

1-1-2010

# The relationship between timed drill practice and the increase of automaticity of basic multiplication facts for regular education sixth graders

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2010

Abstract

The Relationship Between Timed Drill Practice  
and the Increase of Automaticity of Basic Multiplication Facts  
for Regular Education Sixth Graders

by

Nelly P. Knowles

M.A., Brenau University, 1999

B.S., Brenau University, 1994

Doctoral Study Submitted in Partial Fulfillment  
of the Requirements for the Degree of  
Doctor of Education  
Educational Leadership

Walden University

October 2010

## Abstract

By the time students transition from elementary to middle school, many do not demonstrate mastery of recalling basic math facts. This 8-week quasi-experimental quantitative study, based in cognitive development and theories of the construction of memory, used a 3-level independent variable experimental design to determine if there was a relationship between teachers' implementation of timed drill practices and the students' level of automaticity with regard to basic multiplication facts in 9 sixth-grade, regular education math classes. The control group received no intervention, the first treatment group received weekly timed drill practice for 3 minutes, and a second treatment group received daily timed drill practice for 3 minutes. Analysis of variance (ANOVA) procedures were used to measure the differences in pretest and posttest scores among the 3 treatment groups. Although no significant difference was found among the 3 groups' pretest performance, a significant difference among posttest performance was found. Scheffe' post hoc analysis revealed that the students who were administered daily timed practice drills performed statistically higher on the posttest than did the control group and first treatment group. Similarly, students in the weekly timed practice drill group had statistically significant higher gain scores than did students in the no treatment group. This study may lead to a shift in teachers' thought and practice regarding use of timed practice drills with the result of an increase of automaticity of basic math facts. Improved automaticity may lead to positive social changes including superior performance in math for regular education students that can lead to an increased sense of self-efficacy and higher graduation rates.



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## Dedication

This dissertation is dedicated to my brother, Adam Perez (1964 – 1995). Although he left this world entirely too soon, his spirit continues to give me strength, motivation, and inspiration. I love and miss him dearly.

## Acknowledgments

To my husband, Brit – You always encouraged me to chase this dream. Your belief in me is unwavering, and I would not have been able to do this without your love and support. I love you, forever!

To my son, David, and daughter, April – It is a true joy being your Mom! Thank you for believing in me! I am incredibly proud of the wonderful people you have grown up to be. You make my heart smile with joy! Ki – Thanks for being the best daughter-in-law on the planet! Brittan – You are the light of my life! I smile just thinking about you! Bear – I can't wait to welcome you to this world! I love you already!

To my mother, Connie, and my sister, Marilu – We had to put a lot of shopping trips on hold throughout this endeavor, but you were both always there, cheering me on to completion. Your support was appreciated more than you will ever know. Let the shopping marathons resume!

To my best friends, Emily and Laytona – Few people are lucky enough to have one super, great friend in their life, but I am fortunate enough to have two! Thanks for your support, for always being there to listen, and for being in my corner each and every day.

To my committee chair, Dr. Cheryl Keen – Thank you for your continued guidance, advice, and encouragement throughout my dissertation journey. You embody all the qualities of a scholarly mentor, and I am grateful that Walden brought us together.

To my second committee member, Dr. Roberta Liebler – I sincerely appreciate your advice and expertise. Thank you for deepening my methodology understanding.

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## Chapter 1: Introduction to the Study

When students enter the middle school mathematics classroom, many factors contribute to their potential for success. One such factor is the students' ability to recall basic math facts with little effort and a great deal of accuracy. This type of seemingly unconscious recall is often referred to as *automaticity* (Hasselbring, Goin, & Bransford, 1988). Many math educators and researchers believe that automaticity is absolutely essential in order to develop estimation and mental computation skills. It is the essence of overall number sense (Ball et al., 2005; Bratina & Krudwig, 2003; Woodward, 2006).

If students are consistent with their accuracy and speed of computation, they are able to devote more attention to the overall purpose of the problem instead of devoting problem-solving time to basic calculations. Students with the ability to solve problems and reach higher-level math reasoning are sometimes negatively affected by their lack of ability to solve basic computation problems (Isaacs & Carroll, 1999).

Legislative acts such as the No Child Left Behind Act (NCLB, 2002) and the goals of organizations such as the National Council of Teachers of Mathematics (NCTM) are supporting education reform and the way mathematics is being taught to students in the classroom. It is important to look at the impact that NCLB and the NCTM have on student achievement. NCLB is federal legislation is a federal legislation founded on standards-based education reform. The Act incorporates the requirement that basic skills assessments are to be given to all students who are enrolled in federally funded schools at specified grade levels. Beginning in the 2005/2006 school year, all states were required to assess students in both mathematics and reading in grades 3 through 8. States should be

on track to ensure that 100% of students demonstrate proficient levels of achievement on the state created assessments.

High-quality math instruction, according to the NCTM (2000), combines five content standards and five process standards. The five content standards include numbers and operation, algebra, geometry, measurement, and data analysis and probability. Computation skills are critical in each of these five standards in the sixth-grade curriculum, especially numbers and operations, algebra, and geometry, however helping students develop more automatic responses to basic computation problems will aid in their performance within all of the content standards. The five process standards are problem solving, reasoning and proof, communication, connections, and representations. They are designed to increase students' independent effective mathematical thinking, and the NCTM expectation is that they are to be incorporated into content standards lesson planning. The process standards focus on the ways students approach real life problems solving and the ways they are able to justify and communicate their approach. To be successful in the rigorous math curriculum framework established and enacted by the NCTM, students must be proficient at computation involving whole numbers, fractions, and decimals (Bottge, Rueda, Serlin, Hung, & Kwon, 2007). When students develop automaticity, more cognitive capacity is available for solving mathematical problems.

Both content and process standards of the NCTM, as well as those of state and local school systems, imply that automaticity of basic computation fluency should be fully developed before students reach the sixth grade, yet many students have not achieved that fluency when they leave elementary school. The lack of fluency has grave

consequences for high school level math achievement. In a study that examined the development of multiplication facts in students between the ages of 8 and 12 years, Steel and Funnell (2001) found that students who had not become fluent with recalling basic math facts in primary school had few opportunities for the practice, either at school or at home, that is necessary for mastery in secondary school.

Whole number computation is important because it is entrenched in so many other aspects of math performance (Woodward, 2006; Isaacs & Carroll, 1999). According to researchers, the foundation for future advancement in math is automaticity in basic multiplication facts (Wong & Evans, 2007; Steel & Funnell, 2001). In reflecting on their experience and research findings, Bratina and Krudwig (2003) speculated that the importance of automaticity is most obvious when it is, in fact, not present. While some researchers agree that conceptual understanding of mathematical content is necessary (Geary, 2004; Kilpatrick, Swafford, & Findell, 2001), it is not enough. If students have to stop to calculate or to recall a math fact that should be automatically retrieved, they will be more likely to give up before the problem is solved. After conducting an experimental study that measured the impact of computerized drill and practice, Hasselbring, Goin, and Bransford (1988) reported that “the ability to succeed in higher-order skills appears to be directly related to the efficiency at which lower-order processes are executed” (p. 1). A lack of fluency in basic math facts hinders student’s ability to perform more rigorous problems.

Whitehurst (2003), who at the time was Director of Institute of Educational Sciences (IES), stated the following during the start of the Federal Mathematics Summit:

Cognitive psychologists have discovered that humans have fixed limits on the attention and memory that can be used to solve problems. One way around these limits is to have certain components of a task become so routine and over-learned that they become automatic (Conceptual understanding section, ¶ 9).

Some components of problem solving are hard to make routine. Basic math fact retrieval, however, is a component that can be easily be practiced frequently enough to make it become automatic.

### **Problem Statement**

While the benefits of developing automaticity are known, strategies targeted to enhance automaticity are not being executed consistently in the classroom or at home (Woodward, 2006; Hasselbring, Lott, & Zydney, 2005; Burns, 2005; Ashcraft & Christy, 1995). There is no consensus on which strategies are the most effective strategies. Insufficient time is being devoted to helping students enhance their basic recall of math facts. As students who struggle with basic computation skills transition from elementary school to middle school, their area of weakness is often unaddressed. The teachers are pressured to cover the current grade level's objectives and feel they do not have time to teach the students foundational skills that were, according to state and local guidelines, supposed to be learned prior to reaching a middle school classroom. All too often, these students are passed to the next grade level without consideration of their lack of mastery of basic math skills. The inability to retrieve facts in isolation is only accentuated when

these students are presented with tasks that have basic computation embedded in the application of problem solving.

Some of the common strategies thought to help develop automaticity have been researched with learning-disabled participants. Various researchers (Wong & Evans, 2007; Hasselbring, Lott, & Zydney, 2005) have investigated the use of computer programs that expose students to repetitive fact drill and practice. The use of tape-recorded problems were explored by some researchers (McCallum, Skinner, Turner, & Saecker, 2006; Coddling et al., 2007) and were found to be effective when used with students with a learning disability or low cognitive function. The strategy of implementing timed practice drills as a way to help students learn their basic math facts has been investigated (Woodward, 2006; Hasselbring, Lott, & Zydney, 2005; Burns, 2005; Ashcraft & Christy, 1995); however the literature is lacking regarding the specific statistical analysis of the effectiveness of written practice drills as a means to increase automaticity. Another notable point is the existing research does not allow generalization to regular education, middle school students.

The transition between elementary school and middle school is likely to be a difficult turning point for many students (Parker, 2009). Middle school years are often plagued with emotional instability as well as academic turbulence (Barber & Olsen, 2004; Chung, Elias, & Schneider, 1998). Dealing with being promoted to a higher grade and a bigger school, as well as changes in physical maturation and peer influence presents challenges for many learners.

According to Hanich, Jordan, Kaplan, and Dick (2001), students with computational difficulties have not been adequately studied, and the size of population of these students is underestimated. Maccini and Hughes (1997) found that students with math difficulty struggle consistently on basic computation; however, researchers Parmar, Frazita and Cawley (1996) argued that the instructional time used to remediate the lack of proficiency might displace the time that could be devoted to higher-levels of math understanding. It is worth noting little research in the past decade has addressed the argument about this use of instructional time; also, while teaching strategies have been examined with regards to helping students to attain a higher level of automaticity, the evidence needed to support the use or nonuse of instructional time has not been thoroughly measured.

In conclusion, the problem of moving past competence in basic fact retrieval and on to higher level math instruction is illustrated in the story of one high school teacher. Kotsopoulos (2007) was concerned with the reality that many students were not proficient in recalling basic multiplication facts. As a teacher-researcher, Kotsopoulos explored the contributions of cognitive science in understanding fact retrieval to enhance the pedagogical direction of addressing students' struggles with quadratic relations. Factoring quadratics involves finding products within the multiplication table, so when students lack procedural fluency, (i.e. multiplication fact retrieval), their ability to understand and recognize representations of the same quadratic relationship might have been ineffective. Although based on experience rather than research, Kotsopoulos warned educators to not presuppose that conceptual knowledge will build procedural knowledge.

### **Nature of Study**

This quantitative research study employed a quasi-experimental design, specifically, nonequivalent groups design, and was conducted to determine if the use of written, timed practice drills has an impact on the development of automaticity of basic multiplication facts for 227 sixth grade students at one school. Analysis of variance (ANOVA) procedures were utilized to test the null hypothesis that written, timed practice drills do not significantly change the automaticity rate in basic multiplication facts for sixth grade math students. Additionally, ANOVA was used to examine if the frequency of these drills influences students' success. Scheffe' post hoc analysis followed the ANOVA to determine where the relationship exists. Scheffe' post hoc analysis is considered to be conservative and yield more influential results. The study involved the use of a three-level independent variable experiment design with the control group receiving no intervention. One treatment group received the intervention of weekly written, timed practice drills. A second treatment group received the intervention of daily written, timed practice drills. To increase validity and reduce teacher effects, each of the three participating teachers instructed one of each of the three groups, for a total of nine regular education math classes. Teacher constructed automaticity pretests and posttest was administered to each student participant in the study, and the change in mean difference between these scores served as the dependent variable (DV).

### **Research Questions**

The use of three treatment groups guided this study to two research questions:

1. Do written, timed practice drills increase automaticity in basic multiplication facts for sixth-grade math students?

$H_0$ : Use of written, timed practice drills will not be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students.

$H_1$ : Use of written, timed practice drills will be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth grade math students.

*Independent variable*: use of written, timed practice drills

*Dependent variable*: change in mean difference between students' pretest automaticity score and their posttest automaticity score

2. Does frequency of written, timed practice drills significantly change the automaticity rate in basic multiplication facts for sixth-grade math students?

$H_0$ : Frequency of use of written, timed practice drills will not be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students.

$H_1$ : Frequency of use of written, timed practice drills will be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students.

*Independent variable*: frequency of written, timed practice drills

*Dependent variable*: change in mean difference between students' pretest automaticity score and their posttest automaticity score

### **Purpose of the Study**

The purpose of this study was to determine if the use of written, timed practice drills can be linked to a difference in automaticity in basic multiplication facts for sixth-grade math students, and if so, whether the frequency of such drills influences students' performance. The goal was to help students build a strong foundation for successful performance in the mathematics classroom, as well as in their life outside of the classroom. The study's results may have important implications for math teachers, especially in the middle school setting. Educators may be more inclined to incorporate written, timed practice drills into their lesson plans without feeling as if they were not effectively using instructional time.

### **Theoretical Frameworks**

This study finds its theoretical frameworks in Piaget's (1977) cognitive development theory and Baddeley's (1992) working memory model.

#### **Cognitive Development**

Piaget's seminal theory has provided mathematics teachers important analysis of the way children learn mathematical concepts and ideas (Ojose, 2008). This analysis assists educators as they are planning instruction for their students. The theory identifies four primary stages of cognitive development—sensori-motor, preoperational, concrete operational, and formal operational—each characterized by certain ability/maturity levels. Preoperational and concrete operational stages occur during the elementary and middle school ages (Wadsworth, 1996). This research study is primarily concerned with the concrete operational stage, as that is where the concept of math automaticity is

developed (Martini, 2004). According to Piaget (1977), everyone goes through each stage of development before entering the next stage. The preoperational stage occurs between the ages 2 and 6 years, and the concrete operational stage occurs between the ages 7 and 11 years. At the concrete operational stage, organized, logical thought is evident. The ability to perform logical-sequence problem solving and to understand reversibility is possible at this developmental stage. This understanding would include making the arithmetic connection that because  $4 + 5 = 9$  then  $9 - 5 = 4$ .

Adults and adolescents revert to concrete operational thought to address reversibility when working math problems. Phenix and Campbell (2001), two cognitive science researchers, have studied the brain's function with regard to its ability to retrieve numeric facts and whether fact retrieval is order specific. Their research involving basic multiplication facts results indicated that order does matter. These findings suggest that when students are learning multiplication facts, both  $3 \times 7$  and  $7 \times 3$  need to be understood independently of each other. Because the logical reasoning to support such reversibility is not achieved by everyone at the same time, Piagetian theorists argue that educators should not assume that students would automatically make the connection needed to acquire understanding of such relationships.

Many educators revere Piaget's cognitive development theory; however, critics have argued that this theory is incomplete to describing cognitive development (Eggen & Kauchak, 2000). These critics have claimed Piaget underestimated young children with regards to their abilities and the theory leads educators to unfairly draw conclusions about children's performance (Gelman, Meck, & Merkin, 1986). Piaget's theory does not

incorporate the influence of environmental factors. Also, because Piaget's primary subjects were his own three children, his research lacks generalization.

### **Working Memory**

When students employ problem-solving techniques, they are relying on their working memory to assist them. *Working memory* is defined as a processing resource with limited capacity involved in preserving information while, at the same time, processing the same or other information (Baddeley & Logie, 1999). Cognitive psychologists heavily rely on Baddeley's working memory model (1992) to help explain short-term memory. Originally, the working memory model comprised three components. Baddeley identified the *central executive*, described as controlling awareness of information in one's working memory and initiating retrieval of information traveling between three different storage systems. The *visuo-spatial sketchpad*, as termed by Baddeley, is a storage system that controls visual and spatial imagery. The *phonological loop* is a second storage system that controls auditory and linguistics information. Baddeley (2000) later added the third storage system to the model called the *episodic buffer*, which is thought of as a linking system between the other storage systems, integrating visual, verbal, and spatial information.

According to Baddeley's model (2000), working memory is restricted. It can only work with a certain number of resources at one time. This limitation is of notable interest with regards to understanding students' ability to use working memory to help solve problems in mathematics. Many models of arithmetic processing have operated with the general postulation that people rely on their long-term memory when retrieving basic

arithmetic facts (Ashcraft & Battaglia, 1978; Campbell & Oliphant, 1992; Butterworth, Zorzi, Girelli & Jonckheere, 2001). Recently, however, numerous researchers have documented that a large percentage of adults use strategies to solve basic addition, subtraction, multiplication, and division problems rather than relying on long-term memory (Tronsky & Shneyer, 2004; Kirk & Ashcraft, 2002; Hecht, 2002; Campbell & Xue, 2001). Studies have supported the idea that when automaticity increases, reliance on one's working memory decreases; thus fewer functions compete for the limited capacity of working memory (DeStefano & LeFevre, 2004; Tronsky, 2005).

### **Definition of Terms**

- *Algorithm*. A procedure for solving a mathematical problem (.
- *Automaticity*. Automatic recall of facts without conscious control (Hasselbring, Goin, & Bransford, 1988).
- *Scaffolding*. An instructional strategy that is used to support novice learners by gradually adding to their context of knowledge and then removing the level of support as the learners gain confidence and skills in utilizing that knowledge in complex ways (Young, 1993).
- *Working memory*. The capacity to store and maneuver information for short periods of time; information made available to the mind as needed to carry out a mental task or to solve a problem (Tronsky & Royer, 2002).

## **Assumptions, Limitations, Scope, and Delimitations**

### **Assumptions**

This study was based on the assumption that the students who participated performed as well as they could during each administration of a timed practice drill. It was also assumed that the teachers who participated administered the timed practice drills with a positive attitude and strictly followed the time limits for such drills. The difficulty levels of the drills were evenly matched across the three groups, and the drills were conducted within the first five minutes of class of the treatment groups. The ability level and gender ratios were closely matched as well, given the fact that the classes were assigned and balanced by both ability and gender at the start of the school year.

### **Limitations**

This study includes the following limitations:

- The sample population for this study was not randomized and was limited to one geographic location, one school, and one age group. The student data for the school at which the study was conducted showed that 92.4% of current sixth-grade students, including those with disabilities, met or exceeded standards on the state's fifth-grade mathematics test, the Criterion Referenced Competency Test (CRCT). This lack of randomization poses a threat to external validity and generalizability.
- The teachers participating in this study may have potentially differed in the way they presented and administered the written, timed drills. Their differing attitudes and mannerisms may be viewed as a limitation. There is the possibility of a

*Pygmalion effect*, which refers to students in the timed drill groups performing better than students who do not receive the timed drills simply because their teacher expects them to do so (Rosenthal & Jacobson, 1992).

- It is possible that students may have enrolled or withdrawn from the school at which this study was conducted. This transiency may be considered a limitation as it will affect the final data gathered.

### **Scope**

The scope of this study was confined to the topic of basic multiplication facts. Automaticity of addition, subtraction, and division were not evaluated or studied in this research.

### **Delimitations**

This study included the following delimitations:

- The study included only regular education students in sixth-grade math classrooms.
- This study took place during the course of an 8-week period. It is possible that results of a longer study would vary from those of this study.
- Demographic attributes of the students were not collected in order to increase anonymity of the data. It is possible that attributes such as race, socioeconomic level, and health impairments could have had an impact on the data gathered in this study.

### **Significance of the Study**

In order to achieve success in mathematics, students need to be proficient in recalling basic computation facts. Math is a scaffolding subject that requires prior knowledge. The results of this study show a significant difference between the use of daily timed drills versus less frequent timed drills, with regards to increasing automaticity of basic multiplication, therefore, educators are able to make a sound decision, based on research, to incorporate daily timed drills in their classrooms. The impact of the results of this study might empower teachers to justify the use of valuable instructional time for timed practice of basic math facts. This study models a method of implementation that will be easy to follow and will be beneficial to students.

Another significant result of this study is the fact that it contributes to and expands the research literature concerning basic math computation. It offers data regarding the degree of automaticity achieved by rising sixth grade students. This information may be valuable to the education community, may potentially bring about instructional changes in the classroom, and therefore may bring about social change through a better-educated citizenry.

### **Summary**

When students enter the realm of a middle school mathematics classroom, there is a certain expectation with regard to numerical proficiency. Whole number computation is most importance because it is entrenched in so many other aspects of math performance (Woodward, 2006, Isaacs & Carroll, 1999). The foundation for advancement in math is automaticity in basic math facts (Wong & Evans, 2007). If students are accurate and

habitual with their computation, they are able to allocate more attention to the overall purpose of the problem instead of devoting thought to basic calculations. Some research has shown that timed drills help students develop automaticity for students who are academically low achieving (Ashcraft & Christy, 1995; Geary, 1996, Woodward, 2006).

Many aspects of mathematics depend on basic computation. Woodward stated, “Potential difficulties extend well beyond operations on whole numbers. Finding common multiples when adding fractions with unlike denominators or factoring algebraic equations are but two examples from secondary-school mathematics where automaticity in math facts can facilitate successful performance” (2006, p. 269). Students with the ability to solve problems and reach higher-level math reasoning are sometimes negatively affected by the lack of ability to solve basic computation problems (Maccini & Gagnon, 2000).

This study was designed to help determine whether written, timed drills are able to be linked to the development of automaticity among regular education sixth grade students. Chapter 1 introduces the concept of automaticity and has offered reasons for the importance of this study. When students are weak with regards to immediate recall of basic math facts, potential for difficulties extend beyond operations with whole numbers (Woodward, 2006). Chapter 2 examines current research findings and theories that address automaticity, including the importance of it with regards to providing a strong foundation for total math performance of students. Chapter 3 of this study explains the study’s methodology, including a description of the population, sampling procedures, instruments, and data collection procedures. Chapter 4 describes the results of statistical

analysis conducted to answer the research questions that ignited the study, and Chapter 5 summarizes the study, discusses conclusions, and offers recommendations for future studies.

## Chapter 2: Literature Review

It is a scenario that occurs often in mathematics classrooms across the country. Teachers are wondering why many of the students in their classrooms were promoted without the skills necessary to achieve the current grade level's academic expectations. At the forefront of most cases are inconsistent, limited basic computation skills. A generally stated standard to help students develop automaticity in basic math facts is not enough. Teachers need to implement research-based instructional strategies to support and improve their students' fundamental level of proficiency.

This chapter investigates existing research with regards to understanding the process of automaticity in basic computation and how students acquire competency in computation. At the time of this study, little research has evaluated methods designed to help middle school students fine-tune their automaticity of recalling basic math facts. While the scope of literature that explores special education students and their acquisition of automaticity is broad (Burns, 2005; Fuchs, Fuchs, & Prentice, 2004; Hasselbring, Lott, & Zyndey, 2005; Mercer & Miller, 1992), little research to date has investigated methods to assist regular education students who lacking automaticity for basic math facts. Applicability of cognitive development theory and the working memory model will also be reviewed in this chapter. Because this study focuses on how students learn basic math facts, it is important to understand how students learn and at what developmental levels specific learning is expected to take place.

The literature review for this proposal is the result of extensive research utilizing various databases, books, and peer-reviewed journals. Key search terms included

*mathematics instruction, automaticity, basic math skills, fact retrieval, cognitive theory, working memory, teaching methods, math calculations, middle school mathematics, and math difficulty.* The exhaustive search led to a limited amount of research regarding the development of automaticity of basic math facts for regular education middle school students within the past 5 years. Due to the lack of literature on this topic, some references are more than 5 years old.

### **Automaticity of Basic Multiplication Facts**

The NCTM Curriculum Focal Points (2000) recommended fourth-grade students should develop quick recall of whole number multiplication facts and fluency. However, many students find multiplication fact retrieval difficult even by the time they reach high school. According to some researchers, students who develop within the average range with regards to academics, acquire math facts in a progressive manner. They move from deliberate, procedural, and error-prone calculations to accurate and efficient calculations (Hasselbring, Lott, & Zydney, 2005; Ashcraft, 1992; Fuson, 1988). Hasselbring et al. (2005) stated that students who struggled in math showed significant problems with accuracy and timeliness when they tried to retrieve basic math facts. They argued that the “key to making retrieval of basic math facts fluent is to first establish a mental link between the facts and their answers which must be stored in long term memory” (p. 6). Some students encounter their first obstacle in math when they begin learning their multiplication tables. Relying on inaccurate counting methods can lead to difficulties when memorizing tables (Geary, 2004; Kilpatrick et al., 2001). Goldman, Pellegrino, and Mertz (1988) conducted a study that compared students with learning disabilities and

students without learning disabilities with regards fact retrieval methods. The participants in their study ranged from second graders to sixth graders. The students with learning disabilities lean heavily on counting up to find the answer to a basic multiplication problem instead of direct retrieval methods. Students who rely on the use of counting up are often unable to transfer basic facts; for example, the student may have memorized that  $5 \times 8 = 40$  but is not able to transfer the fact that  $8 \times 5 = 40$ . In a study conducted by Steel and Funnell (2001), it was concluded that students who did not acquire basic multiplication facts by age 11 years were not likely to effectively use them in a structured manner in later grades. The participants in the study ranged in age 8 to 12 years and were taught by discovery methods. While retrieval was shown to be the fastest and most error-free strategy, few students used it. The most ineffective and most error prone strategy was the counting in series method.

The foundation for future advancements in math understanding is developing automaticity in basic multiplication facts. The skill of promptly recalling such facts has been shown to be a strong indicator of academic performance on standardized math achievement tests (Royer, Tronsky, Chan, Jackson, & Marchant, 1999). Royer et al. (1999) conducted research involved student participants in the United States and China, ranging from fifth grade to college entrance levels. While much of their research was analyzing gender differences, they found the relationship between math fact retrieval speed and test performance to be significant. Students with the ability to solve problems and reach higher-level math reasoning are sometimes negatively affected by the lack of ability to solve basic computation problems (Maccini & Gagnon, 2000).

Learning higher-level skills, such as multi-digit multiplication, whole number division, fraction computation, decimal computation, and understanding ratios is dependent on the basis of proficiency to work with multiplication facts (Westwood, 2003; Norbury, 2002; Kilpatrick et al., 2001). According to Mercer and Miller (1992), students who struggle with recalling basic facts from memory are unable to perform basic computation, thus they are not as adept with problem-solving tasks. Studies have also found that when students lack the ability to retrieve math facts quickly, they are not as likely to participate in math class discussions (Woodward & Baxter, 1997).

Cognitive psychologists have hypothesized causal relationships between automaticity of basic math facts and students' performance on multiple-step math problem solving (Gersten, Jordan, & Flojo, 2005; Pellegrino & Goldman, 1987). Evidence has shown students' performance with word problems increased as they gain understanding of essential arithmetic operations (Swanson, Jerman, & Zheng, 2008). This understanding of crucial arithmetic operations betters students' ability to distinguish between which mathematical operation is needed in the problem, and as documented in research studies, it leads to students being able to effectively use strategies to attack the word problem successfully (Rasmussen & Bisanz, 2005; Rittle-Johnson, Siegler, & Alibali, 2001).

### **Effective Instruction for the Development of Automaticity**

In the world of education, every student learns in a unique way, however, there are various strategies that have been shown to effectively enhance many students' performance. Strategies to aid in the development of automaticity of basic math facts

have been examined by many researchers (Geary, 2004; Kilpatrick et al., 2001; Wong & Evans, 2007;), however there is a gap in the literature regarding which strategies are most effective in improving student achievement. Some of these strategies, such as timed drill and practice, use of audiotapes, and use of computer programs have been studied, but because the studies were targeting students with diagnosed learning disabilities (McCallum et al., 2006; Coddling et al., 2007; Burns, 2005; Fuchs et al., 2004), their statistical data cannot be generalized to regular education students. Research on effective mathematics development is, according to Whitehurst (2003), in its infancy in comparison to that of reading development. Much of what educators go on is based on educated guessing rather than mathematics research.

In an experimental study that examined the impact of an approach that only used timed practice drills versus an approach that used the integration of fact strategy and timed practice drills to teach multiplication facts and extended multi-digit computation, Woodward (2006) found that both approaches were comparable in increasing automaticity of basic multiplication facts as measured by posttests and maintenance tests. The study duration was four weeks and the timed-drills for both groups were two minutes each. Of the 58 fourth-grade participants, 57% received free or reduced lunch and were an average of 1 year behind grade level in math according to standardized testing. Furthermore, some of the students were learning disabled in math. However, results did indicate that, compared with students who were exposed to only timed drill practice, students who were exposed to an integrated approach incorporating contemporary teaching strategies were able to perform better on posttest assessments, some of which

incorporated skills other than mere computation. In addition, the integrated approach students were also able to retain their skills longer than students who were only exposed to timed drills. Although Woodward (2006) stated, “If educators are only considering facts as a foundation for traditional algorithms, either method would probably suffice” (p. 287), Woodward determined that the students who were taught using the integrated approach were able to extend their application of skills more effectively.

Some educators are relying on the use of methods that originally were designed to help student with their development of language skills. Two such methods are the *taped-problems (TP) intervention* and the *copy-cover-compare (CCC) method*. These methods have been shown to offer promising results for language arts students.

The (TP) intervention was developed by McCallum, Skinner, and Hutchins (2004) and is an adaption of Freeman and McLaughlin’s (1984) taped-words intervention. The intervention includes basic math facts being played on an audiotape. Problems and answers are read and student are challenged to “beat the tape” by writing the answers to the problems before it is heard on the tape. Like TP, the CCC method finds its origins in a language arts intervention. It was originally created to help develop spelling accuracy. Skinner, Turco, Beaty, & Rasavage (1989) transformed CCC to target math facts. Students are taught to study the target problems that are written on the left side of the page, and then they cover the problem and answer. Finally, the students write the problem and answer on the right side of the page. Students are not to move on to the next fact until they successfully write the problem and answer correctly.

The TP and CCC methods were the focus of a study conducted by Poncy, Skinner, and Jaspers (2007). Their study was designed to compare the effects of the two methods in raising fluency and accuracy of recalling math facts. The results of this study showed that both methods increased the performance of the participant, however, TP involved 30% less time than CCC. The researchers admitted that their study was limited with regard to generalization because the only participant was a 10-year old special education student with a full scale IQ of 44, a score that could yield a moderate mental retardation diagnosis.

Other studies have found TP and CCC to be effective in raising math performance for students with learning disabilities with regard to their fluency of basic math facts (McCallum, Skinner, Turner, & Saecker, 2006; Coddington et al., 2007). While their research is valuable in empirically validating these interventions for special education students, there is a significant deficit in current literature regarding interventions for raising fluency of basic math facts for regular education students.

One researcher (Zutaut, 2002) tested the use of mnemonic devices to help a group of fourth-grade students memorize basic multiplication facts. The experimental group, consisting of 11 students, were shown a multiplication fact and then given a mnemonic device for remembering the math fact. The control group consisted of 12 students, who were given the same multiplication facts but were not given the mnemonic device for each fact. The study duration was 3 weeks (12 school days) and each session was ten minutes long. An independent samples *t* test compared post test scores of students who received mnemonic devices along with the multiplication facts and students who did not

receive mnemonic devices with the multiplication facts. The test revealed no significant difference between the mean scores of the two groups.

Wong and Evans (2007) conducted a study that employed a systematic multiplication program that was designed to help students improve their basic multiplication facts. Their study spanned a 4-week period, and the participants were an average age of 10.5 years. Of the four classes involved, two of the classes were given paper and pencil practice and two classes were utilized a computer program to practice facts. While both groups showed an increase in their recall of basic multiplication facts, the study results reveals no statistically significant difference in the maintenance of fact retrieval. Wong and Evans' study included the use of written practice drills, which aligns with the key aspects of the research questions being hypothesized in this study, however it only included 11 practice sessions.

The bank of research studies, scholarly journals, and peer reviewed articles that address the research questions presented in this proposed study is sparse. The databases that were searched include Education Resources Information Center (ERIC), ProQuest Dissertations and Theses, Walden University's Dissertations and Theses, and Education Research Complete. Searches were conducted using keywords such as *math instruction*, *basic multiplication*, *timed tests*, *working memory*, *cognitive development*, and *automaticity*. The literature review performed for this proposed study did not reveal any research studies that incorporated written, timed practice drills as a means to increase automaticity in basic math facts. Few studies included middle grades students, and none were found to specifically target regular education students.

### **Rationale for Quantitative Research Method**

According to Creswell (2003), a quantitative method is best to help understand what factors or variables influence an outcome. This study was conducted to determine if the use of written, timed practice drills impact students' ability to quickly and accurately recall basic multiplication facts. The analysis of this study involved cause and effect thinking, and the fact that the study used predetermined instruments to produce statistical data makes a quantitative method the best approach. There is a gap in the relevant literature that helps mathematics educators make research-based instructional decisions on how to best prepare students to be more competent with regards to immediate recall, or automaticity of basic math facts. The data generated from this study will add to the statistical body of knowledge to help fill that gap.

Crotty (1998) suggested that a researcher consider four questions when determining what research approach to use:

1. What epistemology informs the research?
2. What theoretical perspective lies beneath the methodology in questions?
3. What methodology governs our choice and use of methods?
4. What methods do we propose to use?

When answering these four questions, the researcher opted for an objective approach that centered on experimental research. A pretest/posttest method was decided upon so initial differences between the groups is noted before any subject is exposed to a treatment condition.

## **Summary**

The bank of research studies examining the development of automaticity of basic math facts for regular education students is in short supply. Most studies are limited to the learning disabled population and are, therefore, not able to effectively be generalized for the regular education learner. The lack of statistically significant results in research studies targeting strategies for helping students develop automaticity of basic math facts leads this researcher to propose this quantitative study.

Chapter 3 will describe the methodology components of this research study. The research questions and the hypotheses will be identified, and a description of the research design and approach will be conveyed. Additionally, the participants, the instrumentation and materials, the data collection procedures, the data analysis plan, and the rights of participants, and the role of the researcher will be explained.

### Chapter 3: Research Method

The purpose of this chapter is to provide information about the chosen research design and approach for this proposed study. It will include the research questions driving the study, as well as justification and explanation of the choice of methodology. The setting and participants will be described and an explanation will be given for the instrument that will be used to gather data. Additionally, an explanation and rationale of the data analysis plan will be described, and the rights of the participants will be provided. Finally, the role of the researcher will be explained.

#### **Research Questions and Hypotheses**

1. Do written, timed practice drills increase automaticity in basic multiplication facts for sixth-grade math students?

$H_0$ : Use of written, timed practice drills will not be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students

$H_1$ : Use of written, timed practice drills will be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students

*Independent variable*: use of written, timed practice drills

*Dependent variable*: change in mean difference between students' pretest automaticity score and their posttest automaticity score

2. Does frequency of written, timed practice drills significantly change the automaticity rate in basic multiplication facts for sixth-grade math students?

H<sub>0</sub>: Frequency of use of written, timed practice drills will not be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students.

H<sub>1</sub>: Frequency of use of written, timed practice drills will be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students.

*Independent variable*: frequency of written, timed practice drills

*Dependent variable*: change in mean difference between students' pretest automaticity score and their posttest automaticity score

### **Research Design and Approach**

The purpose of this quantitative study was to determine if there is a significant relationship between the use of written, time practice drills and the change in mean difference between students' pretest automaticity scores and their posttest automaticity scores. The length of the study was eight weeks. This length of study was based on the county school calendar at XYZ Middle School, as well as the structure of the Walden semester system. According to Creswell (2003), a quantitative approach is best if the problem to be researched is to identify factors that have the potential to influence outcome, whether positively or negatively. The researcher employed a quasi-experimental design for this study, specifically the nonequivalent group pretest posttest design. This design was chosen over a true experimental design because of the fact that this study took place in a traditional education setting involving classes that were already intact before the study began. It was not feasible for the researcher to incorporate the use

of random assignment. The use of a pretest posttest design allowed the researcher to partially remove a major limitation by allowing an assessment of the participant groups' initial differences, if any. In this design, the researcher manipulates one of the variables, which in the case of this proposed study is the use of written, timed practice drills, and observes the second variable, in this case participants' posttest performance, to determine if the manipulated variable caused a change in the second variable. This study involved the analysis of three student groups: one with no treatment, one with daily timed practice drills, one with weekly timed practice drills.

To address the research questions previously defined, three analysis of variance (ANOVA) procedures were conducted. The first ANOVA was used to determine whether the three student groups differed in their pretest performance. The second ANOVA was used to determine whether the three groups differed in their posttest performance. The third ANOVA was used to determine whether the three groups differed in their gain scores. Scheffe' post hoc analysis was then utilized to determine which pair of groups differ in their performance.

For each of the three groups, two automaticity pretests (Luce, 2002) were conducted at the beginning of the study, and a similar posttest was conducted after 8-weeks of treatment. According to Gravetter and Wallnau (2005), "the major advantage of ANOVA is that it can be used to compare two or more treatments. Thus, ANOVA provides researchers with much greater flexibility in designing experiments and interpreting results (p. 327)." Additionally, ANOVA reduces the probability of making a Type I error (Patten, 2002). A Type I error occurs when the researcher rejects the null

hypothesis, concluding there was evidence that a treatment led to a change when it really did not. An ANOVA procedure allows the researcher to make statistical comparisons simultaneously, therefore reducing the risk of a false-positive.

For both of the research questions, the dependent variable (DV) is the difference between students' pretest automaticity scores and their posttest automaticity scores. Because the pretests and posttests are to be scored by the number of correct answers on the 111-problem assessments, this variable is considered a discrete variable because the value assigned to it is restricted to whole, countable numbers. While the statistical mean of the scores can have a value between whole numbers, it is not possible for a student's pretest or posttest score to fall between the whole numbers.

The independent variable for the first research question is the use of written, timed practice drills. This variable is considered a categorical variable, in that it has two categories with no intrinsic ordering of the categories. The two categories for this variable are receipt of the treatment of the use of written, timed practice drills or absence of the use of written timed practice drills.

The independent variable for the second research question is the frequency of written timed practice drills. This variable is also considered a categorical variable since it has three categories whose order is not intrinsic. The three categories for this variable are daily use, weekly use, and no use of written, timed practice drills. When considering the analyses of the statistical values for this proposed study, the researcher made the assumption that the intervals between all variable measurements are normally distributed.

According to Campbell and Stanley (1963) there are eight extraneous variables that can interfere with internal validity of a study: history, maturation, testing, instrumentation, statistical regression, selection, experimental mortality, and selection-maturation interaction. One threat to the internal validity of this proposed study is the possibility that students' performance on the pretest and posttests could be affected by the time of the school day when the assessments are given. Research has revealed students' alertness and attention are affected by time of day preferences (Callan, 1999; Dunn & Bruno, 1985). To address this potential threat, this study is designed so that each of the participating teachers will administer each of the three treatments distributed throughout the academic day (See Table 1). This organization of treatments will minimize a potential threat to validity regarding time of day as well as student weariness.

Table 1

*Organization of Treatment Groups for Participating Teachers*

Class Period	Teacher A	Teacher B	Teacher C
1	Daily	Not part of study	None
2	Weekly	Daily	Not part of study
3	None	Weekly	Daily
4	Not part of study	None	Weekly

Additionally, to address the internal validity threat of testing, participants were administered two pretests that were 3 days apart from each other. The participants' two pretest scores were averaged, and that average served as the participants' automaticity

pretest score. Test-taking fatigue will be minimal due to the 3-minute time limit on the pretests, treatments, and posttests. The threat of experimental mortality is possible, yet not likely going to be a factor based on the school's student enrollment history and stability.

With regards to external validity, several aspects of the study have been addressed. To avoid multiple treatment interference, each participant will be assigned to only one of the treatment groups for the entire duration of the study. Given the recent more stringent policy of the institutional review board's requirements for parent consent and student assent when student identifiers are collected, the study was designed to collect data via an activity that was part of the standard curriculum for sixth grade math students at XYZ Middle School. To comply with the institutional review board requirements, this study was conducted without any recording of names or other identifiers, and the school principal had given her approval for the study to be conducted without obtaining parental consent.

The sample population for this study was not randomized and was limited to one geographic location, one school, and one age group. The teachers who participated in this study may have differed in the way they presented and administered the written, timed drills. Their differing attitudes and mannerisms may be viewed as a limitation. Also, the Pygmalion effect is possible, which refers to students in the timed drill groups performing better than students who do not receive the timed drills simply because their teacher expects them to do so (Rosenthal & Jacobson, 1968). It is possible that students may

enroll or withdraw from the school at which this study is being conducted. This transiency may be considered a limitation as it will affect the final data gathered.

### **Setting and Participants**

The population for this research study will be sixth grade regular education math students who attend XYZ Middle School, which is located in a suburban area outside of the Atlanta, Georgia area. According to enrollment data at XYZ Middle School, there are approximately 480 students in this population of sixth graders. The Accountability Reports for the fifth graders in the 2008/2009 school year who are the current sixth grade students at XYZ Middle School showed that 92.4% of all students, including those with disabilities, met or exceeded standards on the state's math Criterion Referenced Competency Test (CRCT). The Mathematics CRCT measures students' performance in six domains (number and operations, measurement, geometry, algebra, and data analysis and probability) and is aligned with the State of Georgia's Performance Standards. The participants come from three feeder elementary schools, and each of those schools has earned adequate yearly progress (AYP) every year. AYP is a series of performance goals that state schools are required to meet by the No Child Left Behind Act (2001). Recent data for the 2009/2010 school year shows that 31% of the study's population receives free or reduced lunches.

Using a sample size calculator with a confidence interval of 95%, a population size of 480 students should have a sample size of 214. The participants for this study was made up of approximately 240 sixth-grade students from the population described above, with each of the three groups containing about 80 students. There were three participating

teachers, each of which instructed one of each of the three groups, for a total of nine regular education math classes. The average class size for these groups was 26. The researcher selected the three participating teachers due to their expressed interest in research-based instruction, as well as their previous experience with sixth-grade math students. The size of the sample in this study was bigger than the samples in any study reviewed in Chapter 2. According to the law of large numbers, having a larger sample size makes it more likely that the sample mean will be closer to the population mean (Gravetter & Wallnau, 2005).

This study was conducted without any recording of names or other personal identifiers. Because of this anonymity and the fact that the study treatments are considered part of the standard curriculum, there was no need to ask for parental consent. The decision to not gather personal descriptive data likely led to an increase in participation and a decrease in the chance that certain subgroups would refuse participation due to fear of embarrassment or judgment. The descriptive data for the study's participating classes are included in the final study results. While administrative measures were taken to ensure that each of the math classes is balanced with regard to student ability, ethnicity, and gender, it is possible that some unbalance may have occurred due to student enrollment or student withdrawal. This study, however, was not designed to analyze any subgroup data.

In general, XYZ Middle School's population has a high attendance rate of 96%. Careful consideration was given to ensure that there were no religious holidays or other

circumstances that might have resulted in high student absences during the study.

Accurate attendance records were kept and are noted in the conclusion of the study.

The participating teachers administered the applicable treatment for each of the three study treatment groups for six weeks. At the end of the six weeks, all participants were given a posttest. The posttests were scored and the mean differences were analyzed. This information could not be traced back to any individual student, yet it will be potentially useful for policy and educational interventions.

### **Instrumentation and Materials**

The written, timed practice drills used in this study were created by a veteran middle school math teacher to address a need she saw with regards to middle school students lacking proficiency and speed when they attempted to recall basic math facts (Luce, 2007). After extensive use and positive results in her own class, Luce shared the drills with colleagues, who then began utilizing the written, timed drills strategy as a method to help students increase their automaticity of basic math facts. Last school year, all of the sixth grade math teachers at the researcher's former school, including gifted and special education teachers, incorporated these drills in their math classes. The drills are known as three minute math. Luce does not have statistically analyzed data to support her own class results, so it is important to address this lack of data support as a threat to the validity and reliability of this research study.

All three participant groups were administered two pretests (see Appendix A) that included 111 basic multiplication fact problems (i.e.  $8 \times 7 = \underline{\quad}$ ;  $11 \times 12 = \underline{\quad}$ ;  $9 \times 6 = \underline{\quad}$ ), the first on the first day of the study, and the second on the third day of the study.

They were given exactly three minutes to correctly answer as many problems as they could. The participants' scores on the two pretests were averaged and this average served as their automaticity pretest scores. Following the pretests, the first experimental group was administered a similar written practice sheet each day for 40 school days (eight weeks). The second experimental group was administered a similar written practice sheet weekly for six weeks. The control group did not receive any written practice during the 8-week study. The time allowed for each group was consistently three minutes. On day 40 of the study, eight weeks after the pretests, all three groups were administered a written posttest (see Appendix B) that also included 111 basic multiplication fact problems.

The written, timed practice drills were designed to be of the same level and difficulty. The multiplication facts used on all of the drills include a random mix ranging from  $1 \times 1$  through  $12 \times 12$ . All of the teachers who taught sixth grade math at the researcher's former school and used the three-minute math drills have agreed that the drills are appropriately leveled and are all equally challenging.

### **Data Collection**

After receiving permission from Walden University's institutional review board, county administration, and the local school principal, data collection began. Participating teachers administered and collected the pretests, all applicable treatment drills, and the posttest. The total number of multiplication facts correct served as the participants' scores. Prior to giving the participants' completed written drills to the researcher, the participating teachers removed the student's name by cutting off the name line at the top

of the drill page. All pretests and posttests were hand scored by the researcher or a participating teacher. An inter-rater reliability random check was employed.

To address the first research question, data was gathered from the control group, the group who received no treatment of additional written, timed practice drills between the time of the pretests and posttest, and the experimental group who received the treatment of daily written, timed practice drills. To address the second research question, the data was gathered from the control group and the experimental groups who received the treatment of either daily or weekly written, timed practice drills.

### **Data Analysis**

The researcher employed quantitative methodology strategies to test the following null and alternative hypotheses:

Null Hypothesis 1: Use of written, timed practice drills will not be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students.

Alternative Hypothesis 1: Use of written, timed practice drills will be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students.

Null Hypothesis 2:  $H_0$ : Frequency of use of written, timed practice drills will not be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students.

Alternative Hypothesis 2: Frequency of use of written, timed practice drills will be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students.

Pretest and posttest scores were based on the participants' total number of correct responses to basic multiplication facts. Gain scores were calculated by subtracting the participants' pretest scores from their posttest scores. The skewness and kurtosis revealed the presence of normally distributed data, which allowed for the effective use of parametric procedures to be conducted. Specifically, three ANOVA procedures were performed to determine if there were differences in pretest performance, posttest performance, and gain scores of the three treatment groups.

According to Gravetter and Wallnau (2005), ANOVA offers more flexibility to the researcher as opposed to *t* tests when designing and interpreting study results if two or more treatments are being compared. If a significant ANOVA is determined, Scheffe' post hoc analysis is conducted. Scheffe' post hoc analysis is considered the "safest of the posttest techniques because it provides the greatest protection from Type I errors" (p. 358).

### **Rights of Participants**

In order to protect the rights of the participants in this research study, the identities of the participants were in no way linked to the data collected for analysis. The participating teachers cut off students' names from the top of the drill pages prior to submitting them to the researcher. The researcher has no access to the students' actual

identities. The drill pages are currently being stored in a locked filing cabinet at the home of the researcher.

The students participating in this study were assigned to one of the treatment groups solely based upon the participating teachers' random assignment regarding treatment type for their particular math class period. Student assignment to a teachers' class was made by the school's administration using a management tool called SASI (School Administration Student Information) to balance student characteristics such as gender, ethnicity, and socioeconomic levels among the sixth grade classes. Since none of the participating teachers have used these timed practice drills in their classes prior to this study, no group is being deprived of meaningful, ongoing instructional practices.

### **Role of Researcher**

This study's researcher is a sixth grade mathematics teacher at XYZ Middle School, the school in which the study will be conducted. The researcher has worked with the participating teachers less than one school year. None of the researcher's students will be participating in the study, and the participating teachers are not subordinates of the researcher. The researcher and the participating teachers meet frequently to discuss and collaborate effective teaching practices and instructional strategies to best meet the needs of their students. These meetings take place at least once a week throughout the school year.

## Chapter 4: Results

This chapter explains the data analysis procedure utilized to answer the research questions that inspired this study. The statistical findings are interpreted and summarized. The null hypothesis for both research questions should be rejected.

### Research Questions

The analysis of data tested the following research questions:

1. Do written, timed practice drills increase automaticity in basic multiplication facts for sixth-grade math students?

$H_0$ : Use of written, timed practice drills will not be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students

$H_1$ : Use of written, timed practice drills will be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students

2. Does frequency of written, timed practice drills significantly change the automaticity rate in basic multiplication facts for sixth-grade math students?

$H_0$ : Frequency of use of written, timed practice drills will not be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students.

$H_1$ : Frequency of use of written, timed practice drills will be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students.

### **Data Analysis**

To analyze the data gathered from this quantitative study, participating students' pretest and posttest scores were entered into SPSS and were assigned a group number. Descriptive statistics were calculated for all 227 participating students' pretest and posttest scores. None of the original sample had to be eliminated due to legibility.

Prior to conducting inferential statistics to address the research questions concerning differences between student performance in the treatment groups, checks were conducted to determine the extent to which student test scores were normally distributed. The skewness and kurtosis for students' pretest scores, posttest scores, and their gain scores were calculated separately for each of the three groups. All 18 skewness and kurtosis values indicated the presence of normally distributed data, thereby permitting the use of parametric procedures, specifically the use of the Analysis of Variance (ANOVA) procedure.

Though the statistical procedure described in the method section of this dissertation involved an independent samples t-test to answer the first research question, and an ANOVA to answer the second research question, the procedure utilized to answer both research questions in this analysis was an analysis of variance procedure. When two groups are present, a t-test would be appropriate, although an ANOVA would be appropriate to utilize, as well (Gravetter & Wallnau, 2005). The researcher opted for an ANOVA procedure because it offers more flexibility by allowing for comparison of the three treatment groups rather than using only the no treatment group and the once a week treatment group as originally designed. When three groups are present, however, as they

are in this study, the t-test becomes an inappropriate statistical procedure. The appropriate statistical procedure for three groups, when normality of data is present and when the data are at the ratio level, is an ANOVA. This avoids the need to conduct separate hypothesis testing, which would result in an increase in a Type I error. As previously mentioned, a Type I error occurs when the null hypothesis is inaccurately rejected, resulting in the researcher concluding that the treatment being tested was found to work when it really did not work. As described by Gravetter and Wallnau (2005), each time a hypothesis is tested at an alpha level of 0.05, the chance of a Type I error is 1 out of 20. Conducting 3 separate hypothesis tests would raise the chance of error to 3 out of 20. Therefore, the ANOVA statistical procedure was used and reported in this chapter.

In the use of an ANOVA procedure, the  $F$  and  $p$  values indicate whether a statistically significant difference is present. When a statistically significant result is present ( $p < .05$ ), then follow up procedures such as Scheffe' post hoc analysis are utilized to determine which pair of groups differ in their test performance. To address the research questions previously delineated, three ANOVA procedures were conducted: (a) to determine whether the three student groups differed in their pretest performance; (b) to determine whether the three student groups differed in their posttest performance; and (c) to determine whether the three student groups differed in their gain scores.

Descriptive statistics were calculated for the gain score, or the difference between students' posttest scores and their pretest scores. A gain score reflects the amount of growth or change in student performance over the duration of the treatment. As evidenced in Table 2, students showed substantial growth in the daily timed practice drill,

an average gain of 32.34 points, compared to a minimal gain of 2.76 points by the no treatment group.

Table 2

*Descriptive Statistics for Students' Pretest and Posttest Scores by Group Membership*

Group	<i>n</i>	Pretest, Posttest, Gain	<i>M</i>	<i>SD</i>
No treatment	76	Pretest	73.41	28.72
		Posttest	76.17	29.27
		Gain	2.76	8.72
Weekly timed practice drill	75	Pretest	66.56	25.79
		Posttest	83.79	24.95
		Gain	17.23	9.04
Daily timed practice drill	76	Pretest	70.22	26.78
		Posttest	102.57	31.86
		Gain	32.34	9.88

The analysis of data to support the rejection of the null hypotheses for the study's two research questions follows.

**Research Question 1: Interpretation of Findings**

For the first research question, the ANOVA did not reveal the presence of a statistically significant difference among the three student groups in their pretest performance,  $F(2, 224) = 1.20, p = .30$ . Table 3 presents the ANOVA summary. As such, the performance of the three groups of students was equivalent, prior to the onset of the treatment. If the three groups of students had demonstrated statistically significant

differences in their pretest scores, then these differences would have had to be accounted for statistically in any other statistical analyses.

Table 3

*Summary of Analysis of Variance (ANOVA) for Pretest*

Source	df	MS	F	<i>p</i>
Between groups	2	886.3	1.20	.30
Within groups	224	736.0		
Total	226			

*Note.* df =degrees of freedom; MS = mean square; F =observed f-value; p = significance level

Because the three groups of students had similar pretest scores, then the ANOVA procedure was an appropriate statistical technique. Additionally, the ANOVA yielded a statistically significant difference among the three student groups in their gain scores,  $F(2, 224) = 195.19, p < .001, \eta^2 = .64$ . Table 4 presents the ANOVA summary. Students showed substantial growth in the daily timed practice drills, an average gain of 32.34 facts correct, compared to a minimal gain of 2.76 facts correct.

Table 4

*Summary of Analysis of Variance (ANOVA) for Gain Score*

Source	df	MS	F	<i>p</i>
Between groups	2	16626.0	195.19	$\leq .0004$
Within groups	224	85.2		
Total	226			

*Note.* df =degrees of freedom; MS = mean square; F =observed f-value; p = significance level

The effect size for this statistically significant difference was extremely large at  $\eta^2 = .64$  (Cohen, 1988) and indicates that the differences between mean scores are very unlikely to be due to simple chance. Scheffe' post hoc analysis revealed that the daily timed practice drill group had statistically higher gain scores than students in either the no treatment group or in the weekly timed practice drill group. Similarly, students in the weekly timed practice drill group had statistically significantly higher gain scores than did students in the no treatment group. Although students in the weekly timed practice drill group began this study with the lowest average score on the pretest, they made statistically higher gains than did students in the no treatment group.

### **Research Question 2: Interpretation of Findings**

Concerning the second research question, the ANOVA yielded a statistically significant difference among the three student groups in their posttest performance,  $F(2, 224) = 16.84, p < .001, \eta^2 = .13$ . Table 5 presents the ANOVA summary.

Table 5

#### *Summary of Analysis of Variance (ANOVA) for Posttest*

Source	df	MS	F	<i>p</i>
Between Groups	2	14019.3	16.8	$\leq .0004$
Within Groups	224	832.3		
Total	226			

*Note.* df =degrees of freedom; MS = mean square; F =observed f-value; *p* = significance level

The effect size for this statistically significant difference was large at  $\eta^2 = .13$  (Cohen, 1988). Scheffe' post hocs revealed that the daily timed practice drill group had

statistically higher posttest scores than students in either the no treatment group or in the weekly timed practice drill group. Students in the no treatment group and students in the weekly timed practice drill group did not differ in their posttest scores. Readers should note, however, that students in the weekly timed practice drill group began this study with the lowest average score on the pretest measure.

### **Summary**

Given the results of the data analysis for this study, the null hypotheses for the two research questions should be rejected. Written, timed practice drills were statistically shown to improve the automaticity rate in basic multiplication facts for sixth-grade math students, and evidence gathered from the data are overwhelmingly convincing that the treatment of the use of daily timed practice drills essentially work to positively change the automaticity rate for sixth-grade math students.

## Chapter 5: Discussion, Conclusions, and Recommendations

This chapter provides an overview of this quantitative quasi-experimental research study's methodology and findings, interpret the findings, examine implications for social change, offer recommendations for action, and present recommendations for further study.

### **Summary of Findings**

This quantitative study was conducted to determine if the use of written, timed practice drills have an impact on students' level of automaticity of basic multiplication facts. The research questions probed in this study were as follows:

1. Do written, timed practice drills increase automaticity in basic multiplication facts for sixth-grade math students?

H<sub>0</sub>: Use of written, timed practice drills will not be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students

H<sub>1</sub>: Use of written, timed practice drills will be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students

2. Does frequency of written, timed practice drills significantly change the automaticity rate in basic multiplication facts for sixth-grade math students?

H<sub>0</sub>: Frequency of use of written, timed practice drills will not be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students.

H<sub>1</sub>: Frequency of use of written, timed practice drills will be significantly associated to a change in the automaticity rate in basic multiplication facts for sixth-grade math students.

Automaticity is defined as the automatic recall of facts without conscious control (Hasselbring, Goin, & Bransford, 1988), and some researchers view it as the core of overall number sense (Wong & Evans, 2007; Woodward, 2006; Ball et al., 2005; Bratina & Krudwig, 2003; Steel & Funnell, 2001).

The 8-week long study involved 227 regular education sixth grade math students from three participating teachers' classes. Each of the three teachers instructed one of three groups. The daily timed practice drill group received the treatment of one 3-minute drill each school day throughout the length of the study; the weekly timed practice drill group received one 3-minute drill per week; the no treatment group received only the pretest and posttest with no timed drills in between.

The analyzed data supports the rejection of both null hypotheses. An ANOVA revealed that the three groups of students were equivalent with regards to their pretest performance prior to the onset of treatment; however after 8 weeks, students who received written, timed practice drills, either daily or weekly, outperformed students who received no treatment,  $F(2, 224) = 195.19, p < .001, \eta^2 = .64$ . These results confirm the alternative hypothesis that written, timed practice drills did significantly change the automaticity rate in basic multiplication facts for sixth-grade math students, and the change is a positive one. With regards to the frequency of written, timed practice drills, an ANOVA showed a significant difference in the gain scores of students who received

daily written, timed practice drills compared to the groups who received weekly drills or no treatment of drills,  $F(2, 224) = 16.84, p < .001, \eta^2 = .13$ .

### **Interpretation of Findings**

The findings of this study strongly support the use of daily written, timed practice drills as a means to improve automaticity of basic multiplication facts for regular education sixth graders. The use of such drills, just 3 minutes a day, does not take up much class time, thus would not be difficult for teachers to incorporate them into a daily routine. When the benefits of automaticity in basic computation have been widely reported (Rasmussen & Bisanz, 2005; Rittle-Johnson et al., 2001; Westwood, 2003; Norbery, 2002, Mercer & Miller, 1992), it is unjustified for mathematics educators to ignore the need for and benefit of short, targeted practice. Research indicates that students who struggle with retrieval of basic math facts have difficulties when solving real life application problems (Hasselbring et al., 2005; Steel & Funnell, 2001; Maccini & Gagnon, 2000; Mercer & Miller, 1992). This study's findings support the use of a strategy to help students increase their successful retrieval rate of such facts.

According to Baddeley (2000), working memory is a limited resource. Studies have supported the idea that as one's automaticity increases, the mind's reliance on working memory decreases, therefore there are fewer functions competing for the limited capacity of working memory when solving problems (Tronsky, 2005; DeStefano & LeFevre, 2004). The findings of this study support the work of DeStefano and LeFevre (2004) and Tronsky (2005), who found when automaticity increases, reliance on one's working memory decreases. Because of this decrease, fewer mental functions are

competing for the limited capacity of working memory needed to analyze and solve problems.

### **Implications for Social Change**

Increasing student achievement in mathematics is an issue that is front-and-center for educators on the national, state, and local levels. Decisions about how to manage the instructional time should be based on research and assessment and be driven by the obligation to help students achieve the highest level of math performance possible. The research findings of this study imply that a decision to utilize short periods of valuable classroom time to reinforce and increase automaticity of basic math facts is indeed justified. According to Wong and Evans (2007), automaticity in basic math facts serves as the foundation for advancement in mathematics. If students are able to accurately and habitually rely on their ability to perform these basic computations, they are able to devote their attention to the overall purpose of the problem they are attempting to solve (Isaacs & Carroll, 1999).

Global economic instability has brought more emphasis to the math performance of the United States' graduating students in regards to their contributions to the country's ability to compete in the world's job markets. By implementing instructional practices that aim to help students increase their level of mathematical automaticity, educators are directly affecting students' future employment opportunities and strengthen the country's economic health.

This study offers a feasible instructional practice that does not require much time, yet yields a statistically significant positive outcome. Another implication for social

change is that this study can serve as a springboard for more research studies to be conducted. There is a huge gap in the literature that addresses basic math facts automaticity for regular education students, and with such strong results, this study may serve as a stepping stone to begin to close that gap.

### **Recommendations for Action**

These results advocate an immediate call for action on the parts of math educators to address the issue of automaticity of basic math facts. While there is immense pressure to cover curricular objectives that fill yearly instructional calendars, educators cannot ignore the reality that many regular education students make it out of elementary school without mastering proficiency with retrieval of basic math facts. Hoping these students will eventually improve their skills without deliberate practice is not in the best interest of students.

These results should be shared with educational leaders as well as classroom teachers to justify the use of a few minutes of class time in order to offer automaticity practice that will potentially lead to freeing up students' working memory. The written, timed practice drills are easy to administer and require little to no training. In addition to Luce's unpublished drills, there are several timed drills books on the market (i.e. Mad Minutes) so teachers do not have to invest time in creating such drills to use with their students to assist with improving automaticity of basic math facts.

While I respect the recommendations of the NCTM, it is important to remember that there are certain levels of computation proficiency necessary for middle school students to utilize when they are exploring and collaborating in order to develop problem-

solving strategies and approaches. According to the NCTM Curriculum Focal Points (2000), by grade 4, students should develop quick recall of whole number multiplication facts and fluency. The use of written, timed practice drills to help increase students' automaticity of basic multiplication facts should be strongly considered as a meaningful tool to aid students in the problem-solving process. School mathematics curriculum leaders should take notice of the results in this study of using timed drills in middle school to help students increase their automaticity.

The unfortunate reality is some teachers are made to feel by state curriculum leaders, as well as those in their own school districts, that having students partake in drill and practice is an antiquated method that does not challenge students to be mathematical thinkers and does not engage students in meaningful learning. The argument, as supported by Royer et al. (1999), is that students who are more proficient at recalling basic math facts in a speedy yet accurate manner are more likely to perform higher on standardized tests than students who struggle with basic computation.

### **Recommendations for Further Study**

The significance of this study's findings for improving students' higher-level math performance calls for this study to be replicated in other schools and districts, making possible more generalization of the findings. It is recommended that future studies gather subgroup data such as gender, ethnicity, and previous performance scores on standardized testing to answer research questions regarding possible differences in outcomes for such groups. Longitudinal studies could determine if the gains last into the next academic year and beyond. Research could also seek to determine how many weeks

of practice is enough. Having only studied one grade level, a multi-grade level study with longitudinal follow-up could help determine what grade is best for this curricular practice. This study also did not test the best time of day or placement in the mix of other math curriculum for the outcome. There was no difference in the outcome among the three participating teachers' implementation of the written, timed practice, but a larger sample from more classrooms might show that certain delivery models of the drills are more effective than others.

Research could be done on the effect of timed practice on other math skills, as well. This study only addressed basic multiplication, as it is at this point that many students run into their first hindrance in math (Geary, 2004; Kilpatrick et al., 2001). Studies could be done on basic addition, subtraction, and division. On a wider spectrum of skills, speedy but accurate computation involving fraction and decimal numbers could be explored. To be a workable classroom activity in a differentiated classroom, would some successful students move on to do 3-minute time drills in other areas of math? Finally, an earlier grade than the sixth grade classrooms used in this study may be a better time for reinforcing automaticity, and research could test the impact of quick daily timed drills with younger children to determine if the use of such drills is statistically justified. According to Steel and Funnell (2001), students who are not able to recall basic multiplication facts by age 11 years are less likely to effectively use them in a structured manner later. Longitudinal studies, starting in earlier grades would also be important. If so, it is possible to have more students entering middle school mathematics classrooms who have already been given enough practice to gain the proficiency needed to be

successful in the higher level problem-solving and the algebraic concepts as anticipated by the NCTM.

Another aspect that could be investigated is whether the use of a self-tracking system for students to record their progress each day as they complete a written, timed practice drill would further increase their number of correct responses. This type of tracking system could potentially motivate students to strive for more proficiency, as they are able to see concrete evidence of improvement. Such a study would be suggested by the self-efficacy component of Bandura's (1986) social cognitive theory. Researchers involved in analyzing this theory find particularly in the area of math, that students' belief in their ability to perform mathematical tasks made a huge difference in their learning (Bong, 1996; Bandalos, Yates, & Thorndike-Christ, 1995). According to Bandura and Adams (1977), "performance accomplishments provide the most influential efficacy information because it is based on personal mastery experiences" (Bandura & Adams, 1977, pg. 297). Additionally, it is recommended that research be done to determine if an increase in students' automaticity of basic computation has any effect on their self-efficacy with regards to their broader math ability. Class participation is also worthy of investigation with regards to students' belief in their own ability of fact retrieval. Woodward and Baxter (1997) found that students who lack the ability to retrieve math facts quickly are not as likely to participate in math discussions in class.

### **Conclusion**

The pretest and posttest data gathered and analyzed in this quantitative research study led to the rejection of the null hypothesis that implementing written, timed practice

drills can be linked to students' increase in automaticity of basic multiplication facts. Daily classroom use of the drills was shown to have more positive influence than only weekly use. While the exercise tested here does require a small amount of class time each day, the benefit of the strategy is apparent when one compares the statistically significant results.

Many regular education math students are entering middle school lacking a necessary skill that allows them to further their ability to learn higher-level skills that are necessary to become effective problem solvers (Westwood, 2003; Norbury, 2002; Kilpatrick et al., 2001). Students' performance has been shown to increase as they become more adept in their understanding of essential arithmetic operations (Swanson, Jerman, & Zheng, 2008).

The attention by middle school math teachers devoted to students obtaining and maintaining automaticity of basic math facts is severely lacking. It is my hope that this study leads to increased opportunities in the classroom for middle school students to improve their retrieval skills of such facts. I encourage teachers to evaluate their students' automaticity ability and to address deficiencies with targeted practice, such as the one described in this study.

## References

- Ashcraft, M. (1992). Cognitive arithmetic: A review of data and theory, *Cognition*, 44, 75-106.
- Ashcraft, M. & Battaglia, J. (1978). Cognitive arithmetic: Evidence for retrieval and decision processes in mental addition. *Journal of Experimental Psychology: Human Learning & Memory*, 4, 527-538.
- Ashcraft, M. & Christy, K. (1995) The frequency of arithmetic facts in elementary texts: Addition and multiplication in Grades 1 – 6. *Journal for Research in Mathematics Education*, 26(5), 396-421.
- Ashcraft, M. (2002). Math anxiety: Personal, educational, and cognitive consequences. *Current Direction in Psychological Science*, 11(5): 181 - 185.
- Baddeley, A. (1992). Working memory. *Science*, 255, 556-559.
- Baddeley, A. (2000). The episodic buffer: A new component of working memory? *Trends in Cognitive Science*, 4, 417-423.
- Baddeley, A. & Logie, R. (1999). The multiple-component model. In A. Miyake & P. Shah (Eds.), *Models of working memory: Mechanisms of active maintenance and executive control* (pp. 28-61). Cambridge, England: Cambridge University Press.
- Ball, D., Ferrini-Mundy, J., Kilpatrick, J., Milgram, J., Schmid, W., & Schaar, R. (2005). Reaching for common ground in K-12 mathematics education. *Notices of the American Mathematical Society*, 52(9), 1055-1058.

- Bandalos, D., Yates, K., & Thorndike-Christ, T. (1995). Effects of math self-concept, perceived self-efficacy, and attributes for failure and success on test anxiety. *Journal of Educational Psychology*, *87*, 611-623.
- Bandura, A., (1997). *Social learning theory*. Englewood Cliffs, NJ: Prentice-Hall.
- Bandura, A., & Adams, N. (1977). Analysis of self-efficacy theory on behavioral change. *Cognitive Therapy and Research*, *1*, 287-308.
- Barber, B. K., & Olsen, J. A. (2004) Assessing the transitions to middle and high school. *Journal of Adolescent Research*, *19*(1), 3-30.
- Bong, M. (1996). Problems in academic motivation research and advantages and disadvantages of solutions. *Contemporary Educational Psychology*, *21*, 149-165.
- Bottge, B.A., Rueda, E., Serlin, R.C., Hung, Y.H., & Kwon, J. M. (2007). Shrinking achievement differences with anchored math problems: Challenges and possibilities. *Journal of Special Education*, *41*(1), 31-49.
- Bratina, T. & Krudwig, K. (2003). Get it right and get it fast! Building automaticity to strengthen mathematical proficiency. *Focus on Learning Problems in Mathematics*, *25*(3), 47-63.
- Burns, M. (2005). Using incremental rehearsal to increase fluency of single digit multiplication facts with children identified as learning disabled in mathematics computation. *Education and Treatment of Children*, *28*(3), 237-249.
- Butterworth, B., Zorzi, M., Girelli, L., & Jonckheere, A. (2001). Storage and retrieval of addition facts: The role of number comparison. *Quarterly Journal of Experimental Psychology*, *54A*, 1005-1029.

- Campbell, D., & Stanley, J. (1963) Experimental and quasi-experimental designs for research. In N. L. Gage (Ed.) *Handbook of research on teaching*. Chicago: Rand-McNally.
- Campbell, J. & Oliphant, M. (1992). Number-fact retrieval: A model and computer simulation. In J. I. D. Campbell (Ed.), *The nature and origin of mathematical skills* (pp. 331-364). Amsterdam: Elsevier.
- Campbell, J. & Xue, Q. (2001). Cognitive arithmetic across cultures. *Journal of Experimental Psychology: General*, 130, 299-315.
- Caron, T. A. (2007). Learning multiplication the easy way. *Clearing House: A Journal of Educational Strategies, Issues and Ideas*, 80(6), 278 – 282.
- Chung, H., Elias, M., & Schneider, K. (1998). Patterns of individual adjustment changes during middle school transition. *Journal of School Psychology*, 36(1), 83-101.
- Codding, R.S., Shiyko, M., Russo, M., Birch, S., Fanning, E., & Jaspen, D. (2007). Comparing mathematics interventions: Does initial level of fluency predict intervention effectiveness? *Journal of School Psychology*, 45(6), 603-617.
- Cohen, J. (1988). *Statistical power analysis for the behavioral sciences*. (2<sup>nd</sup> ed.). Hilldale, NJ: Erlbaum.
- Creswell, J., (2003). *Research design: Qualitative, quantitative, and mixed-methods approaches*. California: Sage.
- Crotty, M. (1998). *The foundations of social research: Meaning and perspective in the research process*. London: Sage.

- DeStefano, D. & LeFevre, J. (2004). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology, 16*, 353-386.
- Eggen, P. & Kauchak, D. (2000). *Educational Psychology: Windows on classrooms* (5<sup>th</sup> ed.) Upper Saddle River, NJ: Prentice Hall.
- Freeman, T.J., & McLaughlin, T.F. (1984). Effects of a taped-words treatment procedure on learning disabled students' sight-word reading. *Learning Disability Quarterly, 7*, 49-54.
- Fuchs, L., Fuchs, D., & Prentice, K. (2004). Responsiveness to mathematical problem-solving instruction: comparing students at risk of mathematics disability with and without reading disability. *Journal of Learning Disabilities, 37*(4), 293-306.
- Fuson, K. C. (1982). An analysis of the counting-on procedure in addition. In T. H. Carpenter, J. M. Moser, & T. H. Romberg (Eds.), *Addition and subtraction: A cognitive perspective* (pp. 67-78). Hillsdale, NJ: Lawrence Erlbaum.
- Fuson, K. C. (1988). *Children's counting and concepts of number*. New York: Springer.
- Geary, D. (1996). The problem-size effect in mental addition: Developmental and cross-national trends. *Mathematical Cognition, 2*(1), 63-93.
- Geary, D. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37*(1), 4-15.
- Georgia Department of Education, (2009). *CRCT content description: Mathematics*. Retrieved July 17, 2009, from <http://www.doe.k12.ga.us/DMGetDocument.aspx/Content%20Descriptors%20Ma>

th%20Grade%2018%20August%2019,%202009.pdf?p=6CC6799F8C1371F63F  
D4A1D950826FBB4D0F1F28D1D3AFF233CB1B89645D975D&Type=D

Gelman, R., Meck, E., & Merkin S. (1986). Young children's numerical competence. *Cognitive Development, 1*, 1-29.

Gersten, R., Jordan, N., & Flojo, J. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities, 38*(4), 293-304.

Goldman, S., Pellegrino, J., & Mertz, D. (1988). Extended practice of basic addition facts: Strategy changes in learning-disabled students. *Cognition & Instruction, 5*(3), 223-265.

Gravetter, F. & Wallnau, L. (2005). *Essentials of statistics for the behavioral sciences*. (5<sup>th</sup> edition). Belmont, CA: Wadsworth/Thompson Learning.

Hanich, L.B., Jordan, N.C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology, 93*(3), 615-626.

Hasselbring, T., Goin, L., & Bransford, J. (1988). Developing math automaticity in learning handicapped children: The role of computerized drill and practice. *Focus on Exceptional Children, 20*(6), 1-7.

Hasselbring, T., Lott, A., & Zydney, J. (2005). *Technology supported math instruction for students with disabilities: Two decades of research and development*.

Retrieved July 3, 2008, from

[http://www.cited.org/library/resourcedocs/Tech-SupportedMathInstruction-FinalPaper\\_early.pdf](http://www.cited.org/library/resourcedocs/Tech-SupportedMathInstruction-FinalPaper_early.pdf)

- Hecht, S. (2002). Counting on working memory in simple arithmetic when counting is used for problem solving. *Memory & Cognition*, 30, 447-455.
- Isaac, S., & Michael, W. (1995). Handbook in research and evaluation. San Diego, CA: EdITS/Educational and Industrial Testing Services.
- Isaacs, A., & Carroll, W. (1999). Strategies for basic fact instruction. *Teaching Children Mathematics*, 5(9), 508-515.
- Kilpatrick, J., Swafford, J., & Findell, B. (2001) *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.
- Kirk, E. P., & Ashcraft, M. H. (2001) Telling stories: The perils and promise of using verbal reports to study math strategies. *Journal of Experimental Psychology: Learning, Memory, & Cognition*. 27, 157-175.
- Kotsopoulos, D. (2007). Unraveling student challenges with quadratics: A cognitive approach. *Australian Mathematics Teacher*, 63(2), 19-24.
- Luce, T. (2002). Three minute math drills. Unpublished manuscript.
- Maccini, P. & Hughes, C. (1997). Mathematics interventions for adolescents with learning disabilities. *Learning Disabilities Research and Practice*, 12(3), 168-176.
- Maccini, P. & Gagnon, J. C. (2000). Best practices for teaching mathematics to secondary students with special needs: Implications from teacher perceptions and a review of the literature. *Focus on Exceptional Children*, 32(5), 1-22.
- Martini, R. (2004). *Metacognitive processes underlying psychomotor performance in*

*children identified as high-skilled, average, and having developmental coordination disorder*. Montreal, Quebec: McGill University.

- Mayer, R. (2004). Should there be a three-strikes rule against pure discovery learning? The case for guided methods of instruction. *American Psychologist*, *59*, 14-19.
- McCall, C.A. (1999). Math computation difficulties in grade 7 and 8 students. (Doctoral dissertation, University of Toronto, 1999). Retrieved from ProQuest (ATTNQ41236.)
- McCallum, E., Skinner, C., & Hutchins, H. (2004). The taped-problem intervention: Increasing division facts fluency using a low-tech self-managed time-delay intervention. *Journal of Applied School Psychology*, *20*(2), 129-147.
- McCallum, E., Skinner, C., Turner, H., & Saecker, L. (2006). The taped-problem intervention: Increasing multiplication fact fluency using a low-tech, classwide, time-delay intervention. *School Psychology Review*, *35*(3), 419-434.
- Mercer, C. & Miller, S. (1992). Teaching students with learning problems in math to acquire, understand, and apply basic math facts. *Remedial and Special Education*, *13*(3), 19-35.
- National Council of Teachers of Mathematics. (2000), *Principles and standards for school mathematics*. Reston, VA: Author.
- No Child Left Behind (NCLB) Act of 2001, 20 U. S. C. A. § 6301 (2002).  
<http://www2.ed.gov/policy/elsec/leg/esea02/107-110.pdf>
- Neuman, W. *Social research methods: Qualitative and quantitative approaches*. Boston: Allyn and Bacon.

- Norbury, H. (2002). The role of models and representation in the development of multiplicative thinking. In B. Barton, K. C. Irwin, M. Pfannkuch, & M. O. J. Thomas (Eds.) *Mathematics Education Research Group of Australasia, Auckland, NZ, Vol. 2, pp. 528-535*. Sydney: MERGA.
- Ojose, B. (2008). Applying Piaget's theory of cognitive development to mathematics instruction. *Mathematics Educator, 18*(1), 26-30.
- Patten, M. L. (2002). Understanding research methods: An overview of the essentials (3<sup>rd</sup> ed.) Los Angeles: Pyrczak Publishing.
- Parker, A. K., (2009). Elementary organizational structures and young adolescents' self-concept and classroom environment perceptions across the transition to middle school. *Journal of Research in Childhood Education, 23*(3), 325-340.
- Parmar, R., Frazita, R., & Cawley, J. (1996). Word problem-solving by students with mild disabilities and normal achieving students. *Exceptional Children, 62*, 415-430.
- Pellegrino, J. W., & Goldman, S. R. (1987). Information processing and elementary mathematics. *Journal of Learning Disabilities, 20*, 23-32.
- Phenix, T.L. & Campbell, J. I. (2001). Fan effects reveal position-specific numerical concepts. *Canadian Journal of Experimental Psychology, 55*(4), 271-276.
- Piaget, J. (1977). *Epistemology and psychology of functions*. Dordrecht, Netherlands: D. Reidel Publishing Company.

- Poncy, B., Skinner, C., & Jaspers, K. (2007). Evaluating and comparing interventions designed to enhance math fact accuracy and fluency: Cover, copy, and compare versus taped problems. *Journal of Behavioral Education, 16*(1), 27-31.
- Rasmussen, C. & Bisanz, J. (2005). Representation and working memory in early arithmetic. *Journal of Experimental Child Psychology, 91*, 137-157.
- Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93*, 346–362.
- Rosenthal, R., & Jacobson, L. (1992) *Pygmalion in the classroom*. Expanded edition: New York, Irvington.
- Royer, J., Tronsky, L. Chan, Y., Jackson, S., & Marchant, H. (1999). Math fact retrieval as the cognitive mechanism underlying gender differences in math test performance. *Contemporary Educational Psychology, 24*, 181-266.
- Rumbaugh, W., & Brown, C. (2000). The impact of reading recovery participation on students' self-concepts. *Reading Psychology, 21*, 13-30.
- Skinner, C. H., Turco, T. L., Beatty, K. L., & Rasavage, C. (1989). Cover, copy, and compare: An intervention for increasing multiplication performance. *School Psychology Review, 18*, 212-220.
- Steel, S., & Funnell, E. (2001). Learning multiplication facts: A study of children taught by discovery methods in England. *Journal of Experimental Child Psychology, 79*(1), 37-55.

- Swanson, H. L., Jerman, O., & Zheng, X. (2008). Growth in working memory and mathematical problem solving in children at risk and not at risk for serious math difficulties. *Journal of Educational Psychology, 100*(2), 343-379.
- Tronsky, L. N. (2005). Strategy use, the development of automaticity, and working memory involvement in complex multiplication. *Memory & Cognition, 33*, 927-940.
- Tronsky, L. N., & Royer, J. M. (2002) Relationships among basic computational automaticity, working memory, and complex mathematical problem solving: What we know and what we need to know. In J. M., Royer (Ed.), Mathematical cognition. Greenwich, CT: Infoage Publishing.
- Tronsky, L. N., & Shneyer, D. (2004). Strategy execution in simple mental division puts a heavy load on the central executive of working memory. Poster presented at the 13<sup>th</sup> Annual Meeting of the New England Psychological Association, Providence, RI.
- Wadsworth, B.J. (1996). Piaget's theory of cognitive and affective development. White Plains, NY: Longman.
- Weinstein, R. (2002). *Reaching higher: The power of expectation in schooling*. Cambridge, MA: Harvard University Press.
- Westwood, P. S. (2003). *Commonsense methods for children with special needs: Strategies for the regular classroom*. London: Routledge.
- Whitehurst, G. (2003, February 6). Speech presented for the U.S. Department of Education, Secretary's Mathematics Summit, Washington, DC.

- Wong, M., & Evans, D. (2007). Improving basic multiplication fact recall for primary school students. *Mathematics Education Research Journal*, 19(1), 89 – 106.
- Woodward, J., & Baxter, J. (1997). The effects of an innovative approach on mathematics on academically low achieving students in inclusive settings. *Focus on Exceptional Children*, 63, 373-388.
- Woodward, J. (2006). Developing automaticity in multiplication facts: Integrating strategy instruction with timed practice drills. *Learning Disability Quarterly*, 29, 269-289.
- Young, M. (1993). Instructional design for situated learning. *Educational Technology research and Development*, 41(1), 43-58.
- Zutaut, A. (2002). *Using mnemonic strategies in fourth grade multiplication*. Retrieved from ERIC database. (ED469632)

## Appendix A: Pretest

Name: \_\_\_\_\_ Ratio: \_\_\_\_\_ Percent: \_\_\_\_\_

**Three Minute Math**

Multiplication Facts

- |                            |                            |                             |
|----------------------------|----------------------------|-----------------------------|
| 1. $7 \times 1 =$ _____    | 38. $4 \times 9 =$ _____   | 75. $6 \times 12 =$ _____   |
| 2. $11 \times 5 =$ _____   | 39. $5 \times 5 =$ _____   | 76. $5 \times 4 =$ _____    |
| 3. $3 \times 11 =$ _____   | 40. $3 \times 7 =$ _____   | 77. $10 \times 12 =$ _____  |
| 4. $10 \times 11 =$ _____  | 41. $9 \times 10 =$ _____  | 78. $4 \times 9 =$ _____    |
| 5. $11 \times 7 =$ _____   | 42. $4 \times 6 =$ _____   | 79. $9 \times 8 =$ _____    |
| 6. $5 \times 5 =$ _____    | 43. $9 \times 8 =$ _____   | 80. $3 \times 9 =$ _____    |
| 7. $11 \times 8 =$ _____   | 44. $5 \times 3 =$ _____   | 81. $8 \times 8 =$ _____    |
| 8. $9 \times 2 =$ _____    | 45. $2 \times 10 =$ _____  | 82. $9 \times 10 =$ _____   |
| 9. $1 \times 6 =$ _____    | 46. $12 \times 6 =$ _____  | 83. $7 \times 7 =$ _____    |
| 10. $5 \times 3 =$ _____   | 47. $8 \times 9 =$ _____   | 84. $9 \times 8 =$ _____    |
| 11. $11 \times 11 =$ _____ | 48. $6 \times 12 =$ _____  | 85. $4 \times 7 =$ _____    |
| 12. $5 \times 9 =$ _____   | 49. $5 \times 3 =$ _____   | 86. $8 \times 12 =$ _____   |
| 13. $9 \times 9 =$ _____   | 50. $12 \times 9 =$ _____  | 87. $9 \times 6 =$ _____    |
| 14. $3 \times 9 =$ _____   | 51. $4 \times 2 =$ _____   | 88. $3 \times 8 =$ _____    |
| 15. $11 \times 3 =$ _____  | 52. $9 \times 6 =$ _____   | 89. $7 \times 5 =$ _____    |
| 16. $5 \times 4 =$ _____   | 53. $12 \times 12 =$ _____ | 90. $5 \times 11 =$ _____   |
| 17. $12 \times 10 =$ _____ | 54. $6 \times 6 =$ _____   | 91. $10 \times 12 =$ _____  |
| 18. $3 \times 3 =$ _____   | 55. $3 \times 6 =$ _____   | 92. $5 \times 5 =$ _____    |
| 19. $11 \times 8 =$ _____  | 56. $7 \times 5 =$ _____   | 93. $7 \times 9 =$ _____    |
| 20. $3 \times 5 =$ _____   | 57. $9 \times 4 =$ _____   | 94. $9 \times 9 =$ _____    |
| 21. $11 \times 8 =$ _____  | 58. $5 \times 12 =$ _____  | 95. $4 \times 11 =$ _____   |
| 22. $7 \times 11 =$ _____  | 59. $8 \times 8 =$ _____   | 96. $3 \times 8 =$ _____    |
| 23. $9 \times 4 =$ _____   | 60. $10 \times 7 =$ _____  | 97. $12 \times 3 =$ _____   |
| 24. $10 \times 8 =$ _____  | 61. $6 \times 10 =$ _____  | 98. $5 \times 12 =$ _____   |
| 25. $7 \times 9 =$ _____   | 62. $9 \times 6 =$ _____   | 99. $4 \times 4 =$ _____    |
| 26. $11 \times 12 =$ _____ | 63. $3 \times 9 =$ _____   | 100. $11 \times 7 =$ _____  |
| 27. $3 \times 9 =$ _____   | 64. $4 \times 6 =$ _____   | 101. $3 \times 4 =$ _____   |
| 28. $8 \times 4 =$ _____   | 65. $12 \times 3 =$ _____  | 102. $7 \times 9 =$ _____   |
| 29. $12 \times 3 =$ _____  | 66. $11 \times 7 =$ _____  | 103. $10 \times 4 =$ _____  |
| 30. $3 \times 9 =$ _____   | 67. $3 \times 7 =$ _____   | 104. $11 \times 12 =$ _____ |
| 31. $5 \times 4 =$ _____   | 68. $8 \times 4 =$ _____   | 105. $9 \times 3 =$ _____   |
| 32. $4 \times 3 =$ _____   | 69. $12 \times 2 =$ _____  | 106. $6 \times 6 =$ _____   |
| 33. $6 \times 2 =$ _____   | 70. $6 \times 10 =$ _____  | 107. $4 \times 9 =$ _____   |
| 34. $12 \times 11 =$ _____ | 71. $7 \times 12 =$ _____  | 108. $12 \times 11 =$ _____ |
| 35. $9 \times 9 =$ _____   | 72. $12 \times 4 =$ _____  | 109. $7 \times 12 =$ _____  |
| 36. $7 \times 10 =$ _____  | 73. $6 \times 6 =$ _____   | 110. $12 \times 9 =$ _____  |
| 37. $11 \times 11 =$ _____ | 74. $7 \times 8 =$ _____   | 111. $10 \times 3 =$ _____  |

## Appendix B: Posttest

Name: \_\_\_\_\_ Ratio: \_\_\_\_\_ Percent: \_\_\_\_\_

**Three Minute Math**

## Multiplication Facts

- |                            |                            |                             |
|----------------------------|----------------------------|-----------------------------|
| 1. $11 \times 12 =$ _____  | 38. $5 \times 4 =$ _____   | 75. $9 \times 6 =$ _____    |
| 2. $8 \times 7 =$ _____    | 39. $6 \times 6 =$ _____   | 76. $4 \times 4 =$ _____    |
| 3. $12 \times 8 =$ _____   | 40. $4 \times 10 =$ _____  | 77. $8 \times 9 =$ _____    |
| 4. $10 \times 10 =$ _____  | 41. $10 \times 5 =$ _____  | 78. $4 \times 12 =$ _____   |
| 5. $5 \times 11 =$ _____   | 42. $7 \times 7 =$ _____   | 79. $10 \times 9 =$ _____   |
| 6. $6 \times 6 =$ _____    | 43. $8 \times 10 =$ _____  | 80. $6 \times 7 =$ _____    |
| 7. $8 \times 12 =$ _____   | 44. $5 \times 11 =$ _____  | 81. $6 \times 11 =$ _____   |
| 8. $6 \times 9 =$ _____    | 45. $4 \times 8 =$ _____   | 82. $7 \times 10 =$ _____   |
| 9. $3 \times 3 =$ _____    | 46. $6 \times 7 =$ _____   | 83. $5 \times 5 =$ _____    |
| 10. $10 \times 11 =$ _____ | 47. $9 \times 7 =$ _____   | 84. $5 \times 9 =$ _____    |
| 11. $9 \times 8 =$ _____   | 48. $11 \times 4 =$ _____  | 85. $3 \times 3 =$ _____    |
| 12. $11 \times 3 =$ _____  | 49. $4 \times 7 =$ _____   | 86. $9 \times 9 =$ _____    |
| 13. $4 \times 6 =$ _____   | 50. $3 \times 9 =$ _____   | 87. $7 \times 11 =$ _____   |
| 14. $7 \times 5 =$ _____   | 51. $5 \times 3 =$ _____   | 88. $5 \times 12 =$ _____   |
| 15. $10 \times 7 =$ _____  | 52. $12 \times 9 =$ _____  | 89. $11 \times 5 =$ _____   |
| 16. $11 \times 11 =$ _____ | 53. $10 \times 8 =$ _____  | 90. $6 \times 10 =$ _____   |
| 17. $4 \times 9 =$ _____   | 54. $8 \times 7 =$ _____   | 91. $8 \times 8 =$ _____    |
| 18. $8 \times 7 =$ _____   | 55. $9 \times 8 =$ _____   | 92. $4 \times 4 =$ _____    |
| 19. $6 \times 12 =$ _____  | 56. $3 \times 7 =$ _____   | 93. $6 \times 5 =$ _____    |
| 20. $10 \times 8 =$ _____  | 57. $5 \times 5 =$ _____   | 94. $6 \times 6 =$ _____    |
| 21. $5 \times 9 =$ _____   | 58. $9 \times 5 =$ _____   | 95. $11 \times 9 =$ _____   |
| 22. $8 \times 6 =$ _____   | 59. $8 \times 9 =$ _____   | 96. $10 \times 4 =$ _____   |
| 23. $10 \times 4 =$ _____  | 60. $4 \times 3 =$ _____   | 97. $7 \times 7 =$ _____    |
| 24. $9 \times 9 =$ _____   | 61. $9 \times 2 =$ _____   | 98. $4 \times 10 =$ _____   |
| 25. $8 \times 5 =$ _____   | 62. $8 \times 7 =$ _____   | 99. $4 \times 8 =$ _____    |
| 26. $12 \times 3 =$ _____  | 63. $3 \times 6 =$ _____   | 100. $9 \times 3 =$ _____   |
| 27. $7 \times 7 =$ _____   | 64. $9 \times 9 =$ _____   | 101. $9 \times 4 =$ _____   |
| 28. $5 \times 11 =$ _____  | 65. $4 \times 7 =$ _____   | 102. $4 \times 11 =$ _____  |
| 29. $8 \times 6 =$ _____   | 66. $3 \times 5 =$ _____   | 103. $8 \times 8 =$ _____   |
| 30. $5 \times 5 =$ _____   | 67. $12 \times 9 =$ _____  | 104. $7 \times 11 =$ _____  |
| 31. $4 \times 10 =$ _____  | 68. $10 \times 3 =$ _____  | 105. $9 \times 10 =$ _____  |
| 32. $9 \times 4 =$ _____   | 69. $10 \times 6 =$ _____  | 106. $5 \times 5 =$ _____   |
| 33. $11 \times 8 =$ _____  | 70. $5 \times 10 =$ _____  | 107. $11 \times 5 =$ _____  |
| 34. $7 \times 6 =$ _____   | 71. $7 \times 12 =$ _____  | 108. $9 \times 7 =$ _____   |
| 35. $8 \times 5 =$ _____   | 72. $10 \times 11 =$ _____ | 109. $8 \times 5 =$ _____   |
| 36. $11 \times 3 =$ _____  | 73. $6 \times 6 =$ _____   | 110. $5 \times 2 =$ _____   |
| 37. $7 \times 7 =$ _____   | 74. $7 \times 4 =$ _____   | 111. $11 \times 11 =$ _____ |

## Curriculum Vitae

**Nelly P. Knowles**[nelly\\_knowles@gwinnett.k12.ga.us](mailto:nelly_knowles@gwinnett.k12.ga.us)**Professional Preparation**

Ed. D.	October 2010	Walden University Minneapolis, Minnesota
M.A	May 1999	Brenau University Gainesville, Georgia
B. S	August 1994	Brenau University Gainesville, Georgia

**Professional Employment History**

2009 – Present	6 <sup>th</sup> Grade Mathematics Teacher	(School Name Withheld) Gwinnett County, Georgia
2004 – 2009	6 <sup>th</sup> Grade Mathematics Teacher	Osborne Middle School Gwinnett County, Georgia
1994 – 2004	6 <sup>th</sup> Grade Mathematics Teacher	Dacula Middle School Gwinnett County, Georgia

**Professional Certification**

Georgia Educator Certificate	Middle Grades (4 – 8)	Mathematics Language Arts Social Science
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**Professional Association**

PAGE	Professional Association of Georgia Educators
NCTM	National Council of Teachers of Mathematics
GC <sup>2</sup> TM	Gwinnett County Council of Teachers of Mathematics