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Exploring Pre-Service Elementary Teachers' After Student-Teaching Content Knowledge and Teaching Knowledge of Fraction Operations

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This is to certify that the doctoral study by

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has been found to be complete and satisfactory in all respects,
and that any and all revisions required by
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Walden University
2024

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MS, Walden University, 2007

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Dissertation Submitted in Partial Fulfillment

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Doctor of Education

Curriculum, Instruction, Assessment

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Abstract

With the changes in Common Core State Standards, it was essential to explore how elementary pre-service teachers' sense of fractions is developing and their thoughts about how to teach fractions. A gap in practice exists because it is unknown how pre-service elementary teachers, after student-teaching, perceive their level of in-practice knowledge in (a) knowledge of content and (b) knowledge of pedagogy in fraction operations. The three research questions centered on content knowledge, pedagogy knowledge, and knowledge of students, and the study was guided by the concepts of Lee Shulman's categories of teacher knowledge. The purpose of this study was to explore pre-service elementary teachers', after student-teaching, perceptions of their knowledge or gaps in knowledge in content and pedagogical approaches to fraction operations. This basic qualitative study had a purposeful sample of 12 pre-service elementary teachers across the United States who had completed student teaching and taught fraction operations. Qualitative interviews and thematic analysis were employed. The results showed that pre-service elementary teachers had knowledge of the addition and subtraction content of fraction operations. Pre-service elementary teachers learned instructional methods to teach addition and subtraction but not for multiplication and division of fractions in their pedagogy classes. Before student teaching, pre-service elementary teachers had non-existent or very minimal pedagogy (student knowledge) in addition, subtraction, multiplication, and division of fractions. Teacher education programs must examine their curriculum outcomes and ensure they provide pre-service elementary teachers with opportunities to learn content and pedagogical knowledge of fraction operations so they will be successful in student teaching and in their future classrooms.

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Table of Contents

List of Tables

List of Figures

Chapter 1: Introduction to the Study

Fraction operations continue to be a research subject with pre-service elementary teachers (Magiera & van den Kieboom, 2021). Pre-service elementary teachers' experiences with fractions during their collegial studies and student teaching were examined in this study based on Shulman's (1986) research. The focal point of my study was to understand how pre-service elementary teachers, after student-teaching, describe their content and pedagogical content knowledge of fraction operations. I examined two types of pre-service teacher knowledge: (a) knowledge of content and (b) knowledge of pedagogy. Pedagogical content knowledge consists of two types of knowledge. It contains knowledge of teaching and the knowledge of students (Shulman, 1987). The study needed to be conducted because teacher education programs in the United States may not sufficiently prepare elementary pre-service teachers in fractions. The study may contribute to positive social change because the research can provide universities with teacher education programs with valuable information on how well-prepared pre-service elementary teachers are in student teaching. Teacher education programs can examine the knowledge or gaps in student teachers' knowledge of content and pedagogical content knowledge of fraction operations and change their courses.

Chapter 1 provides Shulman's (1986) research literature. This chapter includes the problem statement with local evidence and current research showing that the problem is significant to mathematics education. Three research questions are provided. Key concepts are defined, and limitations of transferability are discussed. Lastly, a summary of focal points is provided.

Background

Shulman (1986, 1987) significantly contributed to the examination of teacher knowledge. He examined teacher knowledge in ways that were valuable to teaching content. Shulman combined content knowledge and curriculum (pedagogical knowledge) into pedagogical content knowledge (Shulman, 1986). Pedagogical content knowledge encompasses comprehending a student's prior knowledge, how a student understands pedagogical representations, if a student has any learning difficulties, and the instructional methods teachers use to ensure there are no misunderstandings of topics (Deng, 2018). Shulman (1987) affirmed that teachers can break their subjects down into more straightforward topics so their students can better understand concepts. Moreover, based on the research, pedagogical content knowledge is essential to teaching (Deng, 2018; Shulman, 1986). Therefore, pre-service elementary teachers need pedagogical content knowledge to teach fraction operations.

Adu-Gyamfi et al. (2019) investigated the problem of little fractional pedagogical content knowledge. The authors completed research with 36 pre-service elementary teachers who were enrolled at a university in the southeastern United States. Adu-Gyamfi et al. found that about 67% of the pre-service elementary teachers could complete fraction division problems. However, these students could not analyze student work based on the same problems they solved. Lee and Lee (2020) also completed research with pre-service elementary teachers at two different universities in the United States. The authors used 83 teachers. They discovered that the pre-service elementary teachers had much trouble showcasing fraction addition. This included fractions with

unlike denominators and improper fractions. Furthermore, teachers must develop pedagogical content knowledge to assist students in successfully understanding concepts.

In this research, I describe the gap in practice in how it is unknown how pre-service elementary teachers, after student-teaching, perceive their level of in-practice knowledge in two areas: (a) knowledge of content and (b) knowledge of pedagogy in fraction operations. Pre-service elementary teachers are matriculated in mathematics education courses during their teacher education preparation at their college or university. In these courses, mathematics content knowledge and pedagogical content knowledge are highlighted. As a result of the publication of the Common Core State Standards for Mathematics in 41 states, four territories, and the District of Columbia, expectations for teachers have been set higher (Akkus, 2016). The study was needed because, with the higher expectations placed by the Common Core State Standards, elementary pre-service teachers must comprehend fractions (content knowledge) beyond being able to compute fractions and have the tools and strategies to teach fractions (pedagogical content knowledge). Understanding elementary pre-service teachers' sense of fractions can assist United States teacher education programs in better preparing students' knowledge of fractions. There is personal communication evidence from mathematics education professionals across several states in the United States that there is a problem in that pre-service teachers are not prepared to teach fraction operations. It is essential to hear from the pre-service teachers regarding their knowledge of fraction operations.

Problem Statement

The research problem examined in this study was pre-service elementary teachers after student-teaching, perceptions of their knowledge or gaps in knowledge in content, and pedagogical approaches in fraction operations. The problem is recent, pertinent, and noteworthy to the discipline because pre-service elementary teachers' knowledge of fraction operations remains a research topic (Magiera & van den Kieboom, 2021). Lee and Lee (2020) completed research with pre-service elementary teachers at two different universities in the United States. The authors used 83 teachers. Their research found that it is essential for teacher educators to instruct their pre-service teachers in mathematical reasoning. Another researcher, Kang (2022), completed a study in an elementary teacher preparation program in the western United States that looked at the performance of elementary pre-service teachers with multiplication and division problems of fractions. His research was brought about because of past research on teacher education that continuously reported on the inadequacy of conceptual understanding of fraction multiplication and division within pre-service teachers (Kang, 2022).

Besides these studies, there is confirmation that pre-service elementary teachers entering a teacher education program are not passing Praxis 1. Praxis 1 measures mathematical content knowledge. Research at the University of North Alabama showcased the national scores for Praxis 1 in the United States (The University of North Alabama, n.d.). The passing score for Praxis 1 – The Core Academic Skills for Educators in Mathematics is 150. Table 1 provides the national median scores for the years 2017-

2020. The national median scores show that pre-service teachers were not passing Praxis 1 during these years.

Table 1

National Median Scores of Praxis 1-Core Academic Skills – for the United States

Year	Passing score	Median
2017-2018	150	146
2018-2019	150	146
2019-2020	150	148

Note. The table shows the national median scores of Praxis 1 during the years 2017-2020.

Furthermore, there is evidence from a mathematics education professor at a university in the United States. The mathematics education professor described the first-year pre-service elementary teachers as "having a lack of conceptual knowledge and pedagogical content knowledge of fraction operations." The professor also described the students as "lacking the ability to identify the knowledge elementary students have of fraction operations" (personal communication, April 17, 2023). There is also evidence from elementary educators from the United States. An educator who has previously taught fifth grade described their recent pre-service teacher as "lacking content knowledge of fraction operations." The educator also described their recent pre-service teacher as "not having the knowledge to review prior content for students to build to access the grade level material" (personal communication, April 19, 2023). Another educator who previously taught sixth grade described the pre-service teachers as having "key areas of content knowledge missing." The educator also stated that "delivery was a struggle because the content knowledge was missing" (personal communication, April 19, 2023). Current research, statistics, and personal communication provide evidence that

research in content and pedagogical content knowledge of fractions with pre-service elementary teachers should be explored further in the United States.

With the changes in Common Core State Standards, it was essential to explore how elementary pre-service teachers' sense of fractions is developing and their thoughts about how to teach fractions. My study explored perceptions and experiences regarding pre-service elementary teachers after student teaching, knowledge or gaps in knowledge in content, and pedagogical approaches to fraction operations.

Current research documents early recollections and experiences with mathematics impacting pre-service teachers' view of their capability to work with and teach mathematics (Cruz et al., 2019; Jeffrey et al., 2019). One of these studies included 76 elementary education pre-service teachers considered juniors or seniors (Jeffrey et al., 2019). Findings from this study supplied a more comprehensive range of conversations about pre-service teacher efficacy, which can assist teacher educators and supervisors in ensuring that their programs lay the groundwork for pre-service teachers to flourish in their professions (Jeffrey et al., 2019). Another researcher looked at the self-efficacy of pre-service teachers and how it correlated with mathematical dispositions (Cruz et al., 2019). This study was a step forward in realizing the relationship between mathematical dispositions and self-efficacy, which the National Research Council emphasizes as one of the strands of mathematical proficiency and self-efficacy (Cruz et al., 2019). Therefore, because these studies note the early recollections and experiences with mathematics with pre-service teachers, my study looked at their experiences and knowledge of fraction operations after student teaching.

Many studies document that teachers' math anxiety and perceptions of mathematics may make it challenging to learn fractions (Boyce & Moss, 2022; Ganley et al., 2019). Ganley et al. (2019) investigated 399 lower and upper elementary teachers in Florida using the Math Anxiety Scale for teachers. They found that teachers experienced math anxiety when completing fraction problems on the test (Ganley et al., 2019). Another study examined 76 prospective teachers using a written assessment to see their interpretations of fractions with linear, discrete, and circle representations (Boyce & Moss, 2022). The scholars found that their perceptions of mathematics impacted their approach to making sense of fractions. Therefore, because these studies show that teachers' math anxiety and perceptions of mathematics can influence how they learn fractions, my study examined their attitudes toward fractions after student teaching.

According to current studies, pre-service teachers struggle with fraction knowledge (Lee & Lee, 2020; Morano & Riccomin, 2020). Lee and Lee (2020) completed research with pre-service elementary teachers at two different universities in the United States. The authors used 83 teachers. They discovered that the pre-service elementary teachers had much trouble showcasing fraction addition. This included fractions with unlike denominators and improper fractions. Another study used 55 pre-service elementary teachers to study their capability to showcase fraction multiplication and division. The authors found that pre-service teachers had weak conceptual knowledge in fraction multiplication and division (Morano & Riccomin, 2020). Therefore, because these studies show that pre-service teachers struggle with fraction knowledge, my study

examined whether pre-service elementary teachers struggle with fraction knowledge after student teaching.

Studies document that teachers' attitudes, values, or dispositions have an essential effect on student outcomes (Chen et al., 2022; Tassell et al., 2020). Tassell et al. (2020) completed a study investigating how mathematics anxiety, self-efficacy, mindfulness, and mindset are related among pre-service elementary teachers. The researchers found that by focusing on all four variables, teacher educators could prepare future teachers in the classroom. Another study looked at teaching efficacy. Chen et al. (2022) examined early childhood and elementary pre-service teachers' mathematics and science teaching efficacy to expose paths that would help teacher preparation programs. The researchers found that the number of mathematics and science courses taken was linked to pre-service teachers' teaching efficacy beliefs. Chen et al. argued that pre-service teachers needed pedagogical knowledge and hands-on teaching experience to boost their confidence in influencing their future students' mathematics and science learning. Therefore, because these studies showcase teachers' values, dispositions, and attitudes, my study looked at pre-service teachers' perceptions after student teaching in fraction operations.

Studies indicate the impact teacher education programs have on understanding fractions of pre-service elementary teachers (Reeder & Utley, 2017; Whitehead & Walkowiak, 2017). Reeder and Utley (2017) completed a study looking at pre-service elementary teachers' understanding of fractions. The authors found that pre-service elementary teachers' comprehension of fractions was limited in their final methods

course. They also found that pre-service elementary teachers need to be given opportunities to develop a deep understanding of fractions (Reeder & Utley, 2017). For future elementary teachers, the authors called for more research into how mathematics educators can develop courses regarding fractions. Whitehead and Walkowiak (2017) investigated 48 pre-service elementary teachers to see if there was a change in their understanding of fractions during a Grades 3-5 mathematics methods course. The researchers found that pre-service teachers' level of mathematics was above what they needed to know to instruct elementary mathematics. Even though their mathematics level was above average, this did not improve their explanations of standard fraction algorithms. Therefore, because these studies suggest that an important step to improve pre-service teachers' understanding of fractions is to have been taught content and pedagogy relative to fractions, my study looked at the experiences and perceptions after student teaching from pre-service teachers about fraction operations regarding their own teacher preparatory programs.

Pre-service teachers must have a deep understanding of mathematical content and also possess mathematical discourse (Makowski, 2021). Mathematical content and reasoning are outlined in the Common Core State Standards (Makowski, 2021). Mathematical content knowledge is essential for pre-service teachers to understand how to complete procedures and explain why a procedure makes sense (Lovin et al., 2018). Lovin et al. (2018) completed a study examining which fraction schemes and operations pre-service teachers demonstrated evidence of having. The researchers found that only 27% of pre-service teachers could construct an iterative fraction scheme. Magiera and

Zamback (2020) continued to look at the mathematical discourse and reasoning. They used the written responses of 37 pre-service teachers who were preparing to teach Grades 1-8 mathematics. Results indicated that when pre-service teachers provided commentaries of student explanations, they were weaker than their problem solutions. Therefore, because these studies show a meaningful gap in practice that pre-service teachers are still struggling with the mathematical content and mathematical discourse of fractions outlined by the Common Core State Standards, my study looked at pre-service elementary teachers after student teaching describe their perceptions of their knowledge or gaps in knowledge in content knowledge and pedagogical content knowledge of fractions by the Common Core State Standards.

Purpose of the Study

The purpose of this study was to explore pre-service elementary teachers', after student-teaching, perceptions of their knowledge or gaps in knowledge in content and pedagogical approaches to fraction operations. This study used the interpretivist paradigm. According to Lambert (2012), the interpretivist paradigm is based on the notion that no external reality is waiting to be found. Instead, what one accepts as real comes from interacting with different environments and people. For example, an interpretivist researcher may give a few illustrations of reality from those perspectives and provide possible interpretations of a situation. Another researcher could do something similar but produce a different structure, but if the research was conducted well, it could be as influential as the first (Lambert, 2012). My study emphasized that pre-service elementary teachers share their perceptions and experiences regarding their

knowledge or gaps in knowledge in content and pedagogical approaches to fraction operations. Each pre-service teacher's experience added to the study and benefited in determining how they were or were not equipped with the content and pedagogical approaches to fraction operations. The study explored pre-service elementary teachers' knowledge of content and pedagogy content knowledge of fraction operations. The study explored how prepared pre-service elementary teachers were when instructing students in fraction operations. This phenomenon of interest was pre-service teachers' descriptions of their pedagogical content knowledge and how they developed it in their teacher education program.

Research Questions

The research questions guided the qualitative study. The three research questions involved content and pedagogy content knowledge.

RQ1: (*knowledge of content*) What are pre-service elementary teachers', after student-teaching, perceptions of their knowledge or gaps in knowledge regarding their knowledge of content of fraction operations?

RQ2: (*knowledge of pedagogy*) What are pre-service elementary teachers', after student-teaching, perceptions of their knowledge or gaps in knowledge regarding their knowledge of pedagogy of fraction operations?

RQ3: (*knowledge of students*) What are pre-service elementary teachers', after student-teaching, perceptions of their knowledge or gaps in knowledge regarding their knowledge of students of fraction operations?

Conceptual Framework

Pedagogical content knowledge based on Shulman's (1986) work was the conceptual framework for this study. The framework consists of key elements called Shulman's major categories of teacher knowledge. Common elements for teacher knowledge were the essential components in the teacher education programs during Shulman's time and were the first four categories. Content-specific elements define the last three categories. The content-specific elements are what Shulman believed was still missing in teaching research (Ball et al., 2008). These are logically connected in that the first four are separate forms of knowledge, and pedagogical content knowledge is a mix of content and pedagogical knowledge to form a new entity. Shulman examined earlier studies that looked at the content of mathematics but not how mathematics was taught (Ball et al., 2008). This was a gap in practice that Shulman decided to begin his research on with content knowledge and teaching.

In my study, I focused on content knowledge and pedagogical content knowledge: knowledge of pedagogy and knowledge of students. The research questions were guided by pedagogical content knowledge that focused on what pre-service teachers know and understand about fraction operations, the knowledge of pedagogy for fraction operations, and the knowledge of students for fraction operations.

Research studies have used Shulman's (1986) pedagogical content knowledge as their conceptual framework. Van Garderen et al. (2021) are researchers who used Shulman's framework of pedagogical content knowledge in their study. The researchers investigated special education pre-service teachers. They looked at case-based instruction

using visual representations and how these representations could be helpful for problem-solving. Van Garderen et al. (2021) found many layers to pedagogical content knowledge. Because of this, Van Garderen et al.'s findings applied to instructional ways of acquiring knowledge of pre-service special education teachers and instructional planning. My study focused on pedagogical content knowledge by examining the two components of this knowledge. The two components are content knowledge and pedagogy knowledge. My study focused on how the two components integrate into pedagogical content knowledge.

Shulman's (1986) work was used as the conceptual framework for the study, which was related to the study approach. A basic qualitative study design was used in this study. Denzin and Lincoln (2018) expressed that qualitative methodology allows researchers to see how individuals perceive their experiences and what is the meaning of those experiences. This study examined the perceptions of pre-service teachers and their knowledge or lack of knowledge of content and pedagogy content knowledge in fraction operations. The data collection instrument used was in-depth qualitative interviews, which aided in thematic analysis.

Nature of the Study

A basic qualitative study was used. This type of study was used because it allowed me to look at the perceptions of pre-service elementary teachers' knowledge or lack of knowledge in content and pedagogy content knowledge for fraction operations. The study's focus was to comprehend the participants' experiences in a strong and productive way (Merriam & Tisdell, 2016). Denzin and Lincoln (2018) described a

qualitative study as how individuals perceive their experiences and what is the meaning of those experiences. The pre-service teachers recounted their experiences after student teaching about their knowledge or lack of knowledge in fraction operations.

Pre-service elementary teachers who finished student teaching and had at least taught two different fraction operations were the study participants. There were no prior professional or personal relationships with the participants. In-depth qualitative interviews were used for data collection. The interviews were conducted on Zoom and recorded. This allowed me to transcribe the interview to accurately get a representation of their perceptions and experiences with fraction operations. The qualitative interview was divided into four main categories. Each category had a list of questions. The categories were Student Teaching Details, Content Knowledge of Fractions, Pedagogical Content Knowledge of Fractions (Pedagogy Knowledge), and Pedagogical Content Knowledge of Fractions (Knowledge of Students). The follow-up procedure was emailing participants a codebook, which I asked each participant to check. Thematic analysis was used in this study. The thematic analysis that took place was inductive coding. In this method, the coding and themes were developed through careful analysis of the data content from the transcripts (Braun & Clarke, 2022).

Definitions

Common core state standards: are standards designed to involve students in compounded mathematical practices. These include modeling, reasoning, and argumentation (Schweig et al., 2020).

Content knowledge: refers to knowledge a teacher has. This knowledge may include ideas, facts, theories, vocabulary that teachers must know to be effective in the classroom. (Shulman, 1986).

Pedagogical content knowledge: includes comprehending a student's prior knowledge, how a student understands pedagogical representations, if a student has any learning difficulties, and the instructional methods teachers use to ensure there is no misconceptions of topics (Deng, 2018).

Pedagogy: refers to the most effective frameworks for teaching and learning (Prasad, 2023).

Pre-service teachers: are student teachers who are trying to obtain a teacher certification through a teacher education program (Chand et al., 2022).

Assumptions

Assumptions were critical to my study. First, I assumed the participants would answer the research questions based on their experiences after student teaching. Their responses needed to be their own and not the experiences of others. Second, I assumed they would answer the research questions honestly. I assumed they would not answer the questions based on what they thought I wanted to hear.

Scope and Delimitations

The research problem addressed in this study involved the content knowledge and pedagogical content knowledge of fraction operations for elementary education pre-service teachers after they complete student teaching. This study focused on exploring pre-service teachers' perceptions concerning fraction operations. The qualitative study's

design allowed data to be collected from pre-service teachers across the United States. Through interviews, pre-service elementary teachers could describe their knowledge or gaps in how to instruct students in fraction operations.

The study's boundaries were 12-15 elementary education pre-service teachers who taught in a Grade 1-6 classroom within the past 2 years. They were from the United States. These participants must have taught at least two fraction operations.

Transferability can occur in universities with an elementary education teacher preparatory program, and they may choose to use the study's data to provide positive social change at their universities. The conceptual framework that was not investigated but is most relatable to this study was the conceptual framework that looked at case-based instruction with pre-service special education teachers using visual representations and how these representations could be helpful for problem-solving (Van Garderen et al., 2021). The conceptual framework for these researchers' study was grounded on Shulman's (1986) work. This framework was the emphasis of my study as content knowledge and pedagogical knowledge of fractions with pre-service elementary teachers after student teaching was explored.

Limitations

Having a sample size of 12-15 participants and building trust with participants could have been a limitation of my study. Fortunately, there were no unexpected limitations. My study included 12 participants from all regions of the United States. Ensuring that I had at least 12-15 participants as a sample size was critical to attain data saturation (Guest et al., 2006). Guest et al. (2006) found that data saturation occurred

within the first 12 interviews. I built trust with my participants by sending an introduction to my research that included the consent form. During the interview, my communication skills were excellent because my participants understood all the questions and could answer them thoroughly.

Significance

This study can contribute to the mathematical discipline because it may provide mathematics teacher educators at the university level with information on how prepared pre-service elementary teachers are. The results from this study can provide an understanding of what pre-service elementary teachers know and understand about fraction operations, the knowledge of pedagogy for fraction operations, and the knowledge of students for fraction operations. It can also supply information for universities and colleges to redesign their teacher education programs and inform them of improvements in teaching mathematics content courses and mathematics methods education courses. This study can also improve student teaching experiences with fraction operation teaching. The qualitative study can contribute to positive social change because pre-service elementary mathematics teachers can take the time to ponder their mathematical skills and instructional practice.

Summary

The focus of the study was to explore pre-service elementary teacher's knowledge or lack of knowledge in content and pedagogy content knowledge of fraction operations. The study needed to be conducted because a problem exists nationally. In-depth qualitative interviews were the method used for data collection. The interviews were

transcribed to accurately represent the pre-service teachers' perceptions of fraction operations. This qualitative study has the potential to make a positive impact on society because pre-service elementary mathematics teachers can take the time to ponder their mathematical skills and instructional practice.

In Chapter 2, the conceptual framework is identified and defined based on content knowledge and pedagogical content knowledge. Lee Shulman's (1986) research describes this study's phenomenon. A thorough review of recent literature is used to relate key variables and concepts of the study. The studies described are synthesized and related to the research questions. Lastly, studies on content and pedagogical content knowledge and the methods consistent with the study are discussed.

Chapter 2: Literature Review

The problem of this study was pre-service elementary teachers', after student-teaching, perceptions of their knowledge or gaps in knowledge in practice in content and pedagogical approaches to fraction operations. The purpose of the study was to explore pre-service elementary teachers', after student-teaching, perceptions of their knowledge or gaps in knowledge in practice in two areas: (a) knowledge of content, and (b) knowledge of pedagogy. Zolfaghari et al. (2024) have focused their attention on pre-service teachers' content and pedagogical content knowledge regarding fraction operations. Wicaksono et al. (2024) stated that teachers and prospective teachers must have pedagogical abilities and understand the content. Scholars such as Adu-Gyamfi et al. (2019) have made a pathway for more research with teacher educators and their responsibility to develop the content knowledge needed for effective teaching if pre-service teachers are entering their classes with insufficient understanding of mathematical content. Researchers need to study elementary pre-service teachers' perceptions of content and pedagogical content knowledge because these two entities are essential to quality teacher instruction.

This chapter summarizes the search terms and search engines used to examine research. It examines the conceptual framework of Lee Shulman's (1986) mathematical knowledge for teaching and key definitions essential to the study. It also summarizes recent literature that describes the study and provides a summary of focal points.

Literature Search Strategy

The search engines and library databases used for the literature review were Academic Search Complete, Education Source, ERIC, Google Scholar, SAGE Journals, and the Taylor and Francis Online via the Walden University website. The search yielded in past and current research articles from peer-reviewed journals, books, and educational websites. To find literature, the key search terms used were the following*: pre-service elementary teachers, fractions, conceptual knowledge, pedagogical content knowledge, curriculum knowledge, pedagogical knowledge, Common Core State Standards, mathematics, mathematics anxiety, operations of fractions, Shulman,* and *teacher preparatory programs*.

The literature review starts with Shulman's (1986) work. His work included content knowledge, curriculum (pedagogical) knowledge, and how he grouped the two types of knowledge into a new classification called pedagogical content knowledge. Studies of the effects of mathematics anxiety on pre-service teachers, perceptions of fractions, and pedagogical content knowledge of fractions were obtained. These studies are organized under subheading names below. The literature review then turns to a section that focuses on the context of the investigation: teacher preparatory programs and their effect on pre-service teachers' knowledge of fractions.

Conceptual Framework

The phenomenon that grounds my study is Shulman's (1986) conceptual framework. He was the first researcher to examine and complete research on teachers' knowledge (Shulman, 1986, 1987). Examining teacher knowledge was a significant

contribution that Shulman and his colleagues made. They could examine teacher knowledge essential to the teaching content (Ball et al., 2008). The history is such that Shulman looked at earlier studies that looked at the content of mathematics but not how mathematics was taught (Ball et al., 2008). This was a gap in practice, so Shulman decided to investigate teaching and teacher knowledge. Shulman combined content knowledge and curriculum (pedagogical) knowledge into pedagogical content knowledge (Shulman, 1986). Curriculum (pedagogical) knowledge includes educational programs teachers can use for different subjects at all levels (Ball et al., 2008). Pedagogical content knowledge refers to comprehending a student's prior knowledge, how a student understands pedagogical representations, if a student has any learning difficulties, and the instructional methods teachers use to ensure no misconceptions of topics (Deng, 2018). The types of knowledge are defined in the literature review sections. Shulman supplied a list of the knowledge he looked at before focusing on the three making up pedagogical content knowledge (Ball et al., 2008).

The first four categories were important during Lee Shulman's time in his major categories of teacher knowledge. These four categories are:

- general pedagogical knowledge
- knowledge of learners and their characteristics
- knowledge of educational contexts
- knowledge of educational ends, values, and purposes (Ball et al., 2008)

These four categories highlighted content knowledge; however, they were not the focus of his work (Ball et al., 2008). Lee Shulman acknowledged that "mere content knowledge is likely to be as useless pedagogically as content-free skill" (Shulman, 1986, p. 8). Content-specific elements define the last three categories in Shulman's major categories. These three categories include:

- content knowledge
- curriculum knowledge
- pedagogical content knowledge (Ball et al., 2008)

These elements are what Shulman believed was still missing in teaching research (Ball et al., 2008). My study was grounded on content knowledge, curriculum (pedagogical) knowledge, and pedagogical content knowledge.

Content Knowledge

Content knowledge refers to the knowledge a teacher has (Shulman, 1986). This knowledge may include ideas, facts, theories, and vocabulary that teachers must understand. Shulman (1986) challenged teachers that there is more to teaching than just knowing the topic. They must know the ins and outs of a topic (Ball et al., 2008). Teachers must know why a topic is vital to their discipline (Ball et al., 2008). Understanding a subject is not enough. There is not a significant difference that content knowledge proclaims between the knowledge a teacher has and how that teacher responds in the academic world (Deng, 2018).

Curriculum (Pedagogical) Knowledge

Curriculum (pedagogical) knowledge is the different instructional applications teachers can use for various topics (Ball et al., 2008). Curriculum (pedagogical) knowledge is the different instructional methods and materials teachers can use to instruct (Ball et al., 2008). There are two parts to curriculum knowledge. The first part is a lateral curriculum, which refers to the knowledge students obtain from other subjects (Shulman, 1986). The second part is vertical curriculum knowledge, which refers to the knowledge students obtain from topics taught (Shulman, 1986).

Pedagogical Content Knowledge

Pedagogical content knowledge was designed to institute teaching as a profession (Shulman, 1986). Shulman (1987) affirmed that teachers can break their subjects down into more straightforward topics so their students can better understand concepts. Pedagogical content knowledge includes comprehending a student's prior knowledge, how a student understands pedagogical representations, if a student has any learning difficulties, and the instructional methods teachers use to ensure no misconceptions of topics (Deng, 2018). Pedagogical content knowledge needs to be established by teachers in order to aid in student learning and success.

Key Theorists Views on Teacher Content Knowledge

Other researchers have investigated teacher knowledge, which this study did not use. These researchers have built upon the work of Shulman (1986). However, Shulman's proposed basic theory is at the heart of teacher pedagogical content knowledge in many subject areas. One of these researchers who expanded upon Shulman's work is Grossman

(1990). Grossman began to look at the relationship and connection of teacher knowledge in literature. He believed that teachers must use their knowledge of the subject to select items that best represent that subject for the content to be learned (Grossman, 1990). Grossman's view on literature paved the way for how they handled books with their students. Wilson and Wineburg (1988) continued with the work of Lee Shulman in social studies. They described how a teacher's anthropology, sociology, and political science background would help them represent history meaningfully to their high school students (Wilson & Wineburg, 1988). Ball et al. (2008) was another researcher who expanded upon Shulman's work. She regrouped content knowledge into two groups (subject matter knowledge and pedagogical content knowledge). Therefore, Shulman's (1986 and 1987) framework was essential to this research.

Shulman's Pedagogical Content Knowledge Used by Other Researchers

Other researchers have used Shulman's (1986) pedagogical content knowledge as a framework. Johnson et al. (2018) stated that, according to Shulman, teachers' content knowledge was essential for instruction. These researchers found that pre-service elementary teachers' beliefs about their mathematical ability to instruct are not impacted by the amount of knowledge they have on mathematical content (Johnson et al., 2018). Van Garderen et al. (2021) used pedagogical content knowledge for teaching as the framework for their study. Van Garderen et al. stated that, according to Shulman (1987), pedagogical content knowledge is what makes teachers' knowledge turn into straightforward forms and within reach by their students; however, what makes up pedagogical content knowledge is complicated. Because their study used a robust

theoretical model of pedagogical content knowledge, the researchers could identify where the learning of pre-service teachers enhanced and what was still lacking.

Similarly, Thomson et al. (2022) used Shulman's (1986) pedagogical content knowledge as their framework. Thomson et al. stated that Shulman's introduction to pedagogical content knowledge brought about research that questioned the content and knowledge teachers must have for teaching. Their study's findings can help future researchers and teacher educators better understand teachers' paths in mathematical knowledge and teaching efficacy. Several researchers used Shulman's research on pedagogical content knowledge as the framework for their studies. My study looked at pre-service teachers' content and pedagogical knowledge after student teaching in fraction operations. It focused on pedagogical content knowledge by looking at the two components that make up this knowledge and how the two components blend into pedagogical content knowledge. The conceptual framework of pedagogical content knowledge was used in this study.

Literature Review Related to Key Concepts and Variables

A review of the literature presents six themes, each of which is described and analyzed. Within these themes, I provide the ways researchers approached the problem and the strengths and weaknesses of their approaches. The themes provide different elements as to why pre-service elementary teachers, after student teaching, have knowledge or gaps in knowledge of fraction operations.

Perceptions of Mathematics

Previous research has examined perceptions and attitudes and their effects on mathematics classroom instruction (Rosli et al., 2019). Pre-service elementary teachers' perceptions and attitudes about mathematics will continue throughout their education journey. Fear, anxiety, self-efficacy, and early childhood experiences can affect a pre-service teacher's perception of mathematics.

Mathematics can cause anxiety for pre-service teachers (Jeffery et al., 2019). Jeffrey et al. (2019) investigated across five university campuses in Texas. The researchers surveyed 76 elementary education pre-service teachers who were juniors and seniors. The researchers wanted to examine elementary pre-service teachers' perceptions of confidence and self-efficacy when teaching mathematics (Jeffrey et al., 2019). One idea from this research was that pre-service elementary teachers often ponder if they have the skills to teach mathematics. If pre-service elementary teachers are thinking of this, they may lack the confidence and self-efficacy they need to teach mathematics. A lack of confidence and self-efficacy may hinder their capability of rich instruction.

Having knowledge and confidence in mathematics is essential when instructing mathematics (Stipek et al., 2001). Research has shown that mathematics anxiety can cripple a pre-service teacher's instruction in mathematics (Gresham & Burleigh, 2019). For instance, Stoehr and Olson (2023) completed a study with participants in the western United States. The participants included 48 pre-service elementary teachers. These pre-service elementary teachers were enrolled in a master's program trying to obtain their initial certification. The researchers found that when mathematics teaching anxiety

occurred, about half of the participants shifted their teaching responsibility to their students. They also found that the participants relied on strategies like luck because they encountered their own students who could explain the mathematics. Additionally, Gresham (2021) looked at the mathematics anxiety of 31 pre-service teachers before and after taking a mathematics methods course. The pre-service elementary teachers were working toward a K-6 endorsement during their junior year of the program. Gresham found a substantial difference in the pre-service elementary teachers' mathematics anxiety before and after they took the mathematics methods course. Pre-service teachers' mathematics anxiety levels were lessened at the end of the course. Together, these studies show that math anxiety can affect how a pre-service elementary teacher perceives mathematics and hinder their ability to teach mathematical concepts.

Teacher efficacy is an idea that researchers support that teacher beliefs can affect a student's outcome (Schuck & Grootenboer, 2004; Swars, 2005). Briley (2019) completed a study with elementary pre-service teachers. These teachers were in the southeast region of the United States. Briley used 95 participants who completed three surveys. These surveys measured the correlation between mathematics self and teaching efficacy and mathematical beliefs. The author found that the elementary pre-service teachers who believed they could effectively teach mathematics had more confidence in solving mathematical problems (Briley, 2019). Those elementary pre-service teachers would also have advanced mathematical beliefs. Thomson et al. (2022) investigated 231 elementary pre-service teachers. These pre-service teachers were in a STEM teacher preparation program at a large university in the United States. The researchers examined

the teachers' beliefs about efficacy and mathematical knowledge for teaching. The results indicated that as the pre-service teachers gained assurance in their mathematics teaching efficacy beliefs, there was an improvement in their content knowledge (Thomson et al., 2022). Findings from these two studies provide teacher educators with a better understanding that pre-service elementary teachers need an abundance of self-efficacy in teaching mathematics to aid in student learning.

Pre-service teachers' self-efficacy has been shown to correspond with perseverance and motivation with teacher education occurrences (Clark & Newberry, 2019). These two elements are important because they influence classroom behaviors and educational practices. Brinkmann (2019) investigated an issue at a university in the United States where 60 pre-service teachers completed mathematics courses during their senior practicum experience. The pre-service teachers recounted different levels of self-efficacy in mathematics while teaching mathematics in an elementary classroom. Brinkmann conducted this study to see how teacher preparation programs can boost pre-service teachers' self-efficacy. Findings showed that specific strategies must be used to bolster the self-efficacy of pre-service teacher candidates in teacher education programs. Cruz et al. (2019) completed a study with 238 pre-service teachers at a university in the United States. The researchers looked at the pre-service teachers' self-efficacy and mathematical dispositions and how they were related to prior mathematical experiences and their experiences in the teacher education program. Results displayed that elementary pre-service teachers had much lower mathematical dispositions and self-efficacy than their peers who were enrolled in secondary education (Cruz et al., 2019). The researchers

also found that mathematical dispositions were a strong predictor of self-efficacy and that both mathematical dispositions and self-efficacy were facilitated by the outcome of pre-service teachers of their prior mathematics teachers. Student achievement is correlated to a pre-service teacher's self-efficacy and mathematical dispositions (Cruz et al., 2019).

Clark and Newberry (2019) conducted a quasi-experimental study at a university located in the western United States to see how specific teacher education experiences affected self-efficacy. They also examined scores on a pre-service teacher self-efficacy scale. The researchers found that four causes added to strong pre-service teacher self-efficacy which were vicarious experiences, mastery experiences, verbal persuasion from teacher educators, and verbal persuasion from cooperating teachers (Clark & Newberry, 2019).

Like Clark and Newberry's (2019) research on self-efficacy, Mainali (2022) focused on beliefs. He conducted a case study to examine two pre-service elementary teachers' beliefs before and after starting a mathematics methods course regarding their learning and teaching of mathematics. The researcher used concept maps and interviews to collect data. The researcher found that the two pre-service teachers held negative beliefs about mathematics before the mathematics methods class (Mainali, 2022). The pre-service teachers considered mathematics confusing, tedious, and difficult to understand. They also felt that teaching mathematics was challenging. After completing the mathematics methods class, researchers found that one pre-service teacher changed her beliefs and depicted mathematics as an innovative and practical subject. On the other hand, the second pre-service teacher still held negative beliefs about mathematics

(Mainali, 2022). Researchers found that applying appropriate instructional strategies based on learning styles will help pre-service teachers learn and teach mathematics with ease and interest (Mainali, 2022).

Althauser (2018) also examined self-efficacy when investigating 347 pre-service elementary teachers. These pre-service teachers completed an elementary mathematics methods class at a United States university. The researchers used interviews, observations, and the Mathematics Teaching Efficacy Beliefs instruments. Althauser (2018) used these tools to investigate the pre-service elementary teachers' self-efficacy levels after taking a course on teaching elementary students mathematics. Althauser found that the pre-service teachers' attitudes improved significantly toward mathematics, and their confidence boosted substantially in teaching mathematics after taking the math methods course. These studies suggest that teacher educators must find approaches to improve pre-service teachers' self-efficacy to aid in motivation and perseverance.

A pre-service teacher's early childhood perception of mathematics can affect their belief about teaching mathematics. The level of confidence a pre-service teacher has affects their instructional decision-making (Letwinsky & Cavender, 2018). A qualitative multi-case study that examined this notion was conducted by Letwinsky and Cavender (2018), who examined the instruction procedures for two university instructors and changes in pre-service teachers' beliefs influenced by personal elementary experiences. The study took place in two different states in the northeast region of the United States. Information was collected from 57 undergraduate students in an elementary teacher certification program (Letwinsky & Cavender, 2018). Results showed that out of the 57

pre-service teachers, 50 pre-service teachers' early childhood perceptions of mathematics included a definite and conventional perspective about teaching mathematics. The pre-service teachers believed that students learn through memorization (Letwinsky & Cavender, 2018). The researchers also found that pre-service teachers did express an understanding and desire not to perpetuate similar mentalities per their early childhood perceptions of mathematics (Letwinsky & Cavender, 2018). This study demonstrates how a pre-service teacher's early childhood experiences can affect personal beliefs and future mathematics practice.

Anxiety, self-efficacy, and early childhood experiences can affect a pre-service teacher's perception of mathematics. Negative experiences in a mathematics classroom will influence pre-service teachers' perceptions and cause anxiety or negative feelings toward learning and teaching mathematics (Bekdemir, M. 2010). Moss et al. (2022) investigated how pre-service elementary teachers view themselves as learners during their last year in a teacher education program in mathematics and science. This investigation was conducted at a university in the southeastern United States with 72 students enrolled in a mathematics methods course and 75 in a science methods course. The researchers collected drawings and reflections for their data. The drawings and reflections indicated that when courses were taught from the perspective of social constructivism, the pre-service teachers' perspective of themselves as learners was significantly impacted in a positive manner. This research proposed that pre-service teachers' learning experiences in science and mathematics methods courses can affect how they will teach in the future (Moss et al., 2022).

Like Moss et al. (2022), Burton (2019) used drawings to examine initial perceptions, methods, and field experiences of 62 elementary pre-service teachers at a university located in the southeastern United States. Burton found that the drawings helped explain the implementation of teaching methods with elementary students. The drawings also looked at the mathematics pre-service teachers' vision of mathematics (Burton, 2019). This study provided insight into pre-service elementary teachers' positive and negative experiences and images toward mathematics. The studies support the notion that continued support for pre-service teachers to develop positive experiences in mathematics is needed.

Multiple studies have documented that early recollections and experiences with mathematics impact pre-service teachers' view of their capability to work with and teach mathematics (Cruz et al., 2019; Jeffrey et al., 2019; Letwinsky & Cavender, 2018). One of these studies included seventy-six elementary education pre-service teachers who had a status of junior or senior (Jeffrey et al., 2019). These pre-service teachers came from five university campuses. This study was strong because it provided a more comprehensive range of conversations about pre-service teacher efficacy, which can help teacher educators ensure that their programs prepare pre-service teachers for work in education (Jeffrey et al., 2019). Another strong study was the study that looked at pre-service teachers' self-efficacy and mathematical dispositions and how they were related to prior mathematical experiences and their experiences in the teacher education program (Cruz et al., 2019). The study was strong because survey data from 238 pre-service teachers revealed that mathematical dispositions were a strong predictor of self-

efficacy. This study brought about an understanding of the relationship between mathematical dispositions and self-efficacy, pronounced by the National Research Council as a strand of mathematical proficiency and self-efficacy (Cruz et al., 2019).

There was a study that can be categorized as weak. Lewinsky and Cavender (2018) examined the instruction procedures for two university instructors and changes in pre-service teachers' beliefs that were influenced by personal elementary experiences. This study was weak due to the limited comparable sample of pre-service teachers at each university, and the researchers could not observe pre-service teachers' teaching practice in the field to see if there was transference into practice. This study can only hypothesize likely pedagogical decisions made in the classroom (Letwinsky & Cavender, 2018).

Perceptions of Fractions

Pre-service teachers have described fractions as difficult (Whitacre et al., 2019). Current research confirms that pre-service teachers still have a negative attitude and undesirable perception of fractions (Whitacre et al., 2019). This negative attitude about fractions may impact their teaching when they become elementary teachers.

Ganley et al. (2019) investigated 399 Florida lower and upper elementary teachers. The researchers used the data instrument called the Math Anxiety Scale for Teachers. The researchers found that participants with high math anxiety had much lower mathematical knowledge of teaching. The participants had more traditional beliefs about teaching and learning mathematics (Ganley et al., 2019). The researchers also found that the teachers who had math anxiety experienced even more anxiety when completing fraction problems on the test. Gresham (2018) completed a study first with ten pre-

service teachers to see if completing a mathematics methods course would reduce mathematical anxiety. It did reduce their mathematical anxiety. However, those levels remained high. Gresham decided to take the same ten pre-service teachers and complete a study to see if their mathematical anxiety decreased after five years of teaching. Gresham found that after 5 years of teaching experience, teachers' math anxiety was reduced minimally. With proper instruction in teacher education programs and teaching experience, Gresham found that there would be a decrease in mathematics anxiety, but not substantially.

The mindset that students have about themselves as math students has a direct correlation with their ability to do mathematics. Sidney et al. (2021) studied the attitudes adults and children have when they study fractions and whole numbers. The researchers completed four studies in the United States. In the first study, they examined the attitudes of fifth and sixth graders. In the second and third study, the authors looked at the attitudes in a community. In the last study, the authors examined the attitudes of college-aged adults. Across the four studies, the authors concluded that most adults and children had positive attitudes toward whole numbers but negative attitudes toward fractions (Sidney et al., 2021). These findings significantly associate how adults and children interact with mathematics when showcasing fractions.

Like Sidney et al. (2021), Mielicki et al. (2022) researched college students enrolled in a midwestern United States university. Mielicki et al. used two samples of college students, female and male, who were approximately 19 years of age. Study 1 consisted of 491 participants, and Study 2 consisted of 415 participants. The two

investigations tested if math skills, mindset beliefs, gender, and perceptions correspond to negative math attitudes regarding percentages and fractions related to whole numbers. Mielicki et al. found that the students had more negative attitudes toward fractions than percentages and whole numbers. In conclusion, the two studies reveal students' negative mindsets and beliefs toward fractions.

There is additional evidence that a pre-service teacher's early recollections could impact their learning of fractions. A study was conducted at a university in the United States to examine 76 pre-service teachers' interpretations of fractions using a writing evaluation with linear, discrete, and circle representations (Boyce & Moss, 2022). The researchers designed an instructional unit that introduced and connected circle representations to other representations of fractions (Boyce & Moss, 2022). During the development and implementation of the instructional unit, the scholars found that it was essential to recognize the pre-service teachers' prior mathematics learning experiences, including their experiences with past teachers and mathematics content. They found that their perceptions of mathematics did affect their approach to making sense of fractions. The pre-service teachers also questioned their motivation to learn something new about fractions not taught during their school experience (Boyce & Moss, 2022).

Whitacre et al. (2019) conducted a case study at a university in the southeastern United States. This study comprised 26 pre-service teachers, and ten participated in one-on-one interviews. One participant was selected for the case study because her interview exemplified the four themes the researchers explored. The four themes were the following: (a) creating a safe space that focused on the interviewee's ideas, (b) the

interviewee used prior knowledge to come up with new strategies with fractions, (c) the interviewee activated prior knowledge for cross multiplication, and (d) the interviewee's beliefs and affective factors that allowed the interviewee to study unacquainted pathways through familiar mathematical areas (Whitacre et al., 2019). The researchers used interviews to observe how a pre-service teacher used prior knowledge to create strategies for comparing fractions. One set of results showed that the interviewee expressed that fractions were a "scary monster" and that they should steer clear of them whenever possible. The interviewee also said she considered herself bad at fractions (Whitacre et al., 2019). These reflections connected back to the interviewee's early experiences as a child in school. The interviewee felt she did not have a teacher explaining what a fraction meant. These early recollections conveyed her feelings of anxiety toward fractions. In conclusion, the two studies present evidence that early recollections can hinder a pre-service teacher's ability to perceive and be willing to learn fractions.

Multiple studies document that pre-service teachers described fractions as difficult. One of these studies looked at 399 lower and upper elementary teachers in Florida using the Math Anxiety Scale for teachers. The researchers found that pre-service elementary students had experienced high anxiety when completing fractions. This study was strong because the researchers provided substantive and external validity due to the scale they used to measure responses (Ganley et al., 2019). Another study had one participant selected for the case study because her interview exemplified the four themes the researchers were exploring (Whitacre at al., 2019). Even though one participant was used in this research, the study was strong because the findings were relevant and useful

to the work of mathematics teacher educators. The studies presented show that pre-service teachers look at fractions as a difficult topic to learn (Ganley et al., 2019; Whitacre et al., 2019).

Knowledge of Fractions

As background for this section, it is important to establish that fraction knowledge is complex for pre-service teachers. The PRAXIS teacher exams provide statistical evidence that pre-service elementary teachers are not passing Praxis 1 when entering a teacher education program. One of the areas that Praxis 1 measures is a student's ability to perform fraction operations. The research completed by the University of North Alabama showcased Praxis 1 national scores for the United States (The University of North Alabama, n.d.). Praxis 1-The Core Academic Skills for Educators passing score is 150 (see Table 1). Table 1 shows that between 2017 and 2019, the median score was 146 and the median score for years between 2019 and 2020 was 148. These scores reveal that pre-service teachers are not passing Praxis 1.

Pre-service elementary teachers must have sufficient knowledge of fractions and fraction operations if they are to instruct learners in different ways (Thurtell et al., 2019). This understanding must be instructed in rich conceptual knowledge (Thurtell et al., 2019). Morano and Riccomin (2020) conducted a study at a university in the United States. They used 55 participants who were enrolled in a mathematics methods class. The participants, who were pre-service elementary teachers, completed a test on visual representations and word problems based on multiplication and division. Morano and Riccomin (2020) discovered that the participants were weak in their conceptual

knowledge of fraction multiplication and division because they could not showcase it using visual representations and word problems.

Magiera and van den Keiboom (2021) contributed to the research when they completed their study in the United States with ten pre-service elementary teachers. These participants were enrolled in a teacher education program. The researchers investigated the number and operation sense with fractions when solving problems. Magiera and van den Keiboom found that the participants had difficulty with the representations of fractions and operations and understanding the operations on pairs of fractions. Overall, both studies prove that pre-service teachers struggle to understand fractions.

Pre-service teacher's knowledge of fractions has been the focus of many studies. Ball et al. (2001) stated that during pre-service teachers' "pretraining" phase, developing their understanding of mathematics is essential. A teacher's comprehension of concepts will affect the quality of instruction (Copur-Gencturk, 2021). The effect that the lack of concept knowledge can have on instruction can be seen in the investigation by Lee and Lee (2020), which was completed with 83 elementary pre-service teachers from two universities. They discovered that the pre-service elementary teachers had much trouble showcasing fraction addition. This included fractions that were unlike denominators and improper fractions.

Another study examined the problem of little fractional pedagogical content knowledge. (Adu-Gyamfi et al., 2019). The authors completed their research at a university in the southeastern United States with 36 pre-service elementary teachers.

Adu-Gyamfi et al. (2019) found that 67% of their participants could complete fraction division problems. However, these participants could not analyze student work based on the same problems they solved. Together, these two studies show that pre-service elementary teachers do have insufficient knowledge of conceptual knowledge of fractions, which in turn can affect instruction.

Some studies examine classroom teachers and their understanding of fractions. Copur-Gencturk (2021) researched elementary school teachers across the United States. The researcher examined the knowledge of fraction arithmetic with 303 participants. Coper-Gencturk coded the elementary school teachers' explanations to see their responses' accuracy and representations. The researcher discovered that the participants' knowledge of fraction arithmetic was limited. Coper-Gencturk discovered that 14% of the participants could explain reference units in fraction division. The researcher asked for opportunities for elementary teachers to learn more about fraction arithmetic (Copur-Gencturk, 2021).

Copur-Gencturk and Doleck (2021) continued investigating how 350 teachers responded to four fraction problems with multiple steps. These teachers taught fourth and fifth grade. The researchers investigated how competent these teachers were in solving word problems. They discovered that teachers with solid planned skills used algebraic notations and pictorial representations in their solution procedures (Copur-Gencturk & Duleck, 2021). In summary, the teachers in Copur-Gencturk's first study had a very low understanding of fractions and could not explain the purpose of reference units. Meanwhile, some teachers in Copur-Gencturk and Dulek's second study had solid

strategic competence in their solution procedures. This indicates mixed findings regarding their fraction understanding. These two studies show the critical nature of fractional competence.

There is sufficient research that uses pedagogical content knowledge as the framework. Pedagogical content knowledge represents content and pedagogy. The pedagogical content knowledge of a teacher is developed when pre-service teachers are given opportunities to practice teaching (Kula Unver et al., 2020). Content and pedagogical content knowledge are separate elements of teachers' proficiency in teaching mathematics (Copur-Gencturk & Tolar, 2022). Copur-Gencturk and Tolar (2022) examined how content and pedagogical content knowledge are separate elements in their study with 290 teachers. These teachers came from 48 states in the United States. The researchers found that teachers' expertise in mathematics content for teaching mathematics is intricate. According to Copur-Gencturk and Tolar, teachers' knowledge of events differed from their comprehension of students' mathematical thinking and the math-specific teaching strategies they intended to implement to improve student learning.

Additional research on pedagogical content knowledge looks at content knowledge and why the methods work, which is pedagogical knowledge. To teach fractions efficiently, pre-service teachers must know what methods to use to solve fractions problems and why the methods work, which is pertinent to pedagogical knowledge (Lee, 2017). For instance, if a pre-service teacher does not have the skills to understand fraction addition, they will not be able to articulate to their students why a common denominator is needed. Lee (2017) researched 111 pre-service elementary

teachers investigating their content knowledge using a length model with referent units to solve a fraction division problem. The pre-service teachers wrote responses, which were looked at in terms of strategies and errors. Lee found that the pre-service teachers could make equivalent fractions and complete fraction division. However, they did not understand why to make equivalent fractions and measurement division. In summary, pre-service teachers had content knowledge but needed to gain pedagogical knowledge when making meaning of measurement division.

Zolfaghari et al. (2021) were interested in measuring pre-service teachers' pedagogical content knowledge for fractions. They used cognitive interviews with 85 pre-service and seasoned teachers and Rasch modeling. Results showed from the Rasch modeling that there is a need for items that are less difficult and that pre-service teachers had lower levels of pedagogical content knowledge for fractions. Only 23.5% of participants demonstrated lower-than-average pedagogical content knowledge for fractions. In summary, the participants in this study lacked pedagogical knowledge.

To assist teachers, Hilton and Hilton (2019) completed research using Lee Shulmans' idea of pedagogical content knowledge. These researchers looked at how interventions assist a teacher's knowledge of mathematics for teaching. The interventions included detailed lessons and scripts that aided in students' learning. The researchers found that the interventions supported teachers regarding content and pedagogical knowledge (Hilton & Hilton, 2019). Rosli et al. (2019) completed a quasi-experiment design during pre-and-post tests with 71 female elementary pre-service teachers. The researchers implemented a 3-week fraction unit using problem-solving and models during

mathematical classroom instruction to improve the teachers' perceptions, attitudes, and knowledge of fractions. The researchers found that the combination of concrete models, problem-posing, and problem-solving improved the pre-service teachers' pedagogical content knowledge, attitudes, and perceptions of fractions. Together, these studies provide evidence that there are interventions that can successfully support pre-service teachers' mathematical knowledge of fractions and support their teaching.

Multiple studies document that pre-service teachers struggle with fraction knowledge (Adu-Gyamfi et al., 2019; Lee & Lee, 2020; Morano & Riccomin, 2020). One of these studies used 55 pre-service elementary teachers to see their ability to complete a test to model fraction multiplication and division. This study was strong in terms of the number of participants used. Based on the researchers' findings, mathematics teachers' future effectiveness depends on content and pedagogical content knowledge (Morano & Riccomin, 2020). Another study that was strong used 83 pre-service elementary teachers. These teachers came from two different universities in the United States. This research showed that pre-service teachers still have difficulty using the area model to represent fraction addition, which includes fractions, unlike denominators, and improper fractions (Lee & Lee, 2020). These researchers found that it is essential for teacher educators to instruct their pre-service teachers in mathematical reasoning. It is not enough to be able to compute (Lee & Lee, 2020).

There was a study that can be categorized as weak. Adu-Gyamfi et al. (2019) conducted research with 36 participants. They looked at how the pre-service elementary teachers solved fraction division problems. The limitation of this study was the small

sample used. There were not many participants. The pre-service teachers used in this study may not have represented the population well. The studies presented in this section show that pre-service teachers struggle with fraction knowledge (Adu-Gyamfi et al., 2019; Lee & Lee, 2020; Morano & Riccomin, 2020).

Common Core State Standards

Pre-service elementary teachers are matriculated in mathematics education courses during their teacher education preparation at their college or university. In these courses, mathematics content knowledge and pedagogical content knowledge are highlighted. As a result of the publication of the Common Core State Standards for Mathematics in forty-one states, four territories, and the District of Columbia, expectations for teachers have been set higher (Akkus, 2016). The Common Core State Standards seek to involve students in complex mathematical practices, including reasoning, modeling, and argumentation (Schweig et al., 2020).

Research on Common Core has focused on fractions and fraction operations. For example, Bentley and Bosse (2018) completed a study with eight college students of several majors in a university located in the southeastern United States. They looked at the participants' understanding of fraction operations that were specific objectives from the Common Core State Standards in grades three through six. The researchers found many misconceptions about fraction concepts, such as equivalent fractions, common denominators, and applying incorrect fraction operations. The results showed that college students had the same misunderstandings as elementary students concerning fraction operations (Bentley & Bosse, 2018). Like Bentley and Bosse (2018), Wilkins and Norton

(2018) also looked at fractions per the Common Core State Standards for Mathematics. The researchers had a little over 300 students in grades 5,6,7 and 8 from schools in the Midwest, southeast United States, and China. Wilkins and Norton looked at the hierarchy of fraction schemes. The mental actions of these schemes provided teachers an opportunity to design instructional opportunities for children to have a rich understanding of the measurement concept of fractions (Wilkins & Norton, 2018). The results of the synthesis of research studies suggested that a child's measurement concept of fractions developed through developmental stages with distinct schemes.

Outlined in both the Common Core State Standards for Mathematical Practice and the National Council of Teachers of Mathematics, there are standards in mathematical reasoning and argument (Makowski, 2021). Teaching mathematics requires pre-service teachers to have the ability to reason and engage in mathematical arguments (Makowski, 2021). They must also have content knowledge. Mathematical content knowledge is essential for pre-service teachers because they need to have an understanding of how to complete procedures and explain why a given procedure makes sense to them (Lovin et al., 2018).

Norton and Boyce (2013) affirmed that fourth-grade students must understand improper fractions according to the Common Core State Standards. For example, 5/4 is an improper fraction with five iterations of 1/4. It was essential that Lovin et al. (2018) completed a study that looked at which fraction schemes and operations pre-service teachers demonstrated evidence of having. One hundred nine undergraduate Pre-K-8 pre-service teachers were in their first mathematics course in a university in the mid-Atlantic

United States. Lovin et al. (2018) found that only 27% of pre-service teachers could construct an iterative fraction scheme. Makowski (2021) completed an exploratory examination of the types of justifications pre-service teachers use when making mathematical claims. She used six female pre-service teachers completing a patterns unit in a mathematics methods course at a mid-western university in the United States. Makowski also used 10 pre-service teachers' written work to supplement results with written justifications. The researcher found that the pre-service teachers often used inductive evidence when verifying a final check of an answer. This occurred even when the pre-service teacher would produce several correct intuitive and deductive justifications. This suggested that the pre-service teachers were not trusting their deductive justifications, which could, in turn, make it challenging to facilitate class discussions of mathematical arguments.

Magiera and Zamback (2020) continued to look at the mathematical content knowledge and pedagogical choices outlined by the Common Core State Standards. They looked at the mathematical discourse and reasoning. They used the written responses of 37 pre-service teachers who were preparing to teach Grades 1-8 mathematics. Results indicated that when pre-service teachers provided commentaries of student explanations, they were weaker than their problem solutions. Together, these studies reveal that pedagogical choices outlined by the Common Core State Standards provide pre-service teachers with meaningful and productive ways to deliver fraction concepts.

Measurement and operations with fractions are essential in the Common Core State Standards. This includes representing a fraction on a number line and tasks

involving fraction magnitude comparisons (Fazio & Siegler, 2011). Alqahtani et al. (2022) investigated how reviewing fractions from a measuring perspective could provide information on how pre-service teachers reason about fractions. The researchers utilized participants enrolled in a teacher education program at a university in the United States. There were 46 pre-service teachers in this study. The data collected comprised pre-and post-tests that assessed how the pre-service teachers identified and represented fractions with discrete and continuous models (Algahtani et al., 2022). The researchers found that the participants could provide additional strategies for all fraction models on the post-test.

According to the Common Core State Standards for Mathematics, students should be able to solve problems involving the division of fractions (Common Core State Standards Initiative, n.d.). They should be able to do this with visual fraction models and equations. Sahin et al. (2020) investigated 34 undergraduate pre-service elementary teachers enrolled in a mathematics content course at a university in the southeastern United States. The researchers investigated how the pre-service teachers built conceptual knowledge of fraction division with remainders. Results showed that the pre-service teachers' understanding emerged on three levels. The first level was ignoring the remainder or not putting the remainder in the correct location. The second level was not being able to interpret the remainder. The third level was interpreting the remainder (Sahin et al., 2020). Collectively, these studies show the lack of interpretation of measurement and division of fractions with elementary pre-service teachers.

One feature of the Common Core State Standards for Mathematics is the Standards for Mathematical Practices for each grade level. These practices emphasize

mathematical processes and proficiencies like reasoning, problem-solving, and mathematical modeling (Common Core State Standards Initiative, n.d.). Courtney and Caniglia (2021) examined pre-service teachers in Grades 4-9 and mathematical educators on how they identified and made sense of the work toward solutions to word problems. The researchers also compared the mathematical practices among the pre-service teachers and mathematics teacher educators. Courtney and Caniglia (2021) found that pre-service teachers struggled with six mathematical habits of mind. They could identify which sample problems would support pre-service teachers with mathematical habits of mind. Joshua and Lee (2022) also looked at a feature of mathematical practices that dealt with proportional reasoning. They investigated 199 elementary pre-service teachers. The researchers looked at their proportional reasoning and interpretations of their calculations by providing a 10-problem questionnaire. In Common Core State Standards, the ability to interpret calculations is fundamental to five of the eight mathematical practices (Joshua & Lee, 2022). A fourth of the surveyed pre-service teachers had difficulty interpreting their calculations on a sixth-grade task. Together, these studies show that teacher educators lack the proficiencies like reasoning, problem-solving, and mathematical modeling needed to devise tasks that require them to understand and practice mathematical practices.

Multiple studies have documented that since Common Core State Standards for Mathematics were released, there have been higher expectations for new teachers (Akkus, 2016). One such study was the research conducted by Lovin et al. (2018), where the researchers looked at which fraction schemes and operations pre-service teachers

demonstrated evidence of having. They used 109 undergraduate pre-service teachers at a university. This study was strong because of the number of participants, and it was broadly consistent with the established hierarchy, which showed that lower-level fractional understanding is a prerequisite to higher levels of understanding. Another strong study was conducted by Joshua and Lee (2022), who used 199 elementary pre-service teachers to look at the mathematical practices of the Common Core State Standards. The researchers used qualitative and quantitative analyses, including raw data, creating coding schemes, and coding all the data. The researchers could also provide specific recommendations for teacher educators that originated from their data.

There was a study that can be categorized as weak. Sahin et al. (2020) conducted a case study with 34 students. These 34 students were the case. The results cannot be generalized past the single case being studied. The researchers used a purposive sampling, which can be susceptible to sampling bias (Sahin et al., 2020). The studies presented in this section show that with the higher expectations of Common Core, pre-service teachers are still struggling with fractions.

Pre-Service Teacher Preparation for Mathematics

There has been an emphasis on national education reform on mathematics teacher quality (Jeffery et al., 2019). Attention has been paid to the competence of teacher preparation programs and how undergraduate elementary mathematics teachers are prepared (Jeffery et al., 2019). Empirical evidence shows that teachers' attitudes, values, or dispositions significantly impact student outcomes (Jeffery et al., 2019). Teacher candidates' sense of efficacy in their teacher education programs significantly affects the

program and the teacher candidates themselves. Tassell et al.'s (2020) study examined the relationship between mindfulness, mathematical anxiety, self-efficacy, and mindset among pre-service elementary teachers. With the results from the mixed-methods study, the researchers hoped to address these variables with teacher educators. There were 37 participants recruited from a regional university in the United States enrolled in a mathematics methods class (Tassel et al., 2020). The researchers found that by focusing on all four variables, teacher educators could prepare future teachers in the classroom effectively.

Similarly, Schanke (2023) looked at why pre-service elementary teachers have a higher mathematics anxiety rate compared to other undergraduate students who are not in education. Schanke's participants included seven males and 43 females, all from a university in the United States. These participants were completing their first elementary education mathematics seminar. Schanke investigated the pre-service teachers' self-efficacy before and after completing their seminar. The results showed that the seminar was a success. The seminar had a positive effect on the participants' self-efficacy. Together, these studies suggest that when pre-service teachers are given course activities from teacher educators, they can increase their self-efficacy.

Mathematics teaching efficacy has been linked to how pre-service teachers use instructional practices (Lee et al., 2017). When creating teacher education programs, it's critical to identify the predictors of mathematics teaching efficacy beliefs and the relationships that accompany them. Johnson et al. (2018) investigated 41 elementary pre-service teachers. These participants were enrolled in a mathematics methods course.

Correlation analysis was used to establish if there was a relationship between teaching self-efficacy and content knowledge. Mathematical content knowledge was assessed using the Praxis Elementary Education for Mathematics, and self-efficacy was measured using the Mathematics Teaching Outcome Expectancy and Personal Mathematics Teaching Efficacy. The researchers found no statistically significant relationship. They did, however, find that elementary pre-service teachers' experiences prior to acquiring mathematics content were not as significant regarding efficacy judgments. The participants would gain positive experiences when teaching mathematics (Johnson et al., 2018).

Chen et al. (2022) also looked at teaching efficacy. The researchers looked at early childhood and elementary pre-service teachers' science and mathematics teaching efficacy to expose paths that would help teacher preparation programs. Chen et al. used 180 pre-service teachers to examine their open-ended responses and quantitative data. These teachers were from the United States from four universities. The authors found that the number of mathematics and science courses taken was correlated with their participants' teaching efficacy beliefs. Chen et al. argued that pre-service teachers needed pedagogical knowledge and hands-on teaching experience to increase their confidence in how they will affect their future students' mathematics and science learning.

Like Chen et al. (2022), Brown et al. (2021) determined that teaching self-efficacy increased when students were able to teach mathematics. Brown et al. investigated the perceptions of teaching efficacy of early childhood and elementary pre-service teachers while they were student teachers. The researchers found that

preparedness and teaching efficacy improved considerably during student teaching. Together, these studies provide insight into how teacher preparation programs can better cultivate a pre-service teacher's teaching efficacy in mathematics by increasing their pedagogical knowledge and providing them with hands-on teaching experiences.

Higher education institutions enter a contractual arrangement with individual states that must approve teacher preparation programs that lead to licensure. School districts within individual states rely on higher education institutions to graduate teachers. Publishing data show that many elementary pre-service teacher candidates struggle each year to pass licensing tests (Putnam & Walsh, 2019). Putnam and Walsh (2019) completed research with 817 institutions in the United States. They focused their attention on undergraduate elementary teacher education programs. Putnam and Walsh wanted to look at the statistic that approximately half of all pre-service candidates do not pass their licensing test. Putnam and Walsh looked at teacher education programs in all 50 states, including Washington, D.C. Only one in four programs (27%) covered important elementary math concepts. Only half of the programs covered some concepts, and a quarter of the programs (22%) had no coursework in math content (Putnam & Walsh, 2019). The researchers found that the pre-service candidates had gaps because of their K-12 education. However, the researchers urged state policymakers and teacher education leaders to devise solutions to aid the candidates.

Putnam and Walsh (2021) continued to examine licensure tests. The researchers focused on pre-service elementary teacher candidates who take their licensure tests on the content knowledge outlined by the state. Putnam and Walsh obtained their data from 37

states as of April 13, 2021. The data showed that 55% of those who took the test failed on their first try at taking their licensure test. The data provided by these two studies allows states and teacher preparation programs to see the high number of low passing rates of teacher licensure. With this data, states and teacher preparation programs can see the urgency of supporting pre-service teacher candidates in learning the content the students need to pass their tests and succeed in the classroom.

Multiple studies document that pre-service teachers' attitudes and values are important in student outcomes (Chen et al., 2022; Tassell et al., 2020). One such study was conducted by Tassell et al. (2020), where the researchers investigated how mindfulness, mathematics anxiety, self-efficacy, and mindset are blended within pre-service elementary teachers. The researchers used 37 participants registered in a mathematics methods course at a South Central regional university in the United States. This study was strong because the results informed the teacher educator community. The results give the teacher education community ideas to reexamine their teacher education models in mathematics and join with current elementary teachers (Tassell et al., 2020). Another strong study was conducted by Chen et al. (2022), which used 180 pre-service teachers to examine science and mathematics teaching efficacy to expose paths that would help teacher preparation programs. Chen et al. used a quantitative approach, using a quantitative survey and open-ended response data to provide results that provide teacher preparation programs to better cultivate their pre-service teachers' teaching efficacy in mathematics. The studies presented in this section showcase that teachers' values, dispositions, and attitudes will impact student outcomes.

Pre-Service Teacher Preparation for Fractions

Pre-service elementary teachers need to understand fractions and pedagogic practices to create and investigate ideas about fractions with their students. They must be given learning opportunities to develop content knowledge of fractions. Some learning opportunities include using manipulatives such as fraction tiles (Reeder & Utley, 2017). If pre-service elementary teachers have misconceptions or limited understanding of fractions, this may hinder the knowledge their future students will obtain. Experiences in a teacher education program can impact pre-service elementary teachers' understanding of fractions.

Reeder and Utley (2017) completed a study examining pre-service elementary teachers' understanding of fractions. The authors found that pre-service elementary teachers' comprehension of fractions was limited in their final methods course. They also found that pre-service elementary teachers need to be given opportunities to develop a deep understanding of fractions (Reeder & Utley, 2017). For future elementary teachers, the authors called for more research into how mathematics educators can develop courses regarding fractions.

Stevens et al. (2020), similar to Reeder and Utley (2017), implemented instructional changes to a mathematics course for PreK-8 pre-service teachers. The researchers wanted to see if these changes would help the pre-service teachers with fraction operations and schemes. The researchers found substantial increases in the pre-service teachers' ability to construct fraction schemes and operations.

The study conducted by Whitehead and Walkowiak (2017) suggests that pre-service elementary teachers' ability to understand fractions evolves while taking a mathematics methods course. Whitehead and Walkowiak investigated 48 pre-service elementary teachers to see if their understanding of fractions changed during a Grades 3-5 mathematics methods course. The researchers found that the pre-service teachers' mathematics level was above what they needed to know to instruct elementary mathematics. Even though their mathematics level was above average, this did not improve their explanations of standard fraction algorithms. Together, these studies suggest that teaching content and pedagogy parallel to fractions is essential for teacher preparatory programs to improve pre-service teachers' understanding of fractions.

Multiple studies document teacher education programs' impact on prospective elementary teachers' understanding of fractions (Reeder & Utley, 2017; Whitehead & Walkowiak, 2017). One such study was conducted by Reeder and Utley (2017), in which the researchers used 41 participants to examine pre-service elementary teachers' comprehension of fractions. This investigation was sound because of the number of participants used and the results showcasing that only one in four programs covered in teacher education covered the vital components of elementary mathematics. Reeder and Utley (2017) emphasized that leaders in higher education and state policymakers must work together to help teacher candidates. Another sound study was conducted by Whitehead and Walkowiak (2017), who used 48 participants and found that even though the pre-service teachers' mathematics level was above average, this did not improve their explanations of standard fraction algorithms. Whitehead and Walkowiak's results were

consistent with Ball's (1990) study, which supports that elementary pre-service teachers only restate rules in their mathematical explanations. The studies in this section present how important teacher education programs are when preparing their pre-service elementary teachers' conceptual and pedagogical understandings of fractions.

Summary and Conclusions

To synthesize this literature review, the main convergent findings were that early recollections could impact a pre-service teacher's ability to learn fractions (Boyce & Moss, 2019), pre-service teachers still struggle with the knowledge of fractions (Lee & Lee, 2020), and teacher education programs can impact a pre-service teacher's understanding of fractions (Whitehead & Walkowiak, 2017). Specifically, in the perceptions of fractions subsection, the research indicates that perceptions of mathematics did affect a pre-service teacher's approach to making sense of fractions. The pre-service teachers also questioned their motivation to learn something new about fractions not taught during their school experience (Boyce & Moss, 2022). The research indicates that pre-service teachers still struggle with content knowledge for the knowledge of fractions subsection. Lee and Lee (2020) conducted a study that showed a struggle with the content knowledge of fraction addition with pre-service teachers. In addition, for the pre-service teacher preparation for fractions, a pre-service teacher can develop a much better understanding of fractions when given opportunities. Whitehead and Walkowiak (2017) completed research that showed that when pre-service teachers took a mathematics methods course, their ability to explain fraction algorithms was not

improved. Teacher education programs must create courses that provide opportunities for pre-service teachers to enhance their content and pedagogical knowledge of fractions.

The research provided in this chapter was for pre-service teachers prior to and during student teaching. Research is lacking in pre-service teachers' content knowledge and pedagogical content knowledge after they have completed student teaching. The problem is a gap in practice supported by the research literature, which indicates we are not aware of how pre-service elementary teachers, after student-teaching, perceive their knowledge or gaps in knowledge in practice in two areas: (a) knowledge of content and (b) knowledge of pedagogy. In Chapter 3, the phenomenon and the researcher's role are described and discussed. The topics examined include the population's identity, instrumentation, and the data analysis plan. Lastly, ethical concerns related to the data are reviewed.

Chapter 3: Research Method

The literature review showcased studies that illustrated pre-service elementary teachers' content knowledge or lack of content knowledge of fractions and fraction operations. The literature also emphasized the knowledge of pedagogy of fractions and fraction operations with pre-service elementary teachers. The purpose of the qualitative study was to explore pre-service elementary teachers, after student teaching, perceptions of their knowledge or gaps in knowledge in practice in two areas: (a) knowledge of content and (b) knowledge of pedagogy.

Chapter 3 highlights the phenomenon of the study and explains the role of the researcher. This chapter provides the instrumentation being used and the data analysis plan for the study. Furthermore, the ethical procedures and the trustworthiness of the data is discussed. Lastly, a summary of focal points is provided.

Research Design and Rationale

The research questions addressed the problem and aligned with the purpose statement. The research questions addressed how pre-service elementary teachers describe their perceptions of their own knowledge or gaps in knowledge regarding content and pedagogical content knowledge of fraction operations.

RQ1: (*knowledge of content*) What are pre-service elementary teachers', after student-teaching, perceptions of their knowledge or gaps in knowledge regarding their knowledge of content of fraction operations?

RQ2: (*knowledge of pedagogy*) What are pre-service elementary teachers', after student-teaching, perceptions of their knowledge or gaps in knowledge regarding their knowledge of pedagogy of fraction operations?

RQ3: (*knowledge of students*) What are pre-service elementary teachers', after student-teaching, perceptions of their knowledge or gaps in knowledge regarding their knowledge of students of fraction operations?

This study was based on pedagogical content knowledge, which is both content and pedagogical knowledge that pre-service teachers must have. Lee Shulman (1986) developed three different categories of teacher knowledge: content knowledge, pedagogical content knowledge, and knowledge of students. These three categories guided the research questions.

A basic qualitative study design was used. Denzin and Lincoln (2018) termed a qualitative study as a discovery of how individuals interpret their experiences and their significance. The pre-service teachers recounted their experiences after student teaching about their knowledge or lack of knowledge in fraction operations. A basic qualitative study design was appropriate because the purpose of this study was to explore pre-service elementary teachers, after student-teaching, perceptions of their knowledge or gaps in knowledge in practice in two areas: (a) knowledge of content and (b) knowledge of pedagogy. My study provided an awareness of a particular problem. This methodological design was chosen because this research looked at the perceptions of pre-service elementary teachers and understand their content and pedagogy content knowledge. The

study expected to comprehend the participants' experiences in a strong and productive way (Merriam & Tisdell, 2016).

Role of the Researcher

As the researcher, I had no personal, professional, or supervisory relationships with the participants. The biases of the study were managed using in-depth qualitative interviews. Each participant was asked identical questions. All participants received clear definitions of mathematical terms to ensure understanding. The definitions given to the participants were the same. The open-ended questions allowed each participant to respond in any way they chose. To eliminate bias, the participants were able to disagree with the questions, elaborate on their answers, and even raise new questions (Rubin & Rubin, 2012). I also controlled bias by completing member checking. The follow-up procedure was emailing participants a codebook, which I asked each participant to check for the accuracy of my work. All participants were recruited through social media. I did not use the Walden University participant pool or students from a university in the United States.

Methodology

Participant Selection

Pre-service elementary student teachers were the population for this study. These were people who had engaged in a student teaching exercise under the supervision of a certified teacher. Elementary teacher education programs typically have a described program. There are approximately 1,120 public and private institutions of higher education in the United States preparing pre-service teachers (National Council on

Teacher Quality, n.d.). The sampling procedure was purposeful. I needed to use elementary pre-service teachers as these participants met the standards to answer my research questions. This sampling strategy was chosen so that I would gain detailed knowledge about the experiences pre-service elementary teachers have with fraction operations. The criteria on which participant selection was based were pre-service teachers who had student taught with students that were between the ages of 6-12 in Grades 1-6 classrooms and pre-service teachers who had taught at least two of the following fraction operations: addition, subtraction, multiplication, and division. These participants were from the United States.

There were specific procedures for identifying, contacting, and recruiting participants. Prior to recruiting participants, I obtained IRB approval. Walden University's approval number for this study is 07-31-23-0042447. A flyer was created that described the nature and purpose of the study. It described the activities of the study, eligibility, and the researcher's contact information. If the participant met the requirements, the participant contacted the researcher. The flyer was posted with permission on Facebook educator groups. Plan B would have been in place if not enough participants were obtained. Plan B included having a university director of student teaching send a flyer to the pre-service teachers who had completed student teaching. The director of student teaching was asked to write a letter of intent to participate in the study and agreed to disburse flyers to pre-service teachers who had completed student teaching. Plan B also included the Walden University participant pool.

The participants met the study's criteria because they signed a consent form. I acquired 12 participants for my study. The rationale for 12-15 participants was that 12 interviews should be acceptable for many qualitative studies to attain data saturation (Guest et al., 2006). If saturation had not been reached, additional participants would have been recruited. Having 12-15 participants gave me a fairly accurate idea about pre-service elementary teachers' perceptions of content and pedagogical content knowledge of fraction operations.

Instrumentation

The data collection instrument used in this study were in-depth qualitative interviews. The interviews were conducted on Zoom and recorded. The qualitative interview was divided into four main categories that contained a list of questions (see Appendix A). The categories were Student Teaching Details, Content Knowledge of Fractions, Pedagogical Content Knowledge of Fractions (Pedagogy Knowledge), and Pedagogical Content Knowledge of Fractions (Knowledge of Students). The questions in the interview obtained rich data that answered the research questions. No legal or historical documents were used as a source of data.

The purpose of in-depth qualitative interviews is to look for rich and detailed information in examples, experiences, narratives, and stories of elementary pre-service teachers to explore their knowledge of conceptual and pedagogical content knowledge of fraction operations (Rubin & Rubin, 2012). With in-depth interviews, I did not need to give specific answer questions; instead, because the questions were open-ended, the participants could answer any way they pleased or even pose questions in response to the

interview questions (Rubin & Rubin, 2012). Content validity was established. Two mathematics education professors from the United States provided feedback on the questions created by the interview instrument. The professors ensured that the interview questions answered the research questions (Burkholder et al., 2020). A field test was also conducted to determine content validity. A pre-service elementary teacher who had finished student teaching and had taught fraction operations was asked to review the questions to ensure each question was understood and could be answered.

The research questions and the interview questions were in line. The first section of the interview asked the participants for details on student teaching. The student teaching information included information about the months and year the participant student taught, if the participant taught fraction operations, and which fraction operations the participant taught. This was important to my study because it verified that the participants met the criteria to participate in the study. The second part of the interview asked the participants to review their content knowledge of fraction operations. Content knowledge of fractions aligns with research question one. A teacher's comprehension of concepts will affect the quality of instruction (Copur-Genturk, 2021). For teachers to understand fractions, they must be given opportunities to investigate fractions (Kosheleva & Lyublinskaya, 2007). The third part of the interview was asking questions about pedagogical content knowledge of fractions (pedagogy knowledge). Pedagogical content knowledge of fractions aligns with research question two. To teach fractions efficiently, pre-service teachers must have content knowledge to solve fractions and understand why the methods work, which is pertinent to pedagogical knowledge (Lee, 2017). For

instance, if a pre-service teacher does not have the skills to understand fraction addition, they will not be able to articulate to their students why a common denominator is needed. The last part of the interview was asking questions about students' knowledge. Pedagogical content knowledge (knowledge of students) aligns with research question three. Pedagogical content knowledge in terms of students' knowledge includes understanding students' prior knowledge to comprehend possible misunderstandings of topics (Deng, 2018). Teachers need to acquire pedagogical content knowledge to instruct students so they will be successful in learning concepts.

Procedures for Recruitment, Participation, and Data Collection

Participants were recruited through Facebook educator groups. I obtained permission from each Facebook educator group to post on their site. Plan B would have been in place if not enough participants were obtained. Plan B included having a university director of student teaching send a flyer to the pre-service teachers who had completed student teaching. The director of student teaching was asked to write a letter of intent to participate in the study and agreed to disburse flyers to pre-service teachers who had completed student teaching. Plan B also included the Walden University participant pool.

A flyer was posted to the Facebook educator groups. By reading the flyer, participants could decide if they met the requirements of the study. If participants met the requirements of the study, they contacted me using my Walden email. I sent the approved consent from Walden University. The participants then read the consent form, and if they

wished to volunteer and met the requirements, they replied to the email with the words, "I consent." I contacted the participant, and a date was set for the interview.

The participant had one recorded interview on Zoom that was scheduled for approximately 60-90 minutes. At the start of each interview, I had a copy of the interview questions (See Appendix A). While the participant was speaking during the interview, I had a notebook and took notes as needed. Each participant's copy of the interview questions and notebook was put in a folder locked in a drawer in my home office.

After the completion of the interview, I indicated to each participant that in approximately two weeks, they would receive an email to check over my work. I asked each participant to check over my work and to reply to the email. On the consent form, it indicated to participants that the goal of my study is to help society by providing mathematics teacher educators with data to how prepared pre-service teachers are to teach fraction operations. As a thank you for participating in my study, I emailed each participant an Amazon e-gift card at the end of the interview.

Data Analysis Plan

Data analysis must co-occur with data collection (Green et al., 2007). This analysis involves management and the organization of data, immersive action, writing, and interpretation of the data (Ravitch & Carl, 2021). This is often accomplished by constructing themes from evolving ideas (Creswell, 2016; Saldaña, 2021).

The information each participant provided in this study was kept anonymous and confidential. The participants' answers were not shared among participants. Content validity established the connection of data to a specific research question. The questions

were open-ended, so students could respond in any way they would like or raise questions based on the interview questions (Rubin & Rubin, 2012).

To complete the data analysis, I followed a process that allowed for categories and themes to develop from the data. I conducted and recorded each interview on Zoom. Zoom provided a transcription of each interview. Thematic analysis was used in this study. During thematic analysis, deductive coding did not occur because the interview questions were written based on the conceptual framework of Lee Shulman (1986). Deductive coding is where the analysis is shaped by existing theoretical constructs, providing the focal point to code the data and develop themes (Braun & Clarke, 2022). The thematic analysis that took place was inductive coding. Inductive coding is where the researcher completes the analysis by coding whatever the participant's answers reveal about their perceptions. In this method, the coding and themes are developed through careful analysis of the data content from the transcripts (Braun & Clarke, 2022). Inductive coding was also used because the participants' experiences and prospectives, along with the theory-driven approach, form the starting point of developing codes (Braun & Clarke, 2022).

After each interview, I took the transcription from Zoom and uploaded the document to Taguette. Taguette, a free web-based program, was used for uploading documents, highlighting, and tagging for qualitative data. I used this program to create a codebook. After creating the codes, I created a table (See Appendix B). Each code was listed with a description and a participant's sample quote. After the table of codes was created, I moved to categories. These categories were created based on the frequency of

each code for each question. I took all the categories and created a table (See Appendix C). Each category was listed with a category meaning along with the codes that aligned with the category. Lastly, the themes were developed. I took the categories and grouped similar categories. A theme was developed for each group of categories. I took all the themes and created a table (See Appendix D). Each theme was listed with its meaning and the categories that aligned with the theme.

Discrepant cases may provide unexpected findings or even contradict the data (Creswell, 2016). My study did not find discrepant cases, so if they had, they would have been treated like the rest of the data.

Trustworthiness

Trustworthiness was established early in the study to ensure the validity and reliability of the qualitative methods used in the study. The collection methods were chosen to safeguard the data collected (Burkholder et al., 2020). In this study, four aspects of trustworthiness were examined.

Credibility refers to the collected data that must answer the research question (Burkholder et al., 2020). While the data were being collected, an audit trail was used. An audit trail was used because it describes how the data is collected, decisions made during data collection, and how categories are formed (Burkholder et al., 2020). All interviews were recorded and transcribed for careful analysis using Zoom. A notebook was used to keep all notes during the interview process and when the data was coded. Writing notes allows the researcher to examine any biases and adjustments made to the analysis (Burkholder et al., 2020).

Transferability refers to how the study's results can be generalized to the study population (Burkholder et al., 2020). This study used a thick description. I made sure to include thorough descriptions of the participants and the interview setting, which allowed the readers to understand the meaning of the research (Ravitch & Carl, 2021). I used the notes taken during the interview and transcriptions to write detailed descriptions. The use of transferability in this study upheld the study's trustworthiness.

Dependability is important for the reliability of the study (Burkholder et al., 2020). As previously described, an audit trail was used to ensure that there is uniformity in data collection and the reporting of results (Burkholder et al., 2020). Writing down important information, including how the data was collected and the choices made throughout the data collection process established that this study is dependable.

Lastly, confirmability refers to making sure that the study's results are separated from researcher bias (Burkholder et al., 2020). This was accomplished through reflexivity. I used a notebook and documented any thoughts and feelings that occurred during the interview process and data analysis. The interviews were recorded, and the data analysis was based solely on participants' responses, not my feelings.

Ethical Procedures

Prior to recruiting participants, I obtained IRB approval. During the interview process, I acknowledged each participant's autonomy to make personal choices. I treated each participant with respect and in a way that was morally right. Since I used Facebook educator groups, the selection of the research participants was equitable among groups. I did not know any of the participants personally.

There were no ethical concerns regarding recruitment materials and processes. Flyers were dispersed so that participants had a clear understanding of the study's purpose and details. Participants were able to decide if they met the study's requirements. If they did meet the requirements, they contacted me using my Walden email address. I sent the approved consent form from Walden University. The consent form provided participants with an explanation about their involvement in the study and confirmed their choice to participate. It also informed the participants how the data would be disseminated.

Regarding data collection, there were no ethical concerns. If there were ethical concerns, I would have protected research validity and the rights of research participants and preserved scientific principles. Each participant was informed that their identity would be kept confidential. No participant withdrew from the interview early or refused to participate.

The data collected remained confidential. I safely and securely stored all information. All electronic transcripts, email consents, and approved work emails are locked in a password-protected computer. They are also on a password-protected flash drive and a portable hard drive locked in a drawer in my home office. Disseminating the research include publishing the dissertation and using the research to present at conferences.

Summary

To summarize, this chapter has provided the researcher's role and any researcher biases. It also discussed the population and sampling strategy. This chapter also included

the data analysis plan, which was a thematic analysis. This allowed for coding and themes to develop. The strategies of credibility, transferability, confirmability, and dependability were discussed on how they achieved trustworthiness. Finally, the data treatment, including protections for confidential data, was addressed.

Chapter 4 reviews the purpose and research questions. Data collection and analysis are discussed and analyzed. This includes thematic analysis and its use to create codes, categories, and themes. Results that address each research question are shared.

Chapter 4: Results

The study aimed to examine the perceptions of pre-service elementary teachers' knowledge or gaps in knowledge after student teaching in content and pedagogical approaches to fraction operations. I examined three research questions by asking questions during an interview.

RQ1: (*knowledge of content*) What are pre-service elementary teachers', after student-teaching, perceptions of their knowledge or gaps in knowledge regarding their knowledge of content of fraction operations?

RQ2: (*knowledge of pedagogy*) What are pre-service elementary teachers', after student-teaching, perceptions of their knowledge or gaps in knowledge regarding their knowledge of pedagogy of fraction operations?

RQ3: (*knowledge of students*) What are pre-service elementary teachers', after student-teaching, perceptions of their knowledge or gaps in knowledge regarding their knowledge of students of fraction operations?

This chapter specifies important information about how data was collected and recorded, and how codes and themes were established. It also discusses the study's findings and how they address the research questions. Lastly, a summary of the results is given.

Setting

The study participants completed student teaching in the United States. The participants completed student teaching in a Grade 1-6 classroom within the past 2 years, with a completion date of no earlier than 2021. Since this study focused on fraction

operations, the participants were required to have taught at least two of the following fraction operations: addition, subtraction, multiplication, and division. There were no circumstances that affected the experiences of the participants during the study that would affect the results.

Data Collection

Qualitative interviews were used to collect data from 12 participants. It took approximately 2 months to acquire the participants and complete the interviews. A Zoom subscription was purchased. The participant had one interview on Zoom that was recorded and was scheduled for approximately 60-90 minutes.

The participants were recruited through social media. Facebook educator groups were contacted for permission to post on their site. Each educator group was sent a blurb about the study and a recruitment flyer. The administrators of the groups granted permission in different ways. Some groups granted permission by posting it immediately to the group. I reached out via Instant Messenger for permission, and one group was emailed (See Appendix E- deleted permissions to protect confidentiality). Once permission was given, flyers were posted to the education groups. Participants could read the flyer, and if they met the requirements for the study, they could contact me using my Walden email. I sent the approved consent form from Walden University. The participants then read the consent form, and if they wished to volunteer and met the requirements, they replied to the email with the words, "I consent." I contacted the participant, and a date was set for the interview.

At the start of each interview, I had a copy of the interview questions (See Appendix A). At the top of the copy, I identified each participant with a specific number. For example, the first participant was labeled as Participant 1, the second participant was labeled as Participant 2, and so forth until all 12 participants were completed. While the participant was speaking during the interview, I had a notebook and took notes as needed. Each participant's copy of the interview questions and notebook was put in a folder locked in a drawer in my home office.

After completing the interview, I indicated to each participant that they would receive an email in approximately two weeks to check over my work. I asked each participant to check over my work and to reply to the email. Eleven out of the 12 participants responded to my email and approved my work. I sent two emails to the participant, who did not respond. This participant did not respond to the two emails. As a thank you for participating in my study, I emailed each participant an Amazon e-gift card at the end of the interview.

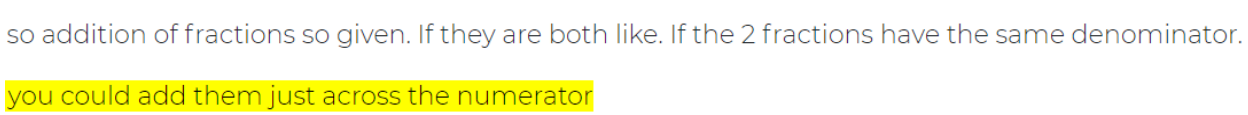
Nothing out of the ordinary occurred during the data collection. As enough participants were obtained through social media, I did not use the Walden University participant pool or the university previously described in Chapter 3. Data saturation was met, as the participants' answers were enough to extract conclusions. All electronic transcripts, email consents, and approved work emails are locked in a password-protected computer. They are also on a password-protected flash drive and a portable hard drive locked in a drawer in my home office.

Data Analysis

To complete the data analysis, I followed a process that allowed for categories and themes to develop from the data. I conducted and recorded each interview on Zoom. Zoom provided a transcription of each interview. After each interview, I took the transcription from Zoom and uploaded the document to Taguette. Taguette, a free web-based program used for uploading documents, highlighting, and tagging for qualitative data, was used to create a codebook. This program allowed me to highlight and code the answers to each question of the 12 participants. An example of this can be seen in three figures. Figure 1 shows how I highlighted a participant's answer from their transcript. Figure 2 shows how a code was assigned to an answer. Figure 3 shows the number of times a code was used for the 12 participants.

Figure 1

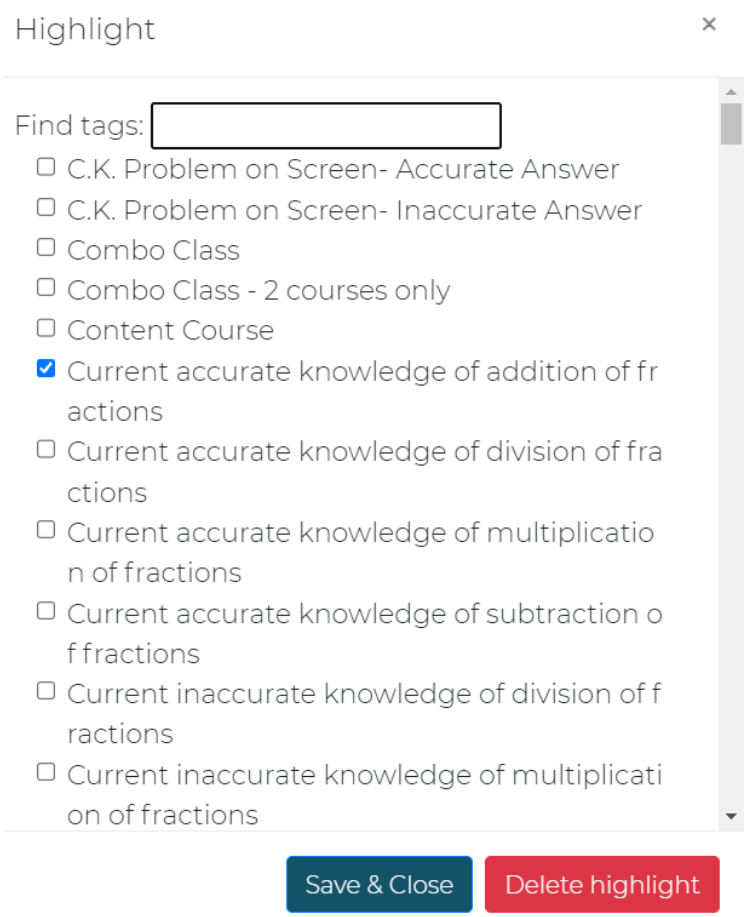
Highlight of a Participant's Answer



Note. The figure shows a participant's answer highlighted on the transcript to create a code.

Figure 2

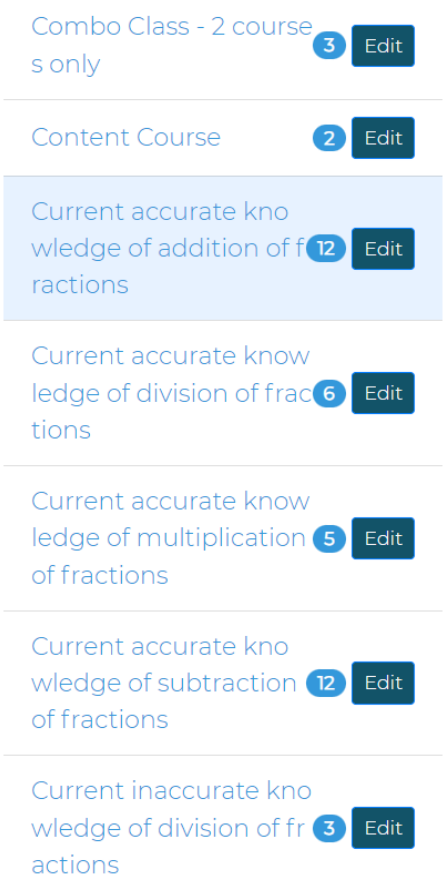
The Development of Codes



Note. The figure shows how a code was developed.

Figure 3

The Frequency of Codes



Note. The figure shows the number of times a code was used for the 12 participants.

After determining the codes, I created a table (See Appendix B). Each code was listed with a description and a sample quote from a participant. Table 2 shows an example of this.

Table 2

Codebook

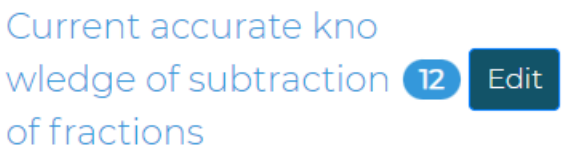
Code	Sample quote	Code description
Combo class	Participant 1 One specific Class	Both content, pedagogy, student knowledge

Note. The table shows an example of a code with its description and a sample quote.

After the table of codes was created, I moved to categories. I took each interview question in Section B. Content Knowledge of Fractions, Section C. Pedagogical Content Knowledge of Fractions (Pedagogy Knowledge), and Section D. Pedagogical Content Knowledge of Fractions (Knowledge of Students) and created categories. These categories were created based on the frequency of each code for each question. If most participants answered the question similarly, a category was created. Figure 4 shows an example of how 12 participants were coded for having current, accurate knowledge of subtraction of fractions. Because of this, a category called Subtraction Content was created.

Figure 4

The Creation of a Category



Note. The figure shows how a category was formed.

I took all the categories and created a table (See Appendix C). Each category was listed with its meaning and the codes that aligned with it. Table 3 provides an example of this.

Table 3

Categories

Category	Category meaning	Aligned codes
Addition content	Accurate knowledge	Current accurate knowledge of addition of fractions

Note. The table shows an example of a category with its meaning and aligned code.

Lastly, the themes were developed. I took the categories and grouped similar categories together. A theme was developed for each group of categories. Each theme that was created answered one of the three research questions for this study. I took all the themes and created a table (See Appendix D). Each theme was listed with its meaning and the categories that aligned with the theme. An example of this can be seen in Table 4.

Table 4

Themes

Theme 1	Theme meaning	Aligned categories
Pre-service elementary teachers have knowledge in addition and subtraction content of fraction operations.	Participants were able to accurately describe addition and subtraction of fractions.	Addition Content Subtraction Content Content of solving an addition of fractions problem

Note. The table shows an example of a theme with its meaning and aligned categories.

The tables show a snapshot of how codes, categories, and themes were developed. The interviews did not produce discrepant cases, and no unexpected findings were analyzed.

Results

The study examined three research questions. The three research questions centered on content knowledge, pedagogy knowledge, and knowledge of students. During the student teaching details of the interview, which transpired at the start of the

interview, each participant was asked what the difference was between content knowledge, pedagogy knowledge, and knowledge of students. All participants had an accurate understanding of content knowledge and knowledge of students. Five participants had an accurate understanding of pedagogy knowledge, and seven participants had inaccurate or no knowledge of pedagogy. Table 5 shows the results of the understanding of these words.

Table 5

Content Knowledge, Pedagogy Knowledge, and Knowledge of Students

Knowledge	Accurate frequency	Inaccurate/no knowledge Frequency
Content	12	0
Pedagogy	5	7
Students	12	0

Note. The table shows the number of participants that could describe aspects of knowledge.

After each participant answered the question, I showed the definition of content knowledge, pedagogy knowledge, and knowledge of students. This was to ensure that the participants understood and clarified any misconceptions they may have had about these words.

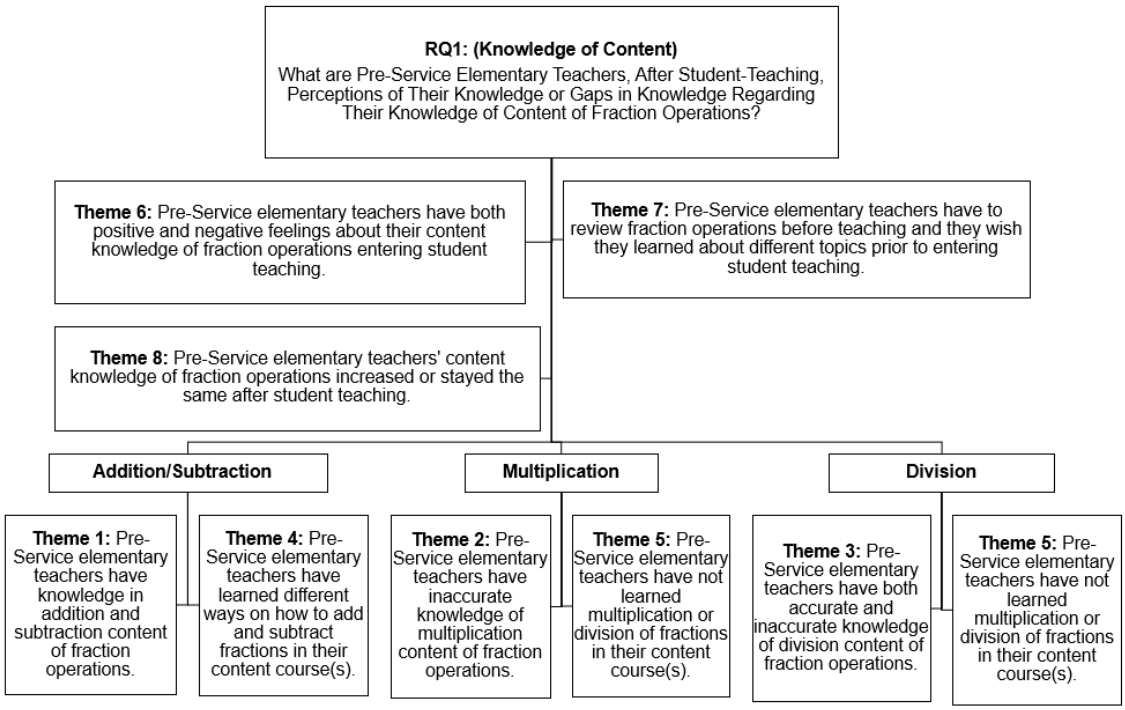
Each research question was addressed and answered by themes developed from codes and categories. Eight themes were created for the first research question, four for the second research question, and four for the third research question. I recognize there are many themes; however, all the themes that were developed answered the research questions.

RQ1: (Knowledge of Content) What are Pre-Service Elementary Teachers', After Student-Teaching, Perceptions of Their Knowledge or Gaps in Knowledge Regarding Their Knowledge of Content of Fraction Operations?

This research question was answered with eight themes. The eight themes centered on content knowledge. Figure 5 shows the organization of the first research question with the eight themes.

Figure 5

Research Question 1



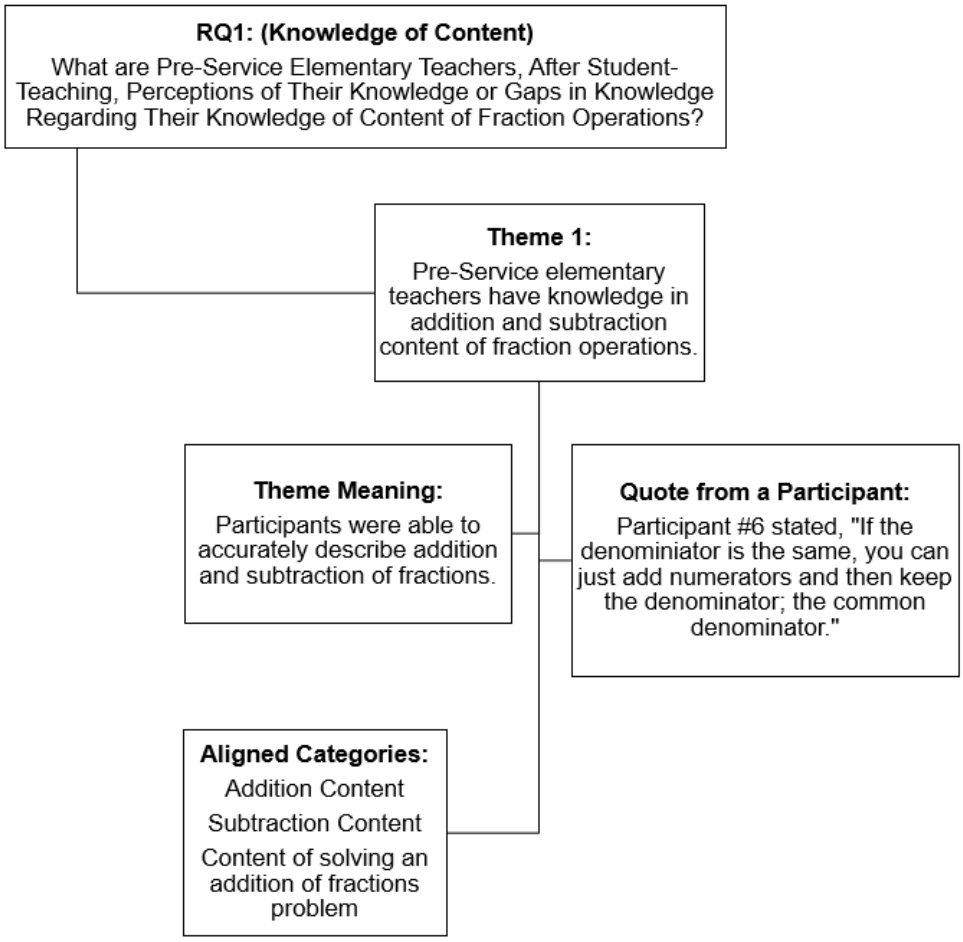
Note. The figure shows the organization of the first research question with the eight themes.

The first theme uncovered that pre-service elementary teachers knew the addition and subtraction content of fraction operations. Participants were able to describe addition and subtraction of fractions accurately. They described needing a common denominator and then adding or subtracting across. Participant 6 stated, “If the denominator is the same, you can just add numerators and then keep the denominator; the common denominator.” Participant 3 stated, “Similar to addition- as long as they’re the same denominator. They can be either added, or I mean they could be subtracted straight

across." The two participants understood that if the denominators are of the same whole, the numerators can be either added or subtracted. This showed an understanding of addition and subtraction of fractions. Figure 6 shows an example of how Theme 1 was developed and how it answers Research Question 1.

Figure 6

Theme 1



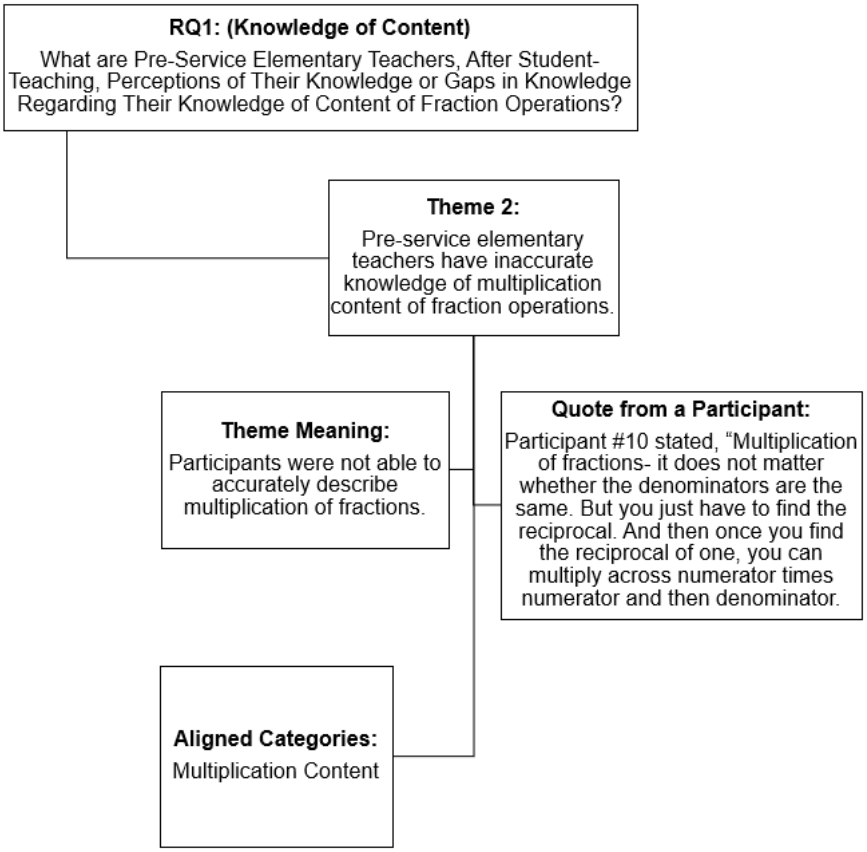
Note. The figure shows the relationship between Theme 1 and Research Question 1.

The second theme showed that more than half of the pre-service elementary teachers had inaccurate knowledge of the multiplication content of fraction operations.

The participants described different procedures than the accurate way. Participant 10 stated, “Multiplication of fractions- it does not matter whether the denominators are the same. But you just have to find the reciprocal. And then once you find the reciprocal of one, you can multiply across numerator times numerator and then denominator.” Participant 4 stated, “You cross multiply.” With these two participants, I saw the inaccuracy in their descriptions of multiplying two fractions. Figure 7 shows an example of how Theme 2 was developed and how it answers Research Question 1.

Figure 7

Theme 2

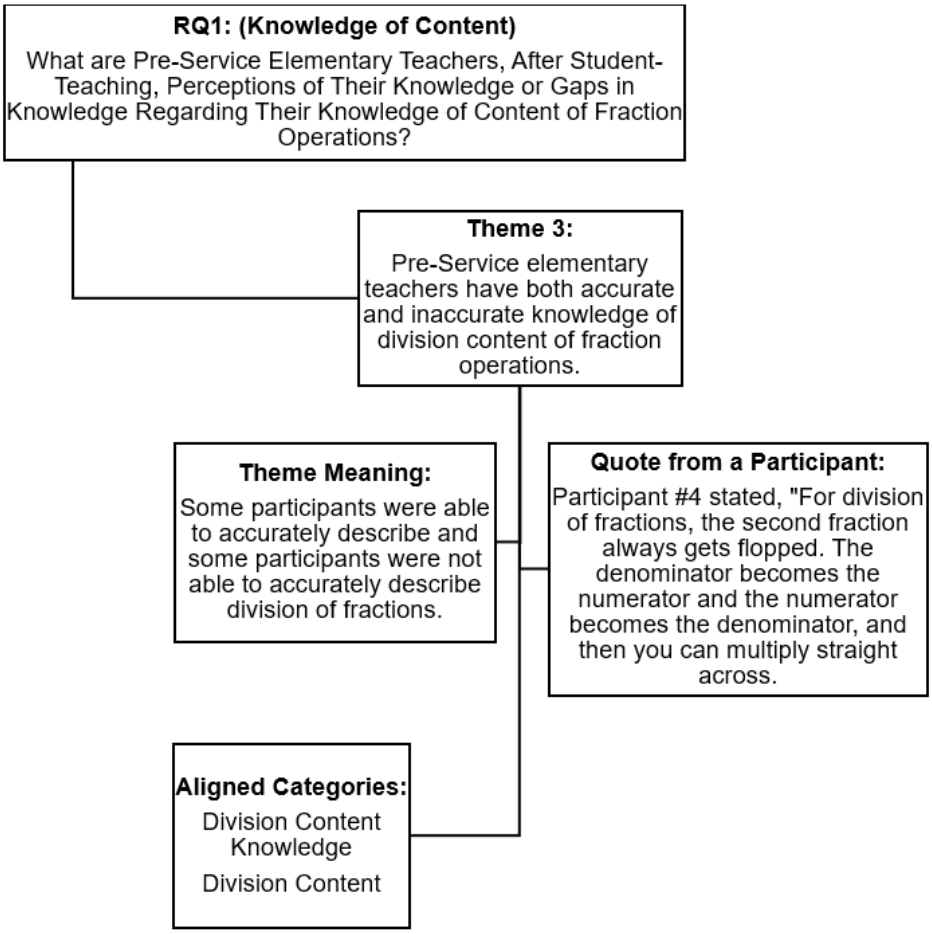


Note. The figure shows the relationship between Theme 2 and Research Question 1.

The third theme revealed that pre-service elementary teachers had accurate and inaccurate knowledge of the division content of fraction operations. Some participants could accurately describe the division of fractions by discussing the reciprocal of the second fraction and then multiplying the fractions. Some participants could not accurately describe the division of fractions because they discussed going from bigger to smaller or having to find a common denominator. Participant 4 accurately described division by stating, “For division of fractions, the second fraction always gets flopped. The denominator becomes the numerator, and the numerator becomes the denominator, and then you can multiply straight across.” Participant 2 did not accurately describe division of fractions. Participant 2 stated, “Going from bigger to smaller. That’s really all I got for that one, we barely did it.” Participant 6 also described division of fractions inaccurately. Participant 6 stated, “Honest, I don’t remember a lot on that. I remember again talking to the kids about having to find the common denominators.” Figure 8 shows an example of how Theme 3 was developed and how it answers Research Question 1.

Figure 8

Theme 3



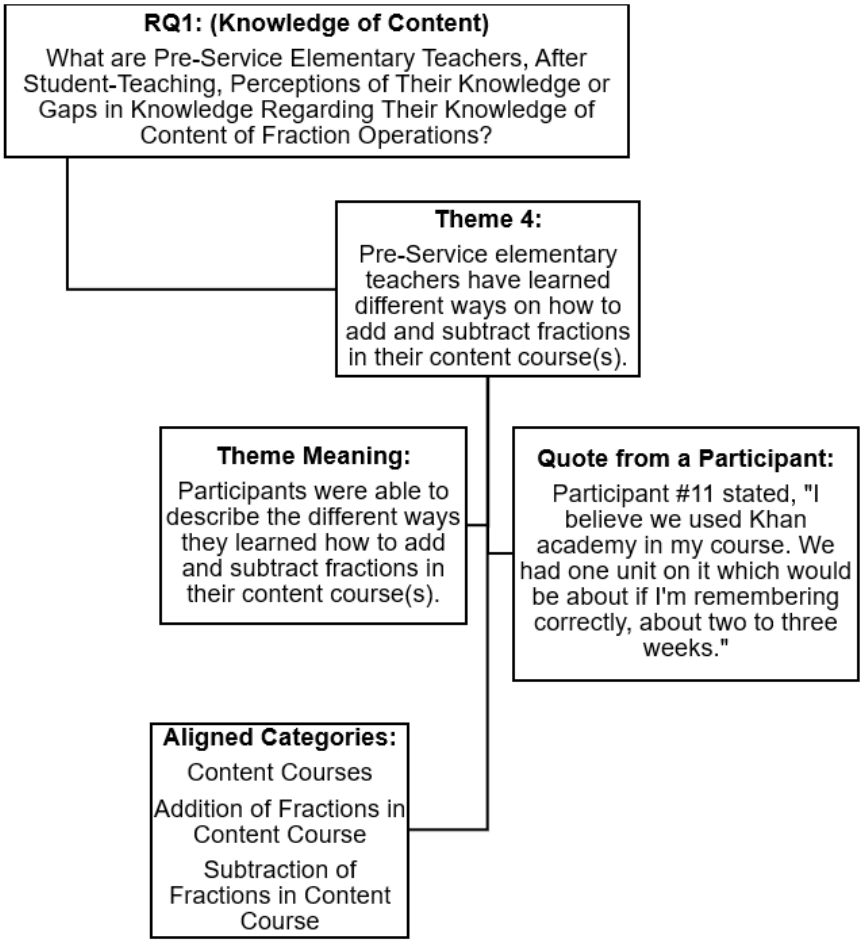
Note. The figure shows the relationship between Theme 3 and Research Question 1.

The fourth theme uncovered that pre-service elementary teachers have learned different ways to add and subtract fractions in their content course(s). Some of the participants described how they learned to add and subtract fractions by using manipulatives, Khan Academy, and lining up fractions. Participant 1 stated, “You just line up, when we did it, we lined it. We lined up the fractions, and then added the top. But you could only do that if the denominator is the same. If the denominator is not the same,

you have to make it the same." Participant 11 stated, "I believe we used Khan academy in my course. We had one unit on it which would be about if I'm remembering correctly, about two to three weeks." Participant 9 stated, "Same thing, using fraction circles and other manipulatives." They could describe how they learned to add and subtract fractions in their content course(s). Figure 9 shows an example of how Theme 4 was developed and how it answers Research Question 1.

Figure 9

Theme 4

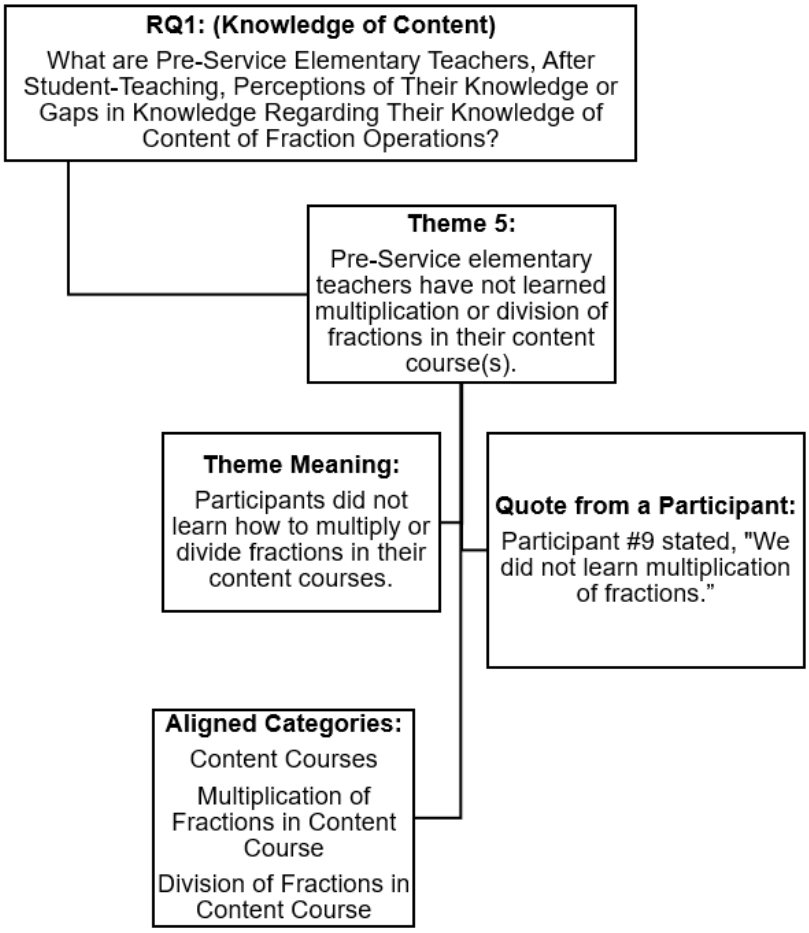


Note. The figure shows the relationship between Theme 4 and Research Question 1.

The fifth theme showed that pre-service elementary teachers have not learned multiplication or division of fractions in their content course(s). Over half of the participants stated they did not learn how to multiply or divide fractions in their content courses. Participant 9 stated, “We did not learn multiplication of fractions.” Participant 6 stated, “I do not remember learning about division of fractions at all.” Figure 10 shows an example of how Theme 5 was developed and how it answers Research Question 1.

Figure 10

Theme 5

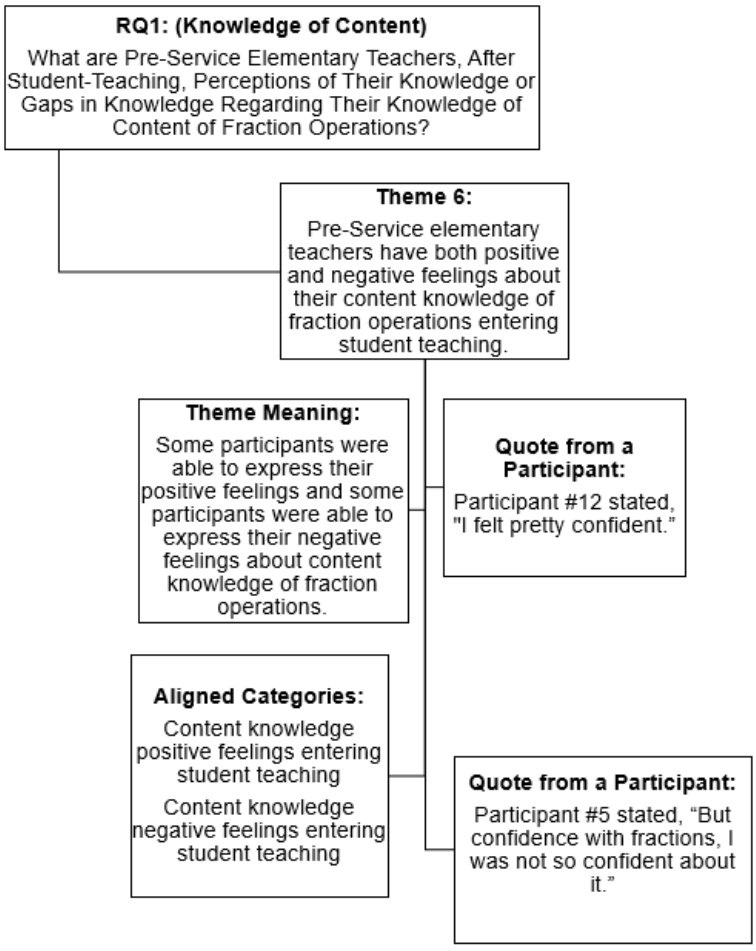


Note. The figure shows the relationship between Theme 5 and Research Question 1.

The sixth theme revealed that pre-service elementary teachers have positive and negative feelings about their content knowledge of fraction operations entering student teaching. About half of the participants stated they were confident in their content knowledge of fraction operations entering student teaching, whereas the other half were not as confident. Participant 3 stated they "Felt confident." Participant 12 stated, "I felt pretty confident." Participant 5 stated, "But confidence with fractions, I was not so confident about it." Participant 7 stated, "I think I was more willing to give it a try, but I was afraid that I wouldn't be able to explain it." Figure 11 shows an example of how Theme 6 was developed and how it answers Research Question 1.

Figure 11

Theme 6



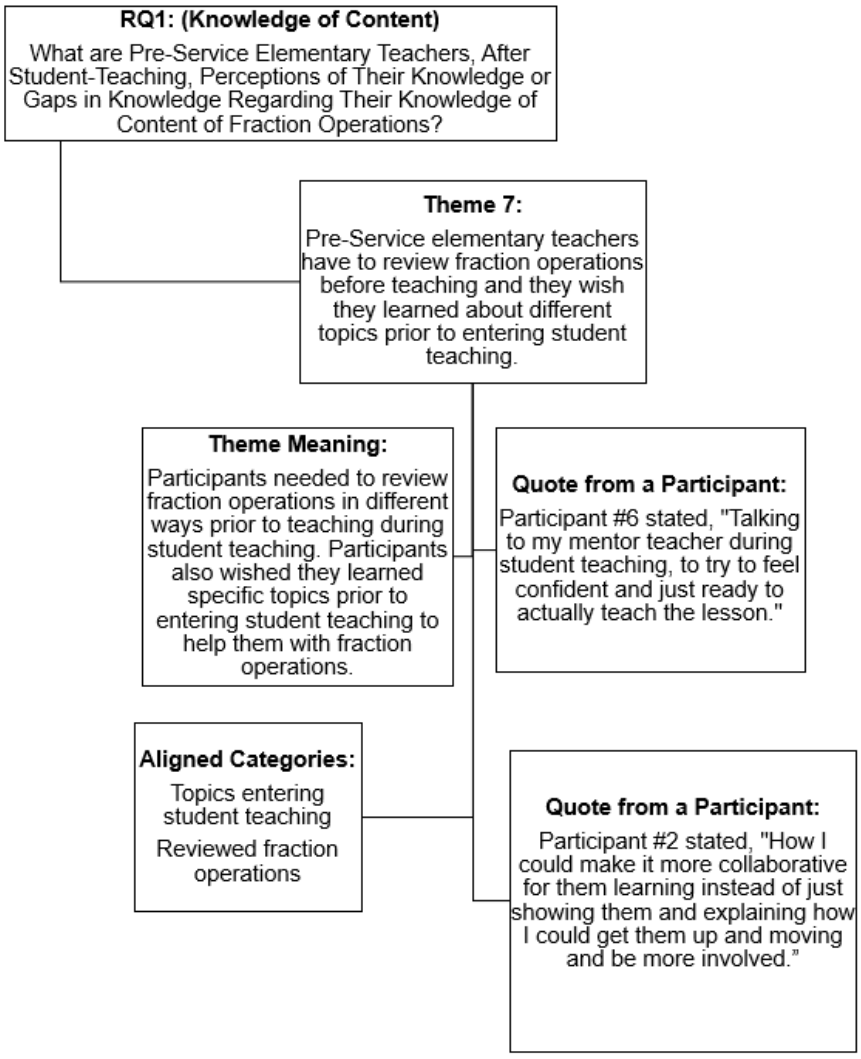
Note. The figure shows the relationship between Theme 6 and Research Question 1.

The seventh theme uncovered that pre-service elementary teachers have to review fraction operations before teaching and wish they had learned about different topics before entering student teaching. Participants needed to review fraction operations in different ways prior to teaching during student teaching. This included using their cooperating teacher, Google, and the teacher manual. Participant 6 stated, "Talking to my

mentor teacher during student teaching, to try to feel confident and just ready to actually teach the lesson." Participant 7 stated, "A lot of conversation with my mentor teacher using the resources like slideshows that were premade for lessons." They also wished they learned specific topics before entering student teaching to help them with fraction operations. This included collaborative learning, differentiation, and how to use manipulatives. Participant 2 stated, "How I could make it more collaborative for them learning instead of just showing them and explaining how I could get them up and moving and be more involved." Participant 1 stated, "How to differentiate with kids that like, were top top and math that knew already how to multiply and divide fractions at second grade." Figure 12 shows an example of how Theme 7 was developed and how it answers Research Question 1.

Figure 12

Theme 7



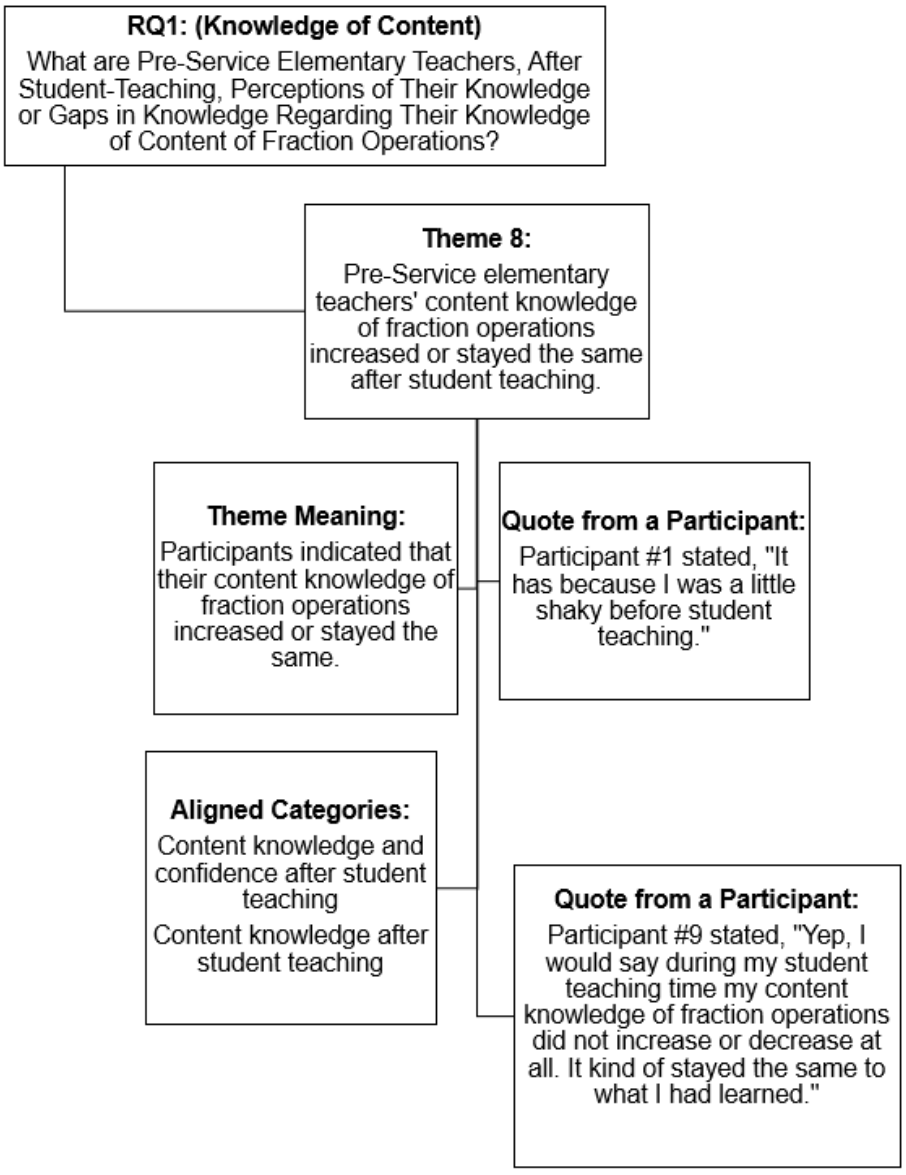
Note. The figure shows the relationship between Theme 7 and Research Question 1.

The eighth theme revealed that pre-service elementary teachers' content knowledge of fraction operations increased or stayed the same after student teaching. More than half of the participants stated that their content knowledge of fractions either increased or stayed the same after student teaching. Participant 1 stated, "It has because I

was a little shaky before student teaching." The participant's knowledge increased after student teaching. Participant 11 stated, "It didn't change. It was the same." Participant 9 also stated, "Yep, I would say during my student teaching time my content knowledge of fraction operations did not increase or decrease at all. It kind of stayed the same to what I had learned." These participants' knowledge stayed the same after student teaching. Figure 13 shows an example of how Theme 8 was developed and how it answers Research Question 1.

Figure 13

Theme 8



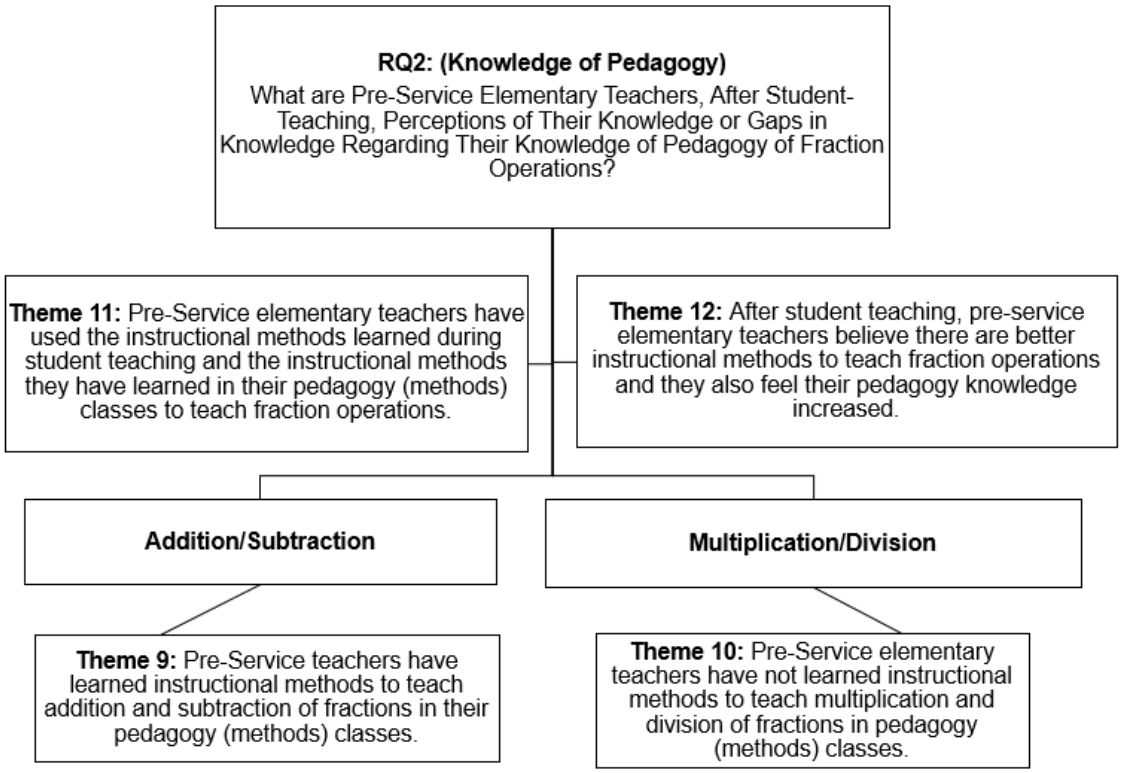
Note. The figure shows the relationship between Theme 8 and Research Question 1.

RQ2: (Knowledge of Pedagogy) What are Pre-Service Elementary Teachers', After Student-Teaching, Perceptions of Their Knowledge or Gaps in Knowledge Regarding Their Knowledge of Pedagogy of Fraction Operations?

This research question was answered with four themes. The four themes centered on pedagogy knowledge. Figure 14 shows the organization of the second research question with the four themes.

Figure 14

Research Question 2

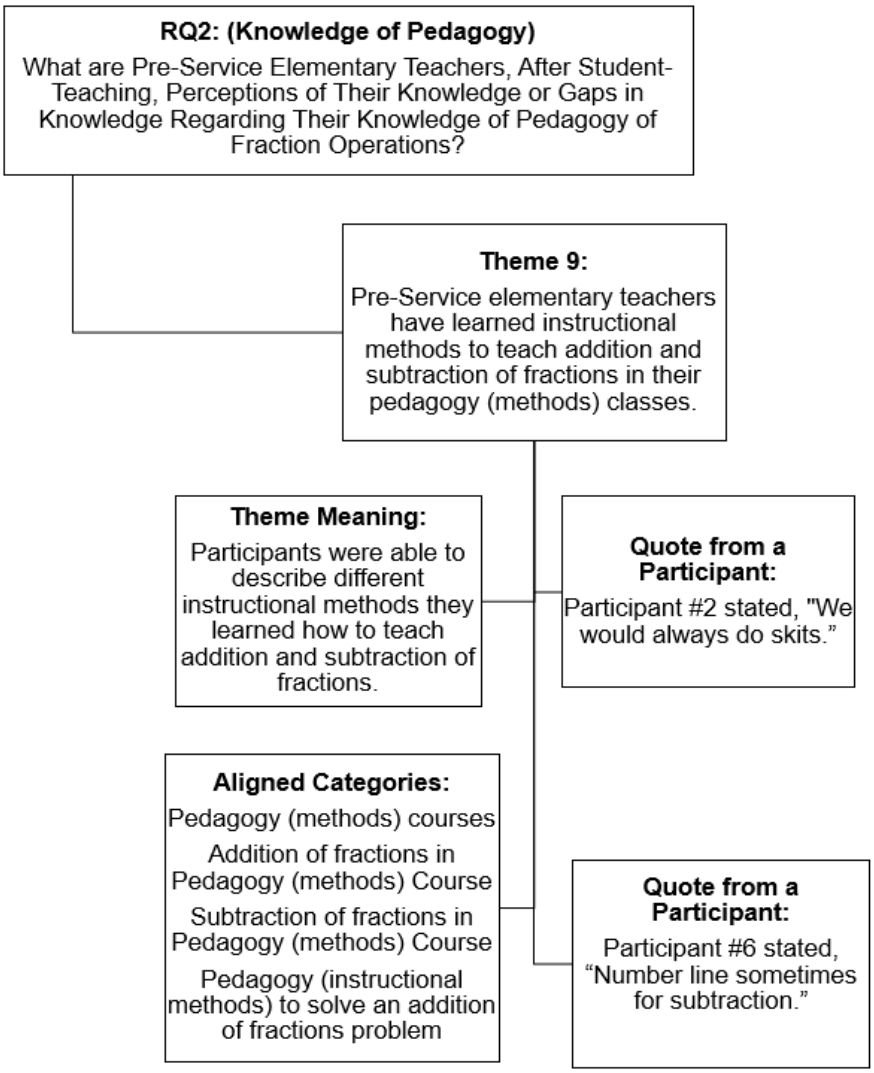


Note. The figure shows the organization of the second research question with the four themes.

The ninth theme uncovered that pre-service teachers have learned instructional methods to teach addition and subtraction of fractions in their pedagogy (methods) classes. This included using skits, manipulatives, and number lines. For addition, Participant 2 stated, “We would always do skits.” Participant 9 stated, “We were instructed to use the manipulatives and hands on materials.” For subtraction, Participant 6 stated, “Number line sometimes for subtraction.” Participants could describe different instructional methods they learned to teach addition and subtraction of fractions. Figure 15 shows an example of how Theme 9 was developed and how it answers Research Question 2.

Figure 15

Theme 9



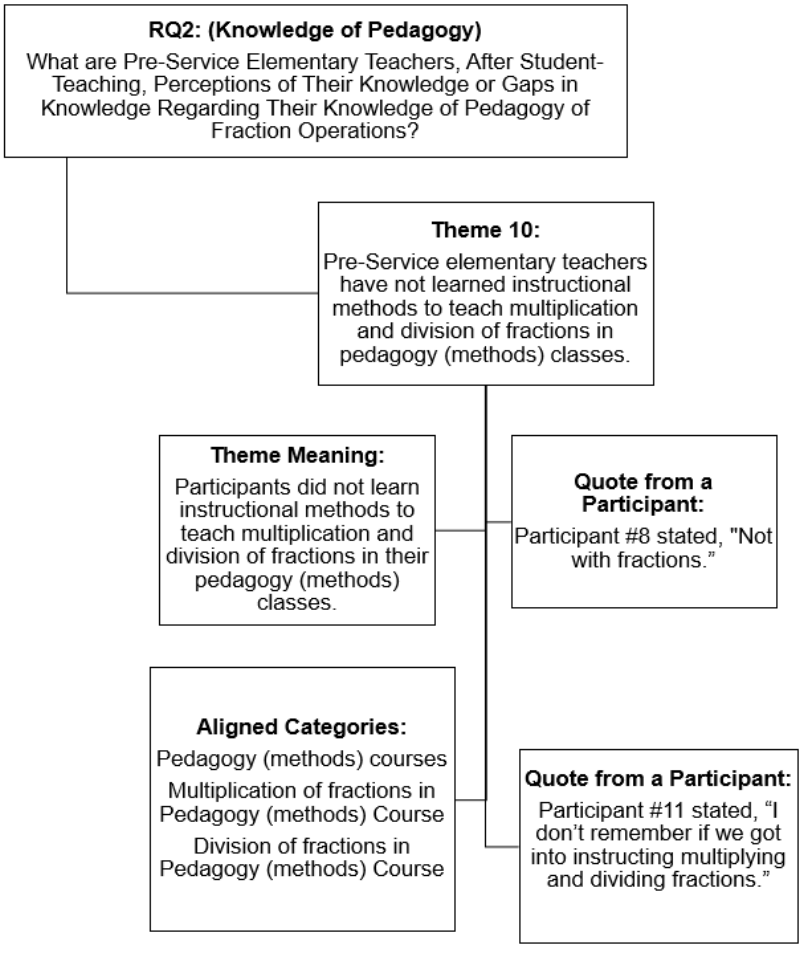
Note. The figure shows the relationship between Theme 9 and Research Question 2.

The 10th theme showed that pre-service elementary teachers had not learned instructional methods to teach multiplication and division of fractions in pedagogy (methods) classes. More than half of the participants indicated this. Participant 6 stated, "I do not remember teaching or learning any strategies to teach multiplication."

Participant 8 stated, “Not with fractions.” Participant 9 stated, “We did not learn how to teach division of fractions.” Participant 11 stated, “I don’t remember if we got into instructing multiplying and dividing fractions.” Participants did not learn instructional methods to teach multiplication and division of fractions in their pedagogy (methods) classes. Figure 16 shows an example of how Theme 10 was developed and how it answers Research Question 2.

Figure 16

Theme 10

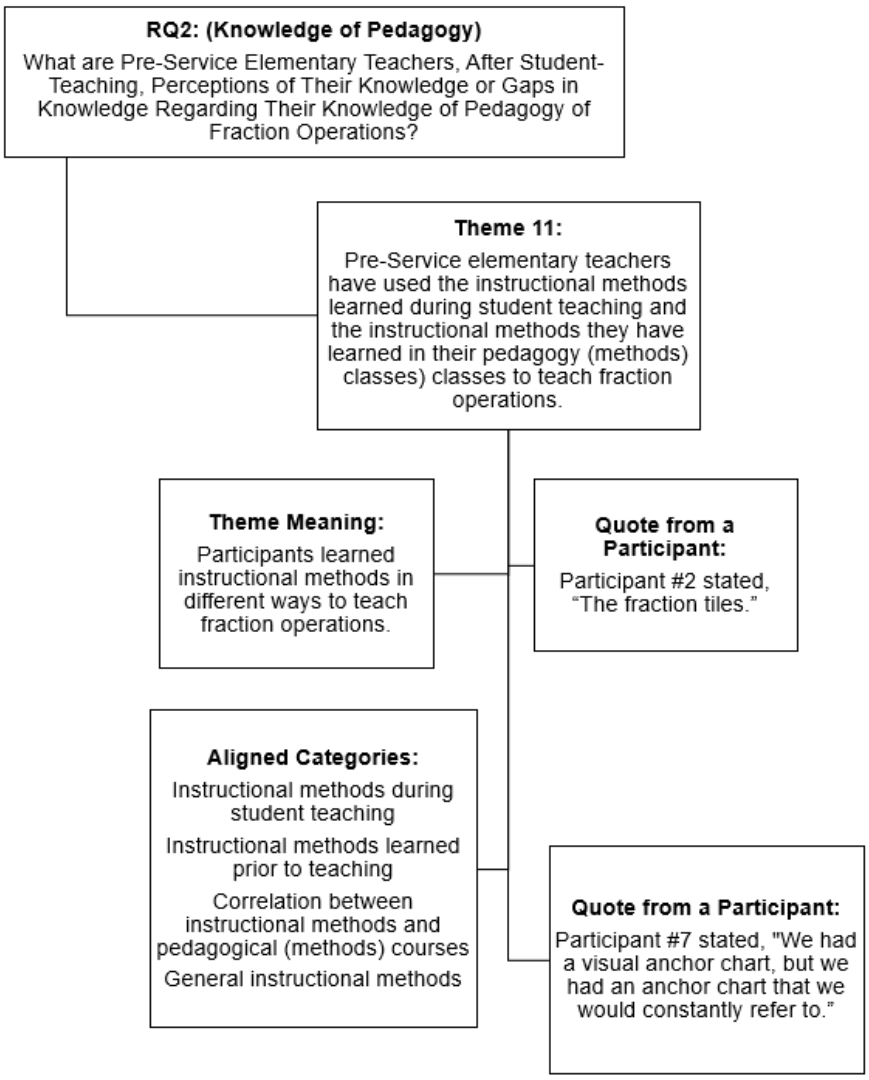


Note. The figure shows the relationship between Theme 10 and Research Question 2.

The 11th theme revealed that pre-service elementary teachers have used the instructional methods learned during student teaching and the instructional methods they have learned in their pedagogy (methods) classes to teach fraction operations. These included using fraction tiles, anchor charts, and manipulatives. Participant 2 stated, “The fraction tiles.” Participant 7 stated, “We had a visual anchor chart, but we had an anchor chart that we would constantly refer to.” Participant 11 stated, “Would be using the manipulatives.” Participants learned instructional methods in different ways to teach fraction operations. Figure 17 shows an example of how Theme 11 was developed and how it answers Research Question 2.

Figure 17

Theme 11



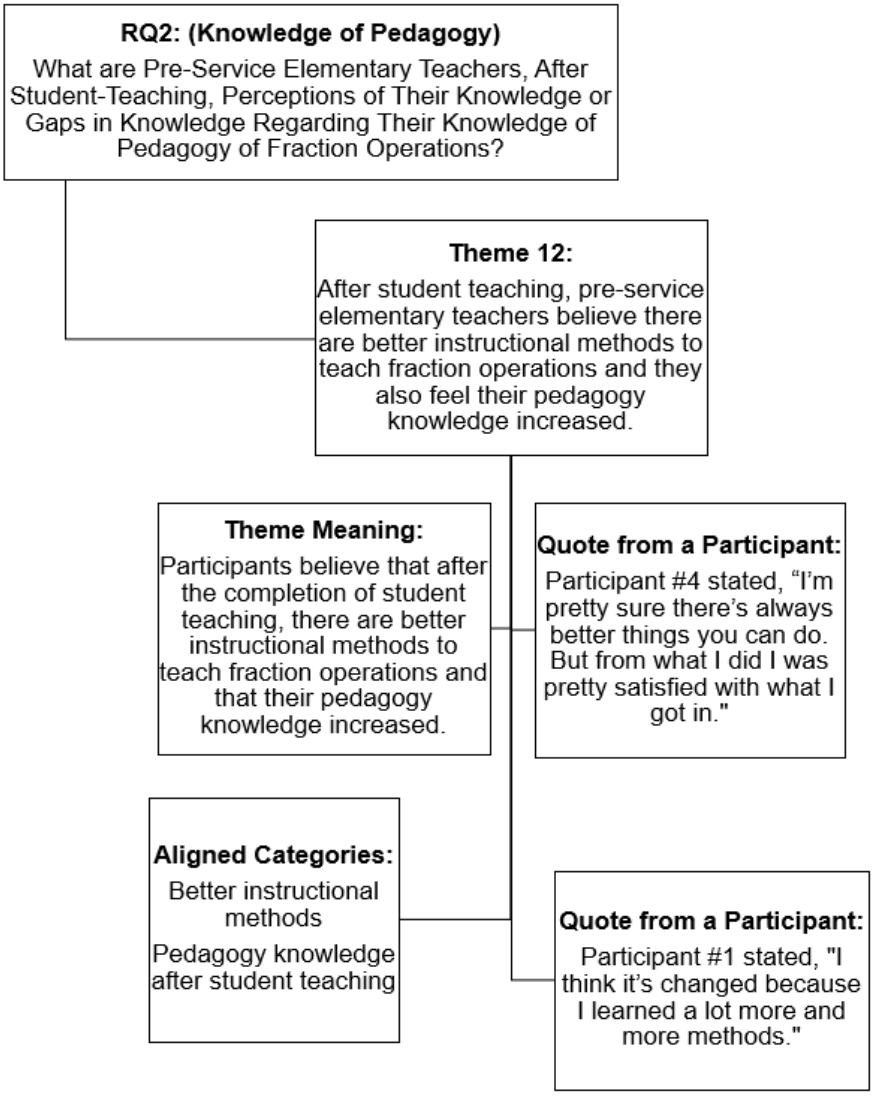
Note. The figure shows the relationship between Theme 11 and Research Question 2.

The 12th theme uncovered that after student teaching, pre-service elementary teachers believe there are better instructional methods to teach fraction operations, and they also feel their pedagogy knowledge has increased. Nearly all participants indicated they believed there were better instructional methods they did not know about.

Participant 4 stated, “I’m pretty sure there’s always better things you can do. But from what I did I was pretty satisfied with what I got in.” Participant 8 stated, “I definitely think that there’s more to be talked about with fractions.” Nearly all participants indicated that their pedagogy knowledge increased after student teaching. Participant 4 stated, “I learned a lot. And I feel a lot more knowledgeable about instructing operations of math with fractions.” Participant 1 stated, “I think it’s changed because I learned a lot more and more methods.” Figure 18 shows an example of how Theme 12 was developed and how it answers Research Question 2.

Figure 18

Theme 12



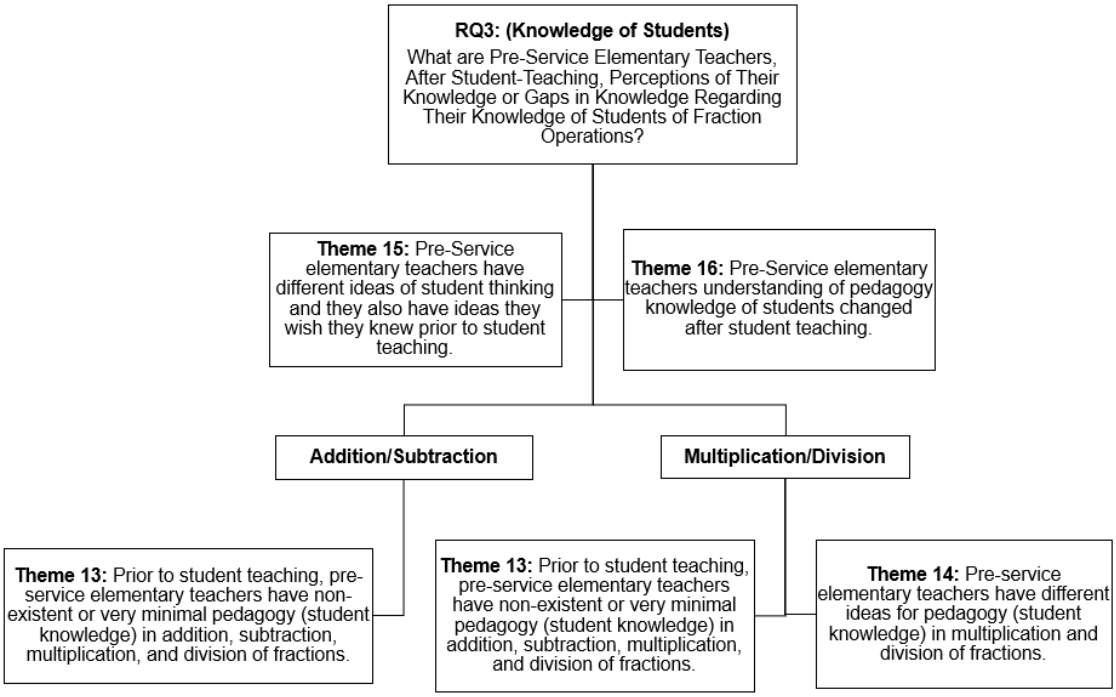
Note. The figure shows the relationship between Theme 12 and Research Question 2.

RQ3: (Knowledge of Students) What are Pre-Service Elementary Teachers', After Student-Teaching, Perceptions of Their Knowledge or Gaps in Knowledge Regarding Their Knowledge of Students of Fraction Operations?

This research question was answered with four themes. The four themes centered on student knowledge. Figure 19 shows the organization of the third research question with the four themes.

Figure 19

Research Question 3

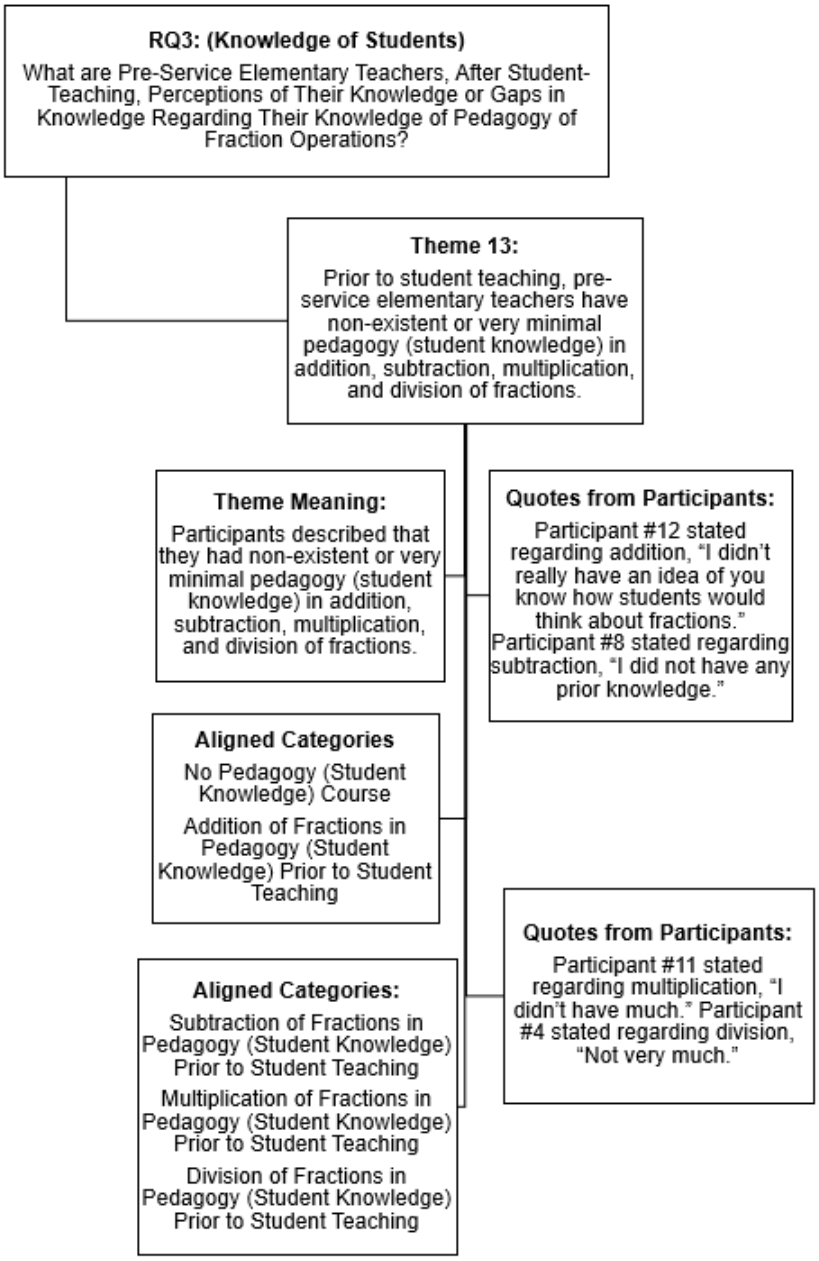


Note. The figure shows the organization of the third research question with the four themes.

The thirteenth theme showed that prior to student teaching, pre-service elementary teachers had non-existent or very minimal pedagogy (student knowledge) in addition, subtraction, multiplication, and division of fractions. Over half of the participants described their non-existent or minimal pedagogy (student knowledge) in these fraction operations. Participant 12 stated regarding addition, “I didn’t really have an idea of you know how students would think about fractions.” Participant 8 stated regarding subtraction, “I did not have any prior knowledge.” Participant 11 stated regarding multiplication, “I didn’t have much.” Participant 4 stated regarding division, “Not very much.” Figure 20 shows an example of how Theme 13 was developed and how it answers Research Question 3.

Figure 20

Theme 13

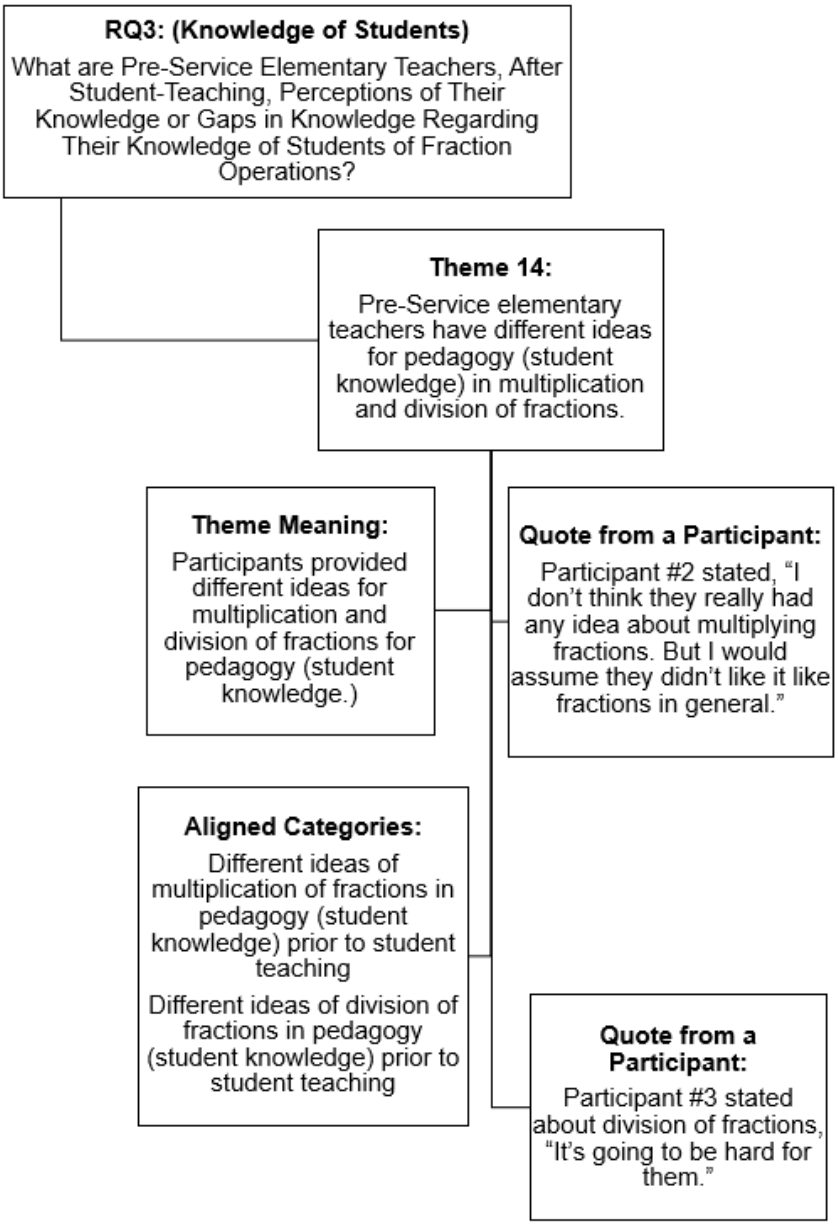


Note. The figure shows the relationship between Theme 13 and Research Question 3.

The 14th theme revealed that pre-service elementary teachers have different ideas for pedagogy (student knowledge) in multiplication and division of fractions. These ideas included that students would not like multiplication/division of fractions, students would think multiplication/division of fractions was hard, and that students would not have any knowledge of multiplication/division of fractions. Participant 2 stated, "I don't think they really had any idea about multiplying fractions. But I would assume they didn't like it like fractions in general." Participant 3 stated about division of fractions, "It's going to be hard for them." Participant 10 stated, "I don't think they have any prior knowledge, or that you can even multiply fractions." Figure 21 shows an example of how Theme 14 was developed and how it answers Research Question 3.

Figure 21

Theme 14



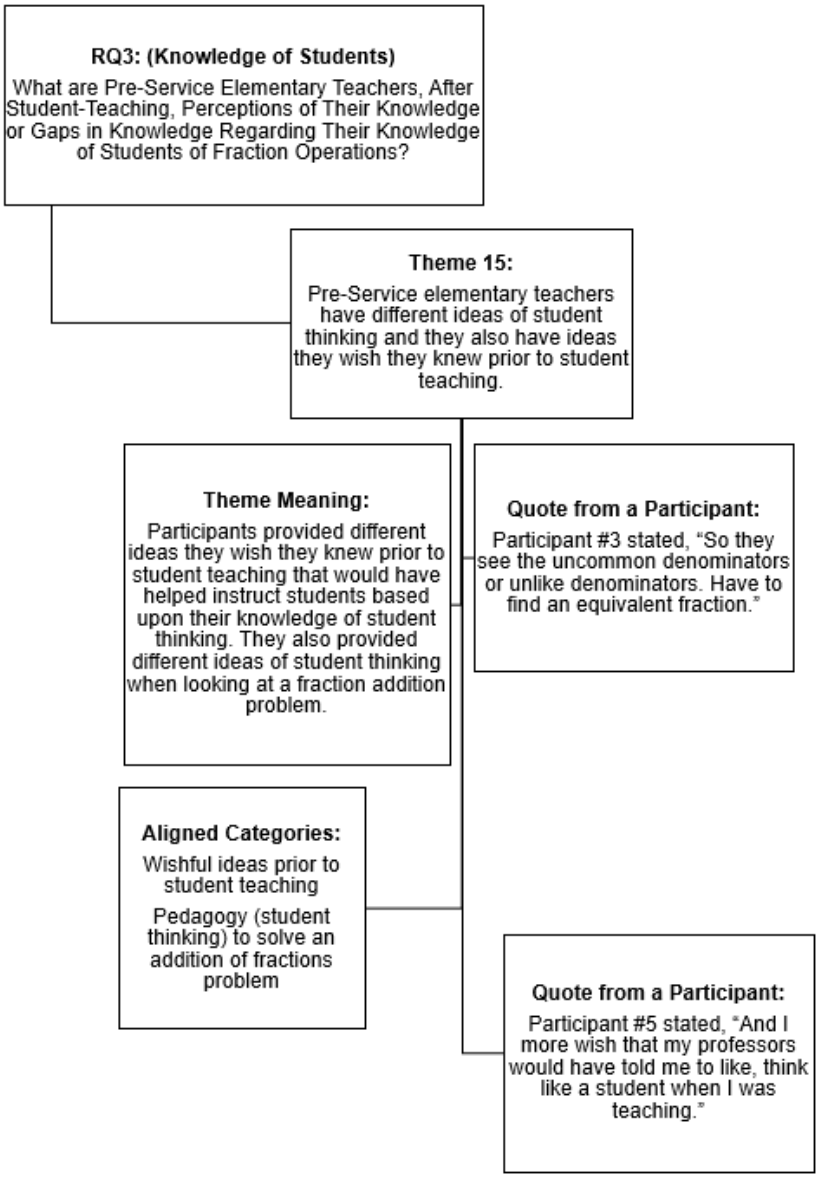
Note. The figure shows the relationship between Theme 14 and Research Question 3.

The 15th theme uncovered that pre-service elementary teachers have different ideas of student thinking and have ideas they wish they knew prior to student thinking.

Participants provided different ideas of student thinking when looking at a fraction addition problem. This included ideas about fraction lines, not the same denominator, and money. Participant 2 stated, “The fraction line is sideways. They would not like that, that it would have to be up and down, but that would bother them so much.” Participant 3 stated, “So they see the uncommon denominators or unlike denominators. Have to find an equivalent fraction.” Participant 7 stated, “If they think about quarters or half dollars, they might be thinking 50 cents plus 75 cents or half dollar, plus 3 quarters.” Participants also provided different ideas they wish they had known prior to student teaching that would have helped instruct students based on their knowledge of student thinking. This included learning more about a student’s conceptual understanding of different topics and wishing professors told them how students think. Participant 9 stated, “I wish I had known more of their conceptual understanding before instructing. So maybe you know how do they understand visuals? And how do they understand models versus how do they understand formulas?” Participant 5 stated, “And I more wish that my professors would have told me to like, think like a student when I was teaching.” Figure 22 shows an example of how Theme 15 was developed and how it answers Research Question 3.

Figure 22

Theme 15



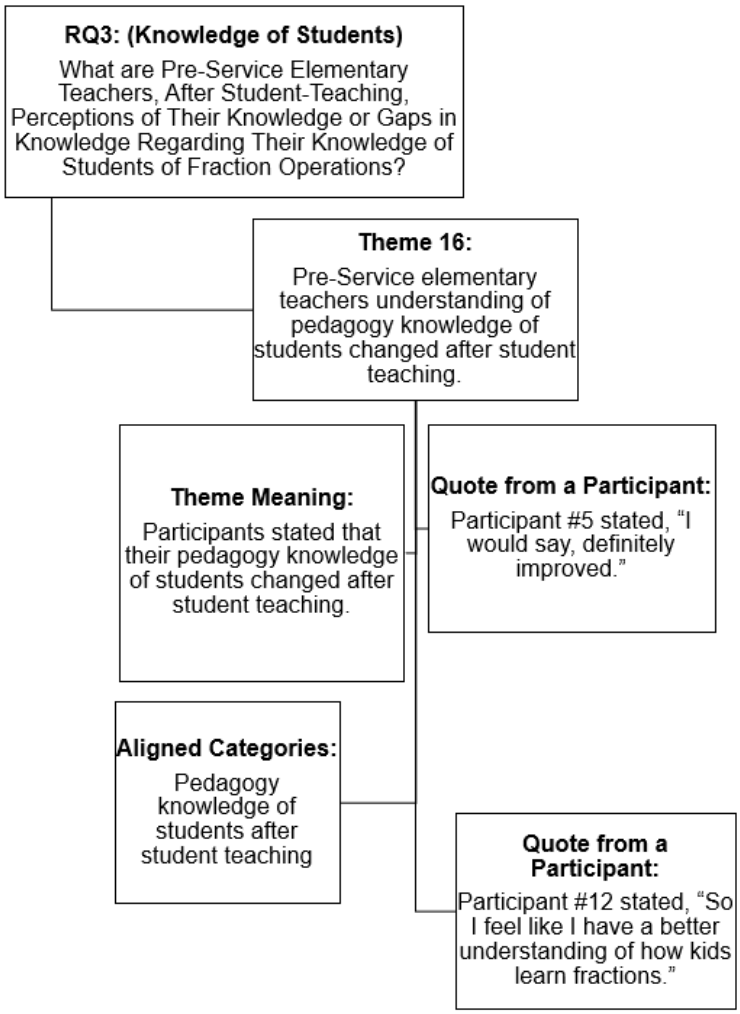
Note. The figure shows the relationship between Theme 15 and Research Question 3.

The last theme showed that pre-service elementary teachers' understanding of pedagogy knowledge of students changed after student teaching. Nearly all participants

stated that their pedagogy knowledge of students changed after student teaching. Participant 5 stated, “I would say, definitely improved.” Participant 12 stated, “So I feel like I have a better understanding of how kids learn fractions.” Figure 23 shows an example of how Theme 16 was developed and how it answers Research Question 3.

Figure 23

Theme 16



Note. The figure shows the relationship between Theme 16 and Research Question 3.

Evidence of Trustworthiness

Trustworthiness was established in this study. The evidence lies within the qualitative methods used to ensure validity and reliability. Four significant features were used to confirm trustworthiness.

An audit trail was used to safeguard the credibility of this study. At the start of the interview process, a notebook was obtained to take notes. For each interview, I took notes on each participant. This included the date and setting of the interview. At times, I needed to ask follow-up questions because the participant did not answer the question fully or I wanted to probe for more information. These questions were written down in the notebook. All interviews were recorded on Zoom, and a transcription was provided. During the data analysis, I created a separate section in the notebook for writing down notes on how codes, categories, and themes were created. Even though a program called Taguette was used to upload the transcription, highlight and tag qualitative data to create codebooks, I wrote descriptions of how the analysis of data took place. Member checking was also used to ensure the credibility of the study. After the codebook was created for the participant, I emailed each participant to check over my work to ensure the participant's work was accurate. The credibility of the study was established by the methods described.

Thick description was used to establish the transferability of this study. Before the interview, the participants were emailed a consent form that explicitly described the requirements of the study. This was to ensure that all participants student taught in a Grade 1-6 classroom within the past 2 years, with a completion date of no earlier than

2021. The notes taken in the notebook during data collection and the transcriptions of each interview allowed me to write specific descriptions in data analysis. The thick description allowed the results of this study to be transferable to the study population.

Dependability was established in this study. As previously described, an audit trail was used. Using an audit trail allowed for the consistency of the data. I took notes during the interview and the analysis of data. Interview questions were created to answer each research question. A field test was conducted to determine the dependability of the research questions. A pre-service elementary teacher who had finished student teaching and had taught fraction operations was asked to review the interview questions to ensure each question was understood and answered. The methods used for dependability contributed to the trustworthiness of the study.

Reflexivity was used to safeguard the confirmability of this study. I kept a notebook where thoughts and feelings that occurred during the interview process and analysis of data was documented. This was to ensure that researcher bias would be separate from the results of the study. The interviews were recorded, and the results were solely based upon participants' responses and not my feelings. Reflexivity was important in this study to warrant trustworthiness.

Summary

The research conducted in this study answered questions about the content, pedagogy, and knowledge of students that pre-service elementary teachers have in fraction operations. The data found that pre-service elementary teachers know the addition and subtraction content of fraction operations, have inaccurate knowledge of the

multiplication content of fraction operations, and have both accurate and inaccurate knowledge of the division content of fraction operations. The results showed that pre-service elementary teachers had learned instructional methods to teach addition and subtraction but not for multiplication and division of fractions in their pedagogy (methods) classes. Lastly, pre-service elementary teachers' understanding of the pedagogy knowledge of students changed after student teaching.

Transitioning to Chapter 5, a review of key findings supports why the study was conducted. Recommendations for further research that arose from the research are provided. Furthermore, implications for positive social change that occurred because of the study are offered.

Chapter 5: Discussion, Conclusions, and Recommendations

The purpose of this study was to explore pre-service elementary teachers', after student-teaching, perceptions of their knowledge or gaps in knowledge in content and pedagogical approaches to fraction operations. A basic qualitative study was used. Qualitative interviews were used for data collection, and thematic analysis was used to develop themes from the data. The data indicated that pre-service elementary teachers knew the addition and subtraction content of fraction operations; however, they had inaccurate knowledge of the multiplication content of fractions and accurate and inaccurate knowledge of the division content of fraction operations. The data also indicated that pre-service elementary teachers had learned instructional methods to teach addition and subtraction but not for multiplication and division of fractions in their pedagogy classes. Pre-service elementary teachers' understanding of the pedagogy knowledge of students changed after student teaching.

Interpretation of the Findings

Morano and Riccomin (2020) found that the participants in their study were weak in their conceptual knowledge of both fraction multiplication and division. I found that pre-service elementary teachers had inaccurate knowledge of the multiplication content of fraction operations and had both accurate and inaccurate knowledge of the division content of fraction operations. The study was convergent because participants in both were weak in multiplication and division. This study had some divergent evidence in that some participants were able to accurately describe the division of fractions, while some of the participants were not.

Reeder and Utley (2017) completed a study examining pre-service elementary teachers' understanding of fractions. The authors found that pre-service elementary teachers' comprehension of fractions was limited in their final methods course. They also found that pre-service elementary teachers need to be given opportunities to develop a deep understanding of fractions (Reeder & Utley, 2017). This confirms and disconfirms the research I conducted. In my research, I found that more than half of the participants had inaccurate knowledge of the multiplication content of fraction operations. Some participants had accurate and inaccurate knowledge of the division content of fraction operations. Pre-service elementary teachers had not learned instructional methods to teach multiplication and division of fractions in methods classes. This confirms Reeder and Utley's research. What I did find in my study also disconfirms Reeder and Utley's research. Pre-service elementary teachers had knowledge of the addition and subtraction content of fraction operations, and they also had learned instructional methods to teach addition and subtraction of fractions in their pedagogy classes.

Rosli et al. (2019) implemented a 3-week fraction unit using problem-solving and models during mathematical classroom instruction to improve the teachers' perceptions, attitudes, and knowledge of fractions. The researchers found that the combination of concrete models, problem-posing, and problem-solving improved the pre-service teachers' pedagogical content knowledge, attitudes, and perceptions of fractions. Stevens et al. (2020) implemented instructional changes to a mathematics course for PreK-8 pre-service teachers. The researchers wanted to see if these changes would help the pre-service teachers with fraction operations and schemes. Stevens et al. found significant

increases in the pre-service teachers' ability to construct fraction schemes and operations. These researchers found significant increases because of the changes they made in the course. A theme that was determined through my research was that pre-service elementary teachers used the instructional methods learned during student teaching and in their pedagogy classes to teach fraction operations. This confirms the research conducted by Rosli et al. and Stevens et al. . The results from their studies show that when students are taught explicitly about fraction operations and instructional methods, there is an improvement in their pedagogical content knowledge. These pre-service teachers will also use these methods to instruct students in fraction operations.

Whitehead and Walkowiak (2017) researched pre-service elementary teachers who took a mathematics methods course to see if there was a change in their understanding of Grades 3-5 concepts of fractions. They were also asked to look at student work and explain why the methods were incorrect. Whitehead and Walkowiak found that even though the pre-service elementary teachers took the course, it did not improve their explanations of common fraction algorithms. This confirms a theme that I found in my research. Before student teaching, pre-service elementary teachers had non-existent or very minimal pedagogy (student knowledge) in addition, subtraction, multiplication, and division of fractions. The participants had non-existent or minimal knowledge of student thinking in fraction operations.

The conceptual framework for my study was grounded by Lee Shulman's (1986) conceptual framework. Shulman combined content and curriculum (pedagogical) knowledge to form pedagogical content knowledge. He believed that content knowledge,

curriculum knowledge, and pedagogical content knowledge were still missing in teaching research (Ball et al., 2008). The history is such that Shulman looked at earlier studies that looked at the content of mathematics but not how mathematics was taught (Ball et al., 2008). This was a gap in practice, so Shulman decided to investigate teaching and teacher knowledge.

Like Lee Shulman's (1986) research, my research found that even though after student teaching, pre-service elementary teachers knew about fraction operations, they also had gaps. Pre-service elementary teachers were found to have knowledge of the addition and subtraction content of fraction operations; however, they had inaccurate knowledge of the multiplication content of fraction operations. The research found that pre-service elementary teachers had accurate and inaccurate knowledge of the division of content of fraction operations. Pre-service elementary teachers were found to have learned instructional methods to teach addition and subtraction of fractions in their pedagogy classes. However, they were found not to have learned instructional methods to teach multiplication and division of fractions in pedagogy classes. I also found that pre-service elementary teachers had non-existent or very minimal pedagogy (student knowledge) in addition, subtraction, multiplication, and division of fractions before student teaching. After student teaching, pre-service elementary teachers' understanding of the pedagogy knowledge of students changed. Even though improvements have been made in pre-service elementary teachers' content and pedagogical content knowledge, gaps remain.

Limitations of the Study

The limitations written in Chapter 1 included transferability, having a sample size of 12-15 participants, and building trust with participants. There were no unexpected limitations. This study included 12 participants from all regions of the United States. I built trust with my participants by sending an introduction to my research that included the consent form. During the interview, my communication skills were excellent because my participants understood all the questions and could answer them thoroughly.

Recommendations

A few recommendations for further research have evolved from my study. First, further research needs to be conducted into why pre-service elementary teachers are not experiencing and receiving the same content and pedagogical content knowledge in fraction operations from their teacher education courses in the United States. For example, my research found that more than half of pre-service elementary teachers had not learned multiplication or division of fractions in their content course(s). Another example is that pre-service elementary teachers had learned different ways to add and subtract fractions in their content course(s). The different approaches were different among all twelve participants. Secondly, further research needs to be conducted into why there is a gap in pedagogy knowledge (student knowledge) in addition, subtraction, multiplication, and division of fractions. My research found that pre-service elementary teachers had non-existent or minimal pedagogy (student knowledge) in the four operations of fractions before student teaching. Lastly, further research needs to be conducted into pre-service elementary teachers' feelings about their content knowledge in

fraction operations entering student teaching. My research found that pre-service elementary teachers had positive and negative feelings about fraction operations entering student teaching. Further research in these three areas can aid our pre-service elementary teachers' confidence in the content and pedagogical content knowledge of fraction operations when entering student teaching.

Implications

My study's results have the potential to make an impact on positive social change. The results of my research can provide universities with teacher education programs with valuable information on how well-prepared pre-service elementary teachers are going into student teaching. Teacher education programs can look at the knowledge or gaps in knowledge of student teachers in content and pedagogical content knowledge of fraction operations and make changes to their courses. The gaps in knowledge include inaccurate knowledge of the multiplication and division content of fraction operations. The research also found that pre-service elementary teachers had not learned instructional methods for multiplication and division of fractions in their pedagogy classes. The results of my study can also allow pre-service elementary teachers time to reflect on their mathematical skills and instructional practice. They can consider what they may not know about fraction operations and what they can do to help be successful in student teaching. This may include working independently or even using YouTube videos to strengthen their understanding of multiplication and division of fractions.

The focus of the study was based on Lee Shulman's (1986) pedagogical content knowledge framework. He believed that content knowledge, curriculum knowledge, and

pedagogical content knowledge were still missing in teaching research (Ball et al., 2008). My results are built on Lee Shulman's existing evidence. I looked at content knowledge and pedagogical content knowledge. I looked at the knowledge of teaching and the knowledge of students. Shulman (1987) affirmed that teachers can break their subjects down into more straightforward topics so their students can better understand concepts. My study found that before student teaching, pre-service elementary teachers had non-existent or very minimal pedagogy (student knowledge) in addition, subtraction, multiplication, and division of fractions. Pre-service elementary teachers need to be given opportunities to learn about student knowledge so that when they are in student teaching, they can break down concepts in a way their students will learn. Shulman (1986) believed that there are two parts to curriculum (pedagogical) knowledge. The second part is what he referred to as vertical curriculum knowledge, which is the knowledge that students are taught (Shulman, 1986). My research found that pre-service elementary teachers had learned instructional methods to teach addition and subtraction of fractions in their pedagogy (methods) classes; however, they had not learned instructional methods to teach multiplication and division of fractions in pedagogy (methods) classes. Shulman (1986) believed that content knowledge was the knowledge a teacher has on a subject. This is vital to have when teaching students. My study found that pre-service elementary teachers had knowledge of the addition and subtraction content but had inaccurate knowledge of the multiplication content of fraction operations. Pre-service elementary teachers had accurate and inaccurate knowledge of the division content of fraction operations. The results show content and pedagogical content

knowledge gaps in teaching fraction operations. These results should be considered when teacher education programs look at their course outcomes to ensure fraction operations are taught in content and methods courses.

Conclusion

Conducting research was essential to explore how pre-service elementary teachers' sense of fractions is developing and their thoughts about how to teach fractions with the changes in Common Core State Standards. By collecting data using qualitative interviews, I determined that pre-service elementary teachers had knowledge of addition and subtraction content of fraction operations. However, they had inaccurate knowledge of the multiplication content of fractions and had accurate and inaccurate knowledge of the division content of fraction operations. I found that pre-service elementary teachers learned instructional methods to teach addition and subtraction but not for multiplication and division of fractions in their pedagogy classes. Lastly, I found that before student teaching, pre-service elementary teachers had non-existent or very minimal pedagogy (student knowledge) in addition, subtraction, multiplication, and division of fractions. Pre-service elementary teachers' understanding of the pedagogy knowledge of students changed after student teaching. They had a better understanding of student knowledge in fraction operations. I concluded that pre-service elementary teachers have knowledge and gaps in content and pedagogical content knowledge. Based on the results of this study, teacher education programs must examine their curriculum outcomes and ensure they equip pre-service elementary teachers with opportunities to learn content and pedagogical

knowledge of fraction operations so they will be successful in their student teaching and eventually in their classrooms.

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Appendix A: Interview Protocol

Participant # _____

Date: _____

Interviewer: Elizabeth Salgado- Doctoral Student

Hello and thank you for taking the time to complete this interview with me. The purpose of this interview is to look at your practice of content and teaching knowledge of fraction operations- addition, subtraction, multiplication, division. You have read and acknowledged the consent form. If you are not comfortable answering a question or want to stop the interview, you may do so at any time. All your responses will be held confidential. Just a reminder as stated on the consent form that I will be recording. Any questions?

A. Student Teaching Details:

1. What state and grade level did you student teach in?
2. What months and year did you student teach?
3. During your student teaching, did you teach fraction operations?
4. Did you teach addition of fractions?
5. Did you teach subtraction of fractions?
6. Did you teach multiplication of fractions?
7. Did you teach division of fractions?
8. What is the difference between content knowledge, pedagogy knowledge and knowledge of students? It is ok if you don't know the definitions. Please try your best. *(After the student finishes their answer, I will provide on the screen the*

definitions of these words, so every participant gets a clear definition.) Content knowledge refers to the amount and organization of knowledge a teacher has. This includes theories, principles, ideas, facts, and vocabulary which teachers must master to be effective (Shulman, 1986). Pedagogy knowledge includes the various instructional methods that can be used by teachers for instructing a concept (Ball et al., 2008). Knowledge of students includes understanding students' prior knowledge and any learning difficulties that allow teachers to tap into students' prior knowledge to understand misconceptions of topics (Deng, 2018). After the definitions are shown, ask the participant the following question: Do these definitions make sense? If the participant was not able to tell the researcher the definitions when first asked, have the participant now state the definitions in their own words to make sure the participant understands the difference between content knowledge, pedagogy knowledge and knowledge of students.

B. Content Knowledge of Fractions

We're going to start by reviewing your knowledge of fraction operations. We will go through each one separately.

9. Tell me what you know about addition of fractions?

10. Tell me what you know about subtraction of fractions?

11. Tell me what you know about multiplication of fractions?

12. Tell me what you know about division of fractions?

13. I want you to think back to the mathematics education courses you took in college. These are not the courses you took like algebra, statistics, calculus. What are the mathematics content courses you took? Please remember that the content courses are the ones that mostly cover mathematical content like how to add fractions, regrouping, place value that you would need to know when you teach elementary students.

Please discuss in detail the lessons you remember from your content courses and we will do each one separately.

14. In your mathematics content courses, how do you remember learning addition of fractions?

15. In your mathematics content courses, how do you remember learning subtraction of fractions?

16. In your mathematics content courses, how do you remember learning multiplication of fractions?

17. In your mathematics content courses, how do you remember learning division of fractions?

18. Now I am going to show you a problem on the screen. The problem is ½ + ¾ = ? What do you need to know to solve this problem?

The next few questions I will ask you about your content knowledge in your student teaching experience.

19. Now, I want you to think about your student teaching experience. When you were told by your cooperating teacher that you needed to teach fraction

operations, how did you feel about this? How confident did you feel in your content knowledge of fraction operations? Was there anything you wish you knew that you didn't know before?

20. Did you need to review fraction operations? If so, what did you do to review this content?

21. Now that you have completed student teaching, how would you describe your content knowledge of fraction operations? How has it changed from before student teaching to after student teaching?

C. Pedagogical Content Knowledge of Fractions (Pedagogy Knowledge)

I am going to show you the definition again on the screen of pedagogy knowledge. *Pedagogy knowledge includes the various instructional methods that can be used by teachers for instructing a concept (Ball et al., 2008).*

22. I want you to think back to the mathematics education courses you took in college again. What are the mathematics pedagogical courses you took? Remember this would be your methods classes- the different ways you learned how to instruct a concept.

23. In your mathematics pedagogical (methods) courses, how do you remember learning how to teach addition of fractions?

24. In your mathematics pedagogical (methods) courses, how do you remember learning how to teach subtraction of fractions?

25. In your mathematics pedagogical (methods) courses, how do you remember learning how to teach multiplication of fractions?

26. In your mathematics pedagogical courses (methods), how do you remember learning how to teach division of fractions?

The next few questions I will ask you about your pedagogy knowledge in your student teaching experience.

27. Now I am going to show you a problem on the screen. The problem is ½ + ¾ = ? What instructional methods would you use to teach this problem?

28. Now, I want you to think about a lesson you taught on fraction operations during student teaching. Can you describe to me how you taught the lesson? What instructional methods did you use to teach? Did you feel you knew instructional methods to teach the lesson, or did you have to learn instructional methods before you taught? Do you think there might be better instructional methods you don't know about or are you satisfied with what you know?

29. On the lesson you described to me, did you use any of the instructional methods you were taught in your mathematics education pedagogical (methods) courses?

30. Now that you have completed student teaching, how would you describe your pedagogical knowledge of fraction operations? How has it changed from before student teaching to after student teaching?

D. Pedagogical Content Knowledge of Fractions (Knowledge of Students)

I am going to show you the definition again on the screen of knowledge of students.

Knowledge of students includes understanding students' prior knowledge and any

learning difficulties that allow teachers to tap into students' prior knowledge to understand misconceptions of topics (Deng, 2018).

31. Again, I want you to think back to the mathematics education courses you took in college again. What are the mathematics pedagogical courses you took that focused on how to know what a student was thinking about a mathematical concept as they learned it?

32. Prior to student teaching, what was your knowledge of student thinking about addition of fractions?

33. Prior to student teaching, what was your knowledge of student thinking about subtraction of fractions?

34. Prior to student teaching, what was your knowledge of student thinking about multiplication of fractions?

35. Prior to student teaching, what was your knowledge of student thinking about division of fractions?

36. Thinking about your knowledge of student thinking, is there anything you wish you knew prior to student teaching that would have helped you instruct students?

37. Now I am going to show you a problem on the screen. The problem is ½ + ¾ = ? What do students think about when they see this problem?

38. In your mathematics pedagogical courses, what were some of the instructional strategies on fraction operations you learned in class? Let me give you some examples: manipulatives, number lines, pictures, drawings, word

problems or anything else you can think of that you used. *Follow up question: Please describe how you can use each instructional strategy you named to teach fraction operations.*

39. What were the instructional strategies you used to teach fraction operations during student teaching?

40. Now that you have completed student teaching, how would you describe your pedagogical knowledge of students in fraction operations? How has it changed from before student teaching to after student teaching?

Thank you for your time. Please remember that I will email you at the end of this interview the Amazon e-gift card. In approximately two weeks, I will send you an email for you to check over my work. If you could please look it over and reply back to my email, I would greatly appreciate it. Do you have any questions for me before we end the interview?

Appendix B: Codebook

Code	Sample Quote	Code Description
C.K. Problem on Screen-Accurate Answer	Participant #2 I need to make 2 equivalent fractions with the same denominator to get to the problem.	Participant stated or shared accurately their knowledge of how to solve the problem.
C.K. Problem on Screen-Inaccurate Answer	Participant #6 So if you want it in the least, wait, we got what isn't it called like least common multiple, or like the most. So I need to know if I need to convert it to like the 2 or the 4 for the denominator. I mean the one and the 3 I would just move over. I guess if you want it to the 4. If you want the denominator 4, I would need to know if you want it to stay as four fourths, or if you want it, to move to like one since four fourths is equivalent to one.	Content Knowledge: Problem of 1/2 + 3/4 was shown and participant stated or showed their inaccurate answer.
Combo Class	Participant #1 One specific class	Both content, pedagogy, student knowledge
Combo Class - 2 courses only	Participant #2 Had 2 separate math courses	Participant stated that they took two courses that combined both content, pedagogy, and knowledge of students.
Content Course	Participant #12 So we had a class that looked over that	Mathematics content course

Code	Sample Quote	Code Description
Current accurate knowledge of addition of fractions	Participant #6 If the denominator is the same, you can just add the numerators and then keep the denominator; the common denominator	Participant stated or showed their accurate knowledge of addition of fractions
Current accurate knowledge of division of fractions	Participant #4 For division of fractions, the second fraction always gets flopped. The denominator becomes the numerator and the numerator becomes the denominator, and then you can multiply straight across.	Participant stated or showed their accurate knowledge of division of fractions.
Current accurate knowledge of multiplication of fractions	Participant #7 When you're multiplying, you can multiply the numerators, multiply the denominators, and then you get your fraction.	Participant stated or showed their accuracy of how to multiply fractions.
Current accurate knowledge of subtraction of fractions	Participant #3 Similar to addition. As long as they're the same denominator. They can be either added, or I mean they could be subtracted straight across.	Participant stated or showed their accurate knowledge of subtraction of fractions.
Current inaccurate knowledge of division of fractions	Participant #2 going from bigger to smaller. That's really all I got for that one we barely did it.	Participant stated or showed inaccurate knowledge of division of fractions

Code	Sample Quote	Code Description
Current inaccurate knowledge of multiplication of fractions	Participant #10 multiplication of fractions. It does not matter whether the denominators are the same. But you just have to find the reciprocal. And then, once you find the reciprocal of one you can multiply across numerator times numerator and then denominator.	Participant stated or showed inaccurate knowledge of multiplication of fractions
Current no knowledge of division of fractions	Participant #11 don't know much about division of fractions. I don't remember much.	Participant stated or showed no knowledge of division of fractions.
Integrated subjects with mathematics courses- 2 courses (content and pedagogy only)	Participant #3 one physical science, math and science literacy; one life science, math, and health literacy.	Participant stated that they took two courses that combined both content and pedagogy, no knowledge of students but these courses were combined with other subjects such as science.
Lesson- S.T.P.K. -step charts	Participant #3 But adding fractions, we would use step charts.	Participant stated they used step charts as a method to instruct fractions.
Lesson- S.T.P.K. collaborative groups	Participant #1 would work through it together	Participant stated they used collaborative groups as a method to instruct fractions.
Lesson- S.T.P.K. stations	Participant #2 We had rotating stations.	Participant stated or showed how they used stations to teach the fraction operation lesson.
Lesson-S.T. P.K- Candy Bar Method	Participant #2 I had another group who was working specifically with me on reviewing the Candy Bar method like	Participant stated or showed how they used the candy bar method to teach fraction operations.

Code	Sample Quote	Code Description
	introducing the candy bar method to them.	
Lesson-S.T. P.K-Manipulatives	Participant #11 given manipulatives that they had the option to use	Participant stated or showed how they used manipulatives to teach fraction operations.
Lesson-S.T. P.K-Traditional Method for addition and subtraction of fractions	Participant #11 I do. We do. You do model	Participant stated or showed how they taught the lesson of addition and subtraction of fractions using a traditional method.
Lesson-S.T. P.K-Traditional Method for addition of fractions	Participant #5 Now I had the kids find the common denominator so like if it was like one half I had gave them like one half and one and three fourths, something like that. They had numbers, and they had to find the common denominator, and there was 2 questions there. And then we went into how to add fractions.	Participant stated or showed how they taught the lesson of addition of fractions using a traditional method.
Lesson-S.T. P.K-Visuals	Participant #9 I'll just start with adding fractions that's the earliest lesson. I can remember, I did use some visuals. So we looked at different fraction circles, and how one half and three fourths and what they look like.	Participant stated or showed how they used visuals to teach fraction operations.
Lesson-S.T. P.K-Whiteboards	Participant #1 Whiteboard	Participant stated whiteboards were used to teach fractions during student teaching.

Code	Sample Quote	Code Description
Lesson-S.T. P.K-fraction blocks	Participant #2 And then I had a final group who had the manipulative, the fraction blocks that I had talked about earlier.	Participant stated or showed how they used fraction blocks to teach fraction operations.
Lesson-S.T. P.K-fraction strips	Participant #4 We would also use fraction strips so that way they had like a visual.	Participant stated or showed they used fraction strips to teach fraction operations.
Lesson-S.T. P.K.-Instructional Videos	Participant #5 So there was like a visual learning video. The kids watch the visual learning video.	Participant stated that they used instructional videos to instruct fractions during student teaching.
Lesson-S.T.P.K. fraction discs	Participant #1 fraction discs	Participant stated they used fraction circles to instruct fractions during student teaching.
Lesson-S.T.P.K. number lines	Participant #8 We had a lot of number lines. There was a lot of number lines in student teaching and having them just to be able to recognize how to label a number line and then to be able to jump back, and then to be able to jump forward, to jump forward as well when you're adding and subtracting.	Participant stated they used number line as a method to instruct fractions.
Lesson-S.T.P.K. tape diagrams	Participant #3 they could connect like the tape diagrams to those number lines	Participant stated they used tape diagrams as a method to instruct fractions.
M.C.C - Subtract - Line Up	Participant #6	Participant stated or showed that fractions need to be lined up and

Code	Sample Quote	Code Description
	like denominators. Just subtract the tops, the numerators.	use a common denominator to subtract.
M.C.C Add- Khan Academy	Participant #11 I believe we used Khan academy in my course. We had one unit on it which would be about if I'm remembering correctly, about two to three weeks.	Participant stated or showed they learned how to add fractions in their content courses by using Khan Academy.
M.C.C Division- Flip fraction and flip operation	Participant #4 You always leave the first fraction alone, and then the second fraction you flop. And so if you have one third, and like the 3, would go on top, and then one would go at the bottom, and then you just multiply straight across so literally, just like the numerators together and the denominators together to get your new answer.	Participant stated or showed accurately how to divide fractions.
M.C.C Division- real world drawings	Participant #12 We would use different foods, for example, and split the food, divide the parts of the whole so that one was a lot of like realistic kind of real world stuff like, I split a pizza or I split a pie stuff like that	Participant stated or showed that during the content course, they learned how to divide fractions using real world food/examples/drawings.

Code	Sample Quote	Code Description
M.C.C Multiply- Butterfly method- Not accurate description	Participant #4 Cross multiplication; So say, if you have one third times one fifth; you would do like the we call it in our classroom the butterfly effect. So you kind of just kind of do the little circles like that; So then it'd be like one times 5, and then one times like three. And then that's how you get your answer for multiplication.	Participant stated or showed how to multiply using the butterfly method, however it was inaccurate.
M.C.C Multiply- Not accurate description	Participant #5 I remember him drawing it on the board and telling us the x was like a, and we would go across, so if it was like one fourth times one fifth he would go across. It'd be like one times five like it was like one fourth. I'm just going to write it. So I see it. He would go across like that and tell us like this line is how you would do it. And he would go like that. And then he, we would get 4 over 5."	Participant stated or showed a description of multiplication of fractions that was inaccurate.
M.C.C. Add - Using manipulatives	Participant #12 fraction blocks to make like parts of a whole. We use just a lot of	Participant stated that during the content course, they learned how to add fractions using manipulatives.

Code	Sample Quote	Code Description
	manipulatives in general.	
M.C.C. Add- Did not learn	Participant #8 I don't remember learning.	Participant stated that they did not learn how to add fractions in their content courses.
M.C.C. Divide- did not learn	Participant #6 I do not remember learning about division of fractions at all.	Participant stated that they did not learn how to divide fractions in their content courses.
M.C.C. Divide- used Khan Academy	Participant #11 We used Khan Academy too.	Participant stated or showed they used Khan Academy to learn how to divide fractions.
M.C.C. Multiply - used Khan Academy	Participant #11 So we were given Khan Academy	Participant stated or showed they used Khan academy to learn how to multiply fractions.
M.C.C. Multiply- did not learn	Participant #9 We did not learn multiplication of fractions.	Participant stated that they did not learn how to multiply fractions in their content courses.
M.C.C. Multiply- manipulatives	Participant #12 so manipulatives	Participant stated that during the content course, they learned how to multiply fractions using manipulatives.
M.C.C. Subtract - Did not learn	Participant #8 No, we didn't learn it.	Participant stated that they did not learn how to subtract fractions in their content courses.
M.C.C. Subtract - Used manipulatives	Participant #9 Same thing, using fraction circles and other manipulatives	Participant stated that they used manipulatives to learn how to subtract fractions.
M.C.C. Subtract- used Khan Academy	Participant #11 we had reviewed through Khan Academy	Participant stated or showed they learned how to subtract fractions by using Khan Academy.
M.C.C Add- Line up	Participant #1 You just line up, when we did it we lined it.	Participant stated how they learned in their mathematical content courses to line up

Code	Sample Quote	Code Description
	We lined up the fractions, and then added the top. But you could only do that if the denominator is the same. If the denominator is not the same. You have to make it the same.	fractions with common denominator to add
No Pedagogy Course	Participant #10 I don't remember taking one.	Participant stated or showed they did not take a pedagogy course.
No Content Course was taken	Participant #10 I don't remember taking any of them. I took one, but it was we didn't focus as much on the strategies to teach students. It was more on the standards of mathematical practice	Participant stated or showed that they did not take a mathematics content course.
No understanding of pedagogy knowledge	Participant #1 So Pedagogy. Honestly, I don't know.	Participant stated or showed they did not understand pedagogy knowledge.
One course- content and pedagogy only	Participant #12 was both content and pedagogy	Participant stated or showed they took one mathematics course that only included math content and pedagogy.
P.K. C.C. Instructional Methods- collaborative groups	Participant #1 divide up like people into groups	Participant stated that they learned about collaborative groups as a method to teach fractions during the pedagogical methods course.
P.K. Problem on Screen-Fraction tiles- Accurate Answer	Participant #7 I would start with the tiles and show them a half. Show them three-fourths. Show them that we could put them	Participant described how to use fraction tiles to add the fractions.

Code	Sample Quote	Code Description
	together and compare it to a whole, and we would know that it's equal to one more than one whole. Then from there I would have them see that one half is equal to a certain number of fourths. I would have them with the tiles figure out how many fourths that half is, and then add that number of fourths to the three fourths and find your total which will be five fourths.	
P.K. Problem on Screen-Tape Diagrams - Accurate Answer	Participant #3 uses of tape diagrams; using the tape diagram showing that the one half of the tape diagram can be cut or can be further divided and seeing that one half is equal to 2, 4 and then, once we have that we can then create either a new tape diagram. If that's the route I would have decided to take. And we would just shade in as we would create one tape diagram and demonstrate that it's one whole. So making it into 4 equal pieces. And then we start. We okay, we know we have 2 fourths we shaded into and above, we saw the additional 3 fourths. Oh,	Participant described how to use tape diagrams to add the fractions.

Code	Sample Quote	Code Description
	we ran out of space here. We need to add the additional fourth and we would have one and one-fourth	
P.K. Problem on Screen-Traditional method - Accurate Answer	Participant #5 Okay, now let's circle, the one half, and under line three fourths and the one we circled, I would say, like, listen one plus three we could do, but when we're adding fractions, this is not possible. We can't do two plus four like that's just not going to be something we can do with fractions. So we have to find a common denominator, and I would say to them, Oh, we have a four here. Does two times anything equal four? Oh, two times two equals four. And I would say, Okay, perfect. So now let's take the one over two, and we know that two times two equals four. So let's put it there and then I would say to them, like, whatever you do to the bottom, you have to do the top, so if you multiply two by the bottom, multiply two by the top, and we multiply two by the top. Oh, we got two fourths	Participant stated or showed how to use a traditional method to teach this problem.

Code	Sample Quote	Code Description
	and then it would be like, Oh, you can't stop there. So now we have to add them together, so can we do two fourths plus three fourths. And then I would just show them how two fourths plus three fourths. I would say, and keep the bottoms the same, and we would just add across the top. So two plus three equals five.	
P.K. Problem on Screen- Traditional method - Inaccurate Answer	Participant #6 If I'm remembering fractions right again, just trying to figure out if you want the 2 or the 4 to be the denominator, and then just adding the numerators.	Participant stated or showed that they would try to find a common denominator, however the explanation was inaccurate.
P.K. Problem on Screen- candy bar method- Accurate description	Participant #2 So we did the candy bar method, so we would draw. We would draw the 2. I would just tell them this whole story about a candy bar, and we would draw 2 boxes, but the boxes would have to be the same size and then I would draw a line down the middle, so it'd be the one half, and I would make sure that I'm shading only one to show them that the numerator was there, and then I would	Participant described how to use the candy bar method to add the fractions.

Code	Sample Quote	Code Description
	draw the lines in the other box to show the 4 fourths, and then I would only shade in 3. And because I have the really fancy Schmancy whiteboard, we would draw the lines up to show that one half is equivalent to 2 fourths and then we would just count the number of fourths that we would have. So then we would get 5 forths. We would do that.	
P.K. Problem on Screen-circles-inaccurate description	Participant #1 I would make like circles. I would do circles honestly, and I would cut, you know, one circle in half, and then I would take another circle and cut it in Thirds Fourths and just add across, and then make sure the bottoms were the same	Participant stated or showed how to use circles to solve the problem 1/2 + 3/4=? inaccurately.
P.K. Problem on Screen-clock method- Accurate description	Participant #10 with the clock model, I know there's 60 min in an hour. One half of an hour's 30 min; one-fourth of an hour is 15 min, so I need 3 sets of 15 to give me 45 min and add 30 min and 45 min	Participant stated or showed accurately the clock method to solve the problem.
P.K. Problem on Screen-money method- Accurate description	Participant #10 we used a money model; and we had	Participant stated or showed accurately the money method to solve the problem.

Code	Sample Quote	Code Description
	students think of fractions as so they would look at that and say, Okay, one half of a dollar, I know is 50 cents and I know three fourths is one-fourth is 25 cents. So three fourths, 75 cents and from there I can add 50 cents and 75 cents.	
P.K. Problem on Screen-Guided Practice-Accurate Answer	Participant #4 Guided practice. So I would do a couple of problems with them, and they would follow with me. So yeah. So it was like, teacher led in front. So then I would show them a couple of problems they were doing with me. And then in their workbook that guide to practice problems.	Participant described how to use whole group to teach the lesson; guided practice to show how to add the fractions the traditional way.
P.K. Problem on Screen-fraction blocks and drawing a picture-Accurate description	Participant #11 We're given fraction blocks and I would work with the fraction blocks as well as drawing a picture on the board. So that way I can attend to our visual and our sensory learners.	Participant stated or showed accurately how to use manipulatives and drawing pictures to solve the problem.
P.K.C Division - Did not learn	Participant #9 We did not learn how to teach division of fractions.	Participant stated they did not learn how to instruct division of fractions in their courses.
P.K.C Instructional Methods- clocks	Participant #7	Participant stated they used clocks to teach fractions that was

Code	Sample Quote	Code Description
	Clocks. Clocks came later.	learned in mathematical pedagogical courses.
P.K.C Instructional Methods- drawing pictures	Participant #5 definitely drawings.	Participant stated they used drawing pictures to teach fractions that was learned in mathematical pedagogical courses.
P.K.C Instructional Methods- manipulatives	Participant #12 We used a lot of manipulatives and a lot of visuals.	Participant stated they used manipulatives to teach fractions that was learned in mathematical pedagogical courses.
P.K.C Instructional Methods- money	Participant #7 and money	Participant stated they used money to teach fractions that was learned in mathematical pedagogical courses.
P.K.C Multiply- Not accurate description	Participant #1 Just if the numerator was the same, or if the numerator is fine and the denominators were different, then you would find the multiplication factor to get to where it needed to be so. If it was 1/8. And then next thing was like times, 2 fourths. How would you get from 2 fourths? How would you make it a 8 on the bottom.	Participant stated or showed a description of multiplication of fractions that was inaccurate.
P.K.C. -instructional methods- traditional method for multiplication	Participant #7 We're going to just multiply straight across.	Participant stated or showed that they learned the traditional method to multiply fractions
P.K.C. -instructional methods- word problems	Participant #10 word problems we talked about	Participant stated how they learned in their mathematical pedagogical courses how to use

Code	Sample Quote	Code Description
		word problems for fraction operations.
P.K.C. Add - Skits	Participant #2 We would always do skits.	Participant stated that that they learned how to use skits to teach addition of fractions.
P.K.C. Add- Basic Instructional Methods to teach Fractions	Participant #3 it's more like you can use this method	Participant stated that instructional methods were taught on how to teach fractions.
P.K.C. Add- Did not Learn	Participant #8 I do not remember learning how to teach the addition of fractions in that course.	Participant stated or showed they did not learn how to add fractions in pedagogy courses.
P.K.C. Add- Line Up	Participant #5 you add the top numbers, you find a common denominator for the bottom ones, and then she just basically did the same exact way	Participant stated how they learned in their mathematical pedagogical courses to line up fractions with common denominator to add
P.K.C. Addition and Subtraction- instructional methods- word problems	Participant #6 addition and subtraction word problems with fractions	Participant stated how they learned in their mathematical pedagogical courses how to use word problems for addition and subtraction of fractions.
P.K.C. Addition- Instructional Methods- Visuals	Participant #11 visuals as well so how to include visuals with our fractions	Participant stated they used visuals to learn how to teach fractions.
P.K.C. Addition- Instructional Methods- manipulatives	Participant #9 We were instructed to use the manipulatives and hands on materials.	Participant stated they used manipulatives to learn how to teach fractions.
P.K.C. Divide- Basic Instructional Methods to teach Fractions	Participant #3 it's more like you can use this method	Participant stated that instructional methods were taught on how to teach fractions.

Code	Sample Quote	Code Description
P.K.C. Divide- Picture based	Participant #7 picture based for division	Participant stated or showed that their pedagogy course(s) showed how to teach division based on pictures.
P.K.C. Flip Fraction and Flip Operation	Participant #1 Keep change, Flip	Participant stated or showed an accurate representation of how to divide fractions.
P.K.C. Instructional Methods - Technology	Participant #3 I think it was Mathigon.org. Like a website where students could use like those fractions. The number lines.	Participant stated that they used Mathigon.org, a technology program.
P.K.C. Instructional Methods- Did not learn in a mathematics pedagogy class	Participant #4 Like I said I didn't really take a math course for it.	Participant stated they did not learn any mathematical instructional methods in their non mathematical pedagogy courses.
P.K.C. Instructional Methods- No fraction operations were taught in a Mathematics Pedagogical Course	Participant #8 that is fair to say that no instructional strategies were taught on fraction operations.	Participant stated or showed that they did not learn instructional methods on how to teach fraction operations in their mathematics pedagogical course.
P.K.C. Instructional Methods- Traditional methods	Participant #12 drill and kill is what we call it, I guess. So you just practice over and over, you know, here's a sheet of it adding fractions, here's the sheet of subtracting fractions.	Participant stated they used traditional methods to teach fractions that was learned in mathematical pedagogical courses.
P.K.C. Instructional Methods- number lines	Participant #9 number lines	Participant stated they used number lines to teach fractions that was learned in mathematical pedagogical courses.

Code	Sample Quote	Code Description
P.K.C. Multiply- Basic Instructional Methods to teach Fractions	Participant #3 it's more like you can use this method	Participant stated that instructional methods were taught on how to teach fractions.
P.K.C. Multiply- Butterfly method	Participant #5 The butterfly method. I remember, for actually drawing the butterfly on the board, and having us like do it ourselves too	Participant stated or showed that they were taught how to show students the butterfly method during their mathematics pedagogical course.
P.K.C. Multiply- Did not learn	Participant #6 I do not remember teaching or learning any strategies to teach multiplication.	Participant stated they did not learn how to instruct multiplication of fractions in their courses.
P.K.C. Subtract - Did Not Learn	Participant #8 No, we didn't learn it.	Participant stated or showed they did not learn subtraction of fractions in pedagogy courses.
P.K.C. Subtract - Skits	Participant #2 We would do skits like addition of fractions.	Participant stated they learned how to use skits to teach subtraction of fractions.
P.K.C. Subtract- Basic Instructional Methods to teach Fractions	Participant #3 it's more like you can use this method	Participant stated that instructional methods were taught on how to teach fractions.
P.K.C. Subtract- Line Up	Participant #1 Just line up your fractions	Participant stated how they learned in their mathematical pedagogical courses to line up fractions with common denominator to subtract
P.K.C. Subtraction-manipulatives	Participant #9 I would say the same thing, using hands-on materials.	Participant stated or showed they used manipulatives to learn how to teach fractions.
P.K.C. Subtraction-number line	Participant #6 number line sometimes for subtraction	Participant stated or showed they used a number line to learn how to teach subtraction of fractions.
P.K.C. Subtraction-visuals	Participant #11 same as addition" (see P.K.C. Addition-	Participant stated or showed they used visuals to learn how to teach fractions.

Code	Sample Quote	Code Description
	Instructional Methods-Visuals)-- visuals as well so how to include visuals with our fractions	
P.K.C. division-manipulatives	Participant #12 So if you are using a manipulative like splitting that up into halves or fourths."	Participant stated or showed they used manipulatives to learn how to teach fractions.
P.K.C. multiplication-manipulatives	Participant #12 Looking at different, like the different types of multiplication of fractions, see like a whole number times fraction, fraction times fraction. All that I really remember like breaking those down and looking at the different types of multiplying fractions. And then how you do that! Something like the manipulatives, the like groupings	Participant stated or showed they used manipulatives to learn how to teach fractions.
P.K.C.- Instructional Methods- Visuals	Participant #12 We used a lot of manipulatives and a lot of visuals.	Participant stated they used visuals to teach fractions that was learned in mathematical pedagogical courses.
Pedagogy Course	Participant #7 one course	Participant stated or showed they took one pedagogy course
Pedagogy Courses- 2 courses includes pedagogy and knowledge of students but no math involved	Participant #4 was 2 different ones; they were not specific to math	Participant stated and showed they took two pedagogy courses which included instructional methods and knowledge of students, however no math involved
Pedagogy and Student Knowledge Course	Participant #5	Student stated that they took a course for Grades 1-6 that

Code	Sample Quote	Code Description
	I took one for grades one to six.	included mathematics pedagogy and student knowledge.
Present Accurate knowledge of content knowledge	Participant #5 So content knowledge is given to like a specific subject, such as, like math, science, social studies.	Participant stated or showed knowledge of content knowledge.
Present Accurate knowledge of pedagogy knowledge	Participant #9 Pedagogical knowledge is how I know how to teach something. The knowledge that I have of teaching something.	Participant stated or showed knowledge of pedagogy knowledge.
Present Accurate knowledge of student knowledge	Participant #10 figuring out what students already know when we teach them	Participant stated or showed knowledge of student knowledge.
S.K. Problem on Screen-Do not know how to add fractions	Participant #12 They look at whatever order of operation is in the middle, and like, disregard all rules. So I feel like they would look at this, and just add one and three and two and four, and put the number that's on top on top, and the number that's on bottom on bottom.	Participant stated or showed that the students would not know how to add fractions.
S.K. Problem on Screen-Fraction Line	Participant #2 the fraction line is sideways. They would not like that, that it would have to be up and down, but that would bother them so much.	Participant stated that the students would notice the fraction line was sideways.

Code	Sample Quote	Code Description
S.K. Problem on Screen- Fractions are not written vertical	Participant #5 Because the 3 fourths and the one for one half are not like vertical."	Participant stated or showed that the students would be confused since the fractions were not written vertically
S.K. Problem on Screen- Misinterpretation from students	Participant #1 but they just think they have to add regular so they just go straight across and no changing of any denominators at all.	Participant stated that students would add the numerators without realizing that the denominators were different.
S.K. Problem on Screen- Not the same denominator	Participant #3 So they see the uncommon denominators or unlike denominators. Having to find an equivalent fraction.	Participant stated or showed that students would see the denominator was different and they would need to make it the same.
S.K. Problem on the Screen- clocks	Participant #7 Clocks think half an hour, plus three quarters of an hour	Participant stated or showed that students may use their knowledge of clocks to solve this problem.
S.K. Problem on the Screen- money	Participant #7 if they think about quarters or half dollars, they might be thinking 50 cents plus 75 cents or half dollar, plus 3 quarters.	Participant stated or showed that students would think of money when solving this problem.
S.T. C.K Not confident and uncomfortable teaching fractions	Participant #6 I was very unconfident, did not feel comfortable.	Participant stated or showed that they were not confident and not comfortable teaching fractions
S.T. C.K- Anxious	Participant #6 I definitely was very anxious. I was freaked out.	Participant stated or showed that they felt anxious when they were told by their cooperating teacher they were teaching fractions.

Code	Sample Quote	Code Description
S.T. C.K- Nervous	Participant #8 I felt very nervous to be honest.	Participant stated or showed that they were nervous to teach fraction operations.
S.T. C.K. Collaborative Learning	Participant #2 how I could make it more collaborative for them learning instead of just showing them and explaining how I could get them up and moving and be more involved.	Participant stated that they wish they knew about collaborative learning before going into teaching fractions during student teaching.
S.T. C.K. Confident on content knowledge of fractions	Participant #3 felt confident	Participant stated or showed confidence in content knowledge of fractions going into student teaching.
S.T. C.K. Content knowledge increased	Participant #1 It has because I was a little shaky before student teaching.”	Participant stated that content knowledge increased after student teaching.
S.T. C.K. Content knowledge stayed the same	Participant #11 it didn't change. It was the same.	Participant stated that during student teaching they were confident in the material and able to teach it; their content knowledge did not change; however now the participant does not feel too confident.
S.T. C.K. Differentiation	Participant #1 how to differentiate with kids that like, we’re top top and math that knew already how to multiply and divide fractions at second grade.	Participant stated that they wish they knew about differentiation before going into teaching fractions during student teaching.
S.T. C.K. Excited about Teaching fractions	Participant #2 I was excited because I liked that. We got to use a lot of manipulatives	Participant stated that they were excited to teach fractions when told by their cooperating teacher.

Code	Sample Quote	Code Description
S.T. C.K. No review	Participant #8 I did not.	Participant stated that they did not need to review fraction operations.
S.T. C.K. Not confident after student teaching	Participant #6 I'm still not confident with fractions.	Participant stated or showed they were not confident about operations of fractions after student teaching.
S.T. C.K. Review division before teaching fractions	Participant #12 The only thing I really reviewed was division of fractions.	Participant stated that they needed to review division prior to teaching fractions.
S.T. C.K. Review multiplication and division before teaching fractions	Participant #2 I probably would have just looked up a Youtube video with the raps.	Participant stated they would need to review multiplication and division before teaching fraction operations.
S.T. C.K. Reviewed addition and subtraction of fractions	Participant #6 Yeah, I did need to review fraction operations. I like, I went to my boyfriend at the time, and was like, I need to teach addition and subtraction of fractions	Participant stated or showed they needed to review addition and subtraction of fractions.
S.T. C.K. having a conceptual knowledge of fractions	Participant #6 I don't feel as if people, my age, and even older, understand the conceptual understanding of fractions. And so I feel like as I'm teaching. Honestly, I'm like, Oh, yeah, that makes sense, because I have no basic understanding of fractions either.	Participant stated or showed they wish they had a better conceptual understanding of fractions.

Code	Sample Quote	Code Description
S.T. P.K. - Better instructional methods	Participant #4 I'm pretty sure there's always better things you can do. But from what I did I was pretty satisfied with what I got in.	Participant stated that there could be better instructional methods.
S.T. P.K. - Confident on instructional methods	Participant #1 I think I'm satisfied with what I know.	Participant stated that they were satisfied with what they knew about instructional methods.
S.T. P.K. - Instructional Methods- did not need to learn any	Participant #9 I felt like I knew instructional methods beforehand.	Participant stated or showed that they did not need to learn any instructional methods to teach fraction operations.
S.T. P.K. Instructional Methods - Did not need to learn them based on cooperating teacher	Participant #6 I feel as if my mentor teacher made me feel as if I did not need to teach, or I did not need to learn any more instructional methods.	Participant stated or showed that they did not need to learn any instructional methods because of the cooperating teacher.
S.T. P.K. Instructional Methods - Needed to learn a think aloud	Participant #5 And then another big one was like a think aloud. Like being able to do a think aloud, and having the students not speak, because I think I was so used to like, oh, I have to let like I have to when I'm doing something. The student input has to be there, and they were like, no, you have to model one question completely on your own thinking aloud how you would do it from a kid's point of view instead of your own point of view, and I	Participant stated or showed they needed to learn how to do a think aloud.

Code	Sample Quote	Code Description
	think that was definitely something that was like kind of a struggle.	
S.T. P.K. Instructional Methods - Needed to learn scaffolding	Participant #5 One of the things that I really had to learn was scaffolding because I think in college they make it sound like every kid knows everything. And no one's going to struggle, and I think I had one of those experiences where I'm like, oh, Eric is going to totally get this.	Participant stated or showed they needed to learn how to scaffold.
S.T. P.K. Instructional Methods - Needed to learn them	Participant #3 I had to learn them. And it was fast, but it was still a learning curve.	Participant stated that they needed to learn the instructional methods before they taught the lesson.
S.T. P.K. Instructional Methods - needed to learn some instructional methods	Participant #7 Probably a little bit of both. Sometimes I felt like I was learning the same moment I was teaching it	Participant stated or showed that they needed to learn some instructional methods and there were some they did not need to learn.
S.T. P.K. Instructional Methods- Learned through cooperating teacher	Participant #2 I think I think I modeled it basically off of my mentor teacher."	Participant stated that they learned the instructional methods they used during student teaching through their cooperating teacher.
S.T. P.K. Instructional Methods- Learned through previous field work and while in grade school	Participant #1 We always did like smart board. So it's kind of different. But I knew the other stuff because I was doing like internships, and I, student taught. You know where I went to	Participant stated that they learned the instructional methods they used during student teaching through previous field work and while in grade school.

Code	Sample Quote	Code Description
	my old school. So I was exposed to that stuff already.	
S.T. P.K. pedagogy knowledge didn't change	Participant #6 pedagogical knowledge of fraction operations didn't change, it stayed the same.	Participant stated and showed that their pedagogy knowledge didn't change; it stayed the same after student teaching
S.T. P.K. pedagogy knowledge increased	Participant #4 I learned a lot. And I feel a lot more knowledgeable about instructing operations of math with fractions.	Participant stated that their pedagogy knowledge increased after student teaching.
S.T. P.K.- Instructional Methods used during student teaching (learned from pedagogical courses (non-math)- Direct Instruction	Participant #4 I did. I did a lot of direct instruction at first.	Participant stated or showed how they used direct instruction which was something they learned in their non-mathematical pedagogy courses.
S.T. P.K.- Instructional Methods used during student teaching (learned from pedagogical courses) -Gradual release	Participant #9 And then did the basic structure of gradual release of responsibility	Participant stated or showed they used gradual release.
S.T. P.K.- Instructional Methods used during student teaching (learned from pedagogical courses) NONE	Participant #8 No, that it was all new to me.	Participant stated they did not use any instructional methods they learned in their courses during student teaching.
S.T. P.K.- Instructional Methods used during student teaching (learned from pedagogical courses) Traditional methods for adding and subtracting fractions	Participant #6 Yes, it was the way I was taught by the professor; finding a common denominator.	Participant stated or showed they used the traditional methods taught to them by their professor in their pedagogical course.
S.T. P.K.- Instructional Methods used during	Participant #1 keep, change, flip	Participant stated that they used flip the fraction and operation

Code	Sample Quote	Code Description
student teaching (learned from pedagogical courses)- Division (Flip Fraction)		while teaching during student teaching.
S.T. P.K.- Instructional Methods used during student teaching (learned from pedagogical courses)- Fraction Tiles	Participant #2 The fraction tiles	Participant stated that they used fraction tiles while teaching during student teaching.
S.T. P.K.- Instructional Methods used during student teaching (learned from pedagogical courses)- anchor charts	Participant #7 we had a visual anchor chart, but we had an anchor chart that we would constantly refer to.	Participant stated that they used anchor charts that they learned in their pedagogy course(s) while teaching during student teaching.
S.T. P.K.- Instructional Methods used during student teaching (learned from pedagogical courses)- manipulatives	Participant #11 would be using the manipulatives	Participant stated that they used manipulatives that they learned from their pedagogy course(s) while teaching during student teaching.
S.T. P.K.- Instructional Methods used during student teaching (learned from pedagogical courses)- visuals	Participant #12 I would say I use like the idea of visual representations because we really talked about that.	Participant stated that they used visuals that they learned from their pedagogy course(s) while teaching during student teaching.
S.T. P.K.- Instructional Methods used during student teaching (learned from pedagogical courses- non-mathematical) - Small Groups	Participant #4 I did a lot of small groups.	Participant stated or showed that they learned from their non-mathematical pedagogy courses about using small groups.
S.T. S.K. Access to Paperwork	Participant #2 I didn't have any access to the 504s or Iep's of my students, so something that may have been some kid might have had a	Participant stated that they would have liked to have had access to paperwork- IEP, 504 plans, etc.

Code	Sample Quote	Code Description
	learning disability, and I didn't know. So some child might have been struggling, and I wouldn't have understood.	
S.T. S.K. Add- Difficult and not like it	Participant #2 they didn't like it. I know that a lot of the fourth grade who I spoke to was like dreading the fraction unit. They thought it'd be difficult. They just didn't like it at all	Participant stated that they thought that the students would not like addition of fractions. It would be difficult.
S.T. S.K. Add- Easy	Participant #3 I assumed it was easy for them.	Participant stated that the students would find addition of fractions easy.
S.T. S.K. Add- Know how to add fractions	Participant #7 I really came in confident, like, okay, you know, this bunch of kids should know that we're talking about less or more than one whole, they should know what a denominator is. We could break things into pieces.	Participant stated that the students come in knowing how to add fractions.
S.T. S.K. Add- No knowledge	Participant #12 I didn't really have an idea of you know how students would think about fractions.	Participant stated that they did not have knowledge on student thinking about addition of fractions.
S.T. S.K. Add- very minimal knowledge	Participant #6 I would say very minimal.	Participant stated they had very minimal knowledge of student knowledge of addition of fractions.
S.T. S.K. Differentiation	Participant #1	Participant stated that they would have liked to have learned

Code	Sample Quote	Code Description
	differentiation of lessons	about differentiation of lessons to help with student thinking.
S.T. S.K. Divide - did not like it	Participant #2 same as multiplication (I don't think they really had any idea about multiplying fractions. But I would assume they didn't like it like fractions in general.)	Participant stated it would be the same as multiplication of fractions.
S.T. S.K. Divide- Hard	Participant #3 It's going to be hard for them.	Participant stated that division of fractions would be hard for students.
S.T. S.K. Division- No Knowledge	Participant #4 Not very much	Participant stated that they did not have knowledge on student thinking about division of fractions.
S.T. S.K. Division- Stuck	Participant #1 stuck in the moment, like they were like. I don't know what to do.	Participant stated that the students would be stuck when it came to division of fractions.
S.T. S.K. Learning Instructional Methods	Participant #3 I think maybe if, like, we had access to like more like manipulatives, or like access to like, easy to use technology for the students, and like those kind of resources.	Participant stated two different instructional methods they wished they learned prior to student teaching.
S.T. S.K. Multiplication- No Knowledge	Participant #9 I have ever thought about that.	Participant stated that they did not have knowledge on student thinking about multiplication of fractions.
S.T. S.K. Multiply - did not like it	Participant #2 I don't think they really had any idea about multiplying fractions. But I would assume	Participant stated that the students would not like multiplication of fractions.

Code	Sample Quote	Code Description
	they didn't like it like fractions in general.	
S.T. S.K. Multiply- Easy	Participant #7 That this would probably be the easiest, because if you know how to multiply, coming from third grade, that you should be able to multiply straight across numerator and denominators.	Participant stated or showed that multiplication for students would be easy.
S.T. S.K. Multiply- Scared	Participant #1 They were scared cause they thought it would be totally different.	Participant stated that the students would be scared.
S.T. S.K. Multiply- hard	Participant #3 It's going to be hard for them.	Participant stated that multiplication of fractions would be hard for students.
S.T. S.K. Student Knowledge has changed compared to before student teaching	Participant #5 I would say, definitely improved.	The participant stated their student knowledge has changed since prior to student teaching.
S.T. S.K. Student Knowledge has not changed compared to before student teaching	Participant #9 It didn't change.	Participant stated or showed that their student knowledge has not changed after student teaching.
S.T. S.K. Subtract- Difficult and not like it	Participant #2 same as addition of fractions—(they didn't like it. I know that a lot of the fourth grade who I spoke to was like dreading the fraction unit. They thought it'd be difficult. They just didn't like it at all.)	Participant stated it would be the same as addition of fractions.

Code	Sample Quote	Code Description
S.T. S.K. Subtract- Easy	Participant #3 subtracting items would be very easy for them.	Participant stated that the students would find subtraction of fractions easy.
S.T. S.K. Subtract- Misconceptions	Participant #7 I knew they wouldn't understand how much we were actually taking away. A lot of the kids tend to think that if the denominator is bigger or higher in value, that we're talking about a larger amount. And that's a misconception. I knew I knew that would be a problem.	Participant stated that the students would have some misconceptions of subtraction of fractions.
S.T. S.K. Subtract- Need addition to be able to subtract	Participant #1 after finding out how to do adding fractions, I think they had more of a knowledge on how to do subtracting.	Participant stated that students would need to know how to add fractions in order to subtract fractions.
S.T. S.K. Subtract- No Knowledge	Participant #8 I did not have any prior knowledge.	Participant stated that they did not have knowledge on student thinking about subtraction of fractions.
S.T. S.K. Subtract- very minimal knowledge	Participant #11 I didn't have much.	Participant stated they had very minimal knowledge of student knowledge of subtraction of fractions.
S.T.C.K. - Training Course	Participant #3 I would have had to taught it like the standard algorithm.	Participant stated that they had a training course that allowed them to understand the content of fractions. This was prior to student teaching.
S.T.C.K. - knew about different mathematical concepts related to fractions	Participant #12 just one thing that I wish I would have like known going into. It was like how they	Participant stated or showed that they wish they knew about how to incorporate different mathematical concepts with fraction operations.

Code	Sample Quote	Code Description
	incorporate like decimals and money, and then time into it.	
S.T.C.K. Confident after student teaching	Participant #4 I feel a lot more confident in it for sure.	Participant stated that they were more confident in their content knowledge after student teaching.
S.T.C.K. Confident in Addition and Subtraction after student teaching	Participant #10 by the end of it I kind of like got the hang of it a little bit more. I still wasn't super confident in multiplication and division, and never really taught division but addition and subtraction, I really got the hang of it.	Participant stated or showed they were confident in addition and subtraction after student teaching but not multiplication.
S.T.C.K. Needed to review virtual manipulatives	Participant #7 virtual manipulatives	Participant stated or showed they needed to review virtual manipulatives
S.T.C.K. Not confident teaching fraction operations	Participant #5 But confidence with fractions, I was not so confident about it.	Participant stated or showed that they were not confident teaching fraction operations.
S.T.C.K. Nothing they wished they knew	Participant #9 Hmm! Not that I can think of. I think I had good I had enough knowledge to teach it.	Participant stated that there was nothing they wished they knew that they didn't know before student teaching.
S.T.C.K. Used Google to review operations of fractions	Participant #7 if I just needed to Google something and I could figure it out.	Participant stated or showed they used Google to review operations of fractions.
S.T.C.K. Used teacher manual to review how to teach fraction operations	Participant #4 the Teacher manual it like literally gave you like lessons. I kind of would just review, like	Participant stated or showed that they needed to use the teacher manual to review how the teacher manual was teaching each fraction operation. They needed to teach it how the

Code	Sample Quote	Code Description
	the Teacher Manual to kind of go over because it shows you what problems you do with the kids, and then on the bottom, it breaks it down of how exactly to solve it. So I kind of would just look at the bottom there, to make sure I'm following it correctly.	textbook was showing how to do it.
S.T.C.K. Wished they had more content courses	Participant #10 I think, having those courses that like taught me the new ways, like common core, how to be able to teach students and having those content courses in college would have helped me be able to teach those lessons.	Participant stated or showed that they wished they had more content courses in college.
S.T.C.K. Wished they knew about curriculum standards and progression of students	Participant #7 I wish I knew the progression and the standards in a certain way, but also knowing that no matter where I would end up, the curriculum would be different. I don't know who starts off with fractions.	The participant stated or showed they would have liked to know the curriculum standards and the progression of how students learn fractions.
S.T.C.K. Wished they knew about manipulatives before student teaching	Participant #5 I would know about, like the types of manipulatives they had in the classroom, like they had like fraction strips for the students,	Participant stated or showed that they wished they knew more about manipulatives before student teaching.

Code	Sample Quote	Code Description
	and like after the lesson was taught, they like open the closet of fraction strips, and I was like, oh, that would have been super helpful if I knew that was there.	
S.T.C.K. Wished they knew vocabulary words and meanings	Participant #8 And the certain vocabulary terms. So, for example, partition came up. I was like, Oh, I had no idea what that meant. So, yeah, just having a better understanding of what it is that they're asking.	Participant stated or showed they wished they knew vocabulary words and meanings when it came to fractions.
S.T.C.K. Worried about teaching fractions	Participant #3 when we started our unit on fractions, and just that whole jump from first grade to fourth was a lot.	Participant was worried about teaching fractions.
S.T.C.K. cooperating teacher was used to review operations of fractions	Participant #6 Talking to my mentor teacher during student teaching, to try to feel confident and just ready to actually teach the lesson.	Participant stated or showed they used their cooperating teacher to review operations of fractions.
S.T.C.K. used people to help review	Participant #6 I like, I went to my boyfriend at the time, and was like, I need to teach addition and subtraction of fractions.	Participant stated or showed they used their boyfriend to help review operations of fractions.
S.T.C.K. used videos to review operations of fractions	Participant #11	Participant used videos to review operations of fractions.

Code	Sample Quote	Code Description
	it would range from Youtube to Khan Academy	
S.T.S.K. - more understanding of student misconceptions	Participant #7 Yeah, I wish I knew more misconceptions. I wish I would have had more time to look over assessments to like let me solve it myself. I know I'll probably get most of the answers right on my own. But then let me try and do it wrong, or see how a kid would have gotten one of the multiple choice answers the wrong ones. What are they thinking about?	Participant stated or showed that they wished they knew the misconceptions of student's thinking when working with fraction operations.
S.T.S.K. Add- Student would not have knowledge of fraction operation	Participant #10 I don't think like looking back. I don't think they really had that much knowledge about fraction operations. I think they had some understanding of what a fraction was.	Participant stated or showed that the student would not have knowledge of addition of fractions.
S.T.S.K. Division- Student would not have knowledge of fraction operation	Participant #10 I don't think they had any like prior knowledge about division.	Participant stated or showed that the student would not have knowledge of division of fractions.
S.T.S.K. Division- very minimal knowledge	Participant #6 Yeah, very minimal.	Participant stated they had very minimal knowledge of student knowledge of division of fractions.

Code	Sample Quote	Code Description
S.T.S.K. Having more mathematical courses in college	Participant #4 I wish they would have offered it specific to the subject so like a course of just math, and figuring out how to push them more in that. So I wish they had a course like that, like the one that you were asking about.	Participant stated or showed they wish they had courses in college that taught about students' knowledge in mathematics.
S.T.S.K. Knowing about IEPs and English Language Learners	Participant #5 My undergrad program was not very focused on Special. Ed, it was all about Gen. Ed, so they never really went into IEP's or English language learners.	Participant stated that they would have liked to have learned about IEPs and English Language Learners prior to student teaching.
S.T.S.K. Knowing more of a student's conceptual understanding of different topics	Participant #9 I wish I had known more of their conceptual understanding before instructing. So maybe you know how do they understand visuals? And how do they understand models versus how do they understand formulas?	Participant stated or showed they wished they knew the topics students had an understanding on
S.T.S.K. Multiplication-Butterfly Method	Participant #6 Butterfly method	Participant stated or showed that their knowledge of student thinking was the butterfly method.
S.T.S.K. Multiplication-Student would have limited knowledge of multiplication	Participant #12 So they may have some prior knowledge on it. But a majority of the kids, you know, it was	Participant stated or showed that students would have minimal knowledge of multiplication of fractions.

Code	Sample Quote	Code Description
	completely new to learning this concept.	
S.T.S.K. Multiplication-very minimal knowledge	Participant #11 I didn't have much.	Participant stated they had very minimal knowledge of student knowledge of multiplication of fractions.
S.T.S.K. Multiply - Student would not have knowledge of the fraction operation	Participant #10 I don't think they have any prior knowledge, or that you can even multiply fractions.	Participant stated or showed that the student would not have knowledge of multiplication of fractions.
S.T.S.K. Subtract- Student would not have knowledge of fraction operation	Participant #10 I don't think they knew like how to do it.	Participant stated or showed that the student would not have knowledge of subtraction of fractions.
S.T.S.K. Wished they knew more about fact fluency	Participant #12 The main thing would have been just like the fact fluency. That comes with it.	Participant stated or showed they wish they knew about fact fluency prior to student teaching.
S.T.S.K. Wishing professors told them how students think	Participant #5 And I more wish that my professors would have told me to like, think like a student when I was teaching.	Participant stated or showed that they wish professors told them how a student was thinking
S.T.S.K. reviewing with students what a fraction is	Participant #6 And so reviewing what a fraction is. And you know this is a fraction with a square. This is a fraction with a circle, and then talking about them, identifying those fractions and talking about the numerators, denominators, I feel like	Participant stated or showed that they wish they knew to review with students what a fraction is and review vocabulary words before jumping into teaching operations of fractions.

Code	Sample Quote	Code Description
	all of that would have been super beneficial.	
Student Knowledge - No Course	Participant #3 They didn't really share; there wasn't much specific.	Participant stated that they did not take a course in student knowledge of mathematics.
Three Classes- Content and Pedagogy	Participant #9 Pedagogy and Mathematics. I took two sections of that section one and section two and then I took a course in mathematical thinking and pedagogy as well; Yes, I would say the undergrad courses were more heavily content than pedagogy, and then the masters was more heavily pedagogy than content. But they were all three of the courses were a mixture of both.	Participant stated or showed that they took three courses- these three courses were a combination of content and pedagogy
Traditional methods in mathematics courses	Participant #1 You just line up, when we did it we lined it. We lined up the fractions, and then added the top. But you could only do that if the denominator is the same. If the denominator is not the same. You have to make it the same.	Standard algorithms
Two Courses - content and pedagogy	Participant #11 I took math for elementary Ed and that was for two semesters; I	Participant stated or showed they took two courses that were both content and pedagogy of mathematics

Code	Sample Quote	Code Description
	learned more content than pedagogy.	
Two Mathematics Content Courses	Participant #5 Two math content courses. One of them was pre-algebra. And then one of them was like theories of algebra	Participant stated or showed they took two mathematics content courses.

Appendix C: Categories, Category Meanings and Aligned Codes

Category	Category Meaning	Aligned Codes
Addition Content	Accurate knowledge	Current accurate knowledge of addition of fractions
Subtraction Content	Accurate knowledge	Current accurate knowledge of subtraction of fractions
Multiplication Content	Inaccurate knowledge	Current inaccurate knowledge of multiplication of fractions
Division Content Knowledge	Accurate knowledge	Current accurate knowledge of division of fractions
Division Content	Inaccurate knowledge and no knowledge	Current inaccurate knowledge of division of fractions Current no knowledge of division of fractions
Content Courses	Participant took at least one mathematics content course or took at least one course that included pedagogy and/or student knowledge while in college	Combo Class Combo Class- 2 courses only Content Course One course- content and pedagogy only Three Classes-Content and Pedagogy Two Courses- content and pedagogy Two Mathematics Content Courses
Addition of fractions in Content Course	Different ways participant learned addition of fractions	M.C.C. Add- Khan Academy M.C.C. Add- Using manipulatives M.C.C. Add- Line Up
Subtraction of fractions in Content Course	Different ways participant learned subtraction of fractions	M.C.C.- Subtract- Line Up

Category	Category Meaning	Aligned Codes
		M.C.C. Subtract- Used manipulatives M.C.C. Subtract- used Khan Academy
Multiplication of fractions in Content Course	Multiplication of fractions was not learned in content course	M.C.C. Multiply- did not learn
Division of fractions in Content Course	Division of fractions was not learned in content course	M.C.C. Divide – did not learn
Content of solving an addition of fractions problem	Accurate knowledge	C.K. Problem on Screen- Accurate Answer
Content knowledge positive feelings entering student teaching	Positive feelings about content knowledge of fraction operations	S.T.C.K. Confident on content knowledge of fractions
Content knowledge negative feelings entering student teaching	Negative feelings about content knowledge of fraction operations	S.T.C.K. Not confident and uncomfortable teaching fractions S.T.C.K. – Anxious S.T.C.K. – Nervous S.T.C.K- Not confident teaching fraction operations S.T.C.K. – Worried about teaching fractions
Topics entering student teaching	Participants wished they knew different topics prior to entering student teaching	S.T.C.K. Collaborative Learning S.T.C.K. Differentiation S.T.C.K having a conceptual knowledge of fractions S.T.C.K.- knew about different mathematical concepts related to fractions S.T.C.K. Wished they knew about manipulatives before student teaching

Category	Category Meaning	Aligned Codes
		S.T.C.K. Wished they knew vocabulary words and meanings S.T.C.K. Wished they had more content courses S.T.C.K. – wished they knew about curriculum standards and profession of students S.T.C.K. Needed to review virtual manipulatives
Reviewed fraction operations	Participants reviewed fraction operations in different ways	S.T.C.K. Review division before teaching fractions S.T.C.K. Review multiplication and division before teaching fractions S.T.C.K. Reviewed addition and subtraction of fractions S.T.C.K. cooperating teacher was used to review operations of fractions S.T.C.K. used people to help review S.T.C.K. used Google to review operations of fractions S.T.C.K. used teacher manual to review how to teach fraction operations S.T.C.K. used videos to review operations of fractions
Content knowledge and confidence after student teaching	Content knowledge and confidence increased after student teaching	S.T.C.K. Content knowledge increased S.T.C.K. Confident after student teaching S.T.C.K. Confident in Addition and Subtraction after student teaching

Category	Category Meaning	Aligned Codes
Content knowledge after student teaching	Content knowledge stayed the same after student teaching	S.T.C.K. Content knowledge stayed the same
Pedagogy (methods) courses	Participant took at least one pedagogy (methods) course or took at least one course that included mathematics content and/or student knowledge while in college	Combo Class Combo Class – 2 courses only One course- content and pedagogy only Pedagogy Course Pedagogy Courses – 2 courses includes pedagogy and knowledge of students but no math involved Pedagogy and Student Knowledge Course Three classes – Content and Pedagogy Two Courses – content and pedagogy
Addition of fractions in Pedagogy (Methods) Course	Different ways participant learned how to teach addition of fractions	P.K.C. Add – Skits P.K.C. Add- Basic Instructional Methods to teach fractions P.K.C. Add – Line Up P.K.C. Addition- Instructional Methods- Visuals P.K.C. Addition- Instructional Methods- manipulatives
Subtraction of fractions in Pedagogy (Methods) Course	Different ways participant learned how to teach subtraction of fractions	P.K.C. Subtract – Skits P.K.C. Subtract- Basic Instructional Methods to teach Fractions P.K.C. Subtract- Line Up P.K.C. Subtraction- manipulatives

Category	Category Meaning	Aligned Codes
		P.K.C. Subtraction – number line P.K.C. Subtraction – visuals
Multiplication of fractions in Pedagogy (Methods) Course	The pedagogy for multiplication of fractions was not learned	P.K.C. Multiply- Did not learn
Division of fractions in Pedagogy (Methods) Course	The pedagogy for division of fractions was not learned	P.K.C. Division- Did not learn
Pedagogy (instructional methods) to solve an addition of fractions problem	Accurate knowledge	P.K. Problem on Screen- Fraction tiles- Accurate Answer P.K. Problem on Screen- Tape Diagrams- Accurate Answer P.K. Problem on Screen- Traditional method- Accurate Answer P.K. Problem on Screen- candy bar method- Accurate description P.K. Problem on Screen- clock method- Accurate description P.K.. Problem on Screen- money method- Accurate description P.K. Problem on Screen- Guided Practice- Accurate Answer P.K. Problem on Screen- fraction blocks and drawing a picture- Accurate description
Instructional methods during student teaching	Instructional methods the participant used during a lesson	Lessons- S.T.P.K. – step charts Lesson – S.T.P.K. collaborative groups

Category	Category Meaning	Aligned Codes
		Lesson-S.T.P.K. stations Lesson- S.T.P.K. Candy Bar Method Lesson-S.T.P.K.- Manipulatives Lesson-S.T.P.K- Traditional Method for addition and subtraction of fractions Lesson-S.T.P.K- Traditional Method for addition of fractions Lesson-S.T.P.K. – Visuals Lesson-S.T.P.K. – Whiteboards Lesson-S.T.P.K- fraction blocks Lesson-S.T.P.K- fraction strips Lesson- S.T.P.K. – Instructional Videos Lesson-S.T.P.K. Fraction discs Lesson-S.T.P.K. number lines Lesson-S.T.P.K. tape diagrams
Instructional methods learned prior to teaching	Participant needed to learn instructional methods prior to teaching a lesson	S.T.P.K. Instructional Methods- needed to learn a think aloud S.T.P.K. Instructional Methods- Needed to learn scaffolding S.T.P.K. Instructional Methods- Needed to learn them S.T.P.K. Instructional Methods- needed to learn some instructional methods

Category	Category Meaning	Aligned Codes
		S.T.P.K. Instructional Methods- Learned through cooperating teacher S.T.P.K. Instructional Methods – Learned through previous field work and while in grade school
Better instructional methods	Participant believes that there are better instructional methods that they don't know about	S.T.P.K. – Better instructional methods
Correlation between instructional methods and pedagogical (methods) courses	Different methods used during student teaching that was learned in pedagogical (methods) courses	S.T.P.K. – Instructional Methods used during student teaching (learned from pedagogical courses) – Gradual release S.T.P.K.- Instructional Methods used during student teaching (learned from pedagogical courses) Traditional methods for adding and subtracting fractions S.T.P.K.- Instructional Methods used during student teaching (learned from pedagogical courses) – Division (Flip Fraction) S.T.P.K. – Instructional Methods used during student teaching (learned from pedagogical courses) – Fraction Tiles S.T.P.K.- Instructional Methods used during student teaching (learned from pedagogical courses) – anchor charts

Category	Category Meaning	Aligned Codes
		S.T.P.K.- Instructional Methods used during student teaching (learned from pedagogical courses)- manipulatives S.T.P.K.- Instructional Methods used during student teaching (learned from pedagogical courses) – visuals
Pedagogy knowledge after student teaching	Pedagogy knowledge increased after student teaching	S.T.P.K. pedagogy knowledge increased
Pedagogy (student knowledge) courses	Participant took at least one pedagogy (student knowledge) course or took at least one course that included mathematics content and/or pedagogy (methods) while in college	Combo Course Combo Class- 2 courses only Pedagogy Courses- 2 courses includes pedagogy and knowledge of students but no math involved Pedagogy and Student Knowledge Course
No Pedagogy (student knowledge) course	Participant did not take a specific student knowledge course	Student Knowledge – No Course
Addition of fractions in Pedagogy (Student Knowledge) prior to student teaching	Participant knowledge of student thinking about addition of fractions prior to student teaching was non-existent or very minimal	S.T.S.K. Add- No knowledge S.T.S.K. Add- very minimal knowledge
Subtraction of fractions in Pedagogy (Student Knowledge) prior to student teaching	Participant knowledge of student thinking about subtraction of fractions prior to student teaching was non-existent or very minimal	S.T.S.K. Subtract – No knowledge S.T.S.K. Subtract – very minimal knowledge
Multiplication of fractions in Pedagogy (Student	Participant knowledge of student thinking about	S.T.S.K. Multiplication – No Knowledge

Category	Category Meaning	Aligned Codes
Knowledge) prior to student teaching	multiplication of fractions prior to student teaching was non-existent or very minimal	S.T.S.K. Multiplication- very minimal knowledge
Different ideas of multiplication of fractions in Pedagogy (Student Knowledge) prior to student teaching	Participant knowledge of student thinking about multiplication of fractions prior to student teaching produced different ideas	S.T.S.K. Multiply- did not like S.T.S.K. Multiply – Easy S.T.S.K. Multiply- Scared S.T.S.K. Multiply – Hard S.T.S.K. Multiplication- Butterfly Method S.T.S.K. Multiplication- Student would have limited knowledge of multiplication S.T.S.K. Multiply- Student would not have knowledge of the fraction operation
Division of fractions in Pedagogy (Student Knowledge) prior to student teaching	Participant knowledge of student thinking about division of fractions prior to student teaching was non-existent or very minimal	S.T.S.K. Division- No knowledge S.T.S.K. Division- very minimal knowledge
Different ideas of division of fractions in Pedagogy (Student Knowledge) prior to student teaching	Participant knowledge of student thinking about division of fractions prior to student teaching produced different ideas	S.T.S.K. Divide- did not like it S.T.S.K. Divide-Hard S.T.S.K. Division- Stuck S.T.S.K. Division- Student would not have knowledge of fraction operation
Wishful ideas prior to student teaching	Ideas participant wished they knew prior to student teaching that would have helped instruct students based upon their knowledge of student thinking	S.T.S.K. Access to Paperwork S.T.S.K. Differentiation S.T.S.K. Learning Instructional Methods S.T.S.K. – more understanding of student misconceptions

Category	Category Meaning	Aligned Codes
		S.T.S.K. – Having more mathematical courses in college S.T.S.K. Knowing about IEPs and English Language Learners S.T.S.K. Knowing more of a student's conceptual understanding of different topics S.T.S.K. Wished they knew more about fact fluency S.T.S.K. Wishing professors told them how students think S.T.S.K. reviewing with students what a fraction is S.T.C.K. Wished they knew about curriculum standards and profession of students
Pedagogy (student thinking) to solve an addition of fractions problem	Participant provided different ideas of student thinking	S.K. Problem on Screen- Do not know how to add fractions S.K. Problem on Screen- Fraction Line S.K. Problem on Screen- Fractions are not written vertical S.K. Problem on Screen- Misinterpretation from students S.K. Problem on Screen- Not the same denominator S.K. Problem on the Screen- clocks S.K. Problem on the Screen- money

Category	Category Meaning	Aligned Codes
General instructional methods	Instructional methods participant learned in their mathematics pedagogical courses	P.K.C Instructional Methods- clocks P.K.C Instructional Methods – drawing pictures P.K.C Instructional Methods- manipulatives P.K.C Instructional Methods- money P.K.C. instructional methods- traditional method for multiplication P.K.C. – instructional methods- word problems P.K.C. Addition and Subtraction- instructional methods- word problems P.K.C. Instructional Methods- Technology P.K.C. Instructional Methods- Did not learn in a mathematics pedagogy class P.K.C. Instructional Methods- No fraction operations were taught in a Mathematics Pedagogical Course P.K.C. Instructional Methods- Traditional methods P.K.C. Instructional Methods- number lines P.K.C.- Instructional Methods- Visuals
Pedagogy knowledge of students after student teaching	Participant's pedagogy knowledge of students after student teaching changed	S.T.S.K. Student Knowledge has changed compared to before student teaching

Appendix D: Research Questions, Themes, Theme Meanings and Aligned Categories

RQ1: (*knowledge of content*) What are pre-service elementary teachers, after student-teaching, perceptions of their knowledge or gaps in knowledge regarding their knowledge of content of fraction operations?	**Theme Meaning**	**Aligned Categories**
Theme 1: Pre-service elementary teachers have knowledge in addition and subtraction content of fraction operations.	Participants were able to accurately describe addition and subtraction of fractions.	Addition Content Subtraction Content Content of solving an addition of fractions problem
Theme 2: Pre-service elementary teachers have inaccurate knowledge of multiplication content of fraction operations.	Participants were not able to accurately describe multiplication of fractions.	Multiplication Content
Theme 3: Pre-service elementary teachers have both accurate and inaccurate knowledge of division content of fraction operations.	Some participants were able to accurately describe and some participants were not able to accurately describe division of fractions.	Division Content Knowledge Division Content
Theme 4: Pre-service elementary teachers have learned different ways on how to add and subtract fractions in their content course(s).	Participants were able to describe the different ways they learned how to add and subtract fractions in their content course(s).	Content Courses Addition of fractions in Content Course Subtraction of fractions in Content Course
Theme 5: Pre-service elementary teachers have not learned multiplication or division	Participants did not learn how to multiply or divide fractions in their content courses.	Content Courses Multiplication of fractions in Content Course

of fractions in their content course(s).		Division of fractions in Content Course
Theme 6: Pre-service elementary teachers have both positive and negative feelings about their content knowledge of fraction operations entering student teaching.	Some participants were able to express their positive feelings and some participants were able to express their negative feelings about content knowledge of fraction operations.	Content knowledge positive feelings entering student teaching Content knowledge negative feelings entering student teaching
Theme 7: Pre-service elementary teachers have to review fraction operations before teaching and they wish they learned about different topics prior to entering student teaching.	Participants needed to review fraction operations in different ways prior to teaching during student teaching. Participants also wished they learned specific topics prior to entering student teaching to help them with fraction operations.	Topics entering student teaching Reviewed fraction operations
Theme 8: Pre-service elementary teachers' content knowledge of fraction operations increased or stayed the same after student teaching.	Participants indicated that their content knowledge of fraction operations increased or stayed the same.	Content knowledge and confidence after student teaching Content knowledge after student teaching
RQ2: (*knowledge of pedagogy*) What are pre-service elementary teachers, after student-teaching, perceptions of their knowledge or gaps in knowledge regarding their knowledge of pedagogy of fraction operations?	**Theme Meaning**	**Aligned Categories**
Theme 9: Pre-service teachers have learned instructional methods to teach addition and subtraction of fractions in their pedagogy (methods) classes.	Participants were able to describe different instructional methods they learned how to teach addition and subtraction of fractions.	Pedagogy (methods) courses Addition of fractions in Pedagogy (Methods) Course Subtraction of fractions in Pedagogy (Methods)

		course Pedagogy (instructional methods) to solve an addition of fractions problem
Theme 10: Pre-service elementary teachers have not learned instructional methods to teach multiplication and division of fractions in pedagogy (methods) classes.	Participants did not learn instructional methods to teach multiplication and division of fractions in their pedagogy (methods) classes.	Pedagogy (methods) courses Multiplication of fractions in Pedagogy (Methods) Course Division of fractions in Pedagogy (Methods) Course
Theme 11: Pre-service elementary teachers have used the instructional methods learned during student teaching and the instructional methods they have learned in their pedagogy (methods) classes to teach fraction operations.	Participants learned instructional methods in different ways to teach fraction operations.	Instructional methods during student teaching Instructional methods learned prior to teaching Correlation between instructional methods and pedagogical (methods) courses General instructional Methods
Theme 12: After student teaching, pre-service elementary teachers believe there are better instructional methods to teach fraction operations and they also feel their pedagogy knowledge increased.	Participants believe that after the completion of student teaching, there are better instructional methods to teach fraction operations and that their pedagogy knowledge increased.	Better instructional methods Pedagogy knowledge after student teaching
RQ3: (*knowledge of students*) What are pre-service elementary teachers, after student-	**Theme Meaning**	**Aligned Categories**

teaching, perceptions of their knowledge or gaps in knowledge regarding their knowledge of students of fraction operations?		
Theme 13: Prior to student teaching, pre-service elementary teachers have non-existent or very minimal pedagogy (student knowledge) in addition, subtraction, multiplication, and division of fractions.	Participants described that they had non-existent or very minimal pedagogy (student knowledge) in addition, subtraction, multiplication, and division of fractions.	No Pedagogy (student knowledge) course Addition of fractions in Pedagogy (Student Knowledge) prior to student teaching Subtraction of fractions in Pedagogy (Student Knowledge) prior to student teaching Multiplication of fractions in Pedagogy (Student Knowledge) prior to student teaching Division of fractions in Pedagogy (Student Knowledge) prior to student teaching
Theme 14: Pre-service elementary teachers have different ideas for pedagogy (student knowledge) in multiplication and division of fractions.	Participants provided different ideas for multiplication and division of fractions for pedagogy (student knowledge).	Different ideas of multiplication of fractions in Pedagogy (Student Knowledge) prior to student teaching Different ideas of division of fractions in Pedagogy (Student Knowledge) prior to student teaching
Theme 15: Pre-service elementary teachers have different ideas of student thinking and they also have ideas they wish they knew prior to student teaching.	Participants provided different ideas they wish they knew prior to student teaching that would have helped instruct students based upon their knowledge of student thinking. They also provided different ideas of student thinking when	Wishful ideas prior to student teaching Pedagogy (student thinking) to solve an addition of fractions problem

	looking at a fraction addition problem.	
Theme 16: Pre-service elementary teachers understanding of pedagogy knowledge of students changed after student teaching.	Participants stated that their pedagogy knowledge of students changed after student teaching.	Pedagogy knowledge of students after student teaching