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Using Reflection to Promote Success in Community College Mathematics

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This is to certify that the doctoral dissertation by

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Abstract

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Dissertation Submitted in Partial Fulfillment

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Abstract

Community college mathematics faculty have instituted changes to address inadequate preparation of incoming students to learn college-level mathematics. However, they could do more to develop student cognitive capabilities to enable underprepared students to successfully learn college mathematics. This basic qualitative study examined the perceptions of community college mathematics instructors who made their classrooms more reflective, including using reflection as an instruction activity for students entering college underprepared to learn college-level mathematics. The conceptual framework consisted of Baxter Magolda's epistemological reflection model of college student development and Schoenfeld's theoretical model for teaching mathematics. Open coding of interviews with 10 community college mathematics instructors in the United States suggested instructors agreed that improvement in teaching mathematics is needed. They reported that reflection fostered development of epistemological and cognitive abilities of underprepared students who were newly successful in learning mathematics. Instructors perceived that reflection activities improved student learning outcomes, with reflection first starting individually and then shared by explaining solutions to the instructor and other students, leading to a growing confidence. In addition, instructors noticed that reflection improved their own teaching practice over time. This study may contribute to positive social change by helping community college faculty and leaders better understand how to improve learning outcomes for underprepared students to become newly successful in learning mathematics and by doing so, remain enrolled in community college and complete their degrees.

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Table of Contents

List of Tables

Chapter 1: Introduction to the Study

Education scholars have advocated reflection to develop underprepared students' cognitive capabilities for achieving success in college mathematics (Baxter Magolda, 2020; Schoenfeld, 2021). Reflection relates to monitoring, regulating, and assessing one's learning and is a factor in feedback, formative assessment, self-regulation, and metacognition (Reinholz et al., 2019). According to Brown (1978), reflection is the process of active thinking that stimulates cognitive development and can ultimately result in metacognition, a reflective practice related to problem solving. As a solitary activity, reflection involves an internal dialogue, fitting ideas together to clarify what is being learned. The reflective process also occurs during external dialogue and conversations. Reflective conversation is known to foster and accelerate cognitive development (Bamberger & Schön, 1983; Erdogan, 2019; Schön, 1992; Williams & Ryan, 2020). Reflection can happen any time a learner puzzles over an interesting question, and it need not happen just in formal settings. Yet, learning interventions have often overlooked reflection, perhaps because it is difficult to define or measure results, or because reflective thinking as not considered worthwhile (see Casey, 2014; Dyment & O'Connell, 2011; Romriell, 2020).

In this chapter, I provide background information and a problem statement to describe the issue's relevance to the success of learners. I present the purpose of the study, research questions, conceptual framework, nature of this study, and definitions of key words and phrases. A discussion of assumptions, scope, delimitations, and limitations of the study follows. I conclude the chapter with an explanation of the significance of the study and a summary of this chapter.

Background of the Study

I proposed to examine the experiences of mathematics instructors applying reflective practices in community college classrooms for students who are entering college underprepared in mathematics. Although community colleges assess most entering high school graduates as needing remediation in mathematics (Nix et al., 2021), many of these students do not even start remediation, let alone complete it, making college graduation or transfer impossible (Logue et al., 2019). Because mathematics requirements are a barrier for many students, some community college mathematics instructors began using new approaches to foster college student intellectual development and mathematics achievement (Barhoum, 2018; Brower et al., 2018; Landry et al., 2018; Perin, 2018; Perin & Holschuh, 2019; Rodriguez et al., 2018; Rutschow, 2018; Vick et al., 2018). To expand the reform, California policymakers enacted legislation in 2018 requiring all California community colleges to reform remediation for mathematics (see California Assembly Bill 705; Rodriguez et al., 2017). However, passing a law did not guarantee students could learn college math successfully (Reyna, 2020).

The problem of accepting entering students who are assessed as underprepared to learn college mathematics needs to be addressed. A possible remedy, epistemological reflection, according to Baxter Magolda (1992), has been shown to foster and even accelerate the development of underprepared college students and is where the developing learner comes to believe some ideas are more valid than others. This has been almost entirely overlooked in studies on college mathematics instruction, according to Schoenfeld (2019a, 2019b, 2020); perhaps this is because epistemological reflection and reflective instruction are difficult to deliver (Schoenfeld, 1988, 2014). Yet, reflective instructional practices have been shown to foster and even accelerate the cognitive understanding of underprepared college students (Baxter Magolda, 2020; Perry, 1970, 1981; Schoenfeld, 2021).

Although reflection is known to benefit the development of students (Baxter Magolda, 2004a, 2004b, 2010, 2014, 2020), only a small percentage of instructors have been found to have tried working with reflection, and who, as a result, have successfully implemented reflective instruction practices as part of the shift to becoming student-centered (Schoenfeld, 2021). Hassi and Laursen (2015) reported that instructors using reflection achieved promising results in fostering successful learning for underprepared students at community college. Schoenfeld's theoretical model of teaching mathematics applies to the role of reflection in support of developing mathematically empowered community college students. Over time, reflection can be internalized by the student and instructor as part of the training of a person's thinking (Dewey, 1933). Ultimately, later in life, as a person's ability to think develops, becoming reflective can become part of a tool kit of capabilities that can be used to solve problems professionally.

There is a scarcity of research on the benefits of reflection and reflective learning in community college mathematics classrooms, which contrasts with a wide variety of research conducted on the power of reflection on learning and improving instruction. For example, qualitative research conducted on the power of reflection for professional students, including nursing students in training in their professional education in the United Kingdom (Bulman et al., 2014); first-year law school students at Cal Western School of Law in San Diego (Casey, 2014); young clergy students in their post-graduate educational training in Australia (Foster, 2018); and student teachers learning the role of reflection in changing their teaching practice in Hong Kong (Cheung & Wong, 2017). In quantitative studies, reflection was shown to improve learning performance by as much as 25%, with learning comprehension lasting longer than without reflection. Di Stefano et al. (2016) found that when new employees who were recent college graduates spent as little as 5 to 10 minutes at the end of each day reflecting on new learning, rather than just continuing to practice a skill, their learning and retention were significantly increased. Another recent quantitative investigation on the role of reflection in student learning outcomes by a researcher at Stanford, Salehi (2018) found undergraduate engineering students significantly improved their learning and retention with reflection. These quantitative and qualitative studies have demonstrated a broad interest in various fields on the role of reflection.

Problem Statement

Mathematics is a gatekeeper for success in college (Douglas & Attewell, 2017), but nearly two-thirds of new high school graduates in the United States enter community college underprepared to learn college-level mathematics. Even worse, students assessed as needing to take remedial math are not likely to start, let alone finish these classes, so they cannot graduate from college (Logue et al., 2019). For this reason, remedial math has been named as the biggest barrier to completing college in the United States (Ganga et al., 2018; McKinney et al., 2019). Students assessed as needing remedial math may have significant cognitive limitations in how they learn (Hunter, 2017). Thus, the research problem addressed in this study pertains to the possible need for greater epistemological reflection and cognitive development of incoming college students to be ready to learn college-level mathematics (Baxter Magolda, 2020; Perry, 1970, 1981; Schoenfeld, 2021). I studied the perceptions of mathematics instructors who use reflective instruction practices in fostering achievement of underprepared students in learning college mathematics. Epistemological reflection and reflective instruction improved young college students' cognitive development and learning outcomes in college mathematics (Baxter Magolda, 2020; Schoenfeld, 2020). As a result of community college instructors' use of reflective instruction for cognitive development, students who otherwise might have had to leave college have been found to have newfound success in learning mathematics (see Cox & Dougherty, 2019; Hassi & Laursen, 2015; Phillips, 2019). Despite these successes, only a small percentage of faculty members have made use of reflective approaches to foster and accelerate student intellectual and cognitive development in college-level mathematics instruction (Anseel & Ong, 2020; Kersey et al., 2018), in part because reflection is difficult to teach and to learn (Schoenfeld, 2019a, 2019b).

Students assessed as needing remedial math may have significant cognitive limitations in how they learn (Hunter, 2017), perhaps indicating a need for epistemological reflection and cognitive development of incoming college students to be ready to learn college-level mathematics (Baxter Magolda, 2020; Perry, 1970, 1981;

Schoenfeld, 2020). In their extensive grounded scholarship, Baxter Magolda and Schoenfeld have demonstrated that college students' cognitive understanding may be improved through reflection. To meet this challenge, some mathematics faculty members at community colleges have recognized the need to change mathematics instruction practices to include reflection to foster student epistemological and cognitive understanding as well as improve student learning outcomes (Kersey et al., 2018).

Purpose of the Study

In this basic qualitative study, I explored perceptions of mathematics instructors at U.S. community colleges on their use of reflection to foster epistemological and cognitive development and academic success of students who entered college underprepared to learn college math. In particular, I explored perceptions of community college mathematics instructors with regard to: (a) what they perceived about how reflection developed epistemological and cognitive abilities of their underprepared students who were newly successful in learning mathematics, and (b) how they used reflection to improve instruction of their underprepared students. Logue et al. (2019) found that many students assigned to take remedial math upon enrollment in community college do not start their remedial courses, let alone finish them. In response, community college faculty have sought ways to improve mathematics instruction to help students overcome the barrier of remediation, and as a result have become successful in learning mathematics (Logue et al., 2019).

Research Questions

Two research questions guided the study with regard to community college students who entered college underprepared to learn college mathematics.

RQ1: What are community college mathematics instructors' perceptions of how reflection developed epistemological and cognitive abilities of their underprepared students newly successful in learning mathematics?

RQ2: How do community college instructors use reflection in instruction in the classroom to improve academic outcomes of their students?

Conceptual Framework

For the conceptual framework, I drew on two theoretical lenses. Baxter Magolda's (2020) epistemological reflection model involves fostering and accelerating recent high school graduates' cognitive understanding. Schoenfeld (2021) conducted extensive analyses of instructional approaches used to foster students' learning to solve problems in college classrooms. Schoenfeld's model of teaching mathematics is domain-specific to instruction of college-level mathematics and problem-solving, as compared to the broader model of Baxter Magolda. I further discuss these models and provide foundational origins with a more detailed analysis in Chapter 2, where I introduce several other supporting theories.

Nature of the Study

Using a basic qualitative approach, I interviewed 10 community college mathematics instructors in the United States to more thoroughly understand how they use reflective instruction practices to foster achievement in underprepared students in college mathematics and their perceptions of how reflection was used to develop cognitive abilities of their underprepared students who were newly successful in learning mathematics. I collected data through semi-structured and individual interviews (Maxwell, 2012; Merriam & Grenier, 2019) using Zoom. Qualitative methodology is considered efficient in acquiring evidence and gaining understanding through description and discovery (Merriam & Grenier, 2019). Percy et al. (2015) noted that basic qualitative inquiries involve examining phenomena that are specific to unique groups, individuals, and research topics, as well as discovering perspectives and worldviews of participants.

During interviews, I asked mathematics instructors what perceptions stood out for them as they used reflective instruction approaches while working with their underprepared students, as well as their perceptions of the role of reflection in fostering learning among their underprepared students, who have achieved newfound success in learning. Participants were recruited from faculty at U.S. community colleges, and members of the American Mathematical Association of Two-Year Colleges (AMATYC), especially those involved with the AMATYC committee on Improving Mathematical Prowess and College Teaching as a way of identifying faculty who are dedicated to using reflection in improving instruction practices, as well as any colleagues they recommended to be interviewed.

Definitions

I adopted the following definitions of key terms:

Cognitive development: Piaget (1972) theorized that a young adult’s intellectual and cognitive development can be accelerated.

Corequisite remediation: Hern (2019) explained this as a strategy involving students assigned to remediation who are also enrolled in for-credit math classes, while simultaneously being provided with additional intensive academic support.

Design experiment or design-based research: Brown's (1992) concept of design-based research in education and the learning sciences is a methodology used by Schoenfeld to investigate complex systems and mechanisms of interactions among students, instructors, and the learning environment in support of student learning, and is used for designing a solution by testing it in a real-world setting and using emerging evidence from studies to revise intervention strategies in real time.

Epistemological development: Pertains to individuals' beliefs about knowledge and cognitive understanding. Kitchener (1984) discussed assumptions of epistemological development, certainty, and sources of knowledge. Baxter Magolda (1992) described student epistemologies and development as a foundation for reshaping pedagogy.

Metacognition: Schoenfeld (1987) described metacognition as a learner management issue related to problem solving where the learner is able to understand a problem to be solved and to bring to bear the problem-solving resources to solve the problem. Developing good metacognition or problem-solving skills depends upon the use of reflection.

Reflection: In general terms, reflection refers to thinking critically and deeply, and evaluating the effectiveness of one's own learning, with an eye toward making changes to improve. Dewey (1933) described reflection as an active consideration for the purpose of learning and improving performance. In this dissertation, reflection is broken down into three aspects:

Epistemological reflection model: When students think deeply about what they have learned, reflecting on their learning to make meaning related to the learned material (Baxter Magolda, 2004a, 2004b, 2010, 2014, 2020).

Reflective instruction: An instructional strategy involving instructor thinking or reflection about what the learner has experienced during the learning process. Learner reflection is an activity that can be prompted by the instructor by having learners talk to each other in pairs or groups, or by writing in a journal, with the instructor guiding the thought process (Schoenfeld, 2015, 2016, 2018, 2019a, 2019b, 2020).

Reflective learning: Schoenfeld (1992, 2010, 2014, 2015, 2016, 2018, 2019a, 2019b, 2020) claimed reflective learning involves a process of reviewing when a new aspect is learned, and seeing what mistakes were made and how to correct or improve them for next time.

Assumptions

I assumed that community college mathematics instructors I interviewed had perceptions that were relevant to my research questions involving how reflection developed epistemological and cognitive abilities of their students and how community college instructors use reflection during instruction in the classroom to improve academic outcomes of their students. I also assumed the instructors would be willing to reflect on these topics and discuss them in response to my interview questions. I entered this study with a conviction that reflective instruction approaches improve student learning outcomes, and that improvements in learning could result from reflective instruction practices (Baxter Magolda, 2020; Schoenfeld, 2021, but I tried to mitigate my biases with open-ended interview questions and careful listening, as well as applying objectivity during data analysis. I assumed that by framing interview questions and pursuing interviews in an inquiring manner rather than as an interrogation, I was able to support my interviewees as they reflected openly about their perceptions, including challenging situations they have faced, and this provided me with in-depth responses to my interview questions.

Scope and Delimitations

This study involved 10 mathematics instructors who have worked with reflective instruction with underprepared college students. I delimited my study to mathematics instructors who had at least 2 years of experience teaching in community colleges.

Limitations

Because results are based on perceptions of a limited number of participants, I did not intend results to be generalized beyond participants in the study. While the study may be of interest and suggest questions for other studies, I did not intend for my study to yield either directly transferable results or be replicated in a subsequent study.

Claims regarding results of qualitative interview studies are confined to the perceptions, views, and understandings of a limited number of interviewed individuals, so although the results may attract attention by others interested in the topic, they are not generalizable beyond the individuals I interviewed in the study. According to Bloomberg and Volpe (2018), transferability in qualitative research is not based on whether a representative sample is included in the study, but rather how closely the sample data can provide readers with an understanding of their own settings. Reliability of results depends on my performance as an interviewer and quality of interview questions I developed, as well as my systematic analyses and interpretations of transcript data I generated based on interviews.

Significance of the Study

Results of this study may be timely and useful as more faculty see the need for and benefits of reflective approaches. Lack of research involving reflection in community college mathematics instruction is necessary to correct because studies on the role of epistemological reflection may provide evidence that is needed to support improvements in instruction. Findings of this study may address a gap in knowledge regarding the role of epistemological reflection in mathematics instruction for young college students. Through my exploration, I sought to generate insights regarding perceptions of mathematics instructors who are making use of reflective instruction in community colleges. This study will contribute to positive social change by better understanding how to improve learning outcomes for underprepared students who need to learn mathematics to remain enrolled in a community college and complete a degree.

Summary

To further understand the complexities of reflective instruction practices which guide student success in mathematics, I explored perceptions of instructors in U.S. community colleges and their perceptions of results among underprepared students due to reflective approaches. Baxter Magolda's (2020) epistemological reflection model and

Schoenfeld's (2021) model of teaching mathematics were the conceptual framework. In this chapter, I explained the background of the issue and associated research concepts, provided a description of the problem, and included the purpose of the study, research questions, nature of the study, and its significance. Results have a potential to address reflective practices of mathematics instructors at community colleges and aid faculty who are facing similar challenges.

In Chapter 2, I explain my literature review strategies, discuss Baxter Magolda's epistemological reflection and Schoenfeld's reflective instruction practices for college mathematics, and review recent empirical research studies I found that were related to the research problem. I conclude Chapter 2 with a discussion of my literature review and how my study addressed a gap in research.

Chapter 2: Literature Review

Students assessed as needing remedial math may have significant cognitive limitations in how they learn (Hunter, 2017), and the research problem addressed in this study pertains to the possible need for greater epistemological reflection and cognitive development of incoming college students in order to be ready to learn college-level mathematics. I studied perceptions of mathematics instructors who use reflective instruction practices for fostering achievement of underprepared students in learning college mathematics. Epistemological reflection and reflective instruction improved young college students' cognitive development and learning outcomes in college mathematics (Baxter Magolda, 2020; Schoenfeld, 2021). As a result of community college instructors' use of reflective instruction for cognitive development, students who otherwise might have had to leave college have been found to have newfound success in learning mathematics (see Cox & Dougherty, 2019; Hassi & Laursen, 2015; Phillips, 2019). Despite these successes, a small percentage of faculty members have made use of reflective approaches to foster and accelerate student intellectual and cognitive development in college-level mathematics instruction (Anseel & Ong, 2020; Kersey et al., 2018), in part because reflection is difficult to teach and to learn (Schoenfeld, 2019a, 2019b). This literature review involved analyzing instructional strategies and outcomes of community college mathematics faculty using reflection and reforming instruction practices for incoming students who were assessed as underprepared and became newly successful mathematics learners.

In Chapter 2, I describe my literature search strategies and conceptual framework. This is followed by a review of additional theories and studies related to the conceptual framework. I then analyze empirical literature addressing teaching practices that fostered underprepared college students' epistemological and cognitive development in terms of mathematical problem-solving. I conclude the chapter with a summary of key factors related to the research problem.

Literature Search Strategy

I reviewed literature that involved fostering achievement in underprepared community college students in learning college-level mathematics, with studies focused on remedial and developmental mathematics. Initially, I conducted a literature search of sources that were published between 2018 and 2022. I searched the following databases using the Walden University Library: Education Source (Education Research Complete), EBSCOHost, ERIC, ProQuest, Teacher Reference Center, and SAGE Premier. In addition, I used Google Scholar for access to databases and scholarly articles.

I used search terms in an iterative manner to identify and compile literature related to the research questions and conceptual framework and review empirical literature. I conducted an initial search using the following search terms: *community college mathematics*, *remedial math in community college*, *community college mathematics success*, *reflective learning in community college mathematics*, *reflective instruction college mathematics*, *reflective learning practices*, *community college mathematics*, *college student cognitive development*, *mathematics education design experiments*, *educational design research to improve community college mathematics* *education*, *research methods in community college mathematics education*, and *improving educational research in community college mathematics education*.

My exhaustive literature search resulted in relatively few studies involving helping new high school graduates' lack of preparation for learning mathematics in community college. Because I found few studies, I expanded my search to include additional aspects of the problem. Revised search terms were *college students*, *experiments in mathematics instruction*, *fostering cognitive development*, *incoming underprepared students*, *student reflection in undergraduate instruction*, *underprepared in mathematics*, and *community college*. More robust results included scholarly studies as well as recent dissertations, conference proceedings, international studies, and studies from parallel fields in education and professional development.

Conceptual Framework

The work of two theorists, Baxter Magolda (2020) and Schoenfeld (2021), guided the study. Specifically, Baxter Magolda's epistemological reflection model emphasizes how learning is intertwined with epistemological beliefs. Baxter Magolda's model illustrates how instructors can create conditions that promote learning, foster reflective development, and accelerate cognitive development by questioning assumptions.

Schoenfeld (2021) introduced a model of teaching in a context specifically in the mathematics classroom with design-based research approaches to fine-tune the model. Brown's concept of design-based research in education and the learning sciences is a methodology used to investigate complex systems and mechanisms of interactions among students, instructors, and learning environments in support of student learning, and is used for designing solutions by testing them in real-world settings with emerging evidence from studies to revise intervention strategies in real time.

Schoenfeld's (2013) model of Teaching for Robust Understanding (TRU) describes quality dimensions for teaching practices. First designed for professional development of mathematics instructors in 2013, TRU was later used as a research tool (Schoenfeld, 2018; Schoenfeld et al., 2018, 2020). TRU teaching model involves reflection as an element of effective mathematics thinking and learning in community college mathematics classrooms. Peer-assisted reflection assists students to learn metacognitive skills that are needed for problem solving during the transition from external feedback to internalized self-regulation.

The focus of Schoenfeld's TRU model is student thinking and use of reflection to address new student learning and questions during each step of problem solving. The TRU framework involves five dimensions in a learning setting. The first dimension is mathematics content that is taught, as well as practices needed for effective learning of mathematical ideas. The remaining four dimensions are cognitive demand, equitable access, agency, ownership, and identity, and formative assessment. These involve what students perceive in the mathematics classroom and impacts of those perceptions.

Schoenfeld et al. (2018) documented a positive association between scores for classroom practices based on the TRU classroom rubric and student performance in math proficiency. Students from classrooms that rate well in the TRU five dimensions became increasingly knowledgeable, flexible, and resourceful as mathematical thinkers and problem solvers. According to Schoenfeld (2020), success or failure in problem solving is not based on what students know but in how they put knowledge to use, meaning that even academically successful high-school students might not be prepared with adequate reflection strategies for college. Schoenfeld identified underprepared students as likely to have preconceptions and misunderstandings that are wrong, and that they consistently misinterpret what they are being taught. Schoenfeld et al. (2020) found student beliefs are an important determinate of students' learning in a college mathematics classroom. For example, some students believe that mathematics is beyond the capability of ordinary individuals like themselves, and so they accept and memorize what is handed to them without trying to make sense of it (see Schoenfeld, 2020). As a result, Schoenfeld found that knowing a lot of mathematics will not do a student very much good if beliefs keep the student from using it. Without good reflection skills, in Schoenfeld's words, students may *go off on wild goose chases* and so, never have the opportunity to make use of the mathematics they have learned.

In his studies using external prompting, Schoenfeld moved students from reflecting after the fact to internalize reflection activities in the moment. He found that a student can acquire higher-order skills with the help of others in small groups by considering and balancing multiple perspectives and then internalizing those skills, which is a justification for the use of small groups. Schoenfeld (2019a, 2019b) found that the small-group approach developed a culture of working together, interactively discussing a problem, explaining it to each other, sharing the false starts and struggles, and enjoying the fun of interacting.

Contributing Theories

Other related theories contributed to the work of Baxter Magolda and Schoenfeld and support how mathematics faculty develop their reflective teaching practice and facilitate student success. Learning through reflection goes back to Dewey (1933) as a response to a learning experience with doubt or conflict and can transform it into a clear and coherent understanding. An instructor creates situations to engage students in problem solving (with doubt and conflict), assisting in their reflective discovery, and in developing their reflective thinking (Dewey, 1933).

Building on these ideas, Brown (1978) and Flavell (1979) and their colleagues researched student reflection and metacognition in the 1970s (see Brown et al., 1983). Brown (1980/2017) described student reflection and metacognition as the knowledge and regulation of an individual's own thought processes, contributing to achievement and learning, where reflection is essential for promoting self-guided learning. Reflection is a process of active thinking that stimulates cognitive development and can ultimately result in metacognition (see Brown, 1978; Brown et al., 1983). Metacognition can be a result of reflective practice and relates to how an individual solves problems (see Brown, 1978).

Schön (1984, 1987), reflecting on Dewey's theory of inquiry, argued that individuals have a large amount of knowledge they can access only by doing something actively (Bamberger & Schön, 1983). In actively doing, the learner has a "reflective conversation" with the situation, testing conjectures, providing information to guide the next decision. Reflection takes place after the fact, in the moment, and even continuously**.** Being reflective means being able to perceive what is going on in the moment and to make needed adjustments in real time. After the fact, it is also important to reflect on what went on, where the learning did not happen as expected, what could be done differently next time, and what worked well.

Schön (1984) discussed reflective conversations as a way for students to understand, define, and solve a problem. Reflective conversations between an instructor and new college students can help develop student cognitive abilities as they think through different perspectives during problem solving, especially for students if they started college underprepared to learn college mathematics (Schoenfeld, 2020). Building on the work of Schön, Schoenfeld (1985) identified the role of reflection in effective mathematics thinking and learning in mathematics classrooms (see for comparison Baxter Magolda & King, 2008, on reflective conversations).

Piaget (1972) posited that cognitive development forms a set of evolving platforms for learning, with an emphasis on development. Piaget saw cognitive development as occurring within the learner as a result of reflective thinking. In this view, an instructor placing an emphasis on educational outcomes can be harmful developmentally to a person. For example, under pressure a student may memorize a formula and sidetrack the cognitive development of the mental capacity necessary for comprehending a formula.

Building on Piaget's (1972) work, Perry (1970, 1981) conducted studies on college students and ways they understand what and how they learned, where cognitive and intellectual development among college undergraduates was observed and delineated, including their reflections and assumptions about their own learning. Perry's (1970, 1981) model stands as a classic model for intellectual and cognitive development in college students, documenting the odyssey of 1960s era undergraduates at Harvard as they moved from what Perry (1981, p.79) termed “dualistic ‘right and wrong’ thinking” (p.79) to the full embrace of “contextual relativism” and the necessity to make “tentative wholehearted commitments” (Perry, 1981, p.79) that evolve in response to engaging in an uncertain and changing world. The cognitive developmental journey Perry described can be emotional, especially as it relates to learning mathematics. Students can experience pain and confusion from a lack of success, or they can get excited, resolve to triumph, and experience the pride that comes with success in learning mathematics.

Building on Perry’s (1970, 1981) pioneering work, Baxter Magolda (2020) integrated faculty and student development theory with Perry’s scheme. Baxter Magolda devised a model of epistemological reflection based on patterns of thinking that promote transformational change in a college student’s developmental journey, including patterns having some association with gender. Students reimagine their expectations for the classroom experience, the authority of the instructor, and of peers, and other factors. Baxter Magolda argued that instructors can design instruction and interaction to reach students at every level of cognitive development to improve learning, develop complex reasoning, and promote skill acquisition. With a reflective instruction orientation, an instructor does not merely lecture to transfer knowledge directly to a college student. Instead, instructors create situations that engage college students in problem solving, which helps develop students’ cognitive thinking capabilities.

Through the preceding analysis of the conceptual framework, I have explored theoretical underpinnings of reflection in regard to the instruction of mathematics. In the following literature review, I report and evaluate current literature related to the conceptual lens I have described. I addressed the role of reflection (a) as an engine of cognitive development in the first 2 years of college, and (b) in learning college mathematics for students who entered college underprepared to learn college mathematics experiencing newfound success learning college mathematics, who might otherwise have had to leave college.

Literature Review

In this section, I review the recent literature on reflection related to two aspects, as suggested by research: reflection fostering cognitive development of underprepared undergraduate students and developing reflective instruction to change how college mathematics is taught at community colleges thereby underprepared students could have newfound success with learning mathematics.

Reflection Fostering Cognitive Development of Underprepared Undergraduate Students

Students entering college underprepared to learn college mathematics may need cognitive development. Traditional approaches to remediation have failed to help underprepared students achieve successful learning outcomes in college mathematics (Reese-Cavanaugh, 2019; Rutschow et al., 2019). Although underprepared students need mathematics instruction that goes beyond simple remediation, they might come to college not cognitively ready to learn college-level mathematics (Joiner, 2020; Sagna, 2019;

Uretsky et al., 2021; Varner, 2018). Iordanou et al. (2019) found that in remediation students memorize rather than understand the math they are trying to learn. Similarly, Sagna (2019) and Er (2018) found that in customary remediation, students merely memorized procedures to satisfy a requirement rather than thinking through the problem solving to understand and learn the mathematics in a meaningful way. Shaw et al. (2020) found that, in part because of rigid approaches to teaching mathematics in K–12 classrooms, many college students were not cognitively ready to learn college mathematics.

Mathematical thinking requires flexibility. Recognizing the problem with traditional remediation, Boylan et al. (2019) studied a different focus for underprepared college students to succeed in math. Ganga et al. (2018) studied the need for a sustained and intensive approach to foster and accelerate student cognitive development. Schoenfeld (2020) studied a dialog-based approach to improve how students learn reflective problem-solving skills in college mathematics. Iordanou et al. (2019) found that arguing enhances student reflection and can improve learning outcomes. McAnally (2019) studied alternative ways of providing support to students, including offering corequisite remediation, providing opportunities for student reflection. Along these same lines, Glen (2019) studied alternatives to the usual remediation for underprepared community college students. To achieve newfound success in learning college-level mathematics, reflection is known to foster, and even accelerate intellectual and cognitive development of underprepared college students (Anseel & Ong, 2020).

Epistemological Reflection: Fostering Cognitive Development of College Students in the First 2 Years

Reflection is known as an engine of student cognitive development and learning success (Bronfenbrenner, 1993). Boylan et al. (2019) found that most instructors do not guide students to plan their tasks or to self-assess their academic work or provide opportunities to work together with study partners (Rutschow et al., 2019). Thanh (2020) demonstrated self-assessment and student reflection activities can be useful as a way to motivate students to reflect and improve learning outcomes, as supported by Baxter Magolda's epistemological reflection model.

For example, instructors seldom give students a choice of tasks to work on (Boylan et al., 2019) or choices of methods for carrying out complex assignments. Most instructors do not guide students with regard to their beliefs about themselves as learners (Baxter Magolda, 2020). Aditomo (2018) surveyed 1,366 Indonesian college students to explore how their epistemic beliefs are foundational to learning outcomes. Findings were that more complex epistemological beliefs were associated with higher grade-point averages. Aditomo also found that epistemic maturity was associated with the practice of student reflection, resulting in better academic performance. Thus, epistemic reflection drove performance in the first 2 years of college and fostered college students' cognitive development (Aditomo, 2018). If students struggle academically, they may have incorrect views (such as that science does not change), rather than having any lack in their ability. For these students, reflection can benefit their learning by either presenting a learning challenge to grapple with, or as a method for gaining a new perspective, which improves cognitive development (Aditomo, 2018). This awareness of beliefs about knowledge can be empowering for both students and instructors. Epistemological reflection and cognitive development and beliefs were found to be a result of education (rather than as a personal deficiency that cannot be remedied) and should be easier to change (Baxter Magolda, 2020). Cognitive development has transdisciplinary benefits for students that go beyond math class (Mevarech et al., 2018). For this reason, epistemic beliefs are good targets of educational interventions to foster college student cognitive development.

As I described earlier, research conducted worldwide in various classrooms in fields other than mathematics has shown the power of reflection in improving student learning. Researchers have examined nursing students in the United Kingdom (Bulman et al., 2014); (b) first-year law school students at Cal Western School of Law in San Diego (Casey, 2014); and (c) young clergy students in Australia (Foster, 2018). Casey (2014) and Di Stefano et al. (2016) reported that students objected to taking the time to reflect, in the belief that the time was not well used, confirming the tendency for students to memorize rather than undertake the hard work of thinking (Salehi, 2018). Di Stefano et al. (2016) found that reflection improved learning by 25%, by adding meaning for the learner, and with learning comprehension lasting longer than without reflection in a quantitative on the power of reflection in examining the professional development of recent college graduates. In this study, when recent college graduates spent as little as 5 to 10 minutes a day in reflection on new learning in their training, rather than spending the same amount of time to continue practicing a skill, learners significantly increased both their learning and retention. For successful learning and problem solving, reflective practices are essential (Schoenfeld, 1992). To test this notion, Salehi (2018) found undergraduate students who were provided with guided reflection training (by inserting guidance on reflection at various stages of problem solving) improved their learning by about 25% in comparison with the control group who practiced self-reflection.

Benefits of Reflecting at Each Step of Problem Solving

Teaching students to think independently through reflection is one objective of education. Past efforts to improve students' reflection were helpful in improving student learning but also had drawbacks, such as Garofalo and Lester's (1985) cognitive-metacognitive learning model (CML), inspired by Schoenfeld's (1983) reflective problem-solving scheme. Schoenfeld defined reflection and metacognition as one's awareness of cognition (i.e., thinking about one's thinking) and regulating reflection continuously during problem solving.

From this, Garofalo and Lester (1985) developed a framework, going beyond previous studies focusing on memory, to get a better understanding of reflection and metacognition skills in problem solving, by having a student reflect on how it went after solving a problem. However, Muhali et al. (2019) noticed that Garofalo and Lester's (1985) CML model conducted reflection only after the learning had occurred; thus, students were limited in their opportunities to reflect on learning in process and to correct misconceptions.

Muhali et al. (2019) were then inspired by the idea of modifying the reflection model to improve learning by increasing student opportunities for reflection, so in their new approach, reflection was inserted at each step of problem-solving and also included social processes with an emphasis on learning through collaboration and interaction with others. The basis of this was the understanding by Muhali et al. (2019) that social learning creates learning conditions needed so students could reflect with others on their thinking processes (Yaacob et al., 2021), and on their own with self-reflection by internalizing the voices of others. Muhali et al. (2019) inserted reflection at every step of problem solving, instead of waiting to reflect until afterwards, and as a result, student learning outcomes were significantly improved (ranging from an improvement of 5% to 9.3% in student learning outcomes, above the improvement of reflection only after problem solving).

Benefit of Reflective Conversations in Accelerating Cognitive Development

Epistemological reflection can mean answering the "how I know" question by listening to and reflecting on one's own voice. However, reflective conversations stimulate cognitive development and learning (see Dewey, 1933; Schön, 1984), where an individual has a conversation in the search for a solution needed to reframe the context needed to better to define a problem. When someone engaged in reflective conversations with others, the teaching changed from a traditional classroom monologue (lecture) to a dialogue or a reflective conversation that powers, fosters, and accelerates college students' cognitive development, as described in recent research (Anseel & Ong, 2020; Iordanou et al., 2019). Ståhl (2020) and Teoh et al. (2020) studied when reflective conversations in the struggle and search to better define or understand a problem, can help stimulate cognitive development and learning to develop young college students' cognitive abilities as they think through different perspectives, especially if students started college underprepared to learn mathematics (see Baxter Magolda, 2004a, 2004b, 2014, 2020; Dewey, 1933; Schön, 1984).

Changing How Mathematics is Taught in the Community College Classroom with Reflective Instruction

There is a recognition for the need to change how math is taught for students who have entered college underprepared. Community colleges can better support undergraduate success with an approach that emphasizes learning a mathematical point of view, where students apply what they have learned in math class with flexibility and resourcefulness, rather than with the traditional approach of passive reception, where the instructor prepares lectures and the student absorbs the material (Iyer, 2020; Schoenfeld, 2020). Traditionally, entering community college students who are assessed as not ready to learn college math have been required to take remedial math (Ganga et al., 2018; Xu & Dadgar, 2018). However, many students fail to take the assigned remedial courses (Fay, 2020), or if they do enroll, they fail and drop out of college (Gewerz, 2018; Rodriguez et al., 2018; Vick et al., 2018). Logue et al. (2019) contended that failure in remedial math classes by students does not necessarily indicate their innate inability to learn college-level mathematics; instead, it reflects a need to develop students' cognitive ability and their motivation to succeed. Logue et al. (2019) along with others (Hern, 2019; Hodara, 2019; Rodriguez et al., 2018), proposed a new strategy called "corequisite remediation" where a student needing remediation is assigned to college-level courses and provided additional academic support at the same time, with the objective of developing student cognitive ability and increasing motivation. Corequisite courses are designed to align to a specific set of other courses, usually in a student's major. In a review by Hodara (2019) for the National Academies of the corequisite model, corequisite remediation resulted in higher passing rates and significantly higher graduation rates than those obtained with traditional remediation.

Although state legislatures and college leaders are implementing acceleration and compression initiatives to redesign mathematics curricula, many faculty members believe that developmental math faculty expertise is being bypassed in this effort, although faculty would be in the front line to implement it (Brower et al., 2018; Edgecombe & Bickerstaff, 2018; Nix et al., 2020). Cafarella (2021) studied implementing acceleration and compression in developmental math classrooms. Cafarella pointed to a need for additional research on requirements of teaching underprepared first-generation students. Such efforts could include expanding the data collected on the traditional and redesigned developmental mathematics programs to determine the relative success of redesigned programs, especially related to the objective of developing student cognitive abilities to learn mathematics in a meaningful way with reflective instruction practices.

Role of Reflection in Learning in Community College Mathematics Classrooms

Aydin (2020) conducted research to determine the beliefs of mathematics instructors (n = 12) at a university in Turkey, such as whether some people are born with mathematics talent (and others are not) using analysis based on the epistemological reflection model of Baxter Magolda. The researchers found that some instructors had negative opinions about the mathematical talent of students, especially underprepared students. The researchers recommended that instructors need to develop epistemological beliefs about students and mathematical talent of students. In Gómez-Chacón and De la Fuente (2019), actions of instructors were explored related to reflection in the mathematics classroom and found to result in improved student understanding of mathematics concepts. Manderfeld and Siller (2018) explored the professional role of reflection in mathematics education by providing feedback on beliefs, motivations, and self-regulation. A reflective approach to mathematics instruction involves having students seeking solutions and exploring patterns rather than just memorizing formulas. It also means having students develop conjectures and do not just do exercises (Schoenfeld, 2020). Mathematics, according to Schoenfeld (2020), is a social activity, with mathematics as a living subject seeking to understand patterns in the world. Students, according to Schoenfeld, need to learn mathematics as a dynamic activity involving patterns rather than as a closed, rigid set of rules that just need to be memorized. From this teaching perspective, students need to become flexible and resourceful problem solvers, empowered and able to interpret quantitative data.

Ahmad and Febryanti (2018) in Indonesia and Daher et al. (2018) in Turkey studied reflection skills of students in STEM classrooms, including epistemological beliefs about the usefulness of mathematics. Student self-regulation skills were studied in group learning settings and found that reflection skills were significant in influencing student mathematical problem-solving and thinking skills. Both studies found an emotional connection with learning mathematics with implications for curriculum design. Callahan and Steiner (2017) studied metacognitive strategies and tools for the undergraduate classroom, including the Think-Work-Pair-Share: Metacognitive

Awareness Inventory and other reflective strategies. Schoenfeld (2019a, 2019b) found reflective process leading to metacognition as integral to learning to think mathematically. In a quantitative study in Indonesia, Bahri (2018) found teaching practices to play a role in enhancing students' reflection and metacognitive regulation skills through guided inquiry. Students taught with guided inquiry developed a higher metacognitive skill in problem solving than students taught by traditional methods.

Making Effective Mathematics Classrooms in Community College

Bieda et al. (2020) studied the challenges of conducting observations in the classroom to measure the quality of classroom instruction practices for mathematics instruction. The study found that guided inquiry strategies of teaching resulted in students having higher levels of metacognitive skills. Di Leo et al. (2019) studied how higher-order thinking was influenced by interactive factors in the mathematics classroom, such as learning strategies and attitudes in the classroom environment. In comparison, Bakar and Ismail (2020) conducted research on student metacognitive skills and student achievement and found that reflection prompted by the instructor in the mathematics classroom improved student self-regulation in mathematics problem solving. Fong and Zientek (2019) studied how to bolster new-found success for underprepared community college students in learning mathematics and the beliefs needed to cultivate student success, found that beliefs were the best predictor of success in remedial mathematics classes (see also Jiang et al., 2021 on student cognitive reflection in developmental mathematics).

Galanti and Miller (2021) studied student readiness for college-level mathematics, including beliefs about mistake-making and mindset attitudes about math in the transformation from high school to learning college math. The authors found a relationship between the interactions of perceived teaching practices and student mathematical mindset with ideas influenced about mathematics indicating complex transitions from high school to college mathematics in terms of student beliefs about success in mathematics. Hammad et al. (2020) studied educational approaches in the classroom to improve college students’ mathematics self-efficacy with 130 first-year students, looking at everyday classroom practice and making use of gamification. The researchers found a positive correlation with student self-efficacy, achievement, and with motivation. Instructor teaching methodology, attitude, and reflection practices with gamification were found to be significant in improving student learning outcomes. Zientek et al. (2019) studied ways for underprepared students (with a sample of 439 students) to improve success in learning developmental mathematics in community college because of improvement in student success. The self-efficacy of students was identified as a predictor of student success. The authors found that little is known about how student self-efficacy can be bolstered for those enrolled in community college. Mastery of skills was found to be the best predictor of student self-efficacy.

Chahine (2018) studied an instructor who was involved in problem solving in a classroom setting and what happened when the instructor failed to solve the problem in the mathematics classroom. The results of reviewing the perceptions of the instructor were to use reflection to reconstruct the session in order to better understand what went on and how to perform better next time. Hwang and Kim (2019) studied the use of the concept of reflection and metacognition in aspects of mathematics education. Although, a center of interest was with teachers attempting to make use of it, the researchers found ambiguity and a lack of commitment in the use of reflection and metacognition in problem solving education. The researchers attempted to identify the core concepts of reflection and meta-cognition.

Desoete and De Craene (2019) studied the assessment and training of metacognition in ways that appear promising for positively influencing the process of learning mathematics and the relationship between reflection, metacognition, and mathematics performance. Jiang et al. (2021) studied how underprepared students tended to rely on only one (perhaps suboptimal) strategy for solving problems, even when they knew how to use more efficient strategies. In Jiang et al.'s study, mathematics anxiety was shown to impair students' ability to engage in problem solving with flexibility and reflection was found to alleviate students' mathematics anxiety and promoted students' strategic flexibility.

Sidhu and Srinivasan (2018) studied undergraduate mathematics classrooms and found that students taught with a reflective learning strategy performed significantly better, with a good understanding of the material. Krouss and Lesseig (2020) studied how traditional lecture courses were adapted to enhance student learning through a learner-centered approach. Litster et al. (2020) explored how student-centered math classrooms incorporated activities that were meaningful in developing student reasoning, thinking, and collaboration (interaction) to promote reflection in mathematics problem solving.

Tachie (2020) studied factors contributing to learners' challenges and opportunities using reflection in learning mathematics to develop metacognitive skills and strategies and ways to improve instructor knowledge with a sample of 87 teachers. The researchers found that instructors reported they fostered reflection and metacognition is various ways, including collaborative ways. The researchers found that instructors perceived they could improve their instruction and student learning outcomes by using strategies, including assessing learners' responses, and asking appropriate questions to promote thinking, to be used during teaching. Kurniati and Nuraeningsih (2019) found reflection as a way to consider mathematics problems in a new and different way and described the process of reflection and associated cycles, vis-à-vis Dewey (1933), as previously discussed.

Misu et al. (2019) studied how reflection and metacognition awareness of mathematics students varied based on mathematical ability. During a mathematics course, Erdogan (2019) studied how reflective thinking activities were found to support cooperative work, or how collaboration supported critical-thinking skill development. In a quantitative inquiry on reflection, Muhali et al. (2019) investigated two models, reflective-metacognitive learning model (with more frequent opportunities for learner reflection) and the cognitive-metacognitive learning model to compare these for designing curriculum to improve students' reflective skills in mathematics problem solving. Both models were inspired by Schoenfeld's (2015) TRU framework (as described earlier), but the reflective-metacognitive model curriculum was intentionally designed with more opportunities for reflection in mathematics problem solving. As a result of Muhali et al.'s (2019) study, the reflective-metacognitive model of instruction was found to be more effective because it offered more opportunities for student reflection than the cognitive-metacognitive model in terms of improving students' reflection and metacognitive ability in problem solving.

Siagan et al. (2019) studied learning materials to improve students' reflective metacognition ability for mathematical problem-solving. The researchers analyzed learning materials for improving student metacognitive abilities. The results showed that reflective learning materials improved student problem-solving and metacognition ability. Pratama et al. (2019) studied the use of reflective metacognitive skills by students in problem solving, as prompted by their instructor. Shida et al. (2019) studied the influence of reflection on mathematical problem-solving of engineering students and found that instructors needed to be more innovative and to vary their teaching practice to be more challenging. Mevarech et al. (2018) studied the effects of reflective metacognitive scaffolding on students in higher education and found that reflective metacognitive strategies enhanced self-regulated mathematics learning shown to be successful in enhancing students' mathematical reasoning in college**.** Ganga et al. (2018) studied a new reflective model for college math classrooms and math "pathways" to improve student success in college math. Using math pathways, college students can learn math that is relevant to their interests. This rethinking of math curriculum involves coordination between 2-year colleges and 4-year transfer colleges. The pathways approach removes barriers that the mathematics obstacle can impose.

Reflective Conversations

As discussed earlier, reflective conversations contribute to learning and cognitive development. Zepeda et al. (2019) studied instructors' use of supports for reflection with classroom talk. Comparisons in instructor talk supporting reflection in 20 classrooms that were tested were found to result in high growth in conceptual mathematics scores compared to 20 classrooms tested with low growth in conceptual mathematics scores, and found reflective conversations supported high-conceptual growth classrooms. Schraeder (2018) studied the impact of question-and-answer instruction in large-enrollment mathematics classes, meaning a lecture dialogue was used with the purpose of turning students into reflective learners.

Aksit et al. (2016) reported on how faculty in a department of education at a Turkish university promoted student-centered learning and reflection based on reforms recently made and what obstacles there were, including student resistance and ways to overcome it. A total of 316 student teachers were surveyed, and the results showed differences in formal and informal learning activities, and many reflective strategies were noted, but there still were many barriers to overcome for successful reflective learning. Comber and Brady-Van den Bos (2018) explored what made reflection effective in student learning. Interviews were conducted with 14 undergraduate students to investigate their perceptions, indicating student resistance to a format that discouraged many from attending. The study found that for those students who did attend valued the learning opportunity, including learning from peers, highlighting the opportunities for peer learning. In another study on the failure of implementation of teaching student reflection by Bharuthram (2018) in a college course, the instructor found that students were able to explain what reflection was, but they could not put reflection into practice. Beyond this, they did not actually view reflection as a learning strategy, perhaps indicating a shortcoming in the course. The instructor concluded that reflection needed to be embedded in the course in a more meaningful and productive way. The paradox of student resistance to strategies that improve learning outcomes was studied by Owens et al. (2020b) and Owens et al. (2020a), where strategies to reduce resistance were explored to assist college mathematics and science instructors in recognizing and reducing resistance in their own classrooms. Students resisted reflective learning despite the benefits of student-centered learning. In a further investigation, Owens et al. (2020b) conducted studies to better understand student resistance to learning with questionnaires and to better understand student resistance to learning with questionnaires and interviews in sections of an introductory undergraduate science course over several weeks, with a pretest and posttest, followed by a delayed posttest to measure learning outcomes.

Silverthorn (2020) investigated instructors who tried and failed to implement a new learning and teaching strategy to teach student reflection and metacognition skills found to be needed for newfound success for students in self-directed learning, with the failures due to obstacles such as student resistance, instructor reluctance, and other barriers. Stover and Holland (2018) found collaborative learning was incorporated into an introductory class but while final grades improved, student resistance to student-centered learning strategies increased, so student satisfaction was reduced. Based on these results, the instructor was able to successfully redesign the course to overcome student resistance by making use of Tolman and Kremling's (2017) integrated model of student resistance as a guide. Andrews et al. (2020) reported on instructors’ concerns about student resistance to reflective teaching practices in the classroom, 27 instructors were surveyed on their attitudes and strategies for reducing student resistance, and 758 of their students were surveyed on their evaluations of their instructors’ teaching. Classroom observations were conducted to supplement the survey data. The results showed a disconnect on perceptions of reflective learning by instructors in comparison to student responses, where faculty overestimated negative student resistance, indicating that instructor fears of adopting new student-centered teaching practices were overstated.

Alsharif and Alamri (2020) evaluated effectiveness of teaching practices by faculty instructors in undergraduate mathematics courses at the King Saud University in Saudi Arabia were evaluated using a teaching practice inventory evaluation model developed by Wieman. Responses were analyzed with findings that the faculty were adept at incorporating in-class activities into their teaching. They were not as skilled in evaluation and guiding teaching assistants. Female instructors were found to be significantly more skilled at collaboration (see Eagle & Pentland, 2002; Howland et al., 2015). Based on these results, the researchers recommended designing professional development programs to help faculty members to improve their teaching practices. Research studies led by Eichhorn, a mathematics lecturer and mathematics education researcher with a female research team of instructors in California and Texas on collaboration strategies (Cung et al., 2018, 2019) found paradoxically that academically underprepared undergraduate students needing remediation in math achieved more success with an Internet-based intelligent math tutoring application developed at University of California, Irvine (ALEKS), which provided instant individualized feedback and guidance in learning math problem solving, when combined with face-to-face collaboration and reflection activities in a physical classroom. Both learning strategies supported cognitive development of reflection and metacognitive problem-solving skills of students needing developmental coursework before starting college-level mathematics.

Measuring Reflection with a Cognitive Reflection Test (CRT)

Some researchers use a CRT instrument test to measure reflection during problem solving, which uses a set of trick questions designed to mislead to test if a student reacts or thinks during problem solving. Similar recent studies (see Jiang et al., 2021; Juanchich et al., 2020; Littrell et al., 2020; Maloney & Retanal, 2020) used a CRT instrument to measure reflection during problem solving in order to determine whether students use initial intuition to solve a set of problems or think (reflect) through the problems to solve them correctly. Studies have shown the usefulness of CRT in predicting behavior. For example, Enke et al. (2021) conducted a study testing 1,236 college students in Nairobi with a CRT to determine the effect of large incentives and found that the cognitive effort was increased by 40% with very high rewards, but performance was not improved at all, in contrast to experts predicting large performance improvements. Brañas-Garza et al. (2019), reported results of 118 CRT studies, comprising 44,558 participants across 21 countries. The researchers found that monetary incentives did not impact performance but instead there was a negative correlation between awareness of correct answers to CRT questions and being female, calling into doubt results of using trick questions to measure reflection.

Experiencing Newfound Success with Their Students Learning College Mathematics Who Might Otherwise Have Had to Leave College. Epistemological changes in instruction are improving student success by changing how mathematics is taught in community colleges. Starting in 2011, a change was undertaken in how mathematics was taught at Cuyamaca College over a period of years (Kersey et al., 2018) resulting in success in facing the challenge of having many newly enrolled students who were underprepared and not passing required college mathematics. Starting early, Cuyamaca College was the first community college in California to transform its mathematics education curriculum in this way, by introducing reforms along with intensive ongoing professional development (Kersey et al., 2018). Before this reform, Kersey et al. (2018) found that only 40% of students who entered underprepared went on to complete college math requirements, in comparison to 70% prepared for college mathematics. If assigned to remediation, only 6% of these students ever succeeded in going on to complete a transfer-level math class. After an extended effort, by 2016, Cuyamaca College was able to eliminate remedial math classes altogether and replace them with a new model of math education, which was an accelerated “math pathways,” developed by the California Acceleration Project (CAP), a faculty-led professional development network to support California's community colleges to reform remedial education and was targeted to students' desired academic and career plans (Ganga et al., 2018). As a result, Cuyamaca College was able to substantially increase the proportion of incoming students – especially across all disproportionately impacted groups – who successfully completed a transferable (for credit) math course in one year. In this way, underprepared students were convinced they belonged in college, and that they had the capability to do college-level mathematics. Reflection offered a way to power up college student epistemological reflection and cognitive development. Innovations implemented at Cuyamaca included having underprepared students take transfer-level math classes along with co-requisite support, and if needed, students could also enroll in an accelerated preparation course for a semester. This was a radical departure to how classes were offered previously. These new courses had to be built one layer at a time by changing to a student-centered classroom and requiring intensive ongoing professional development and training for instructors, as described by Ganga et al. (2018) (also see Hern, 2019 for comparison). On-going research was recommended by Kersey et al. (2018) to assess the continued success of the program.

Instructors need to have reflective skills to demonstrate these skills to students. Reflection is not attained spontaneously, requiring actual teaching to be analyzed to help instructors to direct reflection on their teaching practice in a classroom context. Learning with reflection means being aware of knowledge for a better perspective and to regulate further learning with strategies and to evaluate their effectiveness. Success with problem-solving has been linked to the use of reflection and learning to think mathematically (see Schoenfeld, 2019a, 2019b). Schoenfeld (2019a, 2019b) explored how to create effective classrooms for learning mathematics and found that instructors need help for focusing on classroom practice with the purpose of developing students who emerge successful as knowledgeable and flexible problem solvers.

Changes in How Mathematics is Taught Can Accelerate Learning

Studies in different states in the United States and several nations have explored ways to accelerate how mathematics is taught to underprepared students. Floyd (2017) studied the effect of an accelerated mathematics course on student learning for underprepared students was examined to see if students could complete coursework in a shorter timeframe. In comparison, a study by Finau (2017), and a follow-on study (Finau et al., 2018) in Tonga (an island kingdom in the South Pacific), found a cognitive acceleration program had effects on achievement and self-regulation as part of a world-wide program. Muilenburg (2019) explored practices in Texas for integrating developmental math education for underprepared students into college-level courses as a co-requisite by interviewing 16 instructors regarding pairing the accelerated learning programs at a community college to improve student learning outcomes. Support was provided to instructors for 8 months for improving student thinking skills and mathematics performance. Marzocchi (2019) studied student success in college mathematics in California to determine how working collaboratively with peers was supportive along with ways to seek help. Changes in how students studied were observed with expanded opportunities for students to improve study skills.

Shoji (2019) studied a program in Japan to help students increase their cognitive level in undergraduate mathematics. Students were assessed for early stages of logical thinking and identified. Shoji recommended a check of student level of logic by an assessment to guide instructors in their teaching. Suson (2019) studied improvements in basic mathematics instruction using various acceleration strategies by instructors were assessed for improving learning outcomes. Sole (2020) studied a community college in New York, with alternative accelerated mathematics pathways with just-in-time help were studied, comparing the success of students with traditional courses, to compare time spent and student learning outcomes.

Chase et al. (2021) explored how the teaching of mathematics is accelerated with the pathway model in community colleges, which has emerged as a preferred approach as a solution to low completion rates. This study focused on successful implementations and how department chairs navigated the changes needed in community colleges, nationally. For comparison, Conley (2020) similarly studied implementation of acceleration in mathematics education using co-remediation strategies at a community college in Arkansas, greatly improving achievement and completion rates in comparison to traditional courses in intermediate and College Algebra. Wilkerson (2021) studied a corequisite mathematics course in Kentucky for the efficacy of accelerating underprepared community college students, in comparison to assignment to a long sequence of remedial classes to prepare underprepared students for college-level mathematics. Since most students assigned never complete the developmental sequence, a new approach was studied in comparison to the results of the previous approach.

Results of changes in how math is taught at California community colleges after passage of AB 705. As previously discussed, policy makers in California passed legislation (Assembly Bill 705, called the Seymour-Campbell Student Success Act of

2012) that took effect on January 1, 2018, requiring community colleges to improve the likelihood that underprepared students would complete for-credit (transfer-level) courses for math requirements within 1 year. Since this time, many studies have been conducted to compare results of the new ways of improving college mathematics instruction on student achievement for underprepared students in California community colleges due to Assembly Bill 705 (AB 705), for example, see studies on AB 705 implementation by Sims (2020); Willett et al. (2020), see early evidence of implementation by Albert (2020); Hern (2019); Mejia et al. (2019); Rodriguez et al. (2018); see studies on unprepared students and the transition from high school to college, as related to AB 705, by Armstrong et al. (2020); Buus (2019); Cevallos et al. (2019); Melguizo and Ngo (2020); Park et al. (2020); and on how math instruction is changing related to AB 705, by Ellis et al. (2019), Martinez (2018), and Townsend (2018).

Willett et al. (2020) explored placement accuracy to determine how to maximize the probability that students would complete transfer-level math courses, as required by AB 705, and how to translate the research findings into policy implementation. Albert (2020) examined the effect of revised placement rules related to the reforms instituted by the AB 705 legislation to maximize throughput success rates. The data revealed a statistically significant difference in the throughput success rates by placement levels. The data were impacted by COVID-19 pandemic, but a comparison of results showed that the observed throughput success rates did not meet the State's threshold standard.

In an analysis of early results of the implementation of AB 705, Hern (2019) analyzed efforts at 114 campuses in the state. The study found the proportion of transfer-level classes had doubled and increases in the number of colleges offering corequisite remediation (curricular models where students receive additional support while enrolled in transfer-level classes). Despite this progress, the study identified several areas where implementation was weak, including uneven implementation across the state, for example, at many colleges, remedial classes are still a large portion of the classes offered. Only 13 of the 114 colleges met the benchmark of offering less than 10% in pre-transferable remedial classes. At 49 of the 114 colleges, over 30% of the math classes offered were pre-transfer remedial math classes. Math sections offered for statistics and quantitative reasoning were not balanced between those for business and STEM majors, and those needed by other majors for their degrees. In a similar study, Mejia et al. (2019) studied a small group of community colleges that redesigned and increased access to transfer-level math courses prior to AB 705. The early redesign was in response to the many students who enter California community colleges being placed in development remedial courses, and with relatively few students completing transfer-level math courses required to graduate.

The transition from high school to college for unprepared students can result in a mismatch in math placement. Armstrong et al. (2020) found that most students deemed underprepared by placement tests are, in fact, successful in college-level courses. Standardized placement tests could create false distinctions between prepared and underprepared students. Buus (2019) explored the college readiness epidemic in the California State University system. The growth in the number of students who enter the California State University underprepared continues to increase. The study reviewed proficiency and GPA data across all 23 CSU campuses with variables such as student GPA and college math proficiency reviewed, along with interviews with personnel at three CSU campuses. Themes emerged from the interviews pertaining to the success and shortcomings of implementing college readiness policy. Overall recommendations included creating a mandatory 4th year of math at the high school level.

Cevallos et al. (2019) explored the problem of students starting college underprepared in mathematics. The CSU is the largest 4-year public university system in the nation and has historically struggled with low graduation rates (Johnson et al., 2017) mainly due to the incoming students' academic under-preparation (Millea et al., 2018).

Similarly, the California Community Colleges (CCC), the largest system of higher education in the United States, has faced numerous challenges increasing their completion (associate degree/certificate and/or transfer) rates chiefly as a result of their inherent historical mandate to provide remedial instruction to all students who need it (Beach, 2012). While both systems have cycled through different approaches to increase their completion rates, challenges related to students arriving unprepared for college have continually beleaguered the institutions.

The major roadblock students face on their path toward college graduation is math under preparation. Students who matriculate into college without being fully prepared in mathematics face a higher probability of dropping out in their first year (Scott-Clayton & Rodriguez, 2015). Melguizo and Ngo (2020) studied the extent to which college-ready students, by high school standards, are assigned to remedial courses when they enrolled in community college. They found that misalignment was prevalent and substantial with respect to high school grades and minor to moderate based on standardized test results. The students most affected by this misalignment were female, Black, and Latinx students, and this misalignment in the transition to college was found to be particularly detrimental to Black students. Park et al. (2020) studied the transition from high school to college math, where students are placed lower than needed and found that a majority of students experienced math misalignment in community college. Moreover, math misalignment especially hindered STEM-aspiring students from pursuing STEM pathways.

Martinez (2018) studied outcomes of students placed in the lowest levels of remediation at 2 colleges with different models of course acceleration, from 2013 to 2017. The results of this study suggested students placed in developmental mathematics who were placed in an accelerated pathway have decreased time to complete remediation and a transfer-level math course. A course redesign acceleration model was found to have more improvements in transfer-level math and developmental math completion rates for first-generation students, as well as students placed in both low-level and mid-level remediation. The implications of these results support redesigning academic programs, especially to help students who are most often placed into the lowest levels of remediation. Townsend (2018) studied how reflection is taught. Metacognition, which is one component of self-regulated learning, was significantly correlated to increased academic performance**.**

Summary and Conclusions

In this chapter, I included my literature search strategies, a discussion of conceptual framework for this study, and a concise synopsis of current research that supports the problem’s significance and analysis. I explored current literature that relates to my research questions and conceptual framework. The literature review was focused on reflection as an engine of cognitive development for recent high school graduates who are starting college and the role of reflection in the college mathematics classroom. In Chapter 3, I discuss the methodology for this research.

Chapter 3: Research Method

In this basic qualitative study, I explored perceptions of mathematics instructors at U.S. community colleges of what they perceived about how their use of reflection fostered epistemic and cognitive development and academic success of students who entered college underprepared to learn college math newly successful in learning mathematics. In this chapter, I describe the research design for this study, my rationale for the design, the role of the researcher, the methodological approach for the study, and plans for data collection and analysis. I close with a discussion of issues of trustworthiness and ethical considerations.

Research Design and Rationale

My central research questions were:

RQ1: What are community college mathematics instructors' perceptions of how reflection developed epistemological and cognitive abilities of their underprepared students newly successful in learning mathematics?

RQ2: How do community college instructors use reflection in instruction in the classroom to improve the academic outcomes of their students?

For this study, I used a basic qualitative inquiry approach (Merriam & Tisdell, 2016) with in-depth interviews with participants to examine perceptions of mathematics instructors who apply reflective practices in U.S. community college classrooms for students entering college who are underprepared in mathematics. A basic qualitative interview method was used to support my investigation of participants' subjective perceptions and reflections. I took this approach to aid me in my goal of studying, describing and comparing insights into an issue. Conducting in-depth interviews helped me to understand how community college mathematics instructors perceive and interpret their perceptions of teaching students entering college who are underprepared to learn college mathematics. Merriam and Tisdell (2016) said the qualitative inquiry enables education researchers to understand participants' perceptions. By conducting and recording in-depth interviews, I aimed to collect substantial data from participants in my study.

Role of the Researcher

As the researcher in this qualitative study, I played the main role in framing the study and collecting and analyzing emergent data, with the aim of discovering and interpreting meanings interviewees associate with their perceptions, and then presenting conclusions from this research. I gathered data through semi-structured and one-on-one interviews with participants. To perform this role with integrity, I needed to be self-reflective and proceeded with open personal awareness. Because as the researcher, I needed to consider my bias, I reviewed my own biases and assumptions to clarify my subjectivity . I continuously clarified my own bias and assumptions as I refined my perspectives throughout the research process. As I proceeded with the study, I kept research notes to help assess effects my opinions and perceptions might have on the research in a reflective research journal, and I kept this journal from the start of this study.

To recruit interviewees, I reached out to mathematics faculty of U.S. community colleges who are members of the AMATYC, especially faculty taking part in the

AMATYC committee on improving mathematical prowess and college teaching. I am a member of the AMATYC and have attended conferences, but I have not met instructors whom I interviewed. This study was not done in my work environment. I avoided interviewing anyone with whom I had a current or former supervisory relationship or were current or former students.

Methodology

In this section, I describe the methodology of the study, including participant selection logic, data collection instrument and interview protocol, procedures for participant recruitment, participation, and data collection. Finally, I discuss how I planned to analyze these data.

Participant Selection Logic

For this study's participants, I identified mathematics instructors at U.S. community colleges, preferably who are members of the AMATYC, especially faculty taking part in the AMATYC committee on improving mathematical prowess and college teaching. Participants were currently serving as mathematics instructors, had at least 2 years of teaching experience in mathematics classes at a community college, and specified they made use of student reflection in their teaching. The specific procedure for contacting potential participants was to send a recruitment invitation via email, including qualifications for participation and my contact information. I was able to recruit 10 qualified participants, which was initially considered to be adequate.

Sampling strategy and criteria for selecting the sample in qualitative research typically involves purposeful sampling, which involves finding participants who meet established criteria (Merriam & Grenier, 2019; Patton, 2015). I sought a sample size of 10 interviewees. Patton (2015) suggested saturation (the point at which no new information can be found in qualitative studies) usually occurs after 10 participants. I monitored data saturation as the study progressed. I expected to find repeated information after completing 10 interviews. I determined that I did reach saturation with 10 interviewees, so I did not need additional interviewees. If I had sensed that my data has reached the point of saturation earlier than 10 interviews, I would have considered including fewer interviews, but this was not necessary.

To establish how participants met criteria, I included selection criteria in the invitation and asked potential participants to indicate they made use of reflection during instruction before arranging to interview them. I asked them to address this in their email response. I confirmed at the point of scheduling interviews to make sure I eliminated interview candidates who misunderstood criteria.

Instrumentation

This study involved using a data collection method with oral semi-structured interviews that was conducted remotely. For these interviews, the source for the data collection instrument were interview questions based on my knowledge as the researcher. (see Appendix). For this study, no historical or legal documents were used as sources of data.

I documented interviews with notes I took during interviews and recorded these interviews, later producing transcripts. I formulated interview questions based on research studies I reviewed. I used a preliminary set of questions to conduct several test interviews with mathematics faculty who were unrelated to participants in order to focus on question formulation and revision as my process for developing questions for qualitative interviews. Following practice interviews, after a review, I adjusted interview question responses, and as a result, was better prepared. My notes during interviews, audio recordings, resulting transcripts, and my research journal were the only data sources used in this study. The data collection instrument was sufficient to answer the research questions.

I asked all participants the same set of questions (see Appendix) based on my research questions for the sake of consistency. Rubin and Rubin (2011) addressed the level of control a semi-structured interview allows the researcher to focus on the research questions and to avoid controlling the participant response. If needed, I would have used follow-up (probe) questions providing my interviewees with a broad array of possible responses to expand on answers (Turner, 2010), and to give myself adequate opportunities for follow-up to clarify and probe for meaning and understanding (Merriam & Grenier, 2019), to enable participant flexibility based on my research questions (see Appendix).

Procedures for Recruitment, Participation, and Data Collection

I recruited my initial interviewees from among those available to me through direct contact with the faculty by email from the community college websites and the professional association mentioned above, AMATYC. I e-mailed an informed consent form to those mathematics educators who responded positively to the invitation to participate and who met the selection criteria. I asked them to respond to the e-mail by typing “I consent,” referring to their consent to the steps and procedures for the study explained in the invitation email. With the signed consent form obtained, I scheduled an interview as conveniently as possible for each participant.

To recruit participants, I contacted mathematics faculty members at two California community colleges recognized, according to Ganga et al. (2018) and Kersey et al. (2018), for improved performance in teaching mathematics. However, no faculty from the top two California community colleges responded to my invitation to participate. I extended my search nationwide by inviting mathematics faculty who were members of the AMACTY. In all, I sent out more than 700 invitations to the prospective participants. I stopped recruiting for prospective participants when I was able to recruit and interview 10 qualified participants.

I used the auto-transcript as a starting basis for preparing the final transcript, by listening to the recording in comparison and making any needed corrections to the transcript to accurately capture the actual interview. I also made notes during the interviews, as appropriate, and completed my field notes in a researcher’s journal after each session to help me capture the shared information accurately. I kept this reflective research journal capturing issues in an attempt to remain unbiased in my data analysis.

I notified participants prior to any agreement to participate that they could exit the study at any time. I reminded them of this immediately before starting the interviews, as well. I sent all participants their interview transcripts after the interviews, for them to review for any clarification needed. A few sent corrections and I reviewed them and made the corrections that were requested. I returned transcripts that needed to be revised within one week with a thank-you note. I did not conduct any follow-up interviews. After I complete the study and my dissertation is published, I informed them I would send a summary of the results to each of the participants. These steps supported the dependability of my data.

Several respondents declined participation because of teaching at a 4-year college, although they were members of the association for two-year colleges. Several others responded to my invitation saying they did not have time to participate or were not eligible due to being a student or retired. Of those who declined to be interviewed, one invitee responded to my invitation email in early September with, "Zzzzzzz…" I responded by saying that no matter what his views were about the role of reflection in math instruction, I would like to interview him to learn more about his views. He responded by writing, "I'm sure you're a great person, but this 'affective-factors navel-gazing stuff is just the latest fad in mathematics...not worth it to me." According to the AMATYC member profile, this respondent was a professor of mathematics at a California community college but would not have met my criterion about using reflection, so I would not have interviewed him anyway.

Participation of respondents who did consent was completely voluntary. All the participants were interviewed via Zoom in the privacy of their home office or campus office, and in these interviews, participants shared their perceptions freely and took the time to carefully consider the questions before answering. I interviewed all participants via Zoom and used the Zoom app to capture the audio for the interview, and then used the auto-transcription feature of the Zoom platform to generate an original transcription for each interview. The initial auto-transcripts by Zoom provided a starting point for my final transcription of each interview. I reviewed each initial transcript while listening to the audio recording to make any needed corrections. Along with this, I also reviewed my interview notes to determine if revisions for any inaudible portions were needed. The reviewed transcripts were then sent to each of the participants to see if any changes were requested. Two of the participants had no changes, and eight made a few suggestions for clarification. These changes were made and approval from each participant for the use of the transcripts was received. There was no deviation from my planned data collection procedures, except I requested an IRB change to recruit nationwide, which was approved. I was able to maintain consistency in all of my interviews. At the time of the interviews, early in the fall semester of 2021, faculty had again started teaching in the classroom on campus after teaching previous semesters remotely, due to the COVID-19 pandemic.

I collected data through semi-structured interviews, with a set of questions I developed for data collection (see Appendix). To do this, I recorded each interview in the Zoom app software, listening and taking notes during the interview. After the interview, I reviewed each recording multiple times while reviewing the initial Zoom transcripts, which permitted me to note my own reflections as I listened again to the words of each participant. Data collection began after I received IRB approval from Walden University. Once I sent invitations out to mathematics faculty, and they responded by email with the informed consent, I set up a date and time for the interview, and then conducted the interview. I planned on approximately 60 minutes for each interview as an estimate of time, and some went a bit longer, and with the understanding that a participant could stop the interview at any time and that an interview might extend beyond an hour by mutual agreement as the interview unfolded. I held open the possibility of some follow-up interviews should the need arise for clarification or further elaboration of data collected in one or more of the initial interviews, but no follow-up interviews were conducted. I had the interviews transcribed by the Zoom automated transcription as a starting point to maintain an unbiased processing of the transcription data. Participant identification was kept confidential and interview data was confidential in the published results.

Data Analysis Plan

In a qualitative investigation, data analysis involves searching across the interview data set to find repeated patterns or themes of meaning (Braun & Clarke, 2006, p. 86). I conducted thematic analysis of the interview transcripts employing a flexible approach to accommodate themes and questions that emerged, as well as points that surprised, puzzled, or otherwise challenged me. I was vigilant about tracking divergent as well as convergent data. Because thematic analysis involves inductive thinking, it was helpful to ask questions like: “What do these quotes or observations have in common?” “What's going on here?” “What does this tell me about how people view their world?” “How do these themes relate to each other?” (Taylor & Bogdan, 1998, p. 156).

Inductive Analysis

Inductive analysis does not attempt to fit the data into any preexisting categories. Instead, it takes an appreciative approach in which the researcher holds pre-understandings aside to gain new and fresh insights. In my analysis, I appreciated each informant’s interview as an expression of their individual-self, taking an approach that maintained a sense of each individual narrative as a whole (what Maxwell (2010) termed "syntagmatic analysis") even while I searched for points of comparison and contrast across the set of interviews (what Maxwell termed "paradigmatic analysis"). Once I analyzed the data from all participants, I took a more deductive approach in identifying repeated patterns and themes (as well as data that diverged in interesting ways in relation to these patterns and themes) and composed these into a composite integration that formed the framework for my findings.

Step-By-Step Analysis

In using step-by-step analysis, I first reviewed the data collected from each participant interview. I began my analysis by examining the transcripts recorded from the Zoom interviews provided by the automated Zoom transcription and reviewed by the participants to get an understanding of the data generated. I began by coding each transcript, as described by Saldaña (2021) with an effort to not oversimplify or becoming too narrow in focus, which would have made the coding process more difficult. I highlighted intuitively any sentences, phrases, or paragraphs in the transcript that appeared to be meaningful. During this process, I immersed myself in each participant's data individually. I looked for key words to group the codes together into categories to eventual themes, which addressed a research question. I then put these concepts into similar themes that were related, while looking for perceptions that were alike. I reviewed the transcripts several times to make sure I had not overlooked an idea or miscategorized it. I used a content and thematic analysis to identify themes within the data (Vaismoradi et al., 2013).

Next, I reviewed the highlighted data in relation to the research questions to decide if the highlighted data were related to the research questions. To conduct my data analysis plan, I first organized and condensed, or reduced, the data contained in the interview transcripts, removing information that was not applicable to the research questions (Miles & Huberman, 1994). Next, I eliminated all highlighted data that were not related to the research questions. This procedure helped ensure that the data I processed in subsequent steps was related only to my research questions. I started a separate file to store any unrelated data because I might want to come back to it to reevaluate these data in the future. Next, I assigned categories, or themes, in a procedure called category construction by Merriam and Grenier (2019). I made comments as they occurred to me in places where the theme was underlying or not easily seen (Vaismoradi et al., 2013).

Next, I took each bit of data and coded it. The code used was simple, like a serial number or an address, as a way to keep track of individual items of data. Next, I clustered the data items around those that are related or connected in some way and started to develop patterns. From this point, I identified patterns and irregularities, gathering similar themes together into categories, grouping comments and data pieces that go together (Merriam & Grenier, 2019). As I continued this coding activity, I followed Merriam and Grenier's (2019) recommendation to keep a separate list of themes that seem to go beyond a single participant's interview. As the analysis continued, some themes became subcategories. Once the themes appear to coalesce, I assigned all data pieces to a category. For each distinct pattern that I saw, I described it in a phrase or statement that summed it up. When feasible or useful, I assigned a second-level code to the patterns. At this point, the words I used to describe the patterns were no longer the words of the participants, but my own.

Next, as I started to see patterns, I identified items of data that corresponded to that specific pattern and placed them in the previously assembled clusters that manifested that pattern. Direct quotes taken from these data (from transcribed interviews) support the pattern. The name or descriptor of my identified pattern are a more abstract phrase (vis-à-vis Frye, 1990), whereas the data themselves are direct words from participants. Next, I took all the patterns and looked for the emergence of overarching themes (where a theme is a pattern of patterns). This involved combining and clustering the related patterns into themes. As I saw meaningful themes across patterns, I assigned a yet-more-abstract descriptor to the theme. This was a third level of abstraction, supported by the patterns, in turn illustrated by the direct data.

Next, after I analyzed all the data, I arranged the themes in a matrix with the corresponding supportive patterns. In the matrix, I include the codes or descriptors for each of the data clusters. Thus, the supporting layers of words/text can easily be accessed when discussing an individual theme in my final analysis. Next, for each theme, I wrote a detailed abstract analysis describing the scope and substance of each theme. I completed this process for each participant's data (interview transcript). I did this by hand for the interviews, and it was manageable without software. As the next analytical step in my study, I drew conclusions and created categories (Miles & Huberman, 1994). Next, I continued the analysis of data for all participants, including patterns and themes that were consistent across the participants' data. Finally, I synthesized the themes together to form a composite synthesis of the data collected focused on the research questions. I examined the data coding produced in the previous step to help discover meanings in the data, allowing me to draw conclusions verifiable through the data and to group similar pieces of data. I acknowledged any outlying data collected that varied from the main information. However, finding discrepant data does not negate the themes and conclusions found in the body of the data collected. No outlying data was found. Variants to main themes can clarify and bring completion to the research that would not exist without the discrepancies, if they exist (Creswell & Creswell, 2017). I employed constant-comparison analysis (Glazer & Strauss, 1967) to note themes, questions and divergences that emerge among the interviews. I held open the possibility of brief follow-up interviews if emergent data require clarification. As I did all of this paradigmatic work, I returned occasionally to read full individual interviews to keep alive my sense of the narrative integrity of each interview.

Issues of Trustworthiness

For trustworthiness in the study, I focused on four components: credibility, transferability, dependability, and confirmability. As the researcher, it is important for the integrity of my research to make sure that my work fulfilled these elements of trustworthiness.

Credibility

I ensured credibility by providing each participant an opportunity to review their own transcript to make sure their responses were captured accurately. Once each interviewee reviewed their transcript, some returned them with corrections (and several did), I followed up with any transcript issues that were identified to assure credibility of data. I then returned a revised transcript to the participant within one week. Other well-accepted qualitative research safeguards, such as the ability of potential interviewees to refuse participation and the freely voluntary conditions of participation in my research, contributing to the credibility of my study (Shenton, 2004). No interviewees refused participation.

Establishing a process to maintain consistency in data collection can help to ensure credibility, for example, verifying the qualifications of participants, maintaining consistency in the interview approach and protocol, and in journaling. To establish credibility, during my interviews I took brief notes, as recommended by Saldaña (2021). This helped me to capture words and phrases, and body language that assisted me in coding later. I also reflected systematically after each interview, following a consistent reflective inventory to ensure regularity in my own reflective process (Cummings & Keen, 2008). Among the questions I posed to myself during this post interview phase of reflective journaling, I included ones that addressed my own biases and underlying assumptions in my research journal.

Transferability

Transferability can pose a challenge for qualitative researchers because much of qualitative work involves a small sample size and with unique environments and individuals (Merriam & Grenier, 2019). Hence, basic qualitative research does not aim directly at transferability to other settings. Instead, the goal is to generate findings that may be of interest as elements and points of departure for subsequent researchers in other settings. For this study, qualitative data collection is seeking information from a representative sample of people about their perceptions with real-world events and processes. I prompted my interviewees to provide examples, illustrating their more generalized responses with an aim to generate abundant descriptive narrative material in the hope of generating interest among other researchers and practitioners (Creswell & Miller, 2000).

Dependability

To generate dependability of my study, I maintained a transparent and systematic approach throughout my research. I reviewed my data to make sure participants' views were captured accurately. I documented all generalizations with direct and substantive illustrations from my interviewees, using their own words to breathe life into my findings and analysis. In addition, as has been discussed, each participant had an opportunity to review their transcribed data to make sure I accurately conveyed their perceptions. (If emergent data had required clarification, for example, if a new pattern or question emerges in later interviews that might have been missed in early interviews, or if some important theme needed to be documented with additional examples, I held open the possibility of brief follow-up interviews). No follow-up interviews were conducted.

Confirmability

To ensure confirmability, I implemented measures to safeguard the results of the study, including review by participants for confirmation. I used both an audit trail and self-reflective notes during the data-collection process to clarify my own emotions and beliefs regarding the target subject and to prevent bias in the study distorting results (Merriam & Grenier, 2019). I will ensure confirmability by keeping data gathered in password-protected storage for 5 years to preserve and protect with privacy, after which these data will be destroyed.

Ethical Procedures

Before I started this research, I obtained a review from Walden University's Institutional Review Board (IRB) to proceed with this study. Once I had IRB approval (#08-20-21-0122140) I e-mailed the consent form to participants and had them provide their consent via e-mail. Any individual participating in this research did so voluntarily, stating their informed consent on the form provided. I confirmed the informed consent by email before proceeding with scheduling the interview. In the analysis and conclusion portions of this study, I did seek to make a truthful and clear representation of the interviewees' perceptions and opinions.

To protect participants during recruiting, I sent each participant a separate e-mail with no references to any other participant I attempted to recruit. During the data-collection process, I removed names from any written data, as well as used pseudonyms for any identifying information in the interview transcripts. During the debriefing process, I again sent separate e-mails with no connection to other participants.

Once I received the transcribed interviews and reviewed them for any needed revisions, I sent copies by email to each of the interview participants, giving participants one week to request changes. After this time, I notified them that I would assume the participants agreed with the transcription contents. I sent each participant an email note of thanks with an updated transcription, if changes were requested. Ultimately, I will provide participants with a synopsis of my dissertation after it is published. As all the participants are instructors at the post-secondary level, I did not invite any individuals under age 18 to participate in this research. I have used a laptop with password protection throughout the research process. I have stored recordings and transcriptions of interviews either on my password-protected laptop, within password-protected files, or in the cloud in a password-protected website. I will destroy all the recordings, transcripts, field notes and any other data after 5 years. The files will be securely kept for 5 years on a password-protected hard drive, before being discarded.

Summary

In this chapter, I described and justified my choice of the basic qualitative research method. I also described my role as the researcher. I described the rationale and steps for selection of interviewees. I also described data collection and data analysis processes. I reviewed how I addressed ethical issues and ensured trustworthiness throughout the study. In Chapter 4, I detail findings of my research.

Chapter 4: Results

The purpose of this qualitative study was to better understand perceptions of community college mathematics instructors on what role reflection has on developing epistemological and cognitive abilities of underprepared students newly successful in learning mathematics and how community college instructors use reflection during instruction in the classroom to improve academic outcomes of for their students. I begin this chapter by describing setting and participant demographics. I describe next processes for data collection and analysis, and I review trustworthiness of this study. Finally, I present results of research.

Setting

As planned, I interviewed 10 mathematics professors or instructors for this study. They were in locations across the United States, and all met qualifications to participate in the study by having at least 2 years of teaching experience in community colleges and used reflection during their teaching practices. Interviews were conducted via Zoom, and I suggested participants find private locations. I also conducted interviews in a private location.

Demographics

All 10 participants were faculty members (six men and four women) teaching mathematics at community colleges in the U.S. with at least 2 years of teaching experience. All participants in the study used reflection in their teaching practices. All confirmed they had more than 2 years of teaching, with a minimum of 7 and a maximum of 36 years of teaching at the college level. All had advanced degrees in mathematics, some with PhDs. Six participants were from the west coast, three from the Midwest, and one was from New England.

Teaching experience of participants were varied across a wide spectrum, as were ways of making use of reflection. Perceptions of participants varied, with some commonalities. For example, Grape had never taught math classes online, and Ivy had only taught math classes online, and the rest taught varying degrees of both or a hybrid of remote and in-person teaching, especially during COVID.

Table 1 includes demographic characteristics of participants, including pseudonym names I created based on tree street names in my neighborhood that were assigned to each participant, as well as their gender, work titles at their institution, years of teaching experience in higher education, and highest degree earned. I learned from Elm that some community colleges use the title of professor, but other community colleges instead use the title of instructor, perhaps as a way to distinguish professors teaching math at a nearby state college or university.

Table 1

Participant Demographics

Pseudonym	Gender	Title	Experience	Degree
Ash	Female	Professor	16 years	MA
Beech	Male	Adjunct Professor	25 years	MA
Cedar	Female	Professor	7 years	MA
Date	Female	Instructor	10 years	MA
Elm	Male	Assistant Instructor	7 years	MA
Fir	Male	Assistant Professor	12 years	PhD
Grape	Male	Instructor	36 years	PhD
Hawthorn	Male	Professor	23 years	EdD
Ivy	Female	Professor	9 years	MA
Juniper	Male	Assistant Professor	11 years	PhD

Data Collection

I started collecting data after I received Walden University IRB approval (#08-20-21-0122140). I began recruiting for my target of 10 participants who were experienced mathematics educators in community colleges who used reflection based on number of participants recommended by Patton (2015) for the researcher to end up with valuable, but concise material. I sent out more than 700 invitations, and I accepted the first 10 participants who met the qualification parameters of my study and then scheduled the interviews. After scheduling, participants were assigned a pseudonym to assure confidentiality of their information. I scheduled and conducted interviews at days and times participants chose as most convenient for them.

Interviews were conducted using semistructured interview questions, allowing participants the opportunity to reflect on their perceptions involving in teaching students to learn college mathematics. I used a set of semistructured interview questions (see Appendix) based on Baxter Magolda's epistemological reflection model and Schoenfeld's model for teaching mathematics served as guides for interview questions.

During interviews, I probed situations where I needed more information regarding perceptions of participants until I felt they answered the questions. Due to this, the data collection process flowed smoothly. Based on rich information gathered during interviews, I was satisfied that saturation was met after 10 participants. After the 10th interview, I reached saturation and did not invite or interview more participants.

I began collecting data in the beginning of September 2021 and completed collection by mid-October. I initially designated 60 minutes for each interview, and the majority of interviews lasted approximately this length, but several interviews lasted a little over an hour. After completing interviews, audio recordings were recorded using Zoom, and the auto-transcription feature in Zoom provided me with an initial transcript file Once auto transcripts were received from Zoom, I reviewed transcripts and made any needed edits and corrections due to unintelligible transcriptions. I then sent transcribed interviews to participants to check for accuracy, asking for them to reply to me with any requests for edits within 1 week. I explained to them I would assume that if no response was received from them within a week, they had no issues. After a week of review, several participants sent me back suggested clarifications. Three agreed with transcripts as sent to them and did not request any changes. I then sent each participant a thank you email for their time.

My original plan for data collection was to only interview community college math professors and instructors from several colleges in California, but initially, I did not find any faculty who consented to participate from those colleges. I became concerned about finding enough participants in a timely way, and so I requested IRB approval for a change to extend recruiting to the entire country, which was approved, and this resulted in a more robust study. Other than this approved change, there were no deviations from my planned data collection procedures, and I was able to maintain consistency during all interviews. I did not have need clarifications from participants beyond those provided by participants in response to reviewing interview transcriptions, so no follow up interviews were conducted.

Some participants originated from outside the United States and were not native speakers with heavy foreign accents, including several from Eastern Europe and one from the Caribbean. Foreign accents made accurate transcription more difficult, both because of the potential for misunderstanding words, but also because I noticed that nonnative English speakers seemed to have more challenges in terms of formulating answers to interview questions. They took time to carefully understand and make sense of what they heard before trying to communicate, maybe not unlike what can happen when trying to solve a novel math problem. I noted excitement, enthusiasm, and passion for teaching math among foreign-born instructors, particularly when they expressed their passion for teaching underprepared students who were struggling with learning math at community colleges and helping those students succeed. Hearing about their excitement to teach these underprepared students made my interview experience especially valuable to me.

Data Analysis

To start my analysis, I analyzed data with the intention of supporting dependability of my results, as guided by Braun and Clarke's (2006) six-step approach for investigating the perceptions of diverse individuals, starting with first reading and then manual coding the transcript from the first interview conducted. During and after each interview, I took notes of my impressions. As I conducted the interviews, they were transcribed by the Zoom app. I reviewed the transcripts and made corrections, as needed, and I then asked the participants to review them.

After having each transcription approved as accurate by the participant, I gathered the data to identify, explore, and understand the meanings. Once these steps were completed, I began coding the interview data to identify patterns and themes. The coding process gave me an initial understanding of the perceptions of the instructors and the flow of the narrative. Manual coding allowed me to have accuracy and consistency during the data collection process, as described by Patton (2015). Once I completed the reading and manual coding of each interview transcription, I listened to each audio recording and read each transcript multiple times to guarantee that I captured all themes and patterns. I then gathered all the transcripts into one Microsoft Word document. I reread all the interviews in this unified format while also listening to the audio, noting and tracking keywords and patterns as I noticed them, highlighting words, comparable statements, and perceptions. During this, I wrote up findings that I reached. While listening, I took notes in my researcher's journal whenever I heard vocal inflections or emphasis on a word or phrase, and I noted the overall tone of the participant. I used comments in the Word document as I analyzed the combined transcripts to identify categories and patterns as they emerged for me from the data. I underlined and circled key words. I also used highlighter in the document and sticky notes to highlight information. I condensed sentences and phrases into more compressed categories. I organized categories in my researcher's journal and then organized this information in a file on my password-protected laptop.

Following this documenting, I began the next step of the analysis process by identifying chunks of meaning, then condensing phrases into fewer words while categorizing and compiling. I underlined and circled key words as I gathered my first codes of meaning. I did not find discrepant findings. I found emergent codes for each research question by identifying perceptions participants discussed in their responses in answering my interview questions. These perceptions were their responses to the question and noted in each transcript as an analysis of the data. From here I began to develop a coding scheme of these potential themes by listing key phrases and clustering codes together.

My next step was to analyze the data by applying the codes that emerged from the individual interview questions, by looking for codes occurring across all the interviews and identifying categories. With these keywords and codes identified, I located patterns across the coded data associated with each research question. I combined codes that were similar into categories, and I grouped them with specific words or phrases to align them to the two research questions of my study.

Using the transcripts, I further identified patterns and categories by highlighting words, comparable statements, and related perceptions found in the document. As I reviewed the data, I started to combine codes into similar categories. From the categories I constructed, I identified themes. As themes emerged from my analysis, new codes arose, and I combined them with similar themes. Then, I went back to my interview questions and to my research questions for alignment of my themes.

Once I went back to my interview questions and research questions to make sure my themes were aligned, I reduced the themes even more. As noted in Patton (2015), it is important to identify themes and patterns from the data analysis that may fall into negative cases. I reviewed the data to recognize any outliers that could negatively impact the credibility of the study that should be acknowledged. While reviewing the themes, all the results appeared to relate to one of the two themes that emerged, leaving no discrepant cases. After this, I finalized the themes and determined there were two themes in my data. As I reviewed and analyzed the data, I kept my own words, thoughts, feelings, perceptions, and beliefs bracketed to maintain a reliable interpretation of the data, as much as possible.

The two themes that emerged from participant responses are presented in Table 2, outlining themes that emerged from analysis of the transcript data. I will present the themes with examples from the data in the Results section. See Table 2 for the overview structure of the emergent themes and subthemes.

Table 2

Overview of Thematic Structure

RQs	Themes	Categories/subthemes
RQ1	Reflection developed epistemological and cognitive abilities of underprepared students, newly successful in learning mathematics	Improvement in teaching mathematics was needed to develop student's epistemological and cognitive abilities
		Instructors found reflection improved their own teaching practice over time
		Instructors found that reflection helped their underprepared students develop who were newly successful in learning math
RQ2	Reflection activities improved student learning outcomes	Journals
		Reviewing exams and homework
		Handwriting in solving math problems
		Group projects

Evidence of Trustworthiness

I followed consistent procedures in conducting my interviews, including recording them and keeping them to about one hour each. I asked the participants the same questions, and probed for deeper answers, when needed to more fully understand the participants. I also made interview notes for each interview and spent time thinking about the information provided and points that stood out for me immediately following the interview, and then I reflected on the interview again later.

Credibility

I sought to promote credibility by carefully reviewed the transcripts when I prepared with the audio recordings. The participants made additional clarifications, as needed, so I was able to verify that the transcripts captured the interviews in an accurate way. I then analyzed the transcripts to identify keywords, patterns, trends, categories, and themes. I noted stories about their perceptions.

Transferability

I aimed to promote transferability by documenting the background and years of experience teaching mathematics at the college level for each participant to establish the context. I also asked participants to share stories and memories of specific students who came to mind during the interview. These stories may allow readers to amplify the perceptions and transfer it to their own understanding. Future studies could make use of more participants or use more specific requirements to narrow the perceptions of instructors.

Dependability

I promoted dependability of the study by encouraging participants to review the transcript and make any suggestions for clarifications needed, so they could confirm that their responses were correctly transcribed. I took these steps to ensure dependability and accuracy of the data for the study. As part of this effort, I have described the scope and delimitations of the study and have given detailed steps taken, as I conducted the study to ensure dependability by using participants' responses verbatim. I was cautious not to ignore emergent themes or data that seemed not to fit, as well as not to impose my own thoughts or interpretations of participant responses.

Confirmability

To ensure confirmability, I used a standardized interview guide for the interviews, which allowed for the collected data to relate and correlate to the study's research questions. As the themes and patterns emerged during data analysis, I kept detailed notes of conclusions I derived from these patterns in the data. I used this process to aid in clarifying my own beliefs about the research questions. This allowed me to clarify with consistency as I compared the data between transcripts, my interview notes, and additional information provided by participants by email.

Results

Two themes emerged from the data analysis of the 10 interviews with community college mathematics faculty in the United States. These two themes addressed the research questions of the study, which were:

RQ1: What are community college mathematics instructors' perceptions of how reflection developed epistemological and cognitive abilities of their underprepared students newly successful in learning mathematics?

RQ2: How do community college instructors use reflection in instruction in the classroom to improve academic outcomes of their students?

The first theme addresses the first RQ, on what community college mathematics instructors perceived regarding how reflection developed the epistemological and cognitive abilities of their underprepared students who were newly successful in learning mathematics. There were three subthemes. Before describing the development witnessed in students, participants said they felt it was important to emphasize that improvement in teaching mathematics was needed, particularly the inclusion of reflection. For the second subtheme, participants found reflection improved their own teaching practice over time. In the third subtheme, participants found reflection helped their underdeveloped students become newly successful in learning math by developing their math learning ability.

The second theme addressed RQ2, representing how instructors expressed how they used reflection activities to improve student learning outcomes. The four subthemes illustrate some of the ways participants used reflection to help their students develop who were newly successful in learning math. There were four subthemes. Ways instructors made use of reflection in the classroom included: journals, reviewing exams and homework, handwriting in solving math problems, and group projects, where the instructors perceived that reflection starts individually and then was shared by explaining the "why" to the instructor and to other students. All activities but handwriting were mentioned by all 10 participants. Handwriting math problems was specifically described by four participants. No discrepant or nonconforming cases were found. Presentation of the two themes and their respective subthemes is supported with quotes from the participants.

Theme 1: Reflection Developed Epistemological and Cognitive Abilities of Underprepared Students Newly Successful in Learning Mathematics

The first theme reflects what community college mathematics instructors expressed about how reflection developed the epistemological and cognitive abilities to learn mathematics of their underprepared students. All participants shared the perception that their students were underprepared and needed to develop to become better at learning math and newly successful in learning. There were three subthemes:

- Instructors found that improvement in teaching mathematics was needed.
- Instructors found reflection improved their own teaching practice over time.
- Instructors found that reflection helped their students develop who were newly successful in learning math.

Needed Improvements in Teaching Mathematics Needed to Develop Student Epistemological and Cognitive Abilities

All participants expressed a growing understanding that student development was central to success, as reflected in this first subtheme. Participants all mentioned that math instruction needed to improve to help these students learn math, and this included improving their own teaching practice, which prompted them to seek out ways to improve their instruction. Their perceptions of the need and how to respond varied among them and changed over time, with some commonalities. Facing the reality of underprepared students learning college math, Hawthorn said,

> The challenges that I have are that the students that need help the worst are those who are least likely to describe their process [in a reflection activity]. They'll just give you the bare minimum, the bare bones of what they did. So, they wind up with their math anxiety keeping them from talking about it, basically. And so, that keeps me from being able to help them and see what they need.

Elm had a growing awareness of the need to improve instruction after having 90% of students in a class fail the first exam. Elm said,

> If you are an instructor, and you really do care about this business, about teaching, trust me, you will sit down and wonder what went wrong. Then I started to come up with a lot of scenarios in my mind. You ask yourself a question: Did I give them enough time? Did I make that test too hard?

Instructors expressed the need to reduce student anxiety and develop epistemological reflection and cognitive development of students who were not previously successful in learning math. Date said,

> [Addressing] the anxieties that students have are the types of things that I do through my reflections, so they are then able to move to the next level and not have those same anxieties and concerns. I hope some of the questions I asked in reflections will be things that will get them in the rhythm of being a student. A lot of community college students are first generation, and they don't have people in their lives who can tell them what they need to do at what point. So, I feel that I'm

> teaching the math, yes, but I’m also teaching them student skills in the process, and that's very valuable as they move forward, because then they can focus more on the math rather than on the other things about being a student.

Grape said,

> What is interesting is every student who has a negative view of mathematics, and this happened without exception, they tell me about when they were young their first memory of math was positive, and they loved it. And then they say, “I got to So-and-So's class.” Then there is a clear delineation for them of where they realized that they liked math, and they were good at math until that happened. And they can all identify with the "until this happened" place. But in the reflection, what it does, is it makes them realize that there was a time where they liked math and there was a time when they considered themselves good at it. And, then there was a trigger that changed that, which means it could be changed back the other way.
>
> Ivy asked students to write a math autobiography to learn about their past math experiences in classrooms and found it was a commonality among students with a difficult relationship with math that they did not like math after having a bad experience that changed their math identity.

Underprepared students often have to take placement exams and Grape discussed how placement exams are inadequate as a measure of student ability, because placement exams do not always provide a good indicator of ability to learn mathematics or the level of development of the student, or of motivation. Grape said,

> If someone's properly motivated, and they're willing to meet me outside of class and get extra help, often I would get a student who wouldn't qualify to even get into class to get through with a pretty good grade, but again, it depends on these motivations. The placement exams can identify the weaknesses, but they don't test motivation. I mean those things don't show up on a placement test. And so, a lot of these intangibles are much more important than the prior knowledge to success, in my opinion.

Instructors Found Reflection Improved Their Teaching Practice Over Time

The second subtheme, addressed by all 10 of participants, focused on reflection improving their own teaching practice over time. Seeing reflection as a way to gain insight into his own teaching practice, Elm said, “Reflection for me, it's my way of finding out what my students need, and how I can help them get what they need, so that they can succeed. Okay, that is how I see reflection and how I use it.”

Juniper, when thinking back on a personal school experience, described facing a lot of difficulty in graduate school due to the failures of inadequate teaching methods and the benefits of learning from reflection. Juniper said,

> I think about half my class dropped out after the first year of graduate school, because in a research-oriented institution, the professors teaching the classes are not really good teachers. So ultimately, you're left to really learn everything on your own. I had to struggle to learn and to use reflection. I brought this experience to my teaching.

Hawthorn described the benefits of student reflection to help her decide what to emphasize.

> [There are situations] when they couldn't even get started. Yes, because this problem didn't look like the ones we worked in class or the first step out of the gate, we had to deal with fractions, and so, that tells me, we need to work more on fractions. See, sometimes it means that I forgot to tell them something in the course and to help them see how to work the problem, and so, I'll have to go back and revisit this whole process again because I didn't do something. And that's more than anything the most helpful thing for me is seeing the things that I do give them and the things that I should have emphasized more.

Cedar had a similar analysis of how students need reflection to develop, as Date did, explaining,

> The reason that reflections are important is it because it allows professors to read students' minds, I mean, to put it bluntly, I can tell exactly how effective my teaching is. I can tell exactly to what extent students have learned the material. I can tell exactly whether they're just mimicking my procedures and manipulating symbols at a very shallow level versus understanding the material at a much deeper level, and you can't do that without asking students, tell me, why did you have that, and how did you get that, and what's a good first step, how do you know that's the answer? Explain your thinking, justify your answers." And so that's the reason why I came to realize that the reflection step is critical and, in fact, the more you do it and the less you talk, the more they learn. And reflections

> can be not just in front of the whole class, but also in pairs and small groups. It's a constant everyday pedagogy that I use.

Participants reported a change in their perception over time about reflection being worthwhile with the aim of developing students epistemically and cognitively, despite the extra time required and shared that currently they would not teach a math class without using reflection. Ivy said, “It can be a huge time suck, but it's worth it in the end.” Beech said,

> When I first started teaching, I was like most professors. I figured my job was to deliver content with lectures, and to pause occasionally to see if there are any questions. Of course, there were none, ever. But I just assumed, if there're no questions, I guess they're understanding everything just fine, but over time, I developed a much more effective way to proceed. My role was less of a lecturer and more of a mentor and a coach. I saw that my job was not to deliver content. My job was to change attitudes to inspire, highlight, motivate, and summarize. And I came about that change in my perceived role, mostly by talking to my peers and attending professional development conferences, which I've always done, and I learned so much from my peers and my mentors.

All participants said they were almost entirely on their own in coming to an understanding of what reflection is and how it related to student epistemological and cognitive development, because participants found there is not a lot of guidance. Date said, “There is not a lot of instruction on reflection.” Beech thought math faculty could gain an understanding of what reflection is first by “reading the research to understand, does [reflection] help, what is it? The second thing I advise would be to attend professional development conferences." Beech noted that often the teachers who need to go to those professional conferences the most do not go.

To better understand reflection so as to advance students' development, Ivy made an effort to observe as many teachers in the classroom as possible and invited teachers into Beech's own classroom to observe. Fir became interested in reflection when attending a conference as "a wonderful method for students in the study cycle for learning" and has tried to include reflection in every homework problem in class. Learning how to make use of reflection in class so students would more willingly use it, Ivy said, "Reflection could be a burden on students based on how it is implemented. I think I overloaded them by asking them to reflect on every homework problem. So, I have them reflect on some problems and on the exams."

On what reflection is and how to use it to help students move beyond rote mathematics operations, Juniper, a more experienced faculty, said,

> So, there're various types of reflections that I do…, so, first of all, I am a believer in metacognition, and I want my students to always be thinking about what they're doing and not just doing rote operations and procedures. So, anytime in class that we have a chance when there is an error that happens, and we have a chance to reflect on it, and I give my students a chance to think about something that went wrong and to discuss with each other, so that's one type of reflection that I employ.

For some participants, the understanding of reflection was a bit more limited than Juniper's, such as Fig having students answer several reflective questions at the end of a homework worksheet, such as, "what are the three main points of this worksheet?" More experienced instructors found more ways to utilize reflection. For example, Hawthorn said,

> My understanding of reflection is that it is where I gather information about what a student is learning on a particular topic. So, I would have gone over a lecture, and I would have let them do some practice problems, and then they would come back together, and I would get them to explain exactly their process that they did. I've been doing this kind of thing where I would ask them to tell me about what they've learned, for probably 8 to 10 years now. Reflection helps me to understand what the students are gathering out of my lecture and out of my classes. It helps me to see where I need to adjust and adapt for those students.

All participants agreed that reflection takes time if it is to support student epistemological and cognitive development. Many noted that the time was not a barrier or burden, especially for the benefits attained for underprepared students. For example, Hawthorn said, "Doesn't it [take extra time]? Yes, it does. That would be one of the weaknesses for this for me is that. It for lack of a better word, it can double my time doing instruction." Date stated about her weekly check-in,

> I noticed reflection is a great way to check in with students, a great way for students to really dig into how to be a student. I don't necessarily have them reflect on math as much as I do on their learning processes. I have a little check-in

> weekly between me and students because you don't always have time to check in with each student, but if you have them reflect on their progress during the week and answer some questions. It helps me, and it helps the students, so I can better help them because of it.
>
> The faculty’s understandings of reflection took time to develop. For example, Hawthorn explained that he did not know what he was doing was called reflection.

Instructors Found That Reflection Helped Their Underprepared Students Develop Who Were Newly Successful in Learning Math

The third subtheme captures several of the instructors pointing to getting students thinking about what they are doing and not just doing rote operations and procedures as a way to advance their epistemological and cognitive development. Participants found that many students have a negative view of mathematics. Ash reflected on how sometimes students cannot even get started on solving a problem. Grape and some of the other participants noticed that underprepared students seemed to have something in their school history that made them think they could not learn math and resulted in their having a negative view of math. Hawthorne invited students to share their uncertainties.

> For me, I would say the biggest advantage [of using reflection in the classroom with students] is that I get a direct input as to their weaknesses and things that they need help with. Whether it's the new material or how to handle fractions or whatever it is, I get it direct from their mouth. It's a much cleaner view of what they're thinking about and how they're thinking as far as how to work these

problems. And sometimes even if they can't even get started with a problem, they'll tell me some of those kinds of issues, such as, “I couldn't even figure out how to get started, because of this issue, or that issue,” or whatever it was.

For students underprepared for math, participants found that reflection was a good way to check in with those students. Cedar said,

Reflections can be very helpful so students don't lose hope, because they know that they can succeed, and that they are not alone. Students can reflect in various ways, but it's important to understand our students. We were students ourselves, but time has passed, and it's always good to have that input.

Participants had a somewhat different understanding of reflection, based perhaps on the instructor’s years of experience, education level, and their perceptions of what an instructor’s role is in the mathematics classroom. Participants’ perceptions about what reflection is all about ranged from newer faculty limiting reflection to having students provide feedback at the end of the class or by watching a video, to the most experienced faculty more frequently using it with students in the classroom having transitioned to a deeper understanding of its usefulness. For instance, Juniper said,

At the end of the week, I ask my students to write a reflection of their learning throughout the week., how did they flourish as a mathematician? Were there any places where they were stuck and how did they resolve those situations, and if they have any questions about the course, or how the quarter is progressing.

Ivy, a newer faculty member, said,

> Reflection is when students think back about the work they've done for the week and told me about it, maybe things they could have done better or things they thought they did well or could do better next time, and this kind of thing. I do not ask students to use reflection when they are working on a problem. I started using reflection this past school year. I mostly wanted to give the students an avenue to communicate so I just gave them a formal assignment to share about their failures, and successes gained.

For several faculty, understanding how to use reflection with students included both helping students *connect the dots* as well as getting feedback from them. Ivy said,

> Reflection has two components for me. I want to help the students. I want to give them an opportunity to take the time to connect some dots, whether those dots are content related or putting their schooling and context of their lives and realizing that it's okay not to be perfect and so, not to be really hard on yourself. And, then I also want to gain feedback from the students, and I want to gain feedback on their experience.

Juniper said, "It's important for professors to constantly have students reflect on their knowledge to explain their thinking and justify their answers and communicate in groups and talk to each other. I utilize this in every class I teach using reflection to update teaching, Cedar said, "Using reflection activities provides the potential opportunity to modify my teaching in real time. I think that kind of insight is invaluable," Elm said about using reflection to improve teaching practice,

> What stands out for me is the level I need to adjust my teaching every semester and that's something I started doing roughly 2 or 3 years ago. Sometimes I have a group of students, and you might think they are all the same, meaning that every semester is going to be the same thing, but the reality is, it's not. I might have a group that has a faster pace compared to another one that has a slower pace, okay? But so, I wouldn't know that unless I ask them for reflections, [such as] how am I doing today? How's the homework? Did you miss anything? Do you feel like this grading was fair? Certain types of questions like this usually give me room to change and adapt to their learning.

As a way of finding mentoring to guide instructor teaching strategies, participants suggested observing master teachers and trying out ideas. Hawthorn discussed some things that were helpful to learn from a mentor. Hawthorn said,

> [One thing that a mentor can help with was showing] what reflection looks like. I would tell you that you've got to give a good example of what that looks like… what reflection looks like, what are you talking about, because if they've never seen this before then they don't know what you're doing, and they don't know what you want, so possibly the first one out of the gate you do is show them what you're looking at? Also, you have to make it worthwhile to the student.

Fir said, "I would tell somebody don't be afraid to try things." All participants said that whatever students are asked to write about that the instructor needs to read it fully and respond with feedback. Reflections need to have a purpose for what the instructor wants to achieve.

Theme 2: Reflection Activities Improved Student Learning Outcomes

There were four subthemes related to the second theme, which addresses how community college instructors use reflection activities in instruction in the classroom to improve their students' learning outcomes. The four subthemes are journals, reviewing exams and homework, handwriting in solving math problems, and group projects.

Faculty used several approaches to including student reflection in their instruction, for example, weekly check-ins, journal writing, and the need to handwrite math problems when learning how to solve them. Participants also found ways to interact when instructing in online classes. All 10 participants indicated they asked students what was working and not working. All participants described the reflection activity of reviewing exams with students in class and focusing on what went wrong on problems. In another example, most participants asked for feedback as a reflection activity, with variation about how often this was done and how.

Journals

Several of the participants asked students to keep journals. Ash found that asking students to reflect in a journal helped them understand why they made choices, be it in their personal life or in solving math problems.

> I noticed that what students really had an issue with was with the ability to explain their reasoning. And so, I started using journals at that time, and I would not ask them to write in their journals about math. I asked them things like, "if you could vacation anywhere in the world, where would it be, and why?" They didn't find any problem with where it would be, but they had a terrible time with

why. So, we had a journal every day when they came to class. I read them each completely and offered individual feedback. What struck me was that by the end of the class, the "why" was no longer a problem. And they had also considerably improved their ability to explain their math. So, if I said, “Can you tell me why you took this step,” they could tell me why, where they could not before.

Showing the potential of journal writing as an instructional strategy, one of Cedar’s students wrote about gaining comfort with ‘not knowing.” Grape said,

I had one student who started analyzing a problem and was incorrect in her analysis, and as she was writing, she wrote, "oh no, well, that's not right, that can't be true," and then she gave her second thought. And she said, "oh, wait, that won't work either," and then she wrote, "you know, I guess I don't really know." And I thought that it was really important for her to consider the options and then come to the realization of "I don't know at all."

Grape, again speaking of requiring journals, finds they benefited students by helping them focus and reflect:

I found that reflection pays off so greatly. It's often the only time some students really think deeply, and I don't mean that as a criticism of students, I think that they're very busy, and they're focused on getting the work done, and they don't always stop to reflect, to really think deeply about something. And this gives them the opportunity to do that because it's an assignment, and they get points for doing it, so I’ve actually now made it in my calculus class, so the journals are worth as much of the grade as for the regular assignments.

Participants used several techniques, particularly a weekly written reflection, responding to a prompt. Date said,

> I started a few years ago. At the end of the week, students reflected on that practice. So, their reflection [prompt] is… "what worked for me about this new strategy and what didn't work. What challenges did I have with it, how did this help me overcome a challenge in mathematics? What areas do you think you are strong in now, or do you need to improve," and then the same thing at the end of the semester, and so, so that they can see their own growth in that way. In all my classes, I have some kind of reflective practices. They're done on a weekly basis at a minimum.

Most participants said they made use of reflective journals, including prompts for students to answer. Some of the other prompts were "what scares them about the course?" and "what they expect from the instructor." Participants found journals were helpful to see where students needed more academic support. Juniper, who frequently uses journals, finds they help her get to know her students better, particularly in an online environment. She said,

> With reflection, the main thing is that now I get to know my students much better. So, if someone were to ask me for a letter of recommendation, now I can say, "let me look at your reflections." It makes that process much easier. Usually every quarter, I have 100 students in all my classes, so it's kind of hard, especially now that everything is online, and I don't really see too many faces because most people have the cameras off. [The reflections] make it easier for me to get to

> know my students, so that's the main benefit I am seeing. Also, sometimes the reflections that they do, they just blow me away. I didn't think about it in that way or from that perspective, so I get to learn other interesting things that I didn't know. For example, I had a student from last quarter who wrote this reflection at the end where she detailed the way she was learning, and she put in a concept map about how that she was learning the material, and I was just blown away by the depth of her reflection.

Reviewing Exams and Homework

Other examples of reflection activities used in the classroom by the instructors were reviewing exams and homework to deepen understanding. Using reflection with students after exams was a common starting point for reflection for the 10 faculty. For example, Cedar said,

> I definitely do reflections, which are after the first exam, and maybe after all the exams, with some opportunity for students to do corrections, with a few points extra. I think that's a very good learning tool and motivational for students and keeps our students from dropping.

Cedar also found ways to ask students to reflect on online math homework, after seeing how faculty were using reflection in their online homework at a conference, by using an online adaptive math learning platform developed at the University of California called ALEKS (see www.ALEKS.com), now offered by McGraw Hill. Cedar said,

> They were combining ALEKS with reflection activities, so every student had to reflect on homework. I saw how much using ALEKS and reflections together

made a difference. Instructors would first give students smaller assignments, where students could reflect and then they would see the solution, eventually, and then next comes a bigger assignment. So, now they're a little bit more prepared and then of course it builds up to the exam. I thought, "Wow [said with emphasis], this is making a difference, so let's try something similar."

Handwriting in Solving Math Problems

Beech asked students to use handwriting in solving math problems and developed an approach that also worked in online classrooms. Beech said:

> So, a few years ago, I had a brainstorm, okay, and that was to have students get sharpies and write on paper with a black thick sharpie because a regular pencil or pen won't work. But write it with a nice thick sharpie on paper and hold it up to the webcam, so that they can share their work with me and with each other. It took me a year to come up with that. I was asking all my colleagues, all across the country, "what do you do?" Nobody had a good answer. They were talking about using electronic whiteboards and writing math with a mouse on an electronic whiteboard, [but] it looks terrible.

Instructors found they got better facilitating reflection over time. Hawthorn said:

> My job is not necessarily to deliver lectures. I tried to do that. I see that my job is also to have students engage in what I call productive struggle, so they think about the material. I want students to talk about it with each other, and teach each other, and explain their thinking to me, and to justify their answers to the rest of

> the class, so they can share how they arrived at a solution, and so on. That's my job.

Group Projects

Another teaching strategy that improved reflection was the use of group projects. These were seen by several participants as beneficial for helping students to develop in their ability to do math. For example, Grape said,

> I really do [see a benefit], and I got a lot of feedback from the students that expressed that. Now there's a danger, and I was quite aware of this, that the group project becomes where one does the work and everybody else copies. Well, that did happen from time to time, but generally speaking, I felt very positive about the group projects, and I would get feedback from students that they really learned a lot from this. I always said to them, “what I really want to see, I want to see you argue. I want to see you have a disagreement.” Because often it's from thinking about how you could do the problem and doing it wrong and somebody pointing it out to you that you really learn a lot. And I also said that when you're helping someone, the person who gets the most out of it is the helper, because you have to formulate what you know and try to express it and sometimes in that process, you find out, "Oh, I really don't understand this as well as I thought I did.”

Participants saw that dialogue in small groups is reflection and can be accomplished both in both face to face and virtual settings. However, participants claimed that finding ways of using work groups in math class requires thinking about the size of the group and the value to the students to those who initiate and those who are quieter. Instructors found that reflection starts individually and was then shared by explaining the “why” to the instructor and to other students.

Fir said,

> We had maybe 24 students in a class, and so I’ll have them come to a worktable in groups of three, and they would each get their own set of blocks to work with. Then they’d do this exercise at the back table. The challenge there would be that in a group of three, you know that one of them has to figure it out. But you're never sure if they all understand, and it is definitely helpful (for the one who figures it out), once he figures it out and then explains it to the rest of the group. That's good for that student, but you don't know how many of the partners are sitting there saying, “Okay, that sounds good to me.” I could make smaller groups. I could do twos or even singles, but I do think there's something to be said for discussing a problem instead of just sitting there thinking about it. That would help me know for sure (if I did, say, singles) but then you've missed out on that conversation, so the benefit there is in a group bringing more reflection into it. I ask students to walk me through how they did the calculations two ways, and once they have seen both approaches, which do they prefer?

Summary and Conclusion

In this chapter, I presented results of the study. First, I provided a description of the setting and participant demographics. I described data collection and data analysis processes. I evaluated the study’s trustworthiness and provided results of the study via two themes. In Chapter 5, I interpret key findings, including a comparison of findings involving the conceptual framework and empirical literature related to the research problem. Chapter 5 also includes limitations of findings, recommendations for further research, and possible implications for social change.

Chapter 5: Discussion, Conclusions, and Recommendations

The purpose of this study was to better understand perceptions of mathematics faculty regarding their use of reflection to foster epistemological and cognitive development and academic success of students who entered college underprepared to learn college math and were newly successful in learning math. This study was conducted because mathematics is a gatekeeper for success in college and nearly two-thirds of new high school graduates enter community college underprepared to learn college-level mathematics (Douglas & Attewell, 2017). Epistemological reflection and reflective instruction have been found to foster and even accelerate cognitive development for underprepared students to successfully learn college mathematics, especially among those who otherwise might have to leave college. Open-ended coding of interviews with 10 community college mathematics instructors who use reflection resulted in two themes with associated subthemes that emerged, each addressing one of the two research questions.

The first theme addressed was what instructors perceived regarding how reflection developed epistemological and cognitive abilities of underprepared students who were newly successful in learning mathematics. Instructors found that improvement in teaching mathematics was needed, and reflection improved their own teaching practice over time, and helped their students develop and be newly successful in learning math.

The second theme addressed reflection activities that instructors perceived improved student learning outcomes. The activities, characterized as subthemes, were journals, reviewing exams and homework, handwriting when solving math problems, and group projects. In group projects, instructors found that reflection starts individually and was then shared by explaining the "why" to the instructor and to other students.

Interpretation of the Findings

In this section, I describe in what ways findings confirm, disconfirm, or extend knowledge in the discipline by comparing them with peer-reviewed literature I reviewed in Chapter 2. I also interpret the findings in the context of the conceptual framework.

Interpretation of Empirical Work in the Field

In the empirical research, reflection activities are known as an engine of student cognitive development and learning success (Bronfenbrenner, 1993). Boylan et al. (2019) found that providing reflective activities such as having students work together with study partners improved student success (also see Rutschow et al., 2019). Thanh (2020) demonstrated that student reflection activities can be useful as a way to motivate students to reflect and improve learning outcomes, as supported by Baxter Magolda's epistemological reflection model.

Aditomo (2018) found that students with more complex epistemological beliefs had higher grade-point averages and that epistemic maturity was associated with the practice of student reflection, resulting in better academic performance. If students have incorrect views (such as that science does not change), this may result in learning problems rather than due to their having any lack of ability. For these students, reflection can benefit their learning by gaining a new perspective, which improves cognitive development (Aditomo, 2018). This awareness of beliefs about knowledge can be empowering for both students and instructors. Epistemological reflection and cognitive development and beliefs were found to be a result of education (rather than as a personal deficiency that cannot be remedied) and should be easier to change (Baxter Magolda, 2020).

All 10 participants used reflection in their mathematics classroom and found it developed their students' epistemological and cognitive abilities in learning math, as reflected in the first and second themes. In a confirmation of the first subtheme of the first theme, which reflects the instructors' recognition of the need to improve math instruction, Kersey et al. (2018) suggested that mathematics faculty members at community colleges have recognized the need to foster student cognitive development as well as to change mathematics instruction practices to improve student learning outcomes. Douglas and Attewell's (2017) findings suggested that math instruction needs to be improved and discovered that nearly two-thirds of new high school graduates enter community college underprepared to learn college-level mathematics, confirming the first subtheme of the first theme on the need to improve math instruction, where all 10 participants claimed that improvement in teaching mathematics was needed.

Community colleges assess most of their entering high school graduates as needing remediation in mathematics (Nix et al., 2021), but many of these students do not even start remediation let alone complete it, making college graduation or transfer impossible (Logue et al., 2019), as reflected in the first theme. Logue et al. (2019) found that students in the United States assessed as needing to take remedial math are not likely to start it, let alone finish it, so they cannot graduate from college. For this reason, Ganga et al. (2018) and McKinney et al. (2019) named math as the biggest barrier to students completing college in the United States, confirming the first theme, first subtheme result. Hunter's findings suggested students assessed as underprepared to learn college-level math may have significant cognitive limitations in how they learn, requiring epistemological and cognitive development before becoming ready to learn (Hunter, 2017), confirming the third subtheme of the first theme, where instructors found that reflection helped their underprepared students to develop to become ready to learn college-level math. For example, Hassi and Laursen (2015) indicated that instructors using reflection activities achieved promising results in fostering successful learning for underprepared students at community college, as a confirmation of the two themes that emerged from this study. The power of reflection on learning and improving instruction has been reported in the literature in other fields as well. For example, this includes nursing students in training in their professional education in the United Kingdom (Bulman et al., 2014); first-year law school students at Cal Western School of Law in San Diego (Casey, 2014); young clergy students in their post-graduate educational training in Australia (Foster, 2018); and student teachers learning the role of reflection in changing their teaching practice in Hong Kong (Cheung & Wong, 2017).

Teaching students to think independently through reflection is one objective of education. As a result of using reflection activities, all participants in this study found that reflective activities improved student learning outcomes, reflecting the second theme. However, it is surprising that all the participants in this study conducted reflection activities with students only after the learning had occurred in an effort to correct misconceptions, rather than having a focus on reflecting on learning in process, with the exception of group work, the fourth subtheme of the reflection activities. As has been previously discussed, Muhali et al. (2019), who was inspired by Schoenfeld's (1983) reflective problem-solving scheme, modified the reflection model for teaching math by increasing opportunities for reflection activities for students, so that reflection was inserted at each step of problem-solving and also included social processes with an emphasis on learning through collaboration and interaction with others. Muhali et al. saw that social learning creates learning conditions needed so students could reflect with others on their thinking processes (also see Yaacob et al., 2021), and on their own with self-reflection by internalizing the voices of others. In their study, Muhali et al. inserted reflection at every step of problem solving, instead of waiting to reflect until afterwards, and as a result, student learning outcomes were significantly improved (ranging from an improvement of 5% to 9.3% in student learning outcomes, above the improvement of reflection only after problem solving).

As was found by the participants in the second theme of this study, in group projects reflection starts individually and was then shared by explaining the "why" to the instructor and to other students. Epistemological reflection can mean answering the "how I know" question by listening to and reflecting on one's own voice. Reflective conversations stimulate cognitive development and learning and are an engine of cognitive development for recent high school graduates starting college (see Dewey, 1933; Schön, 1984), where an individual has a conversation in the search for a solution needed to reframe the context needed to better to define a problem. When someone engaged in reflective conversations with others, the teaching changed from a traditional classroom monologue (lecture) to a dialogue or a reflective conversation that powers, fosters, and accelerates college students' cognitive development, as described in recent research (Anseel & Ong, 2020; Iordanou et al., 2019). Ståhl (2020) and Teoh et al. (2020) studied when reflective conversations in the struggle and search to better define or understand a problem, can help stimulate cognitive development and learning to develop young college students' cognitive abilities as they think through different perspectives, especially if students started college underprepared to learn mathematics (see Baxter Magolda, 2004a, 2004b, 2014, 2020; Dewey, 1933; Schön, 1984).

Interpretation of Findings

The two key concepts embodied in the conceptual framework of this study draw on the work of two theorists on reflection, Baxter Magolda (2020) and Schoenfeld (2021) and confirm the two themes captured in data analysis. Baxter Magolda's epistemological reflection model focuses on fostering and accelerating recent high school graduates' success and cognitive development with reflection. Baxter Magolda found reflection is a key for students to learn deeply, and this extends to students who previously were not ready to learn college-level mathematics successfully, specific to the uniqueness of learning math. The other theory, Schoenfeld's model of teaching mathematics, applies to the role of reflection in support of developing mathematically empowered community college students.

The first theme captured how reflection helped students develop, becoming newly successful in learning math, along with the subtheme of needing to improve math instruction, confirming Baxter Magolda's (2020) epistemological reflection model, which refined Perry's (1970, 1981) scheme of cognitive development. Fir said, "I consider reflective practices for students and myself a really important part of the learning process," as captured in the first and second themes.

The first theme captured how instructors can create conditions that promote learning, foster reflective development, and accelerate cognitive development through reflection by questioning assumptions, leading to a more complex perspective for the student from questioning and revising assumptions. In the specific case of learning mathematics, Schoenfeld (2020) confirms reflection helps students learn mathematics with deeper understanding and also provides insight to instructors about how to improve their teaching practice. For example, Fir said, "Reflection helps students understand math…and give us insight to what students think… Reflection can help us to improve our teaching."

Piaget's (1972) views of reflection are also captured in the first and second themes, with an emphasis on student development with cognitive development occurring within the learner as a result of reflective thinking. Instructors in my study found that an instructor placing too much of an emphasis on educational outcomes can be harmful developmentally to a person because under pressure, a student may memorize a formula and sidetrack the cognitive development necessary for comprehending a formula, confirming Piaget. Building on Piaget's (1972) work, Perry (1970, 1981) conducted studies on college students and ways they understand what and how they learned, where cognitive and intellectual development among college undergraduates was observed and delineated, including their reflections and assumptions about their own learning, which the first and second themes capture. The student developmental journey that Perry (1970, 1981) described can be emotional, especially as it relates to learning mathematics. Students can experience pain and confusion from a lack of success, or they can get excited, and perceive the pride that comes with success in learning mathematics. For example, Date said, “I had a student who… was struggling. She wrote in-depth reflections, and those reflections allowed her to know what she was struggling with and allowed me to know how to better help her. Then she moved on to college-level math, which is super exciting.”

Baxter Magolda (2020) built on Perry’s (1970, 1981) pioneering work, integrated faculty and student development theory with Perry’s scheme. The first and second subthemes of the first theme confirm Baxter Magolda suggesting that instructors can design instruction and interaction to reach students at every level of cognitive development to improve learning, develop complex reasoning, and promote skill acquisition. Baxter Magolda suggested that an instructor does not merely lecture to transfer knowledge directly to a college student and instead, create situations that engage college students in problem solving, which helps develop students’ cognitive thinking capabilities, confirming the first theme, capturing that improvement was needed in how mathematics was being taught to bring about the epistemological and cognitive development needed for students to learn.

In a further confirmation of the first and second themes captured, the other theory, Schoenfeld’s (2020) model of teaching mathematics also was confirmed the two themes, that reflection supports development of mathematically empowered community college students. Schoenfeld built on the work of Dewey (1933), finding that over time, reflection can be internalized by the student (and the instructor) as part of the training of a person's thinking. Ultimately, later in life, as a person's thinking develops, reflection can become part of a tool kit of capabilities brought to bear on problems, by becoming "reflective," as described by Schön (1984, 1987). Examples from participants' interviews confirm this, such as Elm, who described an already capable student who doubted herself and who developed as a result of reflection, "The reflective process … helped her to slow down. It helped her to think more deeply about the content. She found that she understood the mathematics better and recognized herself as capable and completely changed how she was able to perform in class."

The first theme confirms Dewey (1933), that learning through reflection is a response to a learning experience with doubt or conflict and can transform it into a clear and coherent understanding. As a prompt for reflection, participants found an instructor can create situations to engage students in problem solving (with doubt and conflict), assisting in their reflective discovery, and in developing their reflective thinking.

The second subtheme of the first theme captured how instructors said they were able to improve their own teaching practice through reflecting on their teaching to help students to learn who were newly successful in learning math. Instructors found ways to improve the math instruction they delivered through the use of reflection. Examples include, Elm said:

> I use reflection in my teaching. I thought every instructor did, but I found out later they don't. After each teaching session, each class period, I make notes of things that work and things that don't work, things I need to change for next time.

Ivy said, “The reflection, the self-assessment, the problem-solving strategies, you need to constantly interrupt your work with self-reflective questions. Its quality, not quantity of time that you spend.” Date said, “I use reflections in my classroom. It helps me, and it helps the students, so I can better help them because of it.” Hawthorn: “I want to teach math. The reflection helped me to teach math. It has helped me to change my instruction.”

Confirming the second theme is Schoenfeld’s (2013) TRU model, fine-tuned by testing in a real-world setting and using emerging evidence from studies to revise intervention strategies in real time. After many years of conducting research in the mathematics classroom and building upon Dewey’s (1933) concept of reflection, Schoenfeld’s TRU model focuses on reflection as an element in effective mathematics thinking and learning in community college mathematics classrooms. TRU uses peer-assisted reflection to assist students to learn metacognitive skills needed for problem solving, in the transition from external feedback to internalized self-regulation. The second theme confirms Schoenfeld’s TRU theory on how reflection starts individually and is shared with the instructor and other students, improving student learning outcomes and learning with deeper understanding. The focus of Schoenfeld’s (2019a, 2019b) teaching framework is on student thinking and the use of reflection to put attention on a student’s new learning and questions at each step of problem solving, focused on what a student experiences in the mathematics classroom and the impact of those perceptions on student learning outcomes.

The second theme captured how participants found reflection starts individually and then is shared by explaining the "why" to the instructor and to other students as a reflective conversation. The second theme confirms the theories of Schön (1984, 1987), reflecting on Dewey, who argued that individuals have a large amount of knowledge they can access only by doing something actively. In actively doing, the learner has a reflective conversation with the situation, testing conjectures, providing information to guide the next decision. Reflection takes place after the fact, in the moment, and even continuously. Being reflective means being able to perceive what is going on in the moment and to make needed adjustments in real time. After the fact, it is also important to reflect on what went on, where the learning did not happen as expected, what could be done differently next time, and what worked well. Schön (1984) discussed reflective conversations as a way for students to understand, define, and solve a problem. Reflective conversations between an instructor and new college students can help develop student cognitive abilities, as they think through different perspectives during problem solving, especially for students if they started college underprepared to learn college mathematics (Schoenfeld, 2021). Building on the work of Schön, Schoenfeld (1985) identified the role of reflection in effective mathematics thinking and learning in mathematics classrooms (see for comparison Baxter Magolda & King, 2008 on reflective conversations). These theories are confirmed in the first theme that emerged in the data analysis on how reflection developed epistemological and cognitive abilities of underprepared college students. Instructors found that reflection helped their students develop who were newly successful in learning math. These theories confirm the second theme captured in the data analysis, where reflection improved student learning outcomes and students learning with deeper understanding. Instructors improved their own teaching practice to improve student learning outcomes.

Limitations of the Study

The most important limitation is that the participants were self-selected in response to my invitation, therefore the results may not apply to those who did not volunteer to participate. I encountered challenges in identifying participants who fit the criteria I had specified, so I extended my search, but I do not think this limited the trustworthiness of my results. The reliability of my results needs to take account of divergent constructions of the term *reflection*, as well as differing understandings of what this term may mean in terms of the perceptions of my interviewees. For some participants, reflections were a way for instructors to have students communicate with them, rather than a way of thinking through problem solving. Another challenge is recognizing and accounting for my own biases, foremost my conviction that reflection improves a student's ability to learn. Although I have not been a student or a faculty member at the colleges involved in the study, I have been an associate professor of mathematics at a private university for 2 years, and I needed to be aware of my own bias, and I tried to mute any biases and assumptions to clarify my subjectivity, as well as applying objectivity in data analysis. I took into account perceptions that diverged from and converged with my own perceptions and practice-based understandings to maintain an unbiased processing.

Recommendations for Future Research

Incoming community college students who are assessed as underprepared to learn college mathematics need improved approaches to mathematics instruction, as suggested by findings of this study and research in the field (see Schoenfeld, 2021). As a possible remedy, the findings of this study suggest epistemological reflection has been largely overlooked in studies on college mathematics instruction, perhaps because reflective instruction is difficult to deliver and to study (Schoenfeld, 2021). Yet, reflective instructional practices have been shown to foster and even accelerate the cognitive development of underprepared college students (Kersey et al., 2018). (Also see Baxter Magolda, 2020; Perry, 1970, 1981; Schoenfeld, 2021).

Further research could be conducted about the perceptions of instructors using reflection with underprepared students learning mathematics in the community colleges because it takes a long time to learn how to think, longitudinal studies are suggested. I recommend the following possible designs for future research studies:

- A study to provide a better understanding of how instructors could implement the use of reflection with their students to improve student learning outcomes in learning college mathematics.
- Cognitive development has transdisciplinary benefits for students that go beyond math class (Mevarech et al., 2018). For this reason, epistemic beliefs

are good targets of educational interventions to foster college student cognitive development.

- A case study on specific ways new instructors could possibly improve their teaching practice in mathematics through the process of professional development to support better practice.
- A qualitative study to understand approaches of how high school mathematics teachers perceive they could successfully teach their underprepared students.
- A study of new teachers being guided in use of reflection in mathematics instruction.
- A quantitative study to understand the effectiveness of ways successful instructors accelerate development of their underprepared students in learning college mathematics using supportive and innovative online technology applications, such as:
    - ALEKS.com (an online tutoring and assessment platform for learning mathematics developed at University of California, Irvine) to help students to fill in gaps of math knowledge by learning individually and in groups (Cung et al., 2018; Cung et al., 2019).
    - VIRBELA.com (a virtual world where students participate as avatars) allowing a reflective perspective from *outside* themselves when solving problems, facilitating reflective learning and ultimately, metacognition (Howland et al., 2015). ALEKS.com and group projects in math could be carried out in VIRBELA.com.

- OneNote Class Notebook by Microsoft provides a personal workspace for every student and a collaborative space for group projects, in support of the second theme of this study, allowing technology for handwritten math problems and support for group projects (Heraty et al. 2021).

Implications

This study provided a glimpse into changes 10 mathematics instructors made during their careers in using reflection to develop student abilities in learning math and as well as influencing changes in their own teaching practices. The implications for change from the findings of this study indicate it takes time for instructors to identify that something in their teaching needs to change, then to pinpoint what needs to change, and then to find ways of changing those things. Better guidance from experienced instructors on the role of reflection could help new instructors to identify and make changes needed to improve college mathematics education. This could include expanded professional development for college-level teaching of mathematics, with mentoring along with the encouragement and willingness to try new things and to be observed by peers and mentors. The focus of my study was on community colleges, but an understanding of high school mathematics instruction could help to improve instruction as a result of the finding that math instruction needs to improve.

College mathematics is learned by doing problems hands on and not so much by watching someone else solve a problem (Foster et al., 2022). Students who struggle to learn college mathematics might not be developmentally ready to learn mathematics

(Burkhardt & Schoenfeld, 2021). My study found that math instructors can be committed to active learning approaches and reflection, but it is a work-in-progress, and sometimes it fails perhaps of student pushback or because of not implementing it successfully (see Lo, 2018). A recommendation for practice is to video teaching sessions and then self-analyze the lesson to evaluate the success of the teaching approaches. Peers could also analyze the successes and troublesome parts together, sharing ideas.

Conclusion

The purpose of this study was to explore the role of epistemological reflection and reflective instruction in fostering and even accelerating cognitive development for underprepared students learning mathematics in community colleges, as perceived by 10 mathematics instructors I interviewed at U.S. community colleges on their use of reflection. The takeaway message from this study is that all the instructors, across a diversity of their understandings of what reflection means, found their approach was helpful in understanding more about what their students needed to help them learn math more deeply and to better understand how they needed to make changes their own teaching practice to improve their students' learning outcomes.

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Appendix: Interview Questions

Interview Guide

Interviewee: Name (and title):

Interviewer: Sherilin Heise via Zoom

Date:

Time:

Introductory Statement:

I am Sherilin Heise, a PhD candidate in Education. Thank you for spending time with me today for this interview. As you saw in the consent form, you were identified as a mathematics instructor in a California community college with at least two years of teaching experience and using reflection. I will record our session and make some notes as we talk, with your permission. This interview is scheduled to last approximately 60 minutes but might last longer. I might also request a follow-up session of approximately 20 minutes, if needed.

After our conversation, I will transcribe our recorded discussion and share a copy of the transcript with you, so I can make sure I have captured your information accurately, with an estimated review time by you of approximately 10 to 15 minutes. Your name and any student's name you mention will not be included in the transcript or in my study, to safeguard your privacy. Do you have any questions before we start? If not, we can begin now.

Background Information

How many years of teaching experience do you have?

What is your title in your current teaching role?

What is your highest degree earned?

Interview Questions

1. What stands out for you about your experience using reflection in your classroom?
2. How do you use reflection with students who are underprepared or unsuccessful or at risk of failure in mathematics?
3. How has reflection prepared your students to meet community college math requirements?
4. What successes can you share with me?
5. What challenges have you encountered?
6. Please describe some key ways you use reflection in your classroom practice?

   Probe: What ways of using reflection seem to work best for you?

   Probe: Are there ways of using reflection that have not worked well?
7. What prompted you begin using reflection in your classroom practice?

   Probe: How did you first learn about reflection?

   Probe: How did you learn about how to use reflection in your classroom practice in mathematics?
8. Imagine that you were asked to be on a panel discussing this topic at a mathematics education conference. What advice would you give to others who are considering introducing reflection into their classrooms?

9. How do you use reflection in your own life and in developing your own teaching practice?

   Probe: How do you connect this to using reflection with your students in your classroom?

10. Is there anything else that you would like to add?
11. Is there a question that I should have asked you about that I did not ask?
12. What stands out for you about this interview?
13. Is there anyone else you think I should interview about this?

Thank you for your time in discussing these questions with me. I appreciate your participation in my study. Your dedication and commitment are a valuable aspect of my study, and I am grateful for your contribution. I will send you the transcript for this interview in several days so you can review it and make any clarifications. You are not required to read and approve the transcript, but the option is open for you to review it. You will have a week from the time I send it to you to reply about any changes you want to make. After a week, if I have not heard back from you, I will assume you have approved the transcript, as is. Once my study is published in several months time, I will send you a brief summary of the results, and the complete dissertation will also be available. Again, I appreciate your participation in my study.

Are there any questions I can answer for you at this time? OK, Thank you.

I will be in touch with the transcript soon. I will now stop the recording and end our meeting. Thank you and Goodbye.