these in-service teachers need to be addressed as they are considered the primary source of knowledge for the students studying in grade six. Therefore, the best choice to provide the necessary support needed by these teachers was mathematics professional development.

The project genre chosen to address the issues identified is the mathematics professional development. Mathematics professional development was designed in three phases. Each of the phases addressed one of the goals of mathematics professional development identified in the previous section.

Rationale for the Content of This Project

The goal of the Mathematics Professional Development Day 1 is to advance algebraic content and pedagogical knowledge of the teachers through developing their conceptual understanding, critical thinking and problem-solving ability. Therefore, the program for the Mathematics Professional Development Day 1 includes activities that focus on improving the participants' algebraic content and pedagogical knowledge. Details of the activities planned and program and the schedule is attached in Appendix A.

Results of this study showed that participants emphasized procedural knowledge and low-level computational skills through rote-learning and repeated practice, instead of developing mathematical discourse among students through contextualized problem-solving and by making explicit connections among concepts. Van Garderen (2008) attributed teachers' lack of conceptual understanding and problem-solving skills to shortcomings in classroom instruction. Krawec and Montague (2014) linked shortcomings in classroom instruction with lack of conceptual understanding and application skills in middle school mathematics students. To address the issue of lack of conceptual understanding and problem-solving skills in middle school mathematics students, Krawec and Montague (2014) conducted mathematics professional development that focused on developing conceptual understanding and problem-solving skills of middle school mathematics teachers. The results showed that professional development that focused on conceptual understanding and problem-solving skills improved the conceptual understanding and problem-solving skills of their students (Krawec & Montague, 2014).

Professional development was found effective when it focused on improving content knowledge, pedagogy, and dispositions (Burrows, Borowczak, Slater, & Haynes, 2012; Burrows, 2015; Crippen, 2012). Moreover, professional development that assigns time for lesson planning and discussions were found to be more practical and effective in supporting implementation (Burrows, 2015). In order to improve teachers' content and pedagogical knowledge and make this pedagogical knowledge permanent effective professional development is essential (Baxter, Ruzicka, Beghetto, & Livelybrooks, 2014; Krawec & Montague, 2014; Polly, 2015). Hence, the Mathematics Professional Development Day 1 is focused on advancing the participants' algebraic content and pedagogical knowledge, their conceptual understanding and problem-solving skills. In addition, it has also included collaboratively planning algebra lessons as one of the activities.

The Mathematics Professional Development Day 2 activities focused on providing teachers with practical application of the knowledge gained in the first day of mathematics professional development. Specifically, the goal of the second day is to expand the capacity of the teachers to detect errors and misconceptions of algebra through students' responses and answer scripts and identify ways to remedy and prevent the formation of these errors and misconceptions. The activities of the second day are designed in a very practical way. Teachers are required to bring the incorrect answers of their students and analyze those for possible errors and misconceptions. In addition, they would be required to identify ways of remedying the errors and misconceptions and prevent the formation of such errors and misconception in the future. A detailed outline of the activities planned for the Mathematics Professional Development Day 2 is also included in the Appendix A.

Effective mathematics professional development should not only focus on increasing teachers' mathematical content and pedagogical knowledge but also it is essential to include understanding how students think about and learn mathematics (Krawec & Montague, 2014; Sowder, 2007). Professional development has been often criticized as ineffective and disconnected (Krawec & Montague, 2014). Thus, the Mathematics Professional Development Day 2 is focused on understanding how students think and learn mathematics through analysis of students' incorrect answers. To make these mathematics professional development sessions as effective and as connected to teachers' classroom experiences, teachers' own students incorrect answers are used.

The goal of Mathematics Professional Development Day 3 is to promote collegial relationships and structures that support continuous mathematics professional development. This session is dedicated to expose the teachers to various strategies that can be used for one's own professional learning in order to empower the teacher to take charge of their own professional growth. Teachers will be guided on how to effectively plan professional learning and also how to effectively collaborate professional learning despite their location and the constraints that arise due to the geographical nature of the islands. Moreover, DTAMS post-test will be administered during this session to assess teachers' algebraic content and pedagogical knowledge strengths and weaknesses. The results will be compared with the results from DTAMS pre-test that was administered earlier during the data collection stage of this study. The detailed program of Mathematics Professional Development Day 3 is attached in Appendix A.

Collaborations serve to expose teachers to new pedagogy and learning strategies for students (Burrows, 2015; Knowlton, Fogleman, Reichsman, & de Oliveira, 2015). Fostering collegial relationships provide participants with access to new perspectives and expertise (Burrows, 2015; Knowlton et al., 2015). Knowlton et al. (2015) testified that participants reported collegial relationships and collaborations contributed to increase their knowledge as well as gain a better perspective on the expectations for their students. Burrows (2015) reported that practices of an effective professional development included clear communications, hands-on activities, reflections and discussions, and intentional collaboration and partnership building. Additionally, inclusion of hands-on activities encouraged collaboration (Burrows, 2015). Hence, the integration of all these practices in the activities planned for Mathematics Professional Development Day 3.

This project is seen as a solution to the problems identified in this study as the project is designed in a way that addresses each and every problem identified in the study. Moreover, by the end of this project teachers will be equipped with the ability to take charge of their own professional growth and professional learning (Baxteret al., 2014; Burrows, 2015; Knowlton et al., 2015; Krawec & Montague, 2014; Polly, 2015). The teachers will be able to work independently as well as collaboratively to enhance their knowledge and skills. These teachers will develop their critical thinking and problem-solving skills (Krawec & Montague, 2014). Last but not least, these teachers will develop a love for learning and become life-long learners who are capable of posing and solving problems. In short, these teachers might be the change agents who will reform the algebra instruction nationally, and in particular, in the schools they work.

Review of the Literature

Search Strategy

Saturation for the literature review was reached after researching peer-reviewed journals in education databases. The databases searched included ERIC, Educational Research Complete, SAGE Premier, ProQuest Central, Science Direct, and Academic Search Complete. Boolean search terms included, but not limited to: *professional development*, *professional development model*, *professional learning standards*, *adult learning theory*, *mathematics professional development*, *teacher knowledge professional* *development*, *professional learning*, *professional development needs*, *enhancing mathematics teaching*, *continuous professional development*, *student achievement professional development*, *effects of teacher professional development*, *mathematics teacher learning, content knowledge professional development*, *pedagogical knowledge professional development*, *algebra teaching professional development*, *algebra professional development*, *collaborative professional leaning*, *collaborative mathematics professional leaning*, *collaborative algebra professional leaning*, and *algebra instruction professional development*.

Models of Professional Development

The literature includes a wide variety for what might be included as professional development. However, it all boils down to any activity that prepares teachers to improve performance of students through teacher learning and enhanced instruction (Desimone, 2009; Killion, & Roy, 2009; Knowlton et al., 2015; Polly 2015; Taton 2015; Trif, 2015). Effective professional development is defined as providing needed support to continuously improve the performance of educators that enable them to successfully reach all students by addressing inequities in teaching quality and educational resources across classrooms through collaborative professional learning which would positively contribute towards continuous improvement of student achievement (Killion & Roy, 2009; Mizell, Hord, Killion & Hirsh, 2011).

Professional development is found effective when it addresses the specific needs of the participants. These specific needs of the participants are mostly a reflection of the needs of their students (Barrett, Cowen, Toma, & Troske, 2015; Polly, 2015; Taton

2015). The heart of professional development should be improving the experiences of teaching and learning. Participants of professional development are practicing adults (Barrett et al., 2015; Lehiste, 2015). Adults only learn what interests them and what they feel is relevant unlike the academic system in which students are forced to study subjects which educators think useful. Adults do not learn anything in the hope that they have to use it someday or that it will become useful one day. Lindeman (as cited in Knowles, Holton III, & Swanson, 2005) identified some key assumptions in adult learning. His assumptions stated that adults are motivated to learn what interests them and what they feel they should know based on their experiences; instead of learning subjects, adults prefer real life situations in learning; in adult learning it is always the life experience not the textbooks; adults prefer to self-direct their learning; and as people age their differences increase due to life experiences (Goldsmith, Doerr, & Lewis, 2014). Therefore, when planning professional development, it is important to be aware of adult learning theories which in turn help in integrating multiple learning styles that cater the needs of the participants.

Specific ways of integrating multiple learning styles and adult learning theory when planning and implementing professional development are listed below:

1. Participants or the audience of professional development should be consulted to identify the areas they need professional help with. In adult education it is the “students” who decide the curriculum (Knowles, Holton III, & Swanson, 2005). Therefore, it is important to do this needs-analysis at the planning stage to ensure the session or sessions conducted are effective to the learners. As

Lindeman stated adults are unlikely to learn something just because it might become useful one day (as cited in Knowles et al., 2005). This will also eliminate the participants' criticism which is always based on the fact that their needs are not addressed in professional development.

2. Facilitators play an important role in setting the mood of the learning experience. Therefore the facilitator needs to be well informed and aware of the adult learning theories and styles. Facilitator should be aware that, just like children, adults also learn in different ways and at different paces. Including scientific stream and artistic stream in professional development will help cater the needs of different learners. Unlike children, adults learn more from the experiences (Knowles et al., 2005). To share their own experiences and learn from others' experiences collaboration is also important in effective professional learning (Killion, & Roy, 2009). To make the professional learning sessions successful it is important that the person who is selected to facilitate is aware of the above mentioned aspects.
3. An important aspect of adult learning is learner being able to examine previous experiences based on the new knowledge acquired, which is the basis of cognitive theory (Jackson, 2009). Therefore, in order to implement professional development it is important to do the follow up. It is important to check if the teachers are reflecting on their previous teaching and learning experiences based on the new knowledge and also whether they are integrating the acquired knowledge in the classrooms. Future sessions should

be informed using the data collected as follow up. This will help identify the effectiveness of the sessions conducted.

4. Behaviors theory states human mind does not function as a change terminal (Jackson, 2009). This is in agreement with one of the aspects of adult learning which emphasizes the fact that adults cannot be passive recipients of knowledge, rather they would actively participate in constructing the knowledge (Knowles, Holton III, & Swanson, 2005). This aspect highlights another orientation in adult learning, which is constructivist theory (Jackson, 2009).

Goldsmith et al. (2014) reviewed 106 refereed articles written on professional learning of practicing mathematics teachers and suggested that effective professional development should cover the following components:

1. Changes in teacher beliefs which included their beliefs about mathematics and its teaching, students and other beliefs related to teaching
2. Changes in teachers' instructional practices which included changes in mathematics content covered in teachers' lessons, changes in the way discussions are carried out, and promoting students' intellectual autonomy
3. Teachers' collaboration which included lesson study groups, video clubs, arranging courses and workshops, online or in person discussions, and one-on-one coaching

4. Teachers attention to students' thinking which included how to look at students' work and explore the students' mathematical thinking and understanding of concepts
5. Enhance teachers' mathematics content
6. Focus on curriculum and instructional tasks

Nathan and Koedinger (2010) found out that teachers' beliefs were prominent in shaping their classroom practices. Improved conceptual understanding leads to better instruction and lack of conceptual understanding of the mathematics teachers teach had been associated with students' poor performance in mathematics (Ma, 1999; Welder, 2012; Tajudin, 2014; Tennant & Colloff, 2014). Moreover, students of teachers who took part in professional development that focused on content knowledge, pedagogical knowledge, and teacher collaboration had shown significant improvement in their performance compared to the students whose teachers did not take part in such professional development (Polly, 2015; Taton, 2015; Zehetmeier, 2014). Judak (2013) discussed that students had difficulties in the areas teachers had difficulties. It is also noteworthy that collaborations contribute towards improvement in teachers' knowledge through shared experiences and learning together (Burrows, 2015; Knowlton et al., 2015). Therefore, planning and designing professional development that covers the components identified by Goldsmith et al. (2014) could contribute towards improving students' performance through improving teachers' knowledge. Moreover, the components discussed by Goldsmith et al. (2014) are in agreement with those discussed by Desimone (2009), Killion and Roy (2009), and Knowles et al. (2005).

Desimone (2009) listed the following steps as the steps a core theory of action for professional development should include (Desimone, 2009, p. 184):

1. Teachers experience effective professional development
2. The professional development increases teachers' knowledge and skills and/or changes their attitudes and beliefs
3. Teachers use their new knowledge and skills, attitudes, and beliefs to improve the content of their instruction or their approach to pedagogy, or both
4. The instructional changes foster increased student learning

Above mentioned steps are reflected in Desimone's (2009) conceptual model for professional development shown in Figure 10. It is noteworthy that this conceptual model also highlighted the components identified by other researchers as critical components of effective professional development. In addition, Desimone's model included an additional feature – measuring the influence of professional development.

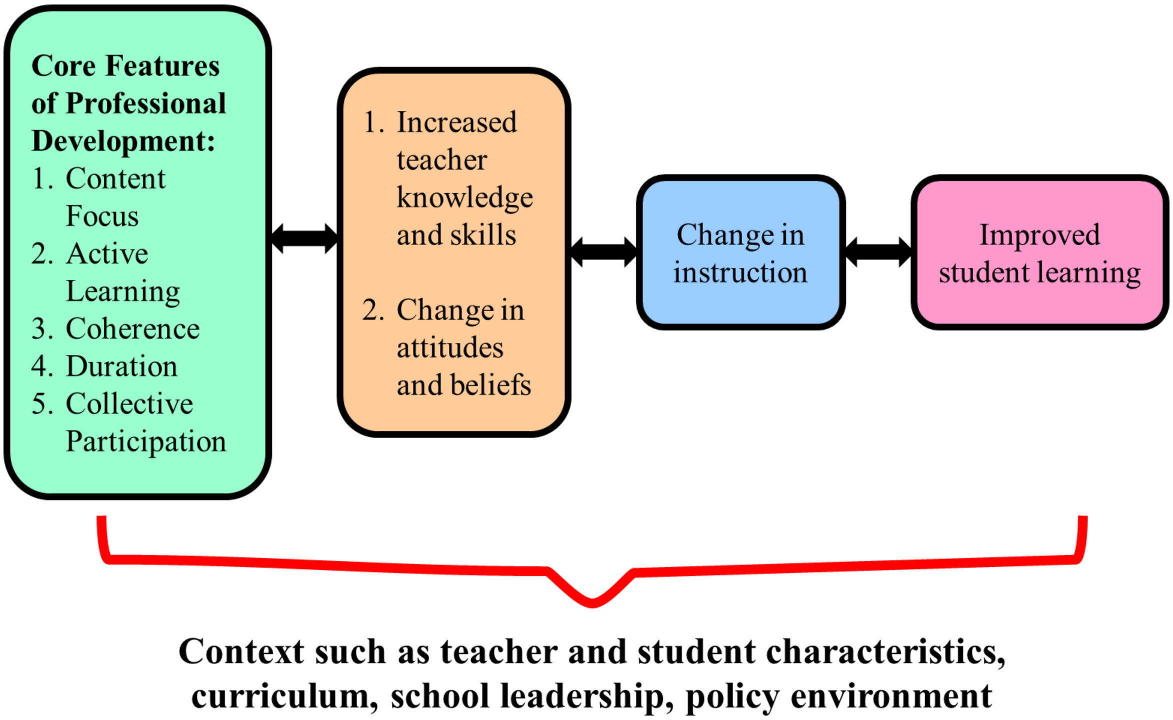


Figure 10. Conceptual model for professional development (Desimone, 2009, p. 185).

The content focus of teacher professional development is the most influential feature. Evidence from past literature showed professional development that focused on subject matter content increased student achievement (Barrett et al., 2015; Desimone, 2009; Polly, 2015; Taton, 2015). These evidences come from case-study data, correlational analysis, quasi-experiments, longitudinal studies, meta-analyses, and experimental designs (Desimone, 2009; Lehiste, 2015). This clearly indicates the importance of focusing on the subject matter knowledge when planning professional development for in-service teachers.

Active learning is also a core feature of effective professional development rather than teachers being passive recipients of knowledge (Desimone, 2009). Desimone (2009) stated that active learning could take different forms. For example, observing expert teachers teach, or letting experts observe one's own teaching and getting interactive feedback, analyzing students' work, and leading and taking part in discussions are considered as active learning (Banilower & Shimkus, 2004; Borko, 2004; Desimone, 2009). Coherence, which is the extent to which teacher learning is consistent with teachers' knowledge and beliefs also plays a vital role in effective professional development as adults do not learn things thinking that they might have to use that knowledge in the future (Desimone, 2009; Goldsmith et al., 2014; Knowles et al., 2005). Research also showed that continuous professional development enhanced student achievement rather than one-time professional development. In fact professional development that spread over a semester or a year and which lasted 20 or more hours showed significant gains in student achievement (Desimone, 2009; Killion & Roy, 2009; Polly, 2015). Collective participation creates room for collaboration. This creates opportunities for teachers to take charge of their own learning and improve areas they need help more through collaboration (Desimone, 2009; Killion & Roy, 2009; Knowlton et al., 2015).

Killion and Roy (2009) and Mizell, Hord, Killion and Hirsh (2011), discussed effective professional development and proposed the following components as key features of effective professional development.

1. Professional development should be aligned with student achievement standards as well as school improvement goals

2. Facilitator of the professional development should not only be well prepared and knowledgeable in the area but also should be aware of the school improvement goals
3. Professional development should not be just a single session which is conducted “once and for all”. Instead it should be conducted often as a continuous process with regular follow-up or evaluation of the results upon the implementation of the strategies learned
4. Most important of all, the professional development should address the specific needs of the students, teachers, and the school
5. The main focus of professional development should be on improving the experiences of teaching and student learning

To sum up, it is clear from the literature that an effective professional development should focus not only on content and pedagogical knowledge but also include active learning, should be coherent, and must lay the foundation for collaboration and continuous professional development among the teachers teaching same grades across the schools (Desimone, 2009; Goldsmith et al., 2014; Killion & Roy, 2009; Knowles et al., 2005; Mizell et al., 2011). The three-day project presented in Appendix A is designed to focus on advancing content and pedagogical knowledge through active learning. Moreover, the project is designed in a way to lay the foundation for collaboration and continuous professional development through lesson study and action research. On the whole, the project includes all the components of effective professional development identified through literature.

Impact of Professional Development on Teaching Quality and Student Achievement

Teacher professional development has been linked to enhancement of teaching quality and student achievement (Goldsmith et al., 2014; Lane et al., 2015; Polly, 2015; Yoon et al., 2007). For example, a meta-analysis conducted by Yoon et al. in 2007 showed that 1,300 studies linked professional development to improved instructional delivery that in turn improved student achievement (Yoon et al., 2015). Polly (2015) reported that mathematics professional development improved instruction which enhanced students understanding of mathematical concepts. Lane et al. (2015) reported that mathematics professional development contributed to a statistically significant improvement in teacher knowledge and confidence. Improved teacher knowledge and confidence has been linked to student achievement in other studies (Polly, 2015; Taton, 2015). For example, Polly (2015) reported that students of teachers who participated in mathematics professional development that focused on developing teachers' knowledge showed a deeper understanding and outperformed their peers in problem-solving.

Professional development and student learning are very much related. Professional development enriches teacher knowledge and skills that in turn improve classroom teaching which results in raising student achievement (Lomos, Hofman & Bosker, 2011). Yoon et al., (2007) stated in their report "teachers who receive substantial professional development - an average of 49 hours in the nine studies – can boost their students' achievement by 221 percentile points" (p. 1). According to Lomos, Hofman and Bosker (2011) there is significant evidence to suggest that professional learning within a school community enhanced student learning. These clearly show that professional development and student learning is directly related in a way that the more professional development received the better the student achievement.

Highlighting the relationship between professional learning and student results, Learning Forward (n. d), explained that standards-based professional development contributed towards frequent use of effective strategies that meet performance expectations and student learning needs. It is worth mentioning that when educators received focused professional development there is a statistically significant improvement in student results (Lomos, Hofman & Bosker, 2011). Mizell et al., (2011) also confirmed that collaboration and professional learning among the educators increased student achievement.

Goldsmith et al. reported that professional development contributed in enhancing the instructional practices. For example, teachers who took part in professional development started recognizing different ways of solving problems and emphasized on students' understanding rather than their ability to answer questions (Goldsmith et al., 2014). Furthermore, it was reported that mathematics professional development changed teachers' beliefs about mathematics teaching and their students, their instructional practices, content of mathematics lessons, and most importantly, the way discussions were carried out – promoting students' intellectual autonomy.

Results of a four-year longitudinal study demonstrated increasing improvements in student literacy learning during the implementation of collaborative professional development (Biancarosa, Bryk & Dexter, 2010). These studies clearly demonstrate the relationship between professional development and student learning, in particular how professional development improves student achievement if implemented effectively.

Teachers shared experiences and discussions and activities that contributed towards improving their knowledge, skills, teaching practice and professional growth are referred to as teacher professional development. In order for professional development to be considered effective it has to lead towards improved student achievement. Research shows evidence that effective professional development leads to improved student achievement (Lomos et al., 2011; Yoon et al., 2007; Learning Forward, n.d; Biancarosa et al., 2010; and Mizell et al., 2011). For example, teachers who took part in professional development that focused on developing teachers' content knowledge and pedagogies related to posing cognitively demanding mathematical tasks, examining students mathematical thinking, making instructional decisions based on student performance showed performance gains in their students compared to students of those teachers who did not take part in professional development (Barrett et al., 2015; Polly, 2015).

Analysis of the component of effective mathematics professional development showed that they addressed the domains identified in Shulman's (1987) and Ball et al.'s (2008) model, namely, subject matter or content knowledge and pedagogical knowledge. Subject matter or content knowledge covered knowledge of mathematics, knowledge of curriculum, and the mathematics skills required for teaching (Ball et al., 2008; Shulman, 1987). Pedagogical knowledge covered knowing how to deliver the content which addressed the individual need of the students, knowing the approaches that could be used to deliver specific content, and being able to select appropriate teaching materials taking into account students' previous learning.

In addition to this, it was also found out that effective professional development laid the ground for teacher collaboration (Knowlton et al, 2015). This helped in shaping the teachers to be lifelong learners who were keen on continuous professional development through collaboration. For example, Knowlton et al. (2015) reported that participating teachers of the study benefited through collaboration that resulted in them continuing to collaborate even a year after the completion of the project. Participants reported that the experienced gained through collaboration was invaluable (Knowlton et al., 2015). For that reason, this project is designed to address teachers' algebraic content and pedagogical knowledge, support collegial relationships and lay the foundation for collaboration.

How Theory Relates to Content of This Project

The Mathematics Professional Development Day 1 includes activities that focus on advancing the algebraic content and pedagogical knowledge of the teachers. The main reason for this is that the related literature showed that the most influential feature of professional development is the focus on enhancing the content knowledge of the teachers (Polly, 2015; Taton, 2015). Content knowledge of the teachers has been linked to improved instruction and enhanced student achievement (Ball et al., 2008; Desimone, 2009; National Council of Teachers of Mathematics, 2010; Polly, 2015; Taton, 2015). The unique feature of this session is that the algebraic content will be covered through varieties of activities incorporating effective pedagogies that could be used to deliver the content. In short, this session would not only enhance the algebraic content knowledge, but also the pedagogical knowledge along with critical-thinking, and problem-solving strategies.

The activities planned for Mathematics Professional Development Day 2 include providing the teachers opportunity to apply what they have learned and actively engage in identifying workable solutions to problems they have identified through the task assigned by the end of the Mathematics Professional Development Day 1. Professional development is effective when the participants see the need for learning and its practical applications (Barrett et al., 2015; Desimone, 2009, Killion & Roy, 2009; Knowles et al., 2005; Lehiste, 2015; National Council of Teachers of Mathematics, 2010). A major task planned for this session is to bring students' work samples and identify the errors and misconceptions from those work samples. In addition, teachers are required to categorize these errors and misconceptions and identify potential causes for these. This practical session that requires active engagement of teachers would enhance their ability to recognize problems and identify ways to remedy and prevent the formation of these errors and misconceptions.

The Mathematics Professional Development Day 3 is planned in a way that lays the foundation for collegial relationships and hence the focus is on identifying ways that promote continuous professional development (Knowlton et al., 2015; Desimone, 2009; Killion & Roy, 2009; National Council of Teachers of Mathematics, 2010). In this session how to conduct research, particularly, action research and also how to conduct lesson study will also be looked at to give them a firm grounding to plan and begin their own professional learning. Most importantly, teachers will come up with a plan for their own professional learning and identify and form groups to work collaboratively to enrich their algebra knowledge and teaching skills.

To sum up, research has indicated that professional development has the potential to influence content and pedagogical knowledge, instructional strategies, and consequently student achievement (Desimone, 2009; Killion & Roy, 2009; Polly, 2015; Taton, 2015). However, researchers also have noted that professional development research projects face the challenge of establishing validity researching the relationship between professional development and students' achievement (Banilower, Boyd, Pasley, & Weiss, 2006; Desimone, 2009; Polly, 2015 Yoon et al., 2007). Some of the research discussed above used self-reporting to evaluate the success of professional development (Polly, 2015; Knowlton et al., 2015). This is a limitation as self-reporting could be subject to response bias (Ebert-May et al., 2011; Knowlton et al., 2015). Other research discussed above used formal assessment data on tests to measure the effectiveness of professional development. It is essential to acknowledge the fact that the improvement in students' performance might not be solely due to professional development (Polly, 2015).

Project Description

This project is designed to address the weaknesses identified through a mixed methods study conducted to examine the algebraic content and pedagogical knowledge of sixth grade mathematics teachers. The results of this study showed that in-service teachers lacked algebraic content and pedagogical knowledge. Based on literature and suggestions from academics and this dissertation committee, the project genre chosen to address the problem identified was mathematics professional development. A thematic professional development was organized to advance algebraic content and pedagogical knowledge of the sixth grade mathematics teachers thereby contributing towards the enhancement of algebra instruction – “Mathematics Professional Development to Increase Algebra Achievement.”

The target population is sixth grade mathematics teachers teaching in the Maldivian public schools as the participants of this study were sixth grade mathematics teachers teaching in public schools of the Maldives. However, all teachers teaching mathematics at K-12 are welcome to attend the sessions as these teachers are responsible for laying a solid foundation to ensure their students’ later success in algebra.

Needed Resources

A variety of resources will be needed to conduct an effective professional development. A facilitator who is well-versed in teaching and learning algebra and adult education is needed to facilitate and guide the sessions. The facilitator could be a teacher educator who has a strong research background. Access to research that discusses the teaching and learning strategies of algebra is also required. Student work samples are also required for the second session in addition to assessment data. Assessment data can be provided by the participating teachers and the school respectively.

Technology needed include computers or laptops and internet connection. Participants will be requested to bring their own laptops for the sessions held in the capital city. That is because, teachers working in the capital city either own or schools provide laptops for them and Wi-Fi spots are available in school. Sessions held outside the capital city will be conducted in computer laboratories in those areas where the sessions are conducted. The reason for this is because outside the capital Wi-Fi spots are hardly available but computer laboratories established in those areas have good internet connection. Moreover, these computer laboratories are for teacher use.

Existing Supports

The main support that exists includes the budget for professional development and teachers being released for three days a year to take part in the mandatory professional development. Funds allocated to professional development could be utilized to cover the expenses of conducting the sessions.

Potential Barriers

Potential barriers include transportation difficulties due to the geographical nature of the Maldives. The main form of transportation used to travel from island to island is watercrafts such as small boats and ferries. Ferry service is provided during the weekends as it was established ease the transportation difficulties faced by those who work in the capital islands of the atolls. So, most of the time ferry leaves from the capital island on Thursday afternoon and returns to the capital island on Saturday morning. Weather is unpredictable and during bad weather transportation halts due to rough seas. In addition, lack of resources such as reference materials other than prescribed textbooks in the majority of the islands is a barrier. Lack of professionals willing to travel to islands to conduct professional development is also a barrier. Participants of the study believed that they had sufficient knowledge to teach sixth grade algebra based on the fact that they were able to solve the problems without any difficulty. Therefore, getting these teachers to consider attending the mathematics professional development would be challenging.

Potential Solutions to Barriers

In order to successfully implement the intended mathematics professional development, it is essential to identify potential solutions to identified barriers. Potential solution for the transportation requirement is to connect all the islands via Internet. At the moment each capital island has a teacher resource center with video conferencing facilities. The challenge is travelling to the capital island during rough weather. Therefore, by connecting all the schools via internet will solve most of the problems faced. It will also provide opportunity for in-service teachers to collaborate and share their knowledge and experiences. Allocating a certain percentage of the education budget for library resources is a potential solution. In addition, use of e-books and journals would be a better option as it will not contribute to maintenance cost of libraries. A potential solution to the problem of getting the participants of this study to attend the mathematics professional development could be overcome by sharing the DTAMS results with the participants. DTAMS results showed that the participants lacked algebraic content and pedagogical knowledge although the participants perceived they had sufficient knowledge to teach sixth grade algebra. It is noteworthy that the participants stated that their belief during the interview which was conducted before the administration of DTAMS. After the administration of DTAMS participants mentioned that they found DTAMS ‘pretty tough.’ Participants also stated that they never got the opportunity to attend any mathematics professional development before and enquired whether I was going to conduct any session on algebra.

Proposal for Implementation and Timetable

Mathematics professional development sessions will be arranged on the days allocated for professional development by the ministry of education. The reason why these days are selected is because teachers will be released only on those days. This will avoid any resistance that might arise if the professional development is scheduled for any other day. The tentative academic calendar for 2017 is not released yet. However, the professional development dates for 2017 is decided based on the academic calendar of 2016. Normally, the first professional development is one month after the beginning of academic year. The second one is scheduled for two months from the first professional development day. The third professional development day is scheduled four months after the second professional development day. Professional development days are always a Thursday mainly because Friday and Saturday are the weekends in the Maldives and that would make travelling easy for the participants.

Dates I propose for professional development are 5 January 2017, 20 April 2017, and 3 August 2017. I proposed 5 January 2017 as Mathematics Professional Development Day 1 because that is the end of the first week of the new academic year 2017. In the Maldives, the first week of the academic year is for staff meetings, department meetings, lesson planning, and any other trainings necessary before for classes commence in the second week of the academic year. During this week schools are encouraged to release teachers for trainings. The reason for choosing that date is to help in-service teachers advance their algebraic content and pedagogical knowledge before the teaching starts (in the Maldives teachers report to work one week before students report to schools). Although, this date is one month ahead of the professional development day allocated by the ministry of education, according to principals, they could release teachers as the actual teaching will begin the following week (M. Majeed, personal communication, 31 March, 2016; W. Waheed, personal communication, April 4, 2016). By arranging the Mathematics Professional Development Day 1 before the teaching begins, teachers could start collecting data for the tasks assigned to them for the Mathematics Professional Development Day 2. Proposed dates for Mathematics Professional Development Day 2 and Mathematics Professional Development Day 3 are the most probable days the ministry of education would declare as professional development days (these days will be finalized once the ministry of education releases the academic calendar).

Professional development session timings will be from 0830 hours to 1130 hours and from 1230 hours to 1430 hours as the official working hours in the Maldives is from 0800 hours till 1400 hours. On each professional development day, the training will last for five hours, so that by the end of the three days the required 15 hours of professional development will be completed. Table 9 shows the proposed timetable for the professional development sessions. Specific hour-by-hour detail of this training is in Appendix A.

Table 9

Proposed Mathematics Professional Development Timetable

Sessions and Date	0830 hours to 1130 hours	1230 hours to 1430 hours
Mathematics Professional Development Day 1 5 January 2017	• DTAMS pre-test • Introduction to algebra • Expansion • Factoring • Posing problems	• Textbook and resource analysis • Identifying and locating resources • Preparation of algebra lessons
Mathematics Professional Development Day 2 20 April 2017	• Analysis of students work • Identifying errors and misconceptions • Categorizing errors and misconceptions	• Identifying teaching strategies to remedy the errors and misconceptions • Identifying teaching strategies to prevent the formation of errors and misconceptions • Planning of lessons that would prevent the formation of errors and misconceptions
Mathematics Professional Development Day 3 3 August 2017	• Action Research • Lesson Study	• Individual professional development plan • DTAMS post-test

Roles and Responsibilities of Student and Others

Students do not play any active role in the professional development training. The roles of teachers include attending the three professional development sessions and collecting data required for the tasks assigned to them. The role of principals includes releasing teachers and providing them the necessary support in the form of funding their transportation and allowing them time to collect data and carry out discussions. Moreover, principals are expected to provide teachers the necessary teaching materials and reference books.

Project Evaluation Plan

The short-term objective of the evaluation is to find out the impact of mathematics professional development on the algebraic content and pedagogical knowledge of sixth grade mathematics teachers working in the public schools of the Maldives. Further to this, the long-term objective of this project is to find out the impact of professional development on the students' performance in algebra as measured by test scores. Specifically, my evaluation seeks to answer the following questions:

1. What is the impact of mathematics professional development on the teachers' algebraic content and pedagogical knowledge as measured by DTAMS post-test?
2. What is the impact of mathematics professional development sessions on the students' performance in algebra as measured by test scores?

A goal of the evaluation plan is to identify the shortcomings in implementing the professional development plan and take timely corrective measures. This would help in improving the implementation process which would result in a more practical professional development plan capable of addressing the teaching and learning needs identified through this study. A limitation of this is that due to the small sample size the evaluation data cannot be generalized outside the participant pool.

The research design employed to evaluate professional development will be a mixed methods practical action research project as the professional development was planned to address an educational issue identified locally within the classrooms (Creswell, 2012). As the professional development aims to improve teaching and learning of algebra, lesson study process will be used to gain a deeper understanding of the teaching and learning process (Killion & Roy, 2009). Data will be collected from the students and teachers. Tools used are adapted from Sanders (2009) with permission. Tools used to collect data include the following.

1. Lesson Study – Preliminary Discussion (Appendix F)
2. Lesson Study – Observation Protocol (Appendix G)
3. Lesson Study – Reflection / Evaluation (Appendix H)
4. Evaluating Success of the Lesson – Student Questionnaire (Appendix I)
5. Evaluating Success of the Lesson – Teaching and Learning Questionnaire (Appendix J)

It is essential to know whether professional development is creating any changes in teacher practice and student learning. According to Killion and Roy (2009) professional development evaluations must focus on four major aspects of the work: team efficiency, teach effectiveness, individual members' contributions, and members' effect on practice and student learning (p. 142). Therefore, to check whether the short-term objective of the mathematics professional development was attained, mathematics professional development will be closely monitored during the sessions, at the end of each session, and at the end of the project using formative evaluation, and summative evaluation methods. Formative evaluations focus on the efficiency, its completion of planned actions, and the outcomes of those actions whereas summative evaluations focus on whether the goal of improving teaching quality and student learning were achieved (Killion & Roy, 2009).

Formative Evaluation Plan and Justification

A formative evaluation assesses how well the participants perform, their actions and the short-term outcomes they produce (Killion & Roy, 2009). In evaluating professional development, actions of participants are often noted in formative evaluations rather than the results (Killion & Roy, 2009). Therefore, the Mathematics Professional Development has been designed in a way that formative evaluation takes place during each and every session.

According to Killion (2008) a logic model is useful in formative evaluations of professional development. A logic model consists of five components, namely, inputs/resources, actions, initial outcomes, intermediate outcomes, and results. The logic model of evaluation is in line with Desimone’s Conceptual Model for Professional Development shown in Figure 10. Tables 10, 11, and 12 show the formative evaluation plan for the Mathematics Professional Development sessions.

Table 10

Formative Evaluation Plan for Mathematics Professional Development Day 1

Inputs/resources	Actions	Initial outcomes	Intermediate outcomes	Results
1. Support from facilitator to increase algebraic content and pedagogical knowledge, and questioning skills	1. Prepare activities 2. Prepare questions 3. Identify appropriate resources	1. Teachers develop conceptual understanding 2. Teachers use the activities and the questions in their classrooms	1. Students develop conceptual understanding and practice applying ideas in solving complex problems	1. 30% increase in teachers' algebraic content and pedagogical knowledge 2. 20% increase in students' assessment scores within a year
Planned actions		Intended results		

Table 11

Formative Evaluation Plan for Mathematics Professional Development Day 2

Inputs/resources	Actions	Initial outcomes	Intermediate outcomes	Results
1. Teachers bring students work samples 2. Facilitator helps teachers in categorizing and analyzing students' work samples	1. Teachers analyze students' work samples 2. Teachers prepare lesson plans to address the issues identified	1. Teachers use the lesson plans in their lessons 2. Students complete activities designed for them based on the analysis of their work	1. Teachers analyze data from assessments to determine who needs help most and what type of help they need	1. 50% increase in teachers' algebraic content and pedagogical knowledge 2. 30% increase in students' assessment scores within a year
Planned actions		Intended results		

Table 12

Formative Evaluation Plan for Mathematics Professional Development Day 3

Inputs/resources	Actions	Initial outcomes	Intermediate outcomes	Results
1. Facilitator explains actions research and lesson study 2. Facilitator helps teachers in creating their own professional development plan	1. Teachers identify the areas they could improve further 2. Teachers prepare an action research plan and a lesson study plan	1. Teachers implement lesson study as collaborative professional development 2. Teachers revise the lesson plans and activities and implement them again	1. Teachers increase their algebraic content and pedagogical knowledge 2. Teachers are flexible in catering the diverse needs of students in a classroom	1. 75% increase in teachers' algebraic content and pedagogical knowledge 2. 40% increase in students' assessment scores within a year
Planned actions		Intended results		

Summative Evaluation Plan and Justification

A summative evaluation helps to determine whether the goals of professional development have been achieved (Killion & Roy, 2009). Summative evaluations occur at the end of a planned action. According to Killion and Roy (2009) a useful form of summative evaluations includes completion of pre- and post-tests upon completion of the planned actions. Therefore, the short-term summative evaluation will be carried out by administration of DTAMS Post-test to determine the change in teachers' algebraic content and pedagogical knowledge right after the completion of 15 hours of mathematics professional development.

The long-term summative evaluation takes place one year after the completion of mathematics professional development, that is, after the teachers implement their action research plan and/or lesson study (Knowlton et al., 2015; Mansour, Albalawi, & Macleod, 2014). At this stage two forms of summative evaluations will take place. Details of the two forms of summative evaluations are provided below:

1. Performance of students will be measured at the end of the year using their end-of-year exam scores
2. DTAMS Post-test will be administered to the participants to measure their algebraic content and pedagogical knowledge

I acknowledge the limitation of using students' end of year exam scores and DTAMS post-test to measure the impact of mathematics professional development as the exam results might not be exclusively influenced by the mathematics professional development. However, literature showed that often student achievement scores and/or pre- and post-test scores of participants are used to measure the effectiveness of professional development (Krawec & Montague, 2014; Lane et al., 2015; Polly, 2015).

Overall Evaluation Goals

The goals of this project were to improve algebraic content and pedagogical knowledge of the sixth grade mathematics teachers by providing them with necessary support. Further to this, this project also aimed at making teachers life-long learners who are in-charge of their own professional growth, and who could effectively collaborate professional learning. The participating teachers will have support for two years from the facilitators. In this project I aim to strengthen algebra instruction and students' classroom experiences through enhancement of algebraic content and pedagogical knowledge of the teachers. On the whole, this project aims to reinforce instructional strategies and questioning techniques used by the teachers. This is believed to positively contribute towards the development of students' creativity, their problem-solving skills, and their analytical abilities (Barrett et al., 2015; Baxter et al., 2014; Krawec & Montague, 2014; Polly, 2015).

Key Stakeholders

Key stakeholders include Ministry of Education, schools, teachers, students, families, and community. Ministry of Education is a key stakeholder as they provide the funding for professional development. Schools are key stakeholders as they release teachers, as well as they give permission to collaboratively work with other schools in the country. Teachers are key stakeholders as they are the ones who are actively involved in the professional development. Students are stakeholders as teachers would be using the answer scripts of their students and asking feedback from their students as to how they could improve classroom instruction. Families play an important role as teachers would take time out of their families to attend professional development and work collaboratively on projects they design for their own professional growth. Community support and understanding is essential especially at the implementation stage as the results of this project might not be visible from the students' results in the short-term, that is, immediately after the initial session of this project.

Project Implications

Possible Social Change Implications

This project is intended to bring a positive social change by addressing the problems identified through this study. One possible social change implication of the project presented in Appendix A includes enhancement of algebra instruction. This might contribute towards the improvement in algebra performance of the students. In turn, this would raise the performance of students in those schools in national exams. Further to this, teachers will be able to take charge of their own professional learning and collaboratively work towards enhancement of algebra instruction in their own schools and the other schools in their atolls.

Importance of the Project to Local Stakeholders

This project is important as this is aimed at improving algebraic content and pedagogical knowledge of sixth grade algebra teachers. During the research it became apparent that stakeholders believed one of the factors contributing to students' low performance in algebra could be due to lack of algebraic content and pedagogical knowledge of the in-service teachers. Results of this study suggested that the teachers who participated in this study lacked algebraic content and pedagogical knowledge. As the issue needed to be addressed immediately, it was found that the best way to address the issue was by conducting professional development. Research has linked professional development to improve teachers' content and pedagogical knowledge, and their confidence (Polly, 2015; Taton, 2015). This project is essential in contributing towards the enhancement of algebraic content and pedagogical knowledge of sixth grade mathematics teachers. Improvement in algebraic content and pedagogical knowledge will positively contribute towards the enhancement of students' algebra performance in these highly populated schools which could contribute to an improvement in students' algebra performance nationally.

Importance of the Project to Broader Community

The project presented in Appendix A could be used in other nations where the problem exists. In addition, this project could be used to develop and strengthen algebra instruction. There is no single answer that would meet all the needs of teachers, however, developing teacher content and pedagogical knowledge and empowering them to take charge of their own professional learning would definitely go a long way.

Conclusion

This study aimed to examine algebraic content and pedagogical knowledge of sixth grade mathematics teachers. The results of the study showed that teachers lacked algebraic content and pedagogical knowledge. After evaluating the possible project options, it was decided to design mathematics professional development to address the problem. A review of literature, best practices, and activities that worked best in mathematics professional development informed the project presented in Appendix A.

Furthermore, project evaluation and possible social change were also discussed in this section. In the following section project development, its strengths and the project's potential impact on social change, and directions for future research are discussed.

Section 4: Reflections and Conclusions

Introduction

This project was aimed at strengthening algebra instruction through enhancement of algebraic content and pedagogical knowledge of sixth grade mathematics teachers. In addition, the project aimed at empowering teachers to take charge of their own learning and laying the foundation for them to be lifelong learners. In this section, the project strengths and limitations are discussed. Furthermore, recommendations and alternative approaches that could be used to address the problem identified in the study are examined. In addition, analysis of self, analysis of the project and its implications, and directions for future research are also presented.

Project Strengths and Limitations

Project Strengths

The project presented in Appendix A was designed to address the needs of the sixth grade mathematics teachers who took part in the study. The project was informed by best practices that have worked well for improving the content and pedagogical knowledge of teachers through professional development (Desimone, 2009; Killion & Roy, 2009; Polly, 2015; Sanders, 2009; Taton, 2015). The strengths of this project are three-fold. First, this project restructures the professional development to focus on subjects and also the areas in which teachers most need help. The participants of this study highlighted that they felt the previous professional development sessions they attended were unproductive as they were focused on generic issues such as classroom management or use of PowerPoint presentations in class. This problem has been addressed by focusing this mathematics professional development on the areas these participants identified as the areas in which they needed help. This is a major change from the professional development the participants have experienced before.

Second, this project is designed to provide opportunities for teachers to step out of their comfort zones and implement various instructional strategies through collaborations among mathematics teachers, peer observations, and common planning of lessons. I observed that participants followed the textbook word-for-word and they failed to identify mistakes in the textbook. This study's mathematics professional development is designed in a way that by the end of the session, the participants will have lesson plans for each of the algebra subtopics they will teach in sixth grade. Collaboratively planning the lesson and activities would help the participants to try new teaching strategies that they have devised. Moreover, they will be confident in trying new strategies instead of following the textbook word-for-word (Lane et al., 2015). Encouraging the participants to try new teaching strategies and providing them a platform on which to collaborate with other teachers is a key strength of this project.

Third, this project is designed to take into account the needs of the in-service teachers and the resources (or lack of resources) available to them. Participants of this study highlighted that they did not have access to resources or reference materials other than the prescribed textbooks. This project is designed to deliver mathematics professional development with minimal resources or no resources except the facilitators. Moreover, through this project the participants will learn how to use the tools available to them such as their smart phones to locate relevant resources. Participants will learn how to make use of what is available rather than focusing on what is not available to them. This is considered a strength because in the Maldives there is unequal distribution of wealth and resources due to the geography (UNDP, 2014). This project is designed in a way that could be delivered in any of the islands in the Maldives, which is also considered as a major strength of this project.

Project Limitations

One of the major limitations of this project would be convincing teachers such as the participants who believed that they did not need any professional development to take part in the mathematics professional development. Participants of this study declared that they had over a decade of experience teaching “the same stuff” and since they were “able to do all the questions in the textbook” they believed that they had sufficient knowledge to teach algebra at the sixth grade level.

Another limitation of this project is the lack of experts in the islands of the Maldives who could provide on-site support to the teachers and facilitate the implementation of their own professional development plan. This gives rise to the problem of not being able to provide assistance when it is needed. For example, a teacher who suddenly needs expert advice would have to contact the facilitator over the phone since there is no one in the school or on the island who could help. The facilitator might not be available at the time of need and this could dishearten the teacher and make the teacher give up.

In some schools of the Maldives there is only one teacher teaching the subject to the same grade due to the small populations on the islands of the Maldives. This is considered a limitation of the project as there is no opportunity for peer observations in these schools. There is a significant difference between planning lessons together and actually being able to observe someone implement the lesson planned together. It is a great learning opportunity to be able to observe someone implement the lesson in a real classroom setting. This presents the observer with the opportunity to learn how to deal with issues that arise unexpectedly. This limitation of not having the opportunity to observe peers presents another disadvantage, that is, the teacher would not get the opportunity to conduct a lesson study even if the teacher so desired.

Recommendations for Alternative Approaches

I recommend that trainers be trained who could conduct the mathematics professional development in various schools. Also, I recommend that the initial group who took part in the mathematics professional development meet regularly to report their progress, share their practices, and plan their own professional learning. Moreover, I recommend that the initial group who took part be the first group of trainers so that they could conduct mathematics professional development for the teachers across the country. I further recommend that each of these trainers be a leader in learning teams to facilitate collaboration and continuous professional development.

An alternative way to address the problem is to allocate an expert for each of the areas to conduct short courses to enhance algebraic content and pedagogical knowledge. However, this approach might not be practical as it would involve substantial travelling. Moreover, this approach would be relatively expensive for the education ministry. Therefore, chances of this alternative working are slim. Another approach would be to conduct short trainings during the school break. The drawback would be that this method might not be popular among the teachers as the breaks teachers get are the only times they can spend with their families.

Another alternative approach would be to use online resources such as those available from the United States National Council of Teachers of Mathematics, Khan Academy, Math Goodies, Math Resources for Teachers, and Math.com as well as a number of popular mathematics teacher blogs. This approach could address the issue of not having enough facilitators across the country to assist these teachers. A major challenge with this alternative is the lack of availability of strong and reliable internet connectivity in the majority of the islands in the Maldives. Also, I would like to acknowledge that for this alternative approach to work, teachers would need to be self-motivated, willing to recognize the need for their self-development, and willing to invest time and effort to improve themselves professionally.

Scholarship, Project Development, and Leadership and Change

As a scholar I have spent a lot of time reading, and also in the field while collecting data. Through the readings I have learned best practices that worked and brought an improvement to the instructional practices. In addition, working in the field during the data collection I have learned firsthand the challenges faced by teachers and schools at various levels. I have gained subject matter knowledge as well as insights into the real problems faced by teachers teaching sixth grade algebra. Moreover, I have come to understand the challenges faced by schools in arranging professional learning sessions and creating a collaborative working environment for the teachers. Throughout this study

I have improved my knowledge and understanding of effective professional development. I have enhanced analytical skills and identified ways of addressing the problem using the minimal resources.

As a practitioner I have come to understand the differences between pedagogy and andragogy. I have learned how challenging it is to get teachers motivated enough to take charge of their own professional learning and change the way they have been teaching their students. There is a huge difference between involving teachers who believe they have sufficient knowledge and those who believe there is room for improvement. I could see that some teachers were reluctant to let go of what they have been doing and what seemed to work well.

The development of this project was challenging, time consuming, exasperating at times. However, this was valuable to understanding processes to implement educational changes based on research-based strategies. As the developer of this project I have learned to take lead and play an active role in planning which included identifying the practices that are suitable to the local settings. This study has contributed towards enhancing my knowledge and understanding of effective professional development. Moreover, I have gained insight into strategies that would encourage collaboration and promote continuous professional development among teachers of mathematics. I have learned how to create a network that would help the teachers overcome the difficulties faced due to the geographical nature of the country.

Leadership does not mean having the ability or the influence to change policies, but it means to bring changes at different levels. Leaders should be able to find ways to overcome challenges. I have learned that the way teachers viewed mandatory professional development could be changed through effective professional development. I have learned that empowering teachers goes a long way. They become reflective practitioners who are able to identify the areas they need help and able to find resources and support they need to grow professionally. Bringing a change to the way teachers have been practicing and providing them the with learning experiences that change their thinking would bring a positive and a lasting change to the profession in general and their teaching practices in particular.

Reflection on the Importance of the Work

This project is without doubt very important as it addressed an urgent issue identified in the education system of the Maldives. Challenges faced by teachers and schools will be different. Moreover, the way these challenges are viewed and dealt with will vary from person to person and school to school. This professional development is designed in a way that could be modified to adapt not only to the schools of the Maldives but also globally. This project could be delivered using minimal resources which makes this project easy to deliver in the developing countries as well as small island nations facing the same problem.

This project was designed after a thorough review of literature and best practices. In addition, the needs of teachers were taken into account. Therefore, this project could play an important role in enhancing algebra instruction through enhancement of algebraic content and pedagogical knowledge of mathematics teachers. This mathematics professional development could also be used as a refresher for in-service teachers.

Collaboration is essential to continuous professional development. This project guides teachers to various ways of continuing their professional learning. In addition, this training lays foundations for teachers to conduct action research and lesson study. Lesson study, which involves groups of teachers collaboratively planning, teaching, observing and analyzing learning and teaching in ‘research lessons’, is a highly specified form of classroom action research focusing on the development of teacher practice knowledge (Li & Huang, 2013). Action research and lesson study are very effective in education settings to improve practice (Berg, Lune, & Lune, 2004; Dudley, 2011). As this project study provides a platform to address or to improve teamwork and collaboration, this project is very important in strengthening algebra instruction through improved teacher content and pedagogical knowledge.

Implications, Applications, and Directions for Future Research

Social Change

A prospective social change at the individual level include enhancing algebraic content and pedagogical knowledge of the participants, improving their instructional practices, empower the participants to step out of their comfort zone and try new teaching strategies, and take ownership of their own learning. The potential impact of this project for families of learners includes improving students’ learning experiences through enhanced classroom instruction, improve students’ performances through improved instruction, develop students’ creativity, analytical ability and their problem-solving skills through implementation of various instructional strategies and best practices. In addition, students will be able to communicate with their peers and develop trust and engage in non-threatening manner. Impact at family level would also include developing well-disciplined children who are ready to face 21st century.

One possibility for social change includes informing teacher training institutions of the results of this study which could inform mathematics teacher training needs. This in turn could inform the teacher training curriculum and mathematics professional development curriculum. Re-designing the teacher training curriculum and the professional development curriculum around the needs of the local teachers would contribute positively to enhance the mathematics education in the country. Moreover, this could contribute to the economy through reducing over dependence on the expatriates.

Future Research

One of the recommendations for future research is to assess algebraic content and pedagogical knowledge of the in-service teachers across the country to identify their algebraic content and pedagogical knowledge strengths and weaknesses. It is essential to identify the algebraic content and pedagogical knowledge strengths and weaknesses of the teachers in order to provide them the necessary training through mathematics professional development. This will also inform the development of mathematics professional development curriculum for the in-service teachers.

Another recommendation for future research is to assess algebraic content and pedagogical knowledge of the pre-service teachers across the country to identify their algebraic content and pedagogical knowledge strengths and weaknesses. Identifying the algebraic content and pedagogical knowledge strengths and weaknesses of pre-service teachers might help determine how effectively teacher training institutions prepare the teachers to teach sixth grade algebra. This could provide invaluable information to inform the development of mathematics teacher training curriculum in the Maldives.

Conclusion

The study presented in this dissertation examined algebraic content and pedagogical knowledge of sixth grade mathematics teachers who consented to take part in this research. Results of this study showed that the participants lacked algebraic content and pedagogical knowledge required to teach algebra at sixth grade. As a result, a project was designed to address the algebraic content and pedagogical knowledge weaknesses of the participants. This mathematics professional development project was designed to strengthen algebra instruction through enrichment of algebraic content and pedagogical knowledge of the participants. Through this project teachers will develop and advance their conceptual understanding, analytical ability, and their problem-solving skills.

Moreover, this project was designed to help participants to come out of their comfort zones and implement various teaching strategies. Participants will be equipped with the knowledge and skills to plan and prepare lessons that address the diverse needs of the students. Instead of following the textbook word-for-word the teachers will be able to locate and adapt various resources to suit the needs of their students. Further to this, teachers will be empowered to take charge of their own professional growth and plan their own professional learning.

To sum up, the participants will strengthen the instructional practices and contribute towards improvement of algebra performance at the school level, and perhaps gradually at the national level through collaborations with teachers across the country. These participants could become the nucleus of a group of regional experts. A positive change will follow as the project is implemented, and the teachers become knowledgeable in the subject matter they teach and confident in their instructional strategies.

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Appendix A: The Project

Mathematics Professional Development Day 1 (MPD D1)

MPD D1 Date and Location

Mathematics Professional Development Day 1 will take place on 5 January 2017 at one of the Schools in the Capital City of the Maldives. Venue will be informed once it is finalized.

MPD D1 Agenda

Time and Duration	***Session Details***	***Activity***
0830-0835 5 minutes	Registration	NA
0835-0840 5 minutes	Welcome remarks	NA
0840-0910 30 minutes	DTAMS pre-test	Participants will do the DTAMS pre-test under examination conditions
0910-0955 45 minutes	Introduction to algebra	Participants in pairs or small groups will prepare lesson plans and a variety of lesson activities to introduce algebra at sixth grade.
0955-1035 40 minutes	Expansion of algebraic expressions	Participants in pairs or small groups will prepare of various activities to teach expansion of algebraic expressions.

Time and Duration	***Session Details***	***Activity***
1035-1045 10 minutes	Coffee break	
1045-1100 15 minutes	Factoring of algebraic expressions	Participants in pairs or small groups will prepare various activities to teach factoring of algebraic expressions.
1100-1130 30 minutes	Posing problems	Participants in pairs or small groups will prepare a list of possible questions of varying levels that could be asked in an algebra lesson.
1130-1230 1 hour	Lunch and prayer break	
1230-1315 45 minutes	Textbook and resource analysis	Participants are expected to bring the textbooks they use to this session. In pairs or small groups will analyze the prescribed textbooks and resources to identified discrepancies (if any) and make suggestions to address those.
1315-1330 15 minutes	Identifying and locating resources*	Participants in pairs or small groups will prepare a list of resources and locate those resources that could be utilized in teaching sixth grade algebra

Time and Duration	***Session Details***	***Activity***
1330-1345 15 minutes	Tea break	
1345-1430 45 minutes	Preparation of lesson plans including a variety of ways of assessing students' performance during a lesson Introduce socrative.com to the participants. ***Note:*** *Socrative.com is a free online tool that can be utilized for preparing assessments. It is user-friendly and fits well to the Maldivian teachers.*	Participants in pairs or small groups will prepare six lesson plans (one for each of the subtopics of algebra taught at sixth grade). Use one online tool to assess students during a lesson (for example, use of socrative.com, which is a free online platform for preparing assessments. Students can work at their own pace and will get live feedback)

During the group work the facilitator observes and checks on the work and contribute to discussions within those groups. After each presentation facilitator as well as participants provide comments and feedback.

*Internet or Wi-Fi, and either laptops, desktop computers, iPads, or smart-phones are required for this activity.

MPD D1 Facilitator Notes

Introduction to Algebra

Before the lesson facilitator communicates the expected learning outcomes to the participants. Facilitator starts the session by asking each of the participants to take a piece of paper. Ask them to:

Think of a number

Add 15 to that number

Multiply the result by 3

Subtract 9 from the result

Multiply the result by 2

Subtract 6 from the result

Divide the result by 6

Add 1 to the result

Subtract 8 from the result

Tell me your answer and I will tell you the number you thought of

When each participant gives their answer, the facilitator subtracts 4 from that and tell the participant the number the participant initially thought of.

The facilitator asks the participants to figure out how the facilitator worked out the numbers they thought of so quickly (because if the facilitator were to mentally work out the process backwards it will take a lot longer). Give the participants three minutes to figure out. They could discuss among themselves. Give participants five minutes to share their answers before explaining the process.

Since there are many of you, each one of you could think of any number. Mentally doing the calculations backwards to get the number each one of you thought would take a lot of time. So what I did it, I let the number you thought be 'n'. And…

Think of a number (n)

Add 15 to that number ($n + 15$)

Multiply the result by 3 ($3n + 45$)

Subtract 9 from the result ($3n + 36$)

Multiply the result by 2 ($6n + 72$)

Subtract 6 from the result ($6n + 66$)

Divide the result by 6 ($n + 11$)

Add 1 to the result ($n + 12$)

Subtract 8 from the result ($n + 4$)

So the number each one of you tells me is four more than the number you thought of. Once you tell me the answer I simply subtract four and tell you the number you thought of.

Facilitator asks "so what do the letters used in algebra, like x and y, actually mean? (Pause to see if anyone responds).

Problem 1: You can exchange 16 rufiyaa for 1 dollar. You have d dollars. You can exchange this money for r rufiyaa. What is the relationship between d and r? (Adapted from Haylock, 2010, p. 249).

Haylock (2010) stated that most of the participants are likely to write down either $16r = 1d$ or $1d = 16r$ or $16r = d$ or $d = 16r$ (which is the same thing written in different ways).

Ask the participants to share their solutions and ask them to explain the reasoning behind their responses.

Facilitator explains that the correct relationship is $r = 16d$. Explain the relationship using a visual such as a table so that it is less abstract.

Number of dollars ***d***	**Number of rufiyaa** ***r***
1	16
2	32
3	48
4	64
5	80

According to Haylock (2010) many people well qualified in mathematics get this relationship wrong because in arithmetic the letters are used as abbreviations (for example, *d* for dollars and *r* for rufiyaa). However, in algebra these letters represent variables. That is *d* does not represent dollars but it represents *number of dollars*. The letter *d* in Problem 1 stands for ‘whatever number of dollars you choose.’

Problem 2: The number of students in a school is *s* and the number of teachers is *t*. There are 20 times as many students as teachers. Write down an equation using *s* and *t* (Extracted from Haylock, 2010, p. 250).

Facilitator asks the participants to explain their responses. Facilitator explains that the correct equation should read $s = 20t$. If need be facilitator uses visual (as with Problem 1) to explain the equation.

Learning and Teaching Points:

1. When you introduce algebra emphasize the idea that a letter in algebra stands for ‘whatever number is chosen’ that is, a variable (Haylock, 2010).
2. Avoid fruit-salad approach to explaining algebraic statements, for example, referring to 3*a* as 3 apples and 5*b* as 5 bananas, or anything that reinforces the ideas that the letters stand for objects or specific numbers (Haylock 2010; Tennant & Colloff, 2014).

Introducing the Idea of Letter being a Variable to Children:

As algebra helps us to express generalizations, students can be exposed to the idea using games such as “what’s my rule?” For example,

A	B
1	3
2	5
3	7
4	9
5	11
6	13
7	15
8	17
9	19
10	21
100	? (201)
x	? ($2x + 1$)

Haylock (2010) stated that "what's my rule?" game could be used to introduce young children to algebraic thinking.

Task: *Participants work in pairs to prepare an introductory algebra lesson including at least one activity that could be used to introduce algebra.* Participants will be given 10 minutes.

Each pair explains how they will carry out the lesson and activity with the entire group. Each pair will be given 3 minutes to do this. This task would help the facilitator to assess whether participants are able to distinguish between variables and abbreviations, and also whether they have correct understanding of the concept to teach the concept to children.

Expansion of Algebraic Expressions

According to Rock and Brumbaugh (2010) students have been encouraged to memorize many of the generalizations which are common in algebra often without any relation to problems. For example, students know $x^2 - y^2 = (x+y)(x-y)$ but they do not know $x^2 - 64 = (x+8)(x-8)$ or $53^2 - 24^2 = (53+24)(53-24)$.

Facilitator explains the concept of difference of the two squares using a visual as follows:

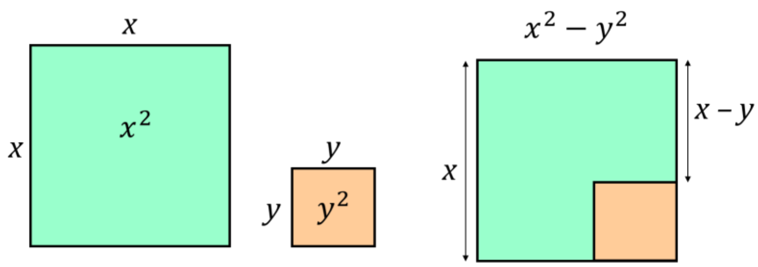

Figure 1

Explain that when a square of dimension y is removed from a square of dimension x it looks like as shown in Figure 1 above. This is followed by cutting of the extra bit at the bottom, and rotating it and putting it together as shown in figure 2.

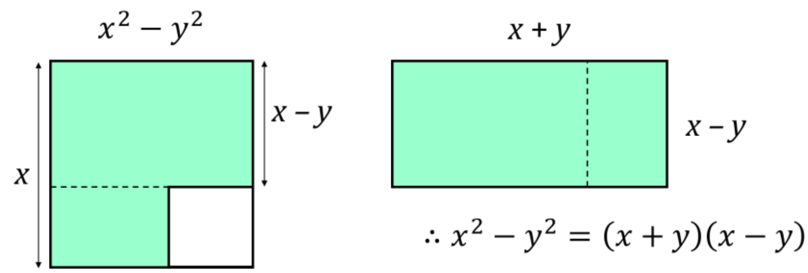

Figure 2

Task: *Describe or create an activity that could be used to show the expansion of* $(x+y)^2$ *and* $(2x+5)(x+3)$. *You may work in pairs.*

Once the task in completed the participants will share their work with the entire group. Facilitator checks how each pair is working while they are at work.

Factoring of Algebraic Expressions

Simply put, when you multiply two numbers, the two numbers are factors of the third number (result). When you factor an algebraic expression, let us say, two terms, we look for what is common in those two terms. For example:

$$x^2y + 2xyz$$

Show that each of the terms can be expressed as follows:

$$x^2y = x \times x \times y \text{ and } 2xyz = 2 \times x \times y \times z$$

When you look at the two terms, what is common to the two terms? *x* and *y* are the only common ones. So once we take the common ones, we are left with x and 2*z*. Therefore,

$$x^2y + 2xyz = xy(x + 2z) \text{ (The Math Forum, 2003; 2004)}$$

Task: *Create an activity to factor the following algebraic expressions. You may work in pairs or groups of three. You have 6 minutes to complete this task.*

a) $8x^3p^2 - 24xpq$

b) $3xyz + 6pqz - 2rz$

c) $ab + 2bc - 5ac$

Each pair explains the activity they have prepared to teach factoring of algebraic expressions.

Posing Problems

Facilitator asks the participants to list the characteristics of a good question or a problem with reasons. Give the participants five minutes to come up with a list either individually or in pairs. Once they have the list ask the participants to share their list with the entire group and discuss. Allow five to ten minutes for discussion.

Task 1: *The length of a garden is f feet. Measured in yards, it is y yards long. What is the relationship between f and y? (There are three feet in one yard.) What criticism could you make of this question?* (Extracted from Haylock, 2010, p. 262)

Task 2: *If I buy a apples at 10p each and b bananas at 12p each, what is the meaning of (a) a + b; (b) 10a; (c) 12b; and (d) 10a +12b? What criticism could you make of this question?* (Extracted from Haylock, 2010, p. 262)

Recall Bloom's Taxonomy and ask the participants to come up with five questions for one of the sub-topics in grade six algebra syllabus. These five questions should be of different levels of Bloom's Taxonomy. In order to avoid repetition, facilitator will assign each group a sub-topic from the grade six algebra syllabus. Allow five minutes for this activity. Once they have completed the task, share with the entire group and discuss the questions.

Textbook and Resource Analysis

Facilitator will ask the participants to work in pairs and go through the prescribed algebra textbooks and resources for sixth grade. Participants are required to check whether the explanations offered are correct and sufficient, whether the examples are appropriate, and any other issue they observe. Participants are required to justify their claims and address the shortcomings in the definitions, examples, and explanations given in the textbooks and resource materials. Participants will develop activities or questions that they would include in the textbook if they were given the opportunity to do so. Once the task is completed it will be shared with the entire group and discussions will take place.

Identifying and Locating Resources

Facilitator asks participants to use any search devise and locate resources (such as websites) that could be used to teach algebraic concepts. This is an individual task. Participants are required to share the resources with the entire group and explain how the identified resource can be used to teach which area of algebra. The purpose of this activity is to ensure that all the participants know where to locate resources and how the resources could be used as teachers might not have the time to locate and find resources once the academic year begins.

Preparation of Lesson Plans

Participants will work in pairs and prepare a full lesson plan for the topic assigned to them. The lesson plan should include details of the questions, activities and any assessment tasks they will give their students. This will be presented and shared among the participants. The purpose of this activity is to ensure that each of the teachers have at least one lesson (the introductory lesson) for each of the sub-topics of algebra before the semester begins. This basically serves as a survival kit for the teachers.

Facilitator explains how scorative.com could be used to assess students and give timely feedback during a lesson. A practical session will be conducted by the facilitator to ensure each of the participants know how to use the tool.

Task: *Participants are required to prepare a quiz using socrative.com to assess the sub-topic they were assigned.*

The session will be concluded by asking the participants to collect students' work samples (particularly the incorrect solutions) for the Mathematics Professional Development Day 2.

MPD D1 Evaluation Tasks

1. Introductory lesson plan and activities for the subtopics identified
2. List of possible questions that could be asked
3. Analysis of textbooks and resources
4. Utilizing online tools to continuously assess the performance of students during a lesson

Mathematics Professional Development Day 2 (MPD D2)

MPD D2 Date and Location

Mathematics Professional Development Day 2 will take place on 20 April 2017 at one of the Schools in the Capital City of the Maldives. Venue will be informed once it is finalized.

MPD D2 Agenda

Time and Duration	***Session Details***	***Activity***
0830-0840 10 minutes	Registration	NA
0840-0845 5 minutes	Welcome remarks	NA
0845-0915 30 minutes	Analysis of students work samples	Participants work in groups to identify the common errors and misconceptions from the work samples of their students.
0915-1015 1 hour	Identifying errors and misconceptions	Facilitator presents the common errors students make and the misconceptions they form in the light of literature. This will be followed by a group discussion. Facilitator also answers questions from the participants (if any).
1015-1030 15 minutes	Coffee break	

Time and Duration	***Session Details***	***Activity***
1030-1130 1 hour	Categorizing errors and misconceptions	Participants work in groups and categorize the errors and misconceptions identified from the students' work samples. Also participants present the common errors and misconceptions they have identified to the audience.
1130-1230 1 hour	Lunch and prayer break	
1230-1315 45 minutes	Identifying teaching strategies to remedy the errors and misconceptions	In groups participants discuss and presents to the audience what they think can be done to remedy the errors and misconceptions. Facilitator explains how formation of these errors and misconceptions can be remedied in the light of literature.

Time and Duration	***Session Details***	***Activity***
1315-1350 35 minutes	Identifying teaching strategies to prevent the formation of errors and misconceptions	In groups participants discuss and presents to the audience what they think can be done to prevent the formation of the errors and misconceptions. Facilitator explains how formation of these errors and misconceptions can be prevented in the light of literature.
1350-1400 10 minutes	Tea break	
1400-1430 30 minutes	Planning of lessons that would prevent the formation of errors and misconceptions	In small groups participants discuss and revise the lessons and activities they prepared during the Mathematics Professional Development Day 1 and present to the audience. Participants also prepare one additional activity for each of the lessons they have prepared earlier.

During the group work the facilitator observes and checks on the work and contribute to discussions within those groups. After each presentation facilitator as well as participants provide comments and feedback.

MPD D2 Facilitator Notes (Presentation)

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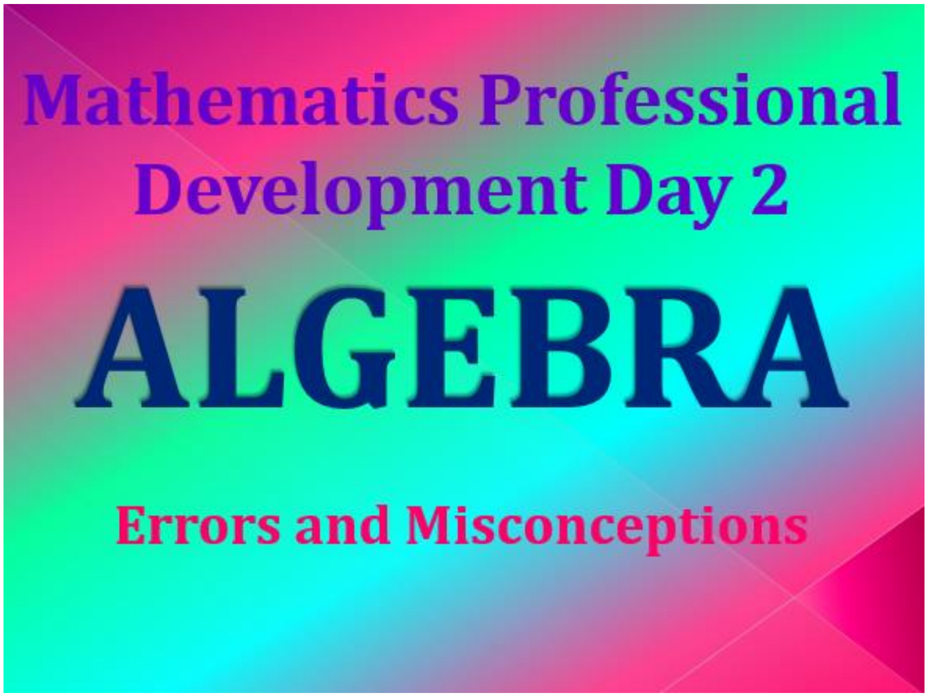
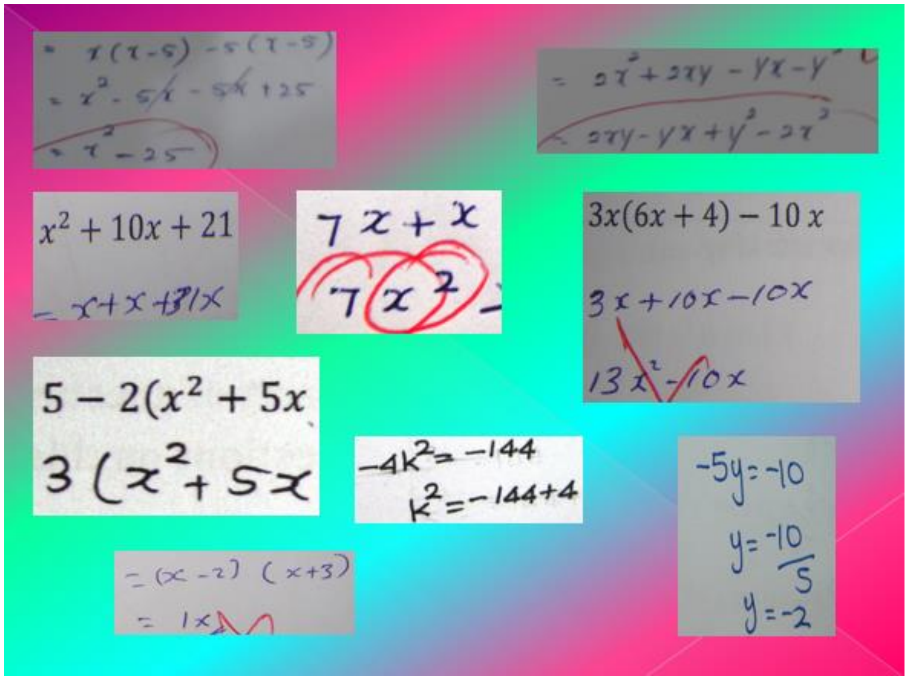


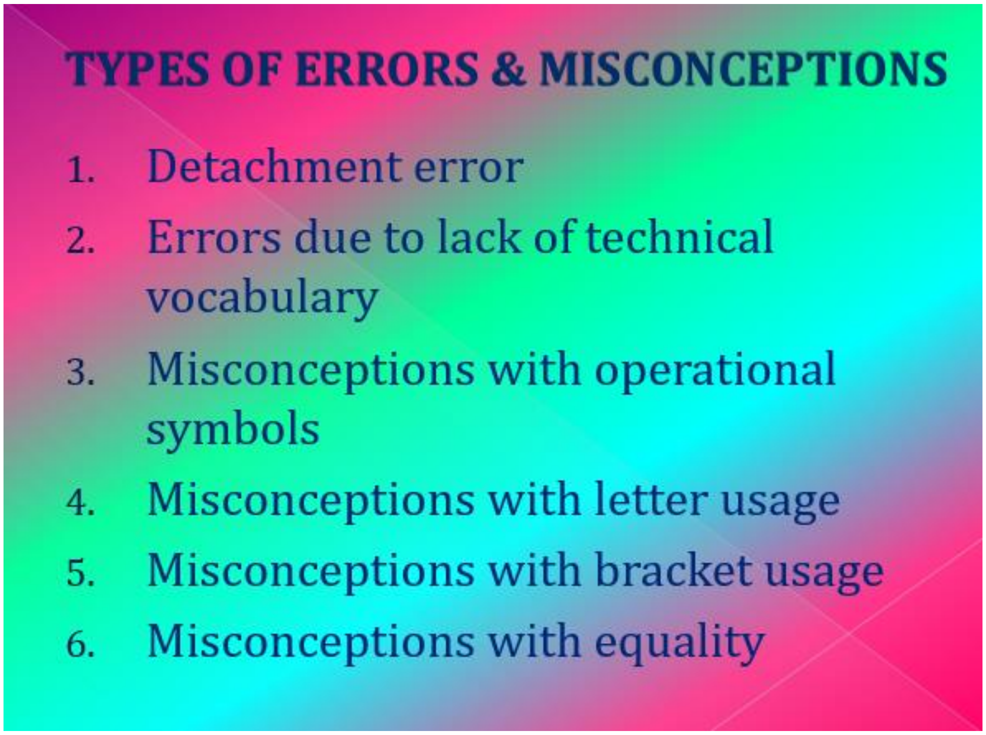
Facilitator Note: Algebra has been considered as an obstruction which prevents students from taking mathematics courses in high school (Welder, 2012; and Brown, Davis, & Kulum, 2011). This prevents students from pursuing careers in science, technology or mathematics.

Slide 2:



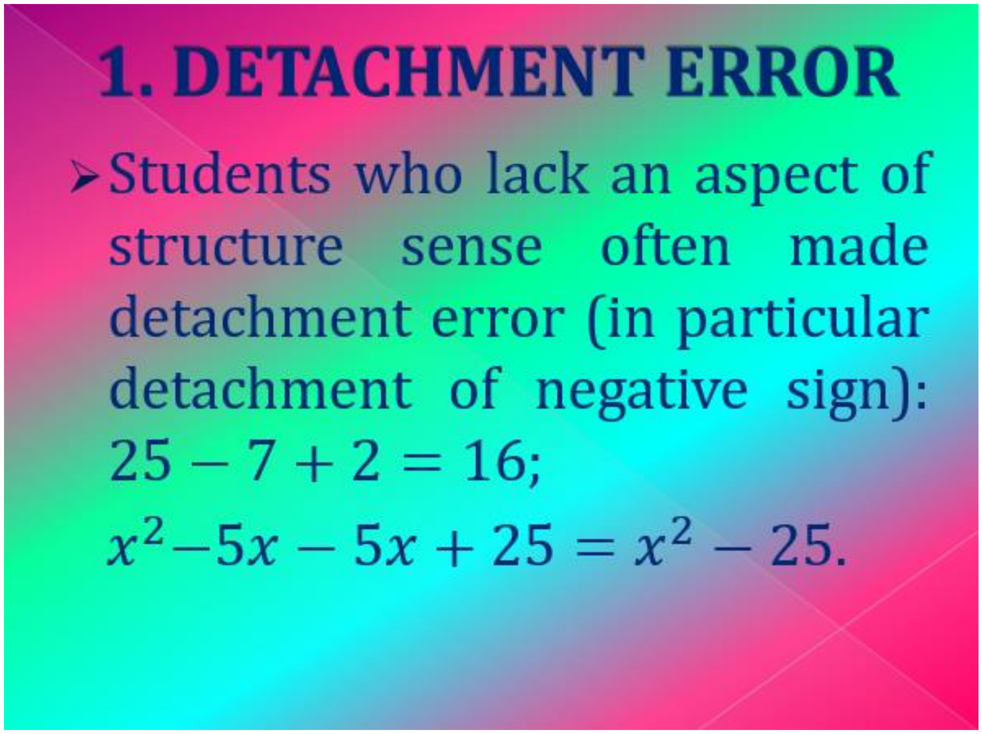
Facilitator Note: Research indicates that students face numerous difficulties in understanding algebra due to lack of understanding of symbols and letters, and of manipulation of algebraic expressions and equations (Kuchemann; Booth; Kieran; and MacGregor & Stacey, as cited in Banerjee, & Subramaniam, 2012). Welder (2012) pointed out that these difficulties could be due to the existing knowledge students have which may be incomplete or misunderstood. Teachers who introduce algebra to the students are responsible for building a solid foundation on which students can later construct their algebra knowledge.

Slide 3:



Facilitator Note: Errors students make and misconceptions they have are not just careless mistakes but are intelligent generalizations that result from their previous learning. In order to build a solid foundation and to prevent and correct misconceptions, it is important that the teachers know these common errors and misconceptions.

Slide 4:



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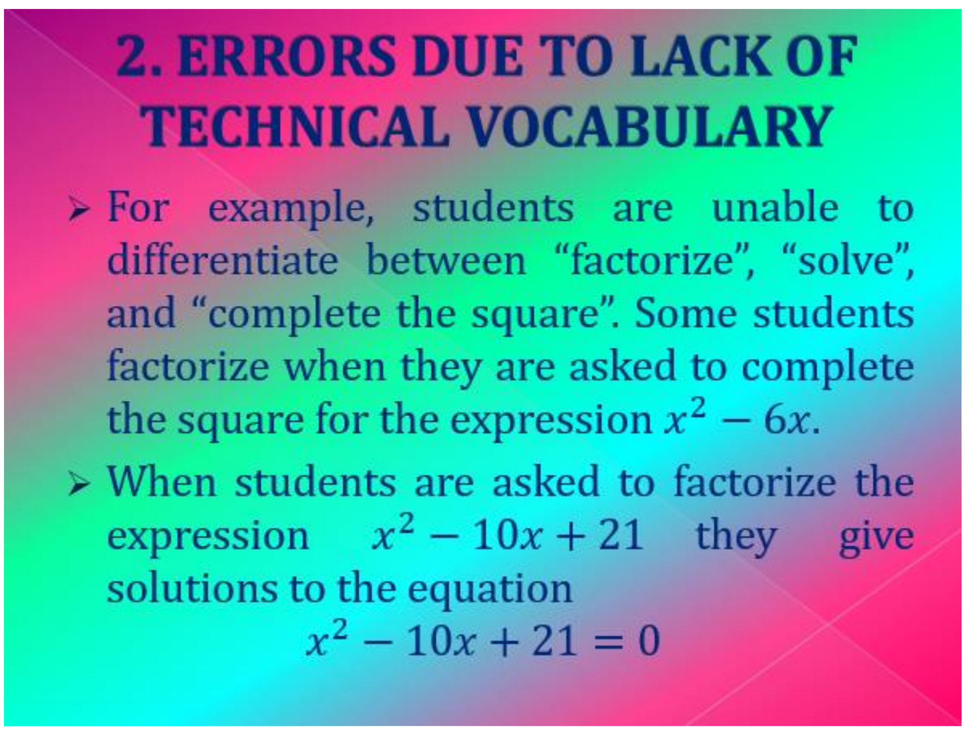

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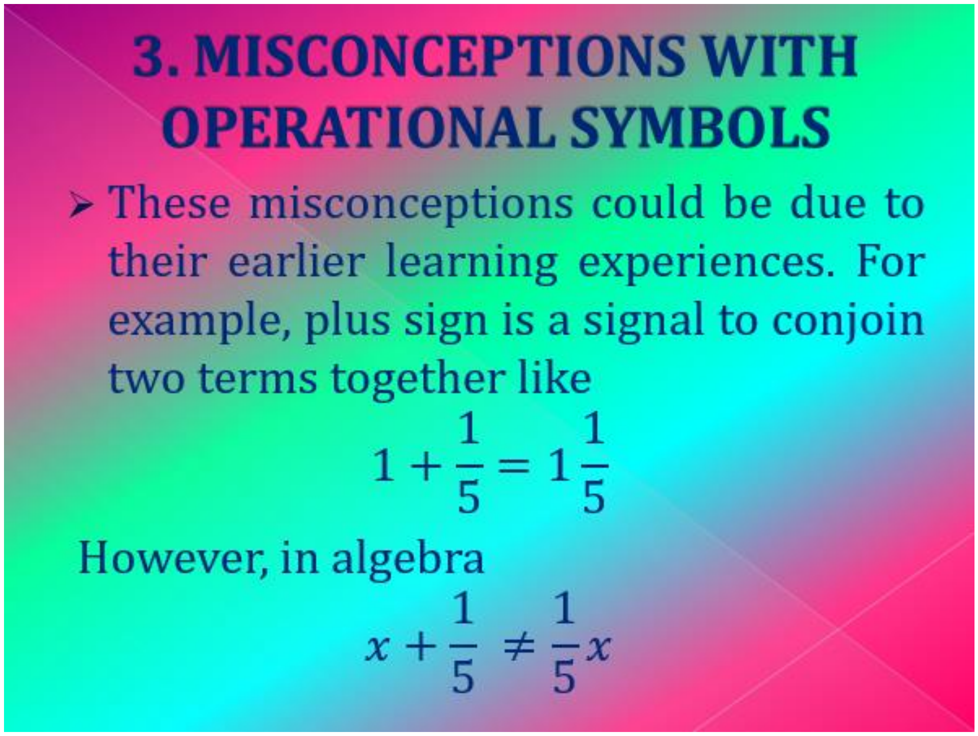

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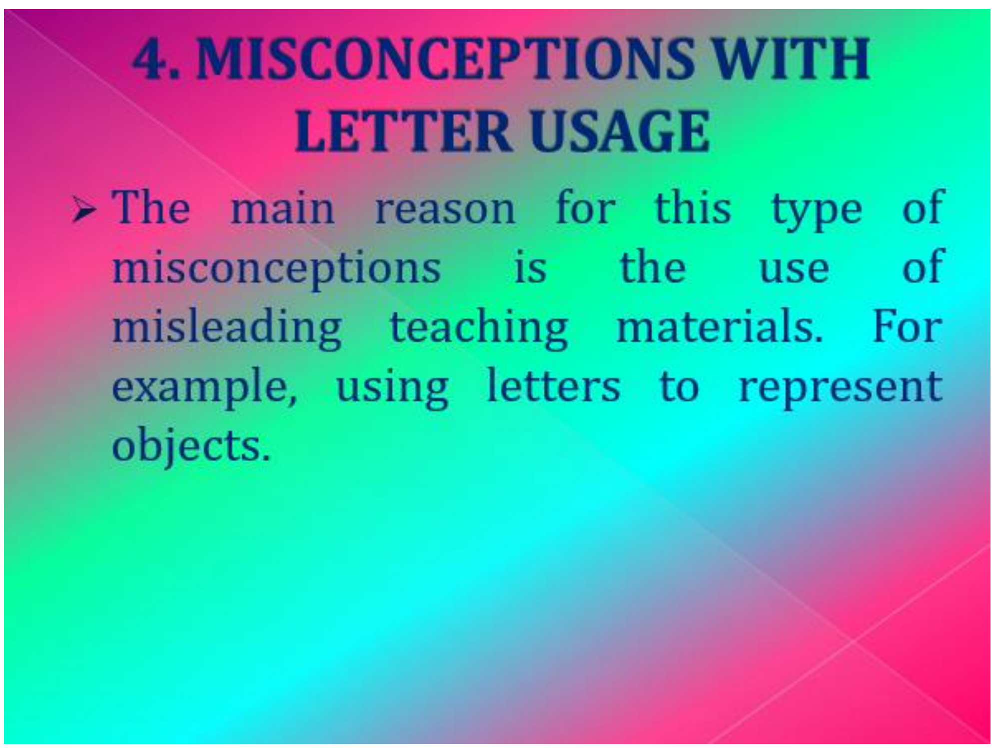

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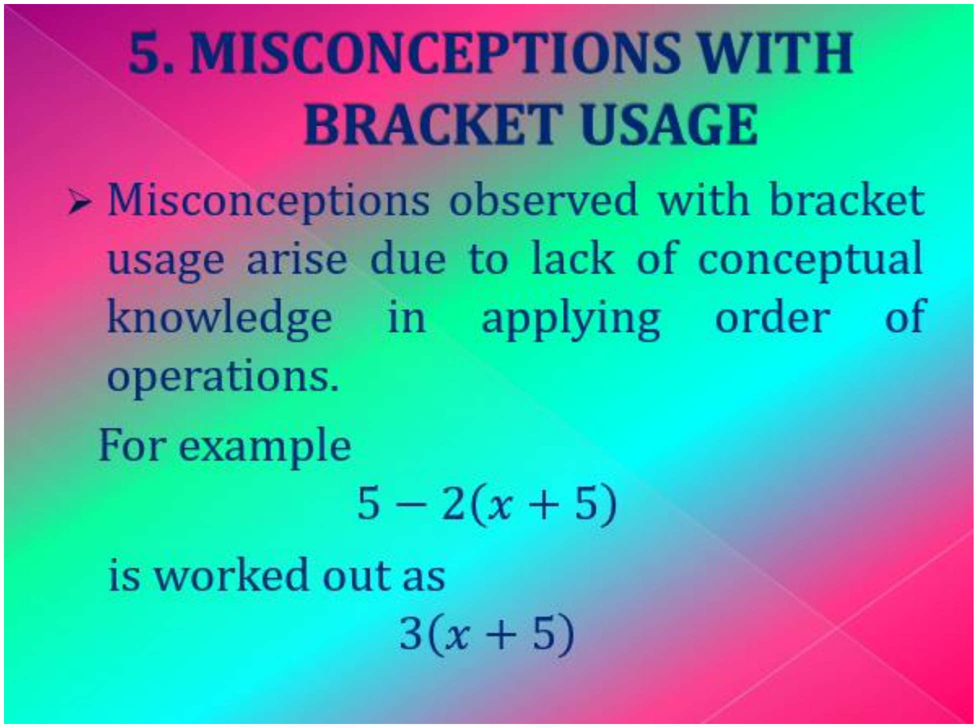

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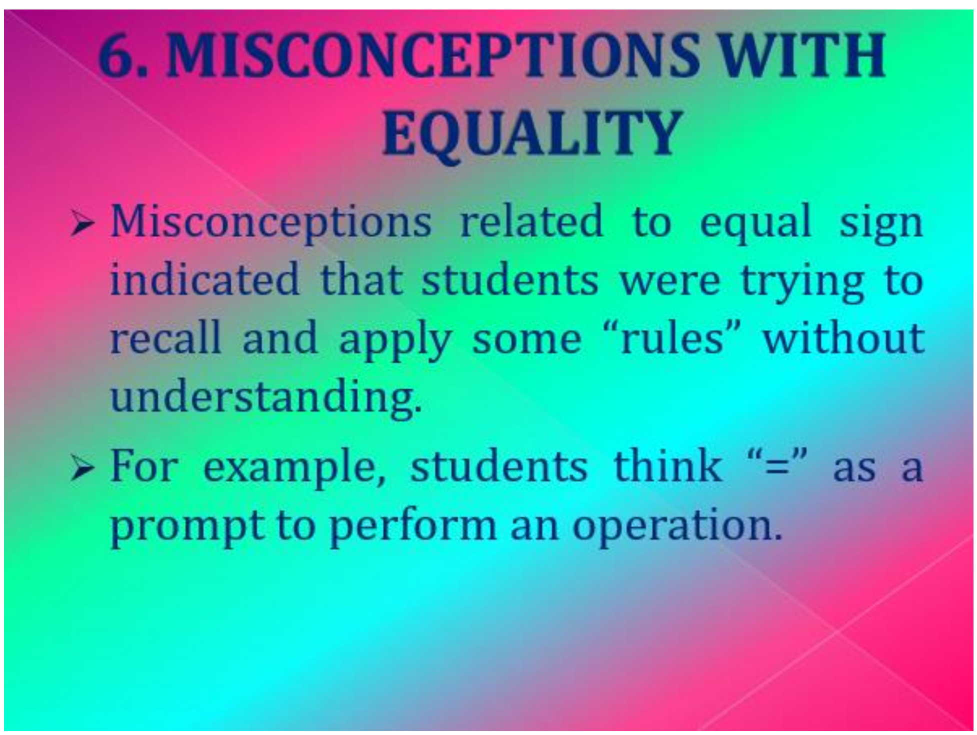

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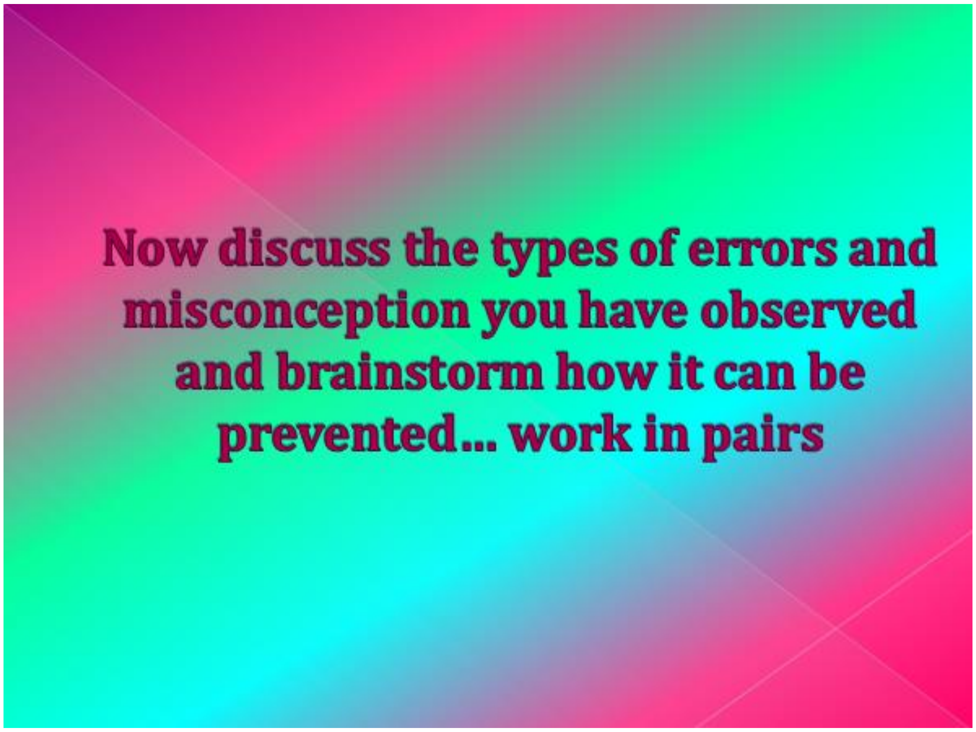
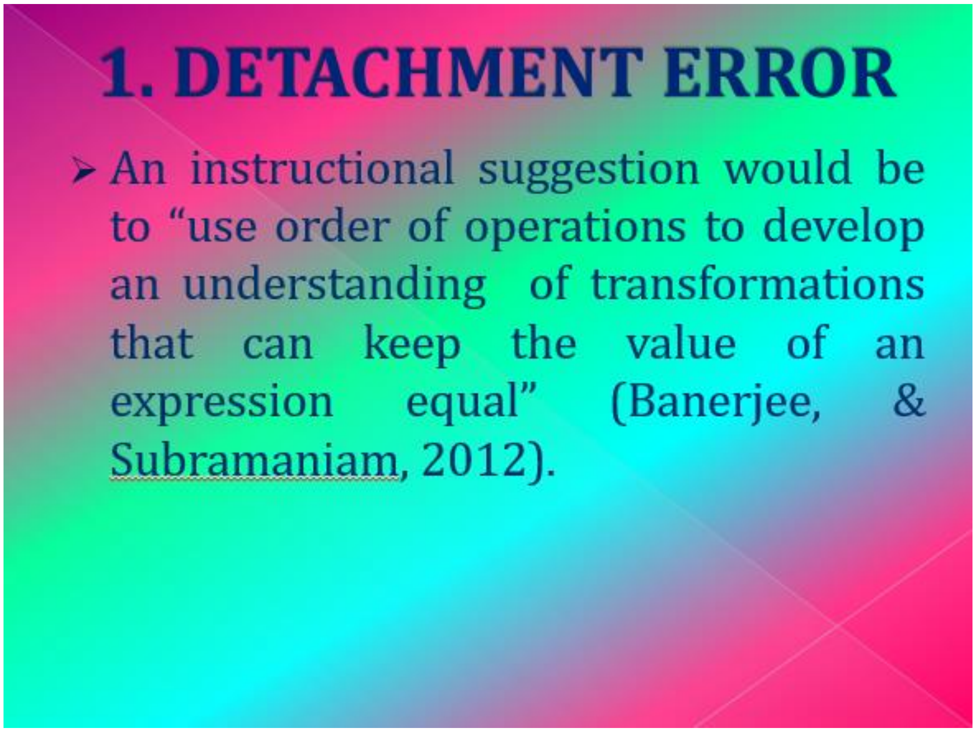


Slide 11:

Facilitator Note: These errors and misconceptions can be used to inform instructional decisions and hence design instruction (Banerjee, & Subramaniam, 2012; Brown, Davis, & Kulum, 2011; Massey, & Riley, 2013; and Welder, 2012). Primary and middle school mathematics teachers are responsible for laying a solid foundation to ensure their students' later success in algebra. Identification of common errors and misconceptions, how they are caused, and how they can be prevented and remedied need to be included in the initial teacher training curriculum to ensure teachers are aware of these. This could contribute positively towards reducing the occurrences of these errors and misconceptions.

Slide 12:



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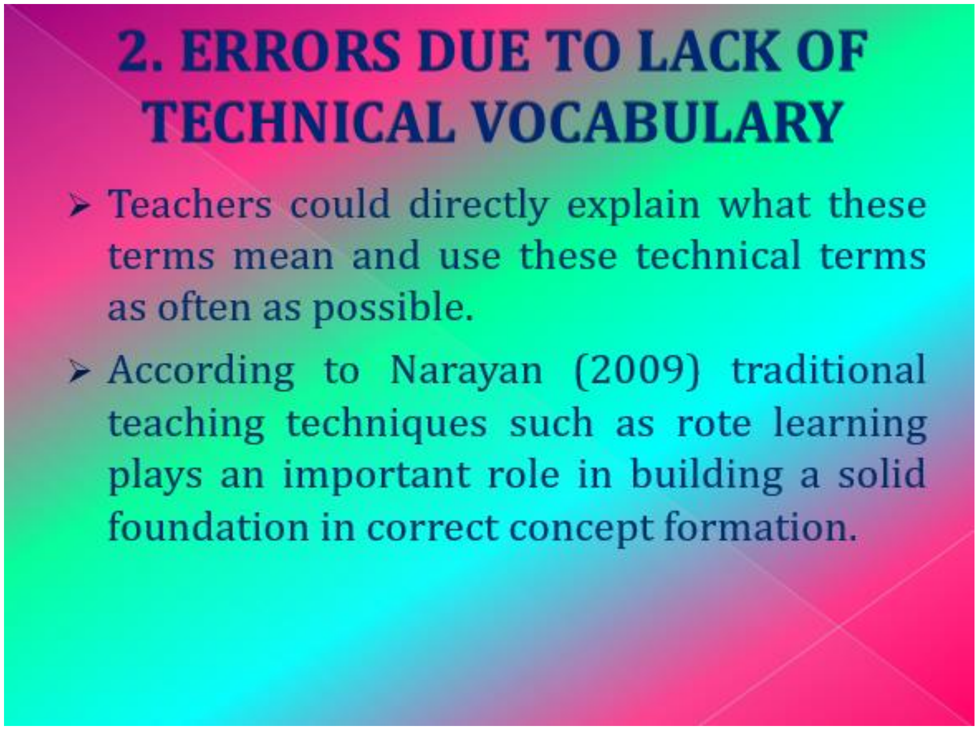

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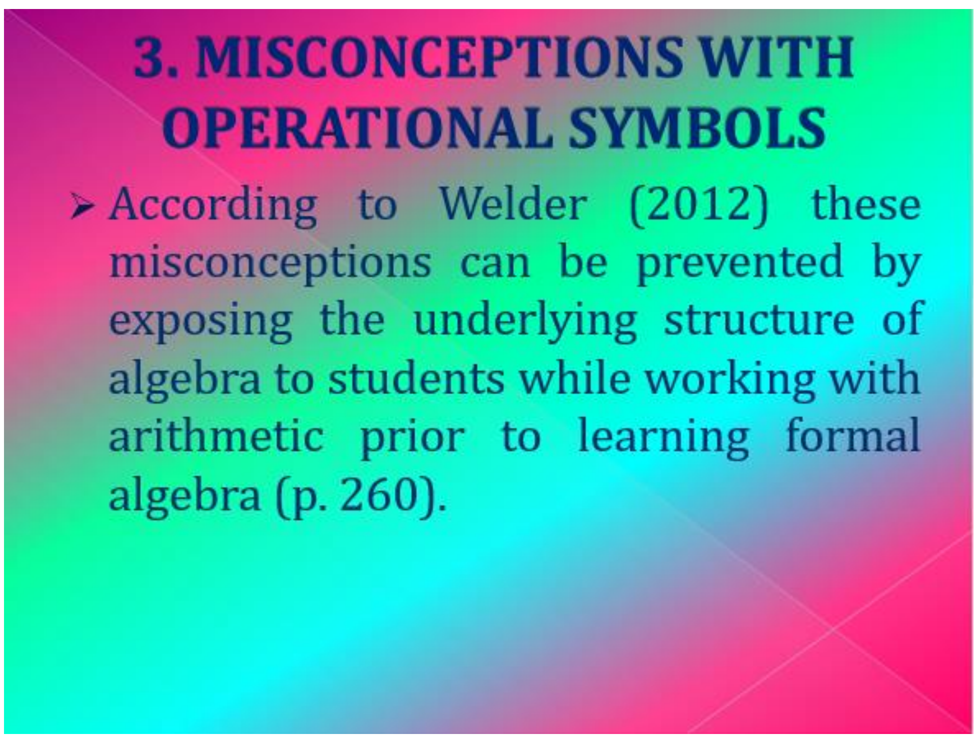

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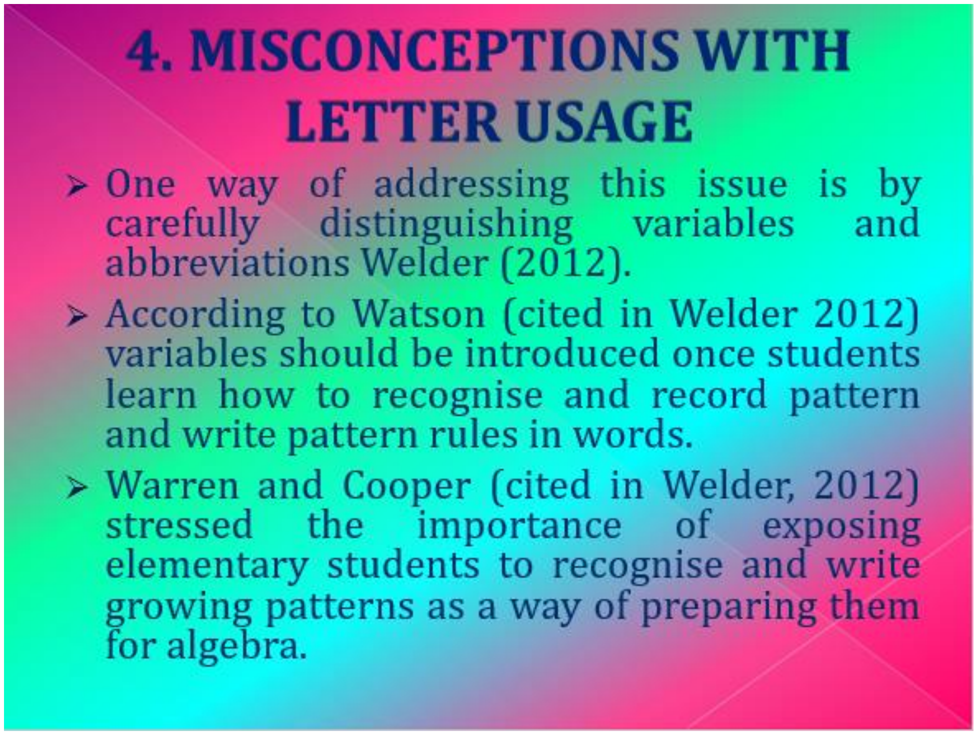

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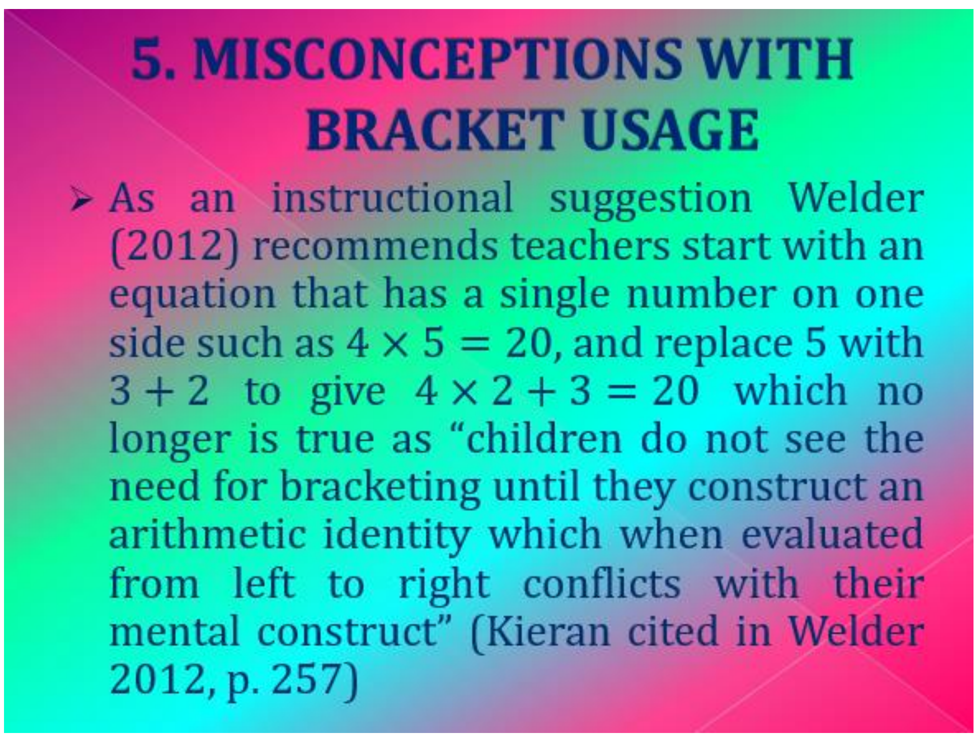

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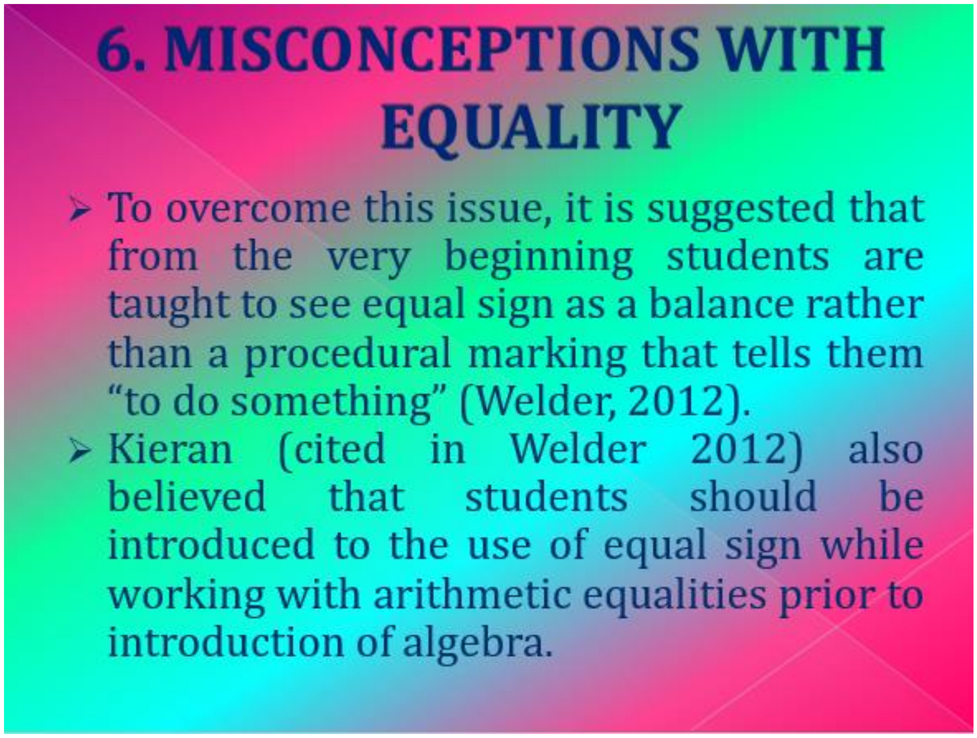

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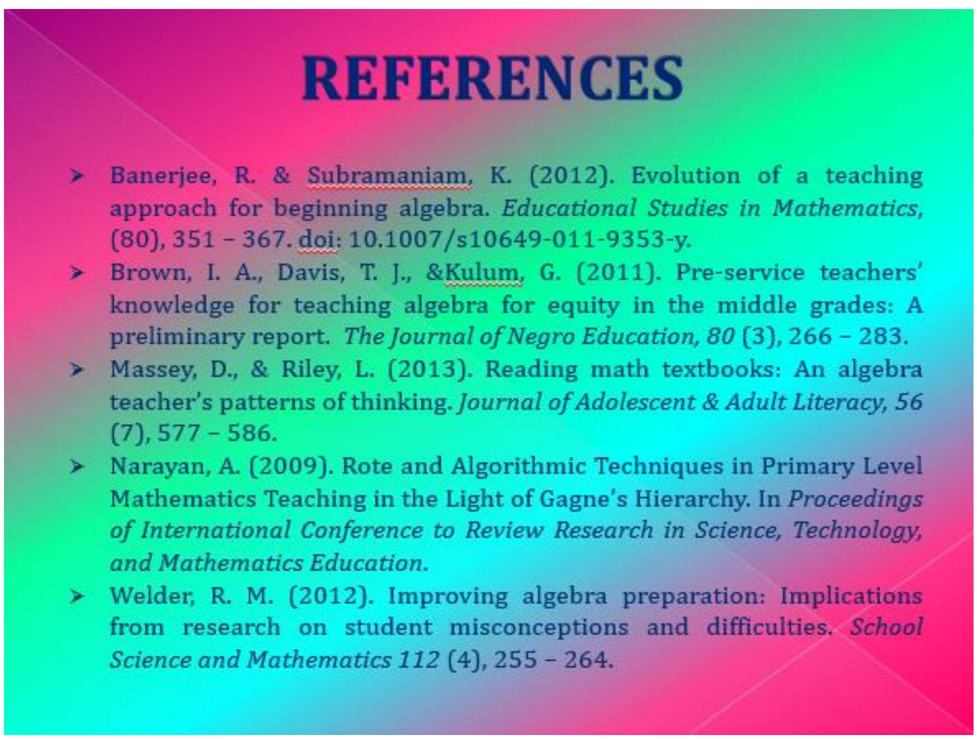

MPD D2 Evaluation Tasks

1. Classification of common errors and misconceptions of students' work samples
2. Identification of ways to remedy and prevent the formation of errors and misconceptions
3. Designing of lessons and activities that would prevent the formation of errors and misconceptions

Mathematics Professional Development Day 3 (MPD D3)

MPD D3 Date and Location

Mathematics Professional Development Day 3 will take place on 3 August 2017 at one of the Schools in the Capital City of the Maldives. Venue will be informed once it is finalized.

MPD D3 Agenda

Time and Duration	***Session Details***	***Activity***
0830-0840 10 minutes	Registration	NA
0840-0845 5 minutes	Welcome remarks	NA
0845-1000 1 hour 15 minutes	Action Research	Participants will prepare an Action Research timeline (a sample is provided under MPD D3 Facilitator Notes)

Time and Duration	***Session Details***	***Activity***
1000-1015 15 minutes	Coffee break	
1015-1130 1 hour 15 minutes	Lesson Study	Participants will prepare a Lesson Study research timeline
1130-1230 1 hour	Lunch and prayer break	
1230-1300 30 minutes	Individual Professional Development Plan	Participants will collaboratively work together to prepare their own professional development plan
1300-1315 15 minutes	Question and answer session	Participants will ask questions and discuss with other participants any questions they might have before the start of DTAMS Post-test
1315-1400 45 minutes	DTAMS Post-test	Participants will take DTAMS Post-test which will be later compared to the DTAMS Pre-test to see whether there is any significant improvement in algebraic content and pedagogical knowledge of the sixth grade mathematics teachers.

Time and Duration	***Session Details***	***Activity***
1345-1400 15 minutes	Tea break	
1400-1430 30 minutes	Concluding remarks and awarding certificate of participation to those who completed 15 hours of professional development	Participants will complete a PD Evaluation form

During the group work the facilitator observes and checks on the work and contribute to discussions within those groups. After each presentation facilitator as well as participants provide comments and feedback.

MPD D3 Facilitator Notes

Action Research

An academic who is proficient in action research will be invited to facilitate the session. As a guide to prepare their action research plan, each group will be provided with a sample plan. This sample plan will consist the essential components of an action research plan. The rationale for providing the participants with such a plan is to guide and motivate them to carry out the action research. Preparing the plan in the presence of the facilitator will provide the participants with the opportunity to share their plans with the facilitator and get feedback from the facilitator.

ACTION RESEARCH PLAN	
1. Write an area of focus statement	
2. Define the variables	
3. Develop research questions	
4. Describe the intervention or innovation	
5. Describe the membership of the action research group	
6. Describe the negotiations that need to be taken	
7. Develop a timeline	
8. Develop a statement of resources	
9. Develop data collection ideas	

Lesson Study

An academic who is proficient in lesson study will be invited to facilitate the session. Lesson study tools that will be provided to the participants include:

1. Lesson Study – Preliminary Discussion (Appendix F)
2. Lesson Study – Observation Protocol (Appendix G)
3. Lesson Study – Reflection / Evaluation (Appendix H)
4. Evaluating Success of the Lesson – Student Questionnaire (Appendix I)
5. Evaluating Success of the Lesson – Teaching and Learning Questionnaire (Appendix J)

MPD D3 Evaluation Tasks

1. Action research plan
2. Lesson study plan
3. DTAMS Post-test

Appendix B: Classroom Observation Checklist

Note: This checklist was developed after analyzing the content of the teaching practice observation forms of teacher training institutions in the Maldives. This checklist was shared with teaching practice supervisors (those who go to the class to evaluate the students' teaching) to establish the construct validity of the instrument.

Algebraic Content Knowledge	**Yes**	**No**	**Comments**
1. Introduction of the topic was clear.			
2. Lesson objectives were clearly communicated to the students.			
3. Teacher has sufficient knowledge in teaching algebra at sixth grade.			
4. Teacher is confident in teaching algebraic concepts.			
5. Materials used are appropriate for the students.			

Pedagogical Knowledge	**Yes**	**No**	**Comments**
1. Teacher related the topic to real-life.			
2. Teacher identified and addressed the potential errors and misconceptions students may have.			
3. Teacher used different teaching strategies to explain the concept to address the diverse needs of the students.			
4. Teacher used different approaches to engage students.			
5. Materials used are appropriate for the lesson / concept taught.			
6. Teacher promotes conceptual understanding, critical thinking, and creativity.			
7. Teacher encouraged students' participation in the lesson.			

Appendix C: Teacher Interview Protocol

Note: The following questions would be used in the interview. Depending on the lesson observed, and the answers given, additional questions may be asked and / or some of the questions may be omitted.

Thank you for letting me observe your classes. I would like to ask a few questions related to the lesson I just observed and some general questions. Would you mind if I record the interview? Recording will actually help me to ensure the accuracy of what we discuss. I assure you, all precautions will be taken not to disclose to anyone else any part of the data that is linkable to a participant's identity. If you have any questions please ask. I would like you to read this consent form and sign it before we begin. If you do not wish to answer any question or if you want to discontinue this interview at any point, feel free to do so. Do you have any questions you would like to ask before we begin?

A. Personal Information:

1. May I know your name and contact number please?
2. How long have you been teaching sixth grade algebra?
3. What is your highest degree achieved, from where, and what is your major?
4. What are the mathematics content courses you took in college?
5. What are the mathematics pedagogy courses you took in college?
6. How many professional development sessions on mathematics have you attended? Why?

7. In your opinion, did any of the professional development sessions you attended contribute towards enhancement of your algebraic content knowledge and pedagogical knowledge? How? Why?

B. Algebraic Content Knowledge:

1. How confident are you in teaching algebra at sixth grade? Why?
2. When a student gives an incorrect answer what do you do? Why?
3. When do you think students are first exposed to algebraic concepts? Why?
4. What teaching strategies and materials do you use to teach specific concepts of algebra? Why?
5. Is there any specific area of algebra that you believe you could use additional help?
6. Do you believe you have sufficient knowledge of algebra to teach at sixth grade? Why do you think so, and what did you base your belief on? (*Note: This question will be asked at the end of the interview as this question may make interviewees defensive which could skew subsequent response data.*)

C. Pedagogical Knowledge:

1. How do you feel about the lesson you just finished? Why?
2. In your opinion, do you think algebraic concepts can be related to real-life? How? Why?
3. Why did you introduce the lesson the way you did?
4. How did you select the teaching materials? Why?

5. Do you think, there were any students who found the lesson difficult, or who did not fully understand what was taught? Why? How did you attend to them?
6. If you get to re-teach the lesson, would you make any changes? Why?
7. If the teacher decides to bring any changes, what kind of changes? Why?
8. For your students, what kind of teaching strategies would best suit them? Why?
9. What are the possible misconception students might have in algebra and how would you overcome those?

Thank you very much for taking part in this study. If there is anything I could do for you, please feel free to contact me.

Appendix D: Data Collection Coordination Request

23 July 2015

Dear Teacher,

I have obtained the Minister of Education's support to collect data for my research project entitled Algebraic Content and Pedagogical Knowledge of Sixth Grade Mathematics Teachers.

I am requesting your cooperation in the data collection process. I propose to collect data on 1 November 2015 through 30 December 2015. I will coordinate the exact times of data collection with you in order to minimize disruption to your instructional activities.

If you agree to be part of this research project, I would ask that you to do a mathematics assessment; let me sit-in your class and observe while you teach algebra; share your lesson plans and notes with me; and take part in an interview.

If you prefer not to be involved in this study, that is not a problem at all. If circumstances change, please contact me via + 960 9631369.

Thank you for your consideration. I would be pleased to share the results of this study with you if you are interested.

I am requesting your signature to document that I have cleared this data collection with you.

Sincerely,

Mariyam Shahuneeza Naseer

Printed Name of Teacher ______________________________

Date ______________________________

Teacher’s Written Signature ______________________________

Researcher’s Written Signature ______________________________

Appendix E: Confidentiality Agreement

Name of signer: Mariyam Shahuneeza Naseer

During the course of my activity in collecting data for this research: “Algebraic Content and Pedagogical Knowledge of Sixth Grade Mathematics Teachers”, I will have access to information, which is confidential and should not be disclosed. I acknowledge that the information must remain confidential, and that improper disclosure of confidential information can be damaging to the participant.

By signing this confidentiality agreement I acknowledge and agree that:

1. I will not disclose or discuss any confidential information with others, including friends or family.
2. I will not in any way divulge, copy, release, sell, loan, alter or destroy any confidential information except as properly authorized.
3. I will not discuss confidential information where others can overhear the conversation. I understand that it is not acceptable to discuss confidential information even if the participant’s name is not used.
4. I will not make any unauthorized transmissions, inquiries, modification or purging of confidential information.
5. I agree that my obligations under this agreement will continue after termination of the job that I will perform.
6. I understand that violation of this agreement will have legal implications.

7. I will only access or use systems or devices I’m officially authorized to access and I will not demonstrate the operation or function of systems or devices to unauthorized individuals.

Signing this document, I acknowledge that I have read the agreement and I agree to comply with all the terms and conditions stated above.

Signature: date:

Appendix F: Lesson Study – Preliminary Discussion

Date		**Lesson Title**	
School		**Teacher**	
The Maths			
Overall Goal:			
Specific Concepts:			
Strategies/skills to be developed:			
Focus on Students			
Relevant concepts/strategies already explored by class. When:			
Familiar/relevant/real-life contexts for this lesson:			
Engagement/thinking that leads to the concepts:			
Predicted misconceptions/difficulties:			
Planned support/interventions:			
Implementation of Lesson			
Introduction/presentation of problem:			
Student Groupings:			
Use of materials/models/visuals:			
Students elaboration of their thinking/explanations:			
Facilitating student discussion of ideas:			
Student Attainment			
Evidence of students understanding:			

Appendix G: Lesson Study – Observation Protocol

<table>
<tr><th>Date</th><td></td><th>Lesson Title</th><td></td></tr>
<tr><th>School</th><td></td><th>Teacher</th><td></td></tr>
<tr><th colspan="4" align="left">Suggested Observation Foci
(to be determined during preliminary discussion)</th></tr>
<tr><td colspan="4">Teacher questioning e.g. open/closed; adequate response time for students, exploring ‘interesting’ comments
Student responses during whole class discussion
Student talk/listening to each other during independent/group activity
Use of materials to support activity
Students use/understanding of presented model. (Task Type 1)
Were predicted misconceptions evident?
Were challenges/extensions offered appropriately?
Did students make/understand appropriate familiar/real-life connections?
Were student groupings appropriate?</td></tr>
<tr><td colspan="4">Teacher One Focus:</td></tr>
<tr><td colspan="4">Teacher Two Focus:</td></tr>
<tr><td colspan="4">Teacher Three Focus:</td></tr>
<tr><td colspan="4">*At least one teacher observer needs to have a digital camera</td></tr>
</table>

Appendix H: Lesson Study – Reflection / Evaluation

Date		**Lesson Title**	
School		**Teacher**	
Teacher Reflection			
How lesson went/problems observed:			
Student Evaluation			
Discussion of student evaluation proforma:			
Teacher One Focus			
Teacher Two Focus			
Teacher Three Focus			
Suggested Lesson Revisions			
Suggested Teaching of Revised Lesson			
When?			

Appendix I: Evaluating Success of the Lesson – Student Questionnaire

Name: _______________________ School: ________________________

Please indicate how strongly you agree or disagree with these statements	**Strongly Disagree**	**Disagree**	**Agree**	**Strongly Agree**
1. I talked about the mathematics using mathematical words				
2. I learnt some mathematics I didn't know				
3. I was thinking about mathematics for most of the lesson				
4. I got started without any help				
5. I saw more than one way of doing the tasks				
6. I tried my hardest				
7. I was challenged				
8. I could now use this mathematics on other problems				

What do you think your teacher wanted you to learn from this lesson?

____

Comments

____

Appendix J: Evaluating Success of the Lesson – Teaching and Learning Questionnaire

Name:_________________ School:____________________

The Lesson: Provide a brief description of the lesson:

____

____

The Teaching: A self-evaluation of achievement of your own goals:

Please indicate the extent of your agreement with the following statements:	**Strongly Disagree**	**Disagree**	**Agree**	**Strongly Agree**
1. I had specific mathematical goals for this lesson(s)				
2. The lesson went as well as I had hoped				
3. My actions matched my goals				
4. I saw an unanticipated opportunity and used it effectively				

The Learning: Your perspectives on student's responses

Please indicate which word best describes the proportion of the class who…	**None**	**Some**	**Half**	**Most**	**All**
1. Talked mathematically to each other					
2. Learnt some new mathematics					
3. Were on task for most of the lesson					
4. Got started without additional help					
5. Saw more than one way of doing the main task(s)					
6. Tried their hardest					
7. Engaged in higher order thinking					
8. Asked meaningful questions					

Comments____