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AN EXPERIMENTAL STUDY OF THE EFFECTIVENESS
OF THE DEVELOPMENTAL MATHEMATICS COURSE
AT LEHIGH COUNTY COMMUNITY COLLEGE

By

Robert G. Clark

B. S., Pennsylvania State University, 1940

M. S., University of Pennsylvania, 1961
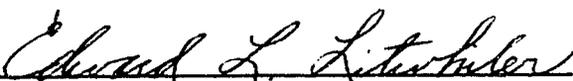
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_____
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A Dissertation Submitted in Partial Fulfillment of
The Requirements for the Degree of
Doctor of Education

Walden University
August, 1972

AN ABSTRACT

of

AN EXPERIMENTAL STUDY OF THE EFFECTIVENESS

OF THE DEVELOPMENTAL MATHEMATICS COURSE

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Robert G. Clark

Walden University

August, 1972

ABSTRACT

The primary purpose of this study was to determine the effectiveness of the Developmental Mathematics program at the Lehigh County Community College.

There was no positive evidence that the existing method of selecting students and/or the material content of the course was effective in achieving its stated objective; that of bringing the skill and ability of weak students needing remedial treatment up to the minimum level required for probable success in first-year college mathematics.

The general hypothesis posed was that the students who took the Developmental Mathematics course would perform better in first-year college mathematics than those students whose ACT scores indicated they needed remedial treatment, but who did not take the Developmental Mathematics course. Four null hypotheses were tested to determine how effective the developmental course was in meeting its objective. One was concerned with the gain scores in the pre- and post-Cooperative Mathematics Test, and another with the performance of the students in first-year college mathematics. The results favored the Experimental group in both cases and indicated the MAT-099, Developmental Mathematics course was doing a good job. The findings of the third hypothesis saw

little relationship between the ACT and Cooperative Mathematics test scores and success in first-year college mathematics, and the findings of the fourth hypothesis indicated that the content of the Developmental Mathematics course correlated reasonably well with the areas of the students mathematical weaknesses, except in several topics such as complex numbers and logarithms.

One limitation of the study was the use of intact groups rather than randomly selected samples and the relatively small size of the sample. To compensate for this, the analysis of covariance procedure was used to test the null hypothesis of no difference in performance in freshman mathematics between the experimental and control groups. The findings again favored the experimental group and the null hypothesis was rejected. For testing all hypotheses the alpha value was selected as the .05 level of significance.

The pre- and post-Cooperative Mathematics Test scores were analyzed and "t" tests used to determine the significance of the difference. The experimental group performed significantly better than the control group.

Multiple correlation techniques were used to examine the relationship between the ACT and Cooperative Mathematics Test scores and success in freshman mathematics; and the

test items were analyzed to determine the students' areas of weaknesses.

A chi square test was used to analyze the frequency distributions of the final grades made by the experimental and control group students in their first-year college mathematics courses. They were found to be significant at the .05 level.

ACKNOWLEDGEMENTS

A sincere expression of appreciation and recognition must properly be given to all the following people who contributed in some way to the completion of this study:

To Dr. Edward L. Litwhiler, who served as Chairman of the Dissertation Committee and provided encouragement and many helpful suggestions throughout the entire year.

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CHAPTER I

THE PROBLEM

When the Pennsylvania General Assembly passed the
Community College Act in August of 1963 it authorized the
development of a state-wide system of comprehensive, public
two-year colleges. This gave each county the right to plan
and establish their own community colleges to meet their
own particular needs and requirements. The Pennsylvania
Community College system has grown from one college with
421 students in 1964 to fourteen colleges in 1971 with an
enrollment of over 42,000 students. This veritable flood
of students has created several difficulties for the col-
leges, and it is the purpose of this study to investigate
one of these problems in considerable detail.

All of the colleges have an "open-door" admissions
policy which, in effect, extends the opportunity for higher
education to all eligible high school graduates. This re-
sults in students being admitted to the community colleges
who differ widely in their academic abilities and prepara-
tion. The colleges have, therefore, mainly through the
efforts of their Admissions and Counseling Divisions,

suggested that students who are insufficiently prepared to meet the objectives of their desired educational programs take remedial or developmental courses. Consequently, all of the Pennsylvania Community Colleges offer developmental programs in mathematics. In general, the existing criterion used by the Guidance Counselors for recommending that a student take developmental mathematics is an ACT (American College Testing) score of 15 or lower. This is equivalent to a standardized test score in the 17-30 percentile range. Roueche, in a national investigation of junior colleges in 1963, identified students in the low-ability group as having standardized test scores in the 10-12 percentile range and below.[1]

The problem investigated in this study, then, was to experimentally determine if the students at the Lehigh County Community College who took the developmental mathematics course actually performed better in their first-year college mathematics course than did another group of students who should have, but did not take the developmental course. In other words, did taking the developmental

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[1]John E. Roueche, The Junior College Remedial Program, Junior College Research Review, 1967, Vol. 2, No. 3, Clearinghouse for Junior College Information.

mathematics course do any good? Did it increase the student's probability of success in the following mathematics course, or was it just a waste of time and effort on the part of both the student and the college?

A related problem was the actual selection process used by the college to determine which students were urged by their counselors and advisors to take the developmental course as compared to those students who elected to take the course on their own decision. Although incidental to the effectiveness of the course, per se, there is always that possibility of having prevented some students from taking the course who might have benefited more from it than those who were urged to take it.

The high degree of diversity in the mathematical backgrounds of entering freshmen has made placement in mathematics classes a matter of concern. Any solution to the problem has been made more difficult by large enrollment increases. In 1959, Rickover wrote:

> The pressure of masses of applicants now knock-
> ing at the college doors is about to have the same
> impact on the colleges that it previously had on
> the high schools. . . . We already have colleges
> which are hardly better than secondary schools.
> We already have state universities which are re-
> quired by law to admit all high school graduates
> from the home state, or all those with a "C" aver-
> age . . . obviously not college material. This
> shows up in the fantastic number of first-year

failures . . . in some instances 40 per cent at
the end of the freshman year.[2]

Admiral Rickover has been a staunch critic of American education, and there are many who would question some of his statements; but the fact remains that some of the problems he has identified have not only continued to plague educators--they have multiplied. Concerning mathematics he wrote:

> Increasingly, the high school must teach elementary subjects because the elementary schools failed to do so. The colleges must give remedial courses in high school subjects because freshmen cannot spell, write grammatically, express themselves or, as Dr. Killian remarked at the recent Senate hearing, because so many of them are mathematical illiterates.[3]

The Need for this Study

Ever since the Lehigh County Community College opened its doors in 1967, it has offered remedial or so-called developmental courses in English, reading skills and mathematics. Since the college does not use placement tests, it relies on the individual one-to-one counseling and advising process to identify and aid the poorly-prepared student in

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[2]Hyman G. Rickover, Education and Freedom (New York: E. P. Dutton and Co., Inc., 1959), pp. 144-45.

[3]Ibid., p. 145.

making his or her own decision as to what course to take.
A large number of today's students do not know what course
they should take or for what they are qualified to study.
The open-door admission policy allows the low-ability stu-
dents to enter, but it should not permit them to enroll in
a course that college officials believe, that in all prob-
ability, they will fail. If the college allows this to
happen, the frequency of failure by these students of low
academic promise will increase.

On the basis of their score on the mathematics sub-
section of the ACT test taken during their senior year in
high school or early in the fall of their freshman year,
students are identified as possible candidates for the de-
velopmental mathematics course. In general, a score of 15
or lower is an indication that they do not have the mathe-
matical background to be successful in first-year college
mathematics. The solution to this problem has been to
strongly recommend they take the developmental mathematics
course. Except for a study by Fadule, in 1969, on the de-
velopmental English course and a brief study by Blyler, in
1970, on the status and evaluation of the developmental
mathematics course, no experimental study has been conducted
on the college to determine the effectiveness of the

developmental mathematics course and to establish cut-off

scores for the college-administered ACT mathematics tests.[4,5]

Presently, subjective cut-off scores are being used by the

guidance counselors and academic advisors.

Concerning the relation between test scores and suc-

cess in college, Froehlich and Darley stated:

> When such relationships are known for specific
> tests in specific situations, their value as pre-
> dictive instruments becomes clearer. . . . the
> counselor should determine the extent of the
> relationship between the test scores obtained and
> marks given in his own school.[6]

There are some members of the administration staff

and faculty who recommend that all developmental courses be

discontinued. This study was not concerned with whether

the college should or should not continue its developmental

mathematics course, but rather with the collection of

objective information concerning the effectiveness of the

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[4]James J. Fadule, "An Experimental Study of the
Effectiveness of the Developmental English Course and the
Selection Process for Classifying Developmental English
Students at Lehigh County Community College," (unpublished
Ed.D. dissertation, Lehigh University, 1969).

[5]George E. Blyler, Developmental Mathematics - A
Retrospective Study, (Departmental Report, Mathematics
Division, Lehigh County Community College, 1970)

[6]Clifford P. Froehlich and John G. Darley, Studying
Students Guidance Methods of Individual Analysis, (Chicago:
Science Research Associates, 1952), p. 240.

developmental mathematics course. However, since there is

no agreement on the part of educators as to what the basic

goals of remedial mathematics programs should be; except,

of course, to improve the student's performance, research

is needed to evaluate these programs regardless of what the

ultimate objectives may be. Even the method of teaching

and the course content are perennial subjects for discussion

at meetings of Mathematical Societies and curriculum commit-

tees.

After describing what he considered to be the desir-

able content of junior college remedial mathematics courses,

Meserve, in an address before a joint meeting of the

American Textbook Publishers Institute and the American

Association of Junior Colleges said he thought most students

were not ready for the courses he had just described. He

further stated:

> These students need at least one additional
> semester of work that has a heavy emphasis upon
> the development of algebraic skills and the under-
> standing of algebraic concepts.[7]

He then went on to say what has become a classic and

often quoted statement with regard to students who never

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[7]Bruce E. Meserve, "The Teaching of Remedial Mathe-
matics," The Mathematics Teacher, 59 (May 1966), pp. 442-43.

had an adequate secondary school mathematics preparation
even though they may have had considerable exposure to such
courses.

> They don't know what mathematics is about. You
> may ask: How can we cover two years of secondary
> school mathematics in one semester? We can't; we
> shouldn't try to; and we don't need to.

He concluded with a challenge to teachers of remedial
mathematics:

> These students have increased their maturity,
> if not their mathematical understanding. We need
> to help them to understand the spirit and power
> of mathematics. We can select from a wide
> variety of topics, but there must be an under-
> lying structure, careful use of definitions, some
> proofs, and, above all, active student partici-
> pation in the growth of his own understanding and
> skill in mathematics.[8]

The Experimental Setting

There is an unmistakable trend today toward account-
ability at all levels of education in both public and
private schools and colleges. Practices that have been
accepted for decades, even generations, are suddently under
attack by critics who would have us change the present
school system. Administrators are being held accountable
for their faculty, faculties are being held accountable for
the performance of their students, and even the students

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[8]Ibid.

are expected to be accountable to the community and to society in general. School boards and taxpayers are asking and demanding the answers to embarrassing questions concerning the expenditures of funds that do not produce effective and measurable results.

Remedial mathematics programs are no exception; they are expected to produce experimentally measurable results in the form of improved performance and/or skills in subsequent mathematics courses. Recently the Mathematics Division at the Lehigh County Community College decided to revise its developmental mathematics course by changing from the traditional text book lecture method to a programmed workbook method in an attempt to better identify the students' areas of mathematical weaknesses and offer some promising improvements.

The central thrust of the study was directed toward establishing the general hypothesis that the students who take the developmental mathematics course will perform better in their initial college mathematics course than those students whose ACT scores indicate they need remedial treatment, but who did not take the developmental course.

Four specific null hypotheses were tested to confirm or reject the general hypothesis.

1.   There is no significant difference in proficiency in elementary algebra of students who took the developmental mathematics course and those who did not, as measured by their gain scores on the pre- and post-Cooperative Mathematics tests.

2.   There is no significant difference in the performance of the students who took the developmental mathematics course and those who did not, as measured by the final grades in their first-year college mathematics course.

3.   There is no relationship between the students' ACT and Cooperative Mathematics tests and their success in first-year college mathematics.

4.   There is no relationship between the course content of the developmental mathematics course and the students' areas of mathematical weaknesses as identified by an item analysis of the results of the Cooperative Mathematics test.

While complete or final solutions to the stated problem areas cannot be offered as a result of this study, it is hoped that much significant and useful information, both general and statistical, will be presented and made available. If the remedial courses are as bad as the critics claim, decisions to improve them should be based on the

results of experimental research.

The mathematics courses offered by the Lehigh County Community College with which this study is primarily concerned appear in the general catalog of the college and are described as follows:[9]

MAT-099   Developmental Mathematics

This course stresses an intensive review and application of basic mathematical concepts to prepare the students to do advanced work in mathematics. It emphasizes fundamental operations, special products and factors, fractions and fractional equations, functions and graphs, systems of equations, integral and fractional exponents, radicals, quadratic equations, and functions. (Fall and Spring Semesters)

MAT-101   Foundations of Mathematics

This course is designed to give basic insight into the nature and structure of modern mathematics. Topics studied include the language of sets, relations and their properties; the systems of whole numbers, integers, rational and real numbers; systems with bases other than ten, and

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[9]Lehigh County Community College Catalog, Vol. 5, No. 1, 1971-1972, p. 111.

selected topics from geometry. (Fall and Spring Semesters)

MAT-103 Algebra and Trigonometry I

This course is designed for students interested in pursuing a technical program stressing applications of basic mathematical concepts. Topics studied include fundamental concepts and operations, linear functions and graphs, trigonometric functions, linear equations, determinants and vectors. (Fall Semester)

Prerequisite: MAT-099 or one year of high school algebra.

MAT-107 College Algebra

This course studies fundamental algebraic operations, exponents, systems of equations, higher degree equations, mathematical induction, determinants, progressions, radicals, inequalities, and the binomial theorem. (Fall and Spring Semesters)

Prerequisite: MAT-099 or two years of high school algebra.

Limitations of the Study

The study was limited in time by the fact that it covered only the summer session and the fall semester of 1971 and did not attempt to examine the students' progress

in mathematics beyond their first-year college mathematics course. The past records of previous classes, however, were recorded and analyzed primarily to serve as a comparison and to substantiate and reinforce, if need be, the findings and results of the relatively small sample.

Another limitation was the use of intact groups rather than randomly selected samples. To allow for this, the analysis of covariance procedure was used to test the null hypotheses of no difference in performance in freshman mathematics between the experimental and control groups.

A final and necessary limitation concerning the time factor, was to use only the ACT test and Cooperative Mathematics Test scores to determine the effectiveness of the development mathematics course.

Definitions

Throughout this study several terms are used repetitively and are defined here to clarify their use and, in some cases, their interchangeability.

1. Remedial students - those enrolled in the developmental mathematics course (MAT-099) on the bases of their scores on the ACT mathematics test.

2. Experimental group - remedial students.

3. Non-remedial students - those enrolled in one of

the three first-year college mathematics courses.

    4. <u>Control group</u> - non-remedial students.

    5. <u>First-year college mathematics courses</u> - MAT-101, MAT-103, MAT-107.

    6. <u>ACT test</u> - American College Testing Program test, with particular reference to the mathematics sub-section.

    7. <u>Cooperative Mathematics Test</u> - an achievement test designed by the Educational Testing Service.

    8. <u>Proficiency</u> - satisfactory performance in first-year college mathematics.

    9. <u>Satisfactory performance</u> - those who received a final grade of either A, B, or C.

    10. <u>Unsatisfactory</u> - those who received a final grade of D or F.

CHAPTER II

REVIEW OF RELATED LITERATURE AND RESEARCH

Introduction

Shortly after World War II, remedial or developmental mathematics courses were introduced into the curricula of many colleges and universities. The purpose of these courses was to prepare the large number of returning veterans who were entering college, and who had not had the prerequisites or the required geometry and algebra in high school and needed a refresher course. These courses were to be terminated after serving their purpose, but many schools found that the remedial courses also served a high percentage of students just completing high school, i.e. the slow learners and underachievers.

In 1953, Hunter completed a comprehensive study on the status of remedial mathematics courses in 269 universities and state colleges and found at least one remedial course in 74 per cent of the schools.[1] Hunter states:

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[1]Louise S. Hunter, "Pre-Freshman Mathematics in State Colleges and Universities" (Unpublished Ph.D. dissertation, University of Virginia, 1953), p. 154.

The institutions who reported years of ex-
perience and research in remedial mathematics
were the ones who expressed satisfaction with
the results in offering such courses and from
the high percentage of students who pass col-
lege mathematics after taking pre-freshman
mathematics.[2]

This review of research is limited in general, to

studies completed since 1952 with the majority, by far,

completed during the past ten years. An attempt has also

been made to consider as acceptable those studies or

sources which were conducted on the basis of experimental

research and/or statistical analyses. Consequently, the

major sources of the related literature and research re-

viewed in this chapter are as follows: (1) unpublished

doctoral dissertations, (2) microfilms of Dissertation

Abstracts, (3) reviews of educational research published by

ERIC (Educational Research Information Center), and (4) ar-

ticles by mathematicians, educators and teachers appearing

in journals and periodicals.

A search of the literature on the general subject of

remedial mathematics in the colleges and universities re-

vealed that there are many studies on the background and

philosophy of remedial work; many studies concerned with the

pros and cons of remedial programs; many articles dealing

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[2]Ibid., p. 193.

with the advantages and/or disadvantages of remedial

courses; many studies comparing two different methods of

teaching remedial mathematics; but very few experimental

studies on the statistical effectiveness or value of the

remedial mathematics course at a particular institution.

It seems appropriate, then, to categorize the review of the

literature into groups according to subject rather than

chronologically. The studies concerned with effectiveness

and achievement, both at the four-year college and two-year

college, will be reviewed first, followed by the research

on programmed instruction, the prediction of success and

placement procedures.

The Effectiveness of Remedial Mathematics

In order to determine or evaluate the effectiveness

of any procedure or treatment it is necessary to measure

any change or difference that may or may not have taken

place during or immediately following an effort designed to

bring about the prescribed change. Bradley, in 1960, evalu-

ated the effectiveness of the general mathematics course at

Texas College and Tyler District College by determining the

extent that the course objectives were being achieved by the

students. An analysis of the pre- and post-test scores of

eighty-five students indicated that seventy-nine students

grew mathematically, while six lost ground. The greatest amount of growth appeared to be on the group of items classified under "proof-deductive and inferential reasoning," while the least amount of growth appeared to be on "symbolism." The author was able to conclude that the course was more effective for the poorer student concerning "proof-deductive and inferential reasoning."[3] This study was considered to be significant, because placement tests had indicated that almost thirty per cent of the students did not have the prerequisites for first-year college mathematics.

Zwick, in 1964, studied the effectiveness of the remedial mathematics program at the Ohio State University and recommended that the present remedial program be retained, because "it was relatively effective in accomplishing its purpose of preparing students who are deficient in mathematical background to compete in college-level mathematics courses." Recognizing that the college freshman who is deficient in mathematical background has created several difficulties for colleges and universities, Zwick was also

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[3]Lillian K. Bradley, "An Evaluation of the Effectiveness of a Collegiate General Mathematics Course," University of Texas, Dissertation Abstracts, 21, 9:2528, 1960.

concerned with the problem of selection and placement. He found that the placement tests significantly differentiated between the students who were likely to pass first-year college mathematics with a satisfactory grade and those who were not likely to pass. His conclusion was that the remedial group, averaging 2.21, performed slightly but not significantly better than the non-remedial group which averaged 2.09 in the course.[4]

At the Virginia State College, Clark studied the academic performance of 854 entering freshmen who had completed the remedial mathematics course and who then enrolled in the first-year college mathematics course as compared with the performance of non-remedial students in their initial college mathematics course.[5] The relationship between academic performance and placement tests results was also of interest to the investigator. No statistical study had ever been conducted at the college to establish cut-off scores on the College-administered mathematics placement

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[4]Earl J. Zwick, "An Evaluation of the Effectiveness of the Remedial Mathematics Program at the Ohio State University," (Unpublished Ph.D. dissertation, Ohio State University, 1964), p. 60.

[5]Lawrence M. Clark, "An Evaluation of the Remedial Mathematics Program at Virginia State College," (Unpublished Ed.D. dissertation, University of Virginia, 1967), p. 40.

tests, and more information was needed to replace the

presently used subjective cut-off scores. Another problem

investigated was the relationship between the remedial

students' area of mathematical weakness and the remedial

mathematics course content. Clark reported that there were

many faculty members who advocated discontinuation of the

remedial programs, claiming that remedial courses should

not be offered at the college and university level.[6] He

cited Drasgow, who claimed that remedial courses were the

product of "misplaced pedagogic emphasis," and that prep-

aration for higher education is the business of the high

school, the preparatory school, and the junior college.

Drasgow doubted that the good preparatory schools and junior

colleges would accept some of the students who are assigned

to remedial classes in our colleges and universities.[7] The

study concluded that the remedial course was relatively

effective in accomplishing its purpose of preparing students

who were deficient in mathematics to compete successfully

with non-remedial students in two of the three initial

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[6]Ibid., p. 11.

[7]James Drasgow, "College How-to-do-it Courses,"
Journal of Higher Education, pp. 156-58, as cited by Clark,
op. cit., p. 25.

college mathematics courses. The placement tests were in-efficient and it was recommended they be deleted from the entering student's testing program. It is interesting to note that a recommendation was made for the present remedial program to contain a Level II course designed to strengthen those remedial students who are to later enroll in the pre-calculus course. It was further recommended, and this investigator is in full accord, that the Virginia State College work closely with the high schools of which a large percentage of the entering deficient students are products, in order that a joint effort may be made to strengthen the students in secondary school mathematics before they enter the College.[8]

Schremmer, conducted a short but interesting experi-ment at the Philadelphia Community College in 1971 where he attempted to teach abstract mathematics to college freshmen with ACT scores less than 15. The course, a three semester terminal sequence, included formal mathematical language, set theory, Boolean Algebra, relations and functions, oper-ations, cardinals and ordinals, the rational numbers, and college algebra. The tests for this course consisted of problems not previously encountered in class as well as

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[8]Clark, op. cit., p. 128.

"open questions" of the "prove or disprove" nature. The

students in this experimental course were compared to stu-

dents in the traditional three semester terminal sequence

with respect to passing and failing rates. The results

indicated that the students in the experimental course fared

consistently better and caused the author to conclude that

"abstract mathematics can be taught to almost anybody will-

ing to try, at no other cost than time and rigor."[9] These

results would seem to strongly support Bruner's claim that

"any subject can be taught effectively in some intellectu-

ally honest form to any child at any stage of development."[10]

O'Regan, in 1966, tested the theory that a college

freshman's performance in mathematics depends on his current

level of proficiency in elementary school mathematics, by

studying the effectiveness of a programmed remedial course

in algebra, taken just prior to Freshman Mathematics. He

was interested in determining possible sources of difficulty

other than an obvious lack of subject-matter background. An

experimental group who had taken the summer remedial course

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[9]A. G. Schremmer, "A Preliminary Attempt at Teaching
Abstract Mathematics to Freshmen with an ACT Score of less
than 15," Research Studies in Education, Vol. 6, No. 6,
(June 1971).

[10]Jerome S. Bruner, The Process of Education
(Cambridge: Harvard University Press, 1966), p. 33.

and then enrolled in Freshman Mathematics was matched with

two groups of freshmen who had not taken the summer course.

The results indicated that the experimental group did

not perform better than the control groups, over the full

year of Freshman Mathematics or in either of the two indivi-

dual semesters. All differences favored the control groups.

Over the full year all students made significant gains in

algebraic proficiency. Based on the major theories of

learning, the study revealed that all considered the student

to be in a difficult learning situation, whenever there were

severe discontinuities between his expectations about a course

and what it really was. O'Regan concludes that not only

does remedial work in algebra appear to make no contribution

toward success, it may actually reduce the student's prob-

ability of success in Freshman Mathematics. This unusual,

but significant finding indicates that the many studies

concerned with the prediction of success in college mathe-

matics should be re-evaluated.[11]

An informative and comprehensive study by Schenz, in

1963, of over 200 public and private junior colleges

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[11]Patrick J. O'Regan, "Freshman Mathematics and a
College Student's Current Level of Proficiency in Elementary
Secondary School Mathematics," (unpublished Doctor's thesis,
New York University, 1966), p. 72.

revealed that junior colleges report very little research

regarding the success or failure of their students with low

ability. The remedial function, however, is widely accepted

by junior college administrators as a legitimate function of

their institutions.[12]

Sharon, in 1970, conducted a short study to determine

the effectiveness of remedial courses and placement policies

and instruments. He found that the mathematics remedial

course eliminated some of the students' dissatisfaction with

the regular course and had a significant effect on subse-

quent course work. The placement procedures appeared to be

more effective in assigning students to appropriate mathe-

matics courses than to English courses.[13]

Beal, surveyed the remedial mathematics programs of

ninety-eight community-junior colleges during the fall and

winter of 1969. Approximately twenty-five per cent of those

originally contacted indicated they had no such program, and

three per cent questioned their value. The most often indi-

cated reasons for the existence of remedial programs were to

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[12]Robert F. Schenz, "An Investigation of Junior Col-
lege Courses and Curricula for Students with Low Ability,"
University of California, Los Angeles, 1963, Dissertation
Abstracts, 24, 5:1889-90, 1963.

[13]Amiel T. Sharon, "Effectiveness of Remediation in
Junior College," Educational Testing Service, Princeton,
N. J., 1970.

enable students to continue in regular college mathematics,
or to satisfy prerequisites for other courses. In selecting
students for remedial programs, standardized tests, previous
grades, and counselor recommendation were the most often
used criteria. Enrollment in remedial courses included at
least twenty per cent of all mathematics students at fifty
per cent of the schools. Some indication of the effective-
ness can be concluded from the fact that in fifty-one of the
ninety-eight colleges, almost forty per cent of the remedial
students enrolled in subsequent mathematics courses. The
most surprising conclusion was that only twenty-six respon-
dents indicated an effort to evaluate their program.[14]

Blyler and others, in 1970, made a brief but compre-
hensive study of the developmental mathematics program at
the Lehigh County Community College. Conceived as an out-
growth of the college's open-door philosophy, developmental
mathematics was intended to provide the student with mathe-
matical skills and techniques that may aid in future
educational or vocational endeavor. Based on four factors:
ACT scores, high school grade and courses, time interval
since last related educational experience, and college

[14] Jack Beal, "An Analysis of Remedial Mathematics
Programs in Junior and Community Colleges," Research in
Education, Vol. 6, No. 2 (February 1971), p. 15.

program; incoming freshmen are recommended to the non-credit
developmental mathematics course. The statistical findings
were based on five full semesters and two summer sessions of
experience. Members of the mathematics department made the
following recommendations on the basis of a critical evalu-
ation of the available statistics:

1. Developmental mathematics is not recommended as a
prerequisite for the Foundations of Mathematics course (MAT-
101) for students with an ACT score below 15.

2. Any student with a grade of "B" or lower in de-
velopmental mathematics should be discouraged from enrolling
in College Algebra (MAT-107).

3. Students with low ACT scores, but with a back-
ground in algebra and/or a strong incentive, can do well in
Algebra and Trigonometry I (MAT-103). The ACT scores alone
were not adequate predictors.[15]

An interesting joint project of the Northampton Area
Community College and Lehigh University, conducted by Krupka
in 1969, examined the college's General Studies Program for

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[15]"Developmental Mathematics - A Retrospective
Study," A departmental report on the status of the develop-
mental mathematics program, Lehigh County Community College,
1970; George E. Blyler and others, (in the files of the
department).

the student scoring below the twelfth percentile on the ACT

mathematics or English tests. A unique feature of the re-

medial program combines programmed self-study and individual

instruction with a Programmed Materials Learning Lab in

English, or mathematics. The college staff judges the pro-

gram's effectiveness by subsequent course success, pre-post

ACT gain score, grade-point average, dropout rate and

achievement in the program. Krupka reported that the per-

centage of enrollment in the program is low, but further

added that the dropout rate is also correspondingly low.

Most likely to drop out are the seriously deficient students

who usually stay in school only three semesters.[16]

Early in his study, Krupka identified five success-

failure factors established by the college to evaluate the

effectiveness of the remedial program. He then clearly and

logically stated the typical randomly selected experimental-

control group situation well known to all research workers.

Expressing what must be the consensus of the majority of all

investigators, he further stated:

> But the college will not run this true and
> sensible experiment. Why? Because based on

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[16]John G. Krupka, "A Community College Remedial Pro-
gram: A Description and Evaluation," Research in Education,
Vol. 4, No. 11 (November, 1969), p. 9.

national norms and research findings, such students belong in the remedial program, and the college would be neglecting its responsibility to the student and might even ruin his collegiate career if it did experiment with him. So the question will not be answered as to whether or not the student would succeed without this program. It is felt that this program will certainly not hurt the student and may, at worst, delay the date of failing or dropping out of school. Those students who fall 3 or 6 semester hours behind their fellow classmates can make this up during the summer sessions.[17]

The Prediction of Success or Achievement

Closely related to the studies on effectiveness are those concerned with success and achievement or, in many cases, the factors associated with their prediction and/or measurement. Most of these studies, however, give little evidence of research on the actual effectiveness of the remedial mathematics programs.

A comprehensive study by Wick, in 1963, was designed to determine the factors significantly associated with success in first-year college mathematics (first semester) at six Minnesota and Wisconsin colleges and universities. He was not concerned with the effectiveness of remedial mathematics at the college level, but rather the effectiveness of experimental SMSG (School Mathematics Study Group) secondary

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[17]Ibid.

school mathematics program on first-year college mathematics. The prediction of success was investigated in terms of student achievement in each course at each of the six colleges.

His results suggested that there is little difference between the achievement of students with experimental (SMSG) or traditional mathematics backgrounds in first-year college mathematics. Correlations between the factors analyzed and success in the courses were low, (.35-.45). Some aspect of the high school record consistently gave the highest correlation, usually high school achievement (grades) or high school rank.[18]

Graybeal, in a similar study at the University of North Carolina, in 1958, found that the best single predictor of either achievement or success in college algebra was the measure of incidental or residual knowledge of fundamental algebraic processes as indicated by scores on a diagnostic or pre-test in the subject. Contrary to Wick's finding, and several other investigators, Graybeal found that rank in the high school graduating class was only of

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[18]Marshall E. Wick, "A Study of the Factors Associated with Achievement in First-Year College Mathematics" (unpublished Ph.D. dissertation, University of Minnesota, 1963), p. 274.

secondary importance for the prediction of either achievement or success. High school grades in mathematics carry the greatest predictive weight for success, while intelligence test scores are more influential in predicting achievement. Surprisingly, personal interests play non-existent roles in predicting achievement, but rather influential supporting roles in predicting success. Vocational interests, rarely ever included in a study of factors associated with academic achievement and success in mathematics, were found to play minor roles in the prediction of achievement and non-existent roles in predicting success.[19]

A recent study by Edwards, in 1971, led to the conclusion that success in remedial mathematics can be predicted by using a multiple regression equation with five select factors as predictors. The study involved 359 remedial students in seven community colleges. Significant differences were found between the means of the independent variables for male and female groups, and the successful and unsuccessful groups. Edwards reports that correct predictions were made seventy-one per cent of the time. The biserial

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[19]Walter T. Graybeal, "Predictive Factors Associated with Achievement and Success in College Algebra," The University of North Carolina, 1958, Dissertation Abstracts, 19, 10:2539, 1959.

correlation between success in remedial mathematics and achievement in first-year college mathematics was found to be significant at the .05 level. A disappointing and questionable statistic reported was that fifty-seven per cent of those tested failed to exceed the score which determined their placement in remedial mathematics. This would seem to indicate that the remedial mathematics program was not very effective in bringing about the desired result.[20]

If by success in first-year college mathematics we mean the achievement of something desired or hoped for, then a study by Fournet, in 1963, identified certain selected measurable factors that were effective in producing the desired result. Although the expectation for success in general college mathematics for a randomly selected beginning freshman student is low, Fournet found that it was closely related to general academic success during the first semester in college, and, strangely enough, to success in freshman English.[21]

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[20]Ronald R. Edwards, "Predicting Success in Remediation Programs in Mathematics for the Public Community Junior College," University of Connecticut, 1971, Dissertation Abstracts, 32, 5A:2432, 1971.

[21]Francis G. Fournet, Jr., "A Study of Various Factors Related to Success in College General Mathematics," Louisiana State University, 1963, Dissertation Abstracts, 24, 12:5239-40, 1964.

At the New York City Community College, Brodsky investigated the effect of a pre-technology remedial semester on the academic competence of students with marginal qualifications for admission to technical curricula. Forty pairs of subjects were matched on three academic variables: high school grade average, high school diploma type, and engineering technician curriculum. The experimental group took the non-credit, pre-technology semester while the control group did not. Brodsky reported statistically significant differences between the groups in the hypothesized direction after the first technical curriculum semester.[22]

In contrast to the findings of Wick,[23] Graybeal[24] and Morgenfeld,[25] who concluded that some aspects of the high school were the best predictors of college success, Brodsky found that the best individual predictors from each set of variables were the Cooperative School and College Ability

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[22]Stanley M. Brodsky, "Predicting the Academic Competence of Students in Certain Technical Curricula at the New York City Community College of Applied Arts and Sciences after an Experimental, Preliminary, Remedial Semester," New York University, 1964, Dissertation Abstracts, 25, 2:928, 1964.

[23]Wick, loc. cit.   [24]Graybeal, loc. cit.

[25]George R. Morganfeld, "The Prediction of Junior College Achievement from Adjusted Secondary School Grade Average," University of Arizona, 1967, Dissertation Abstracts, 28, 8A:2987, 1968.

Tests and the pre-technology grade-point average. Variables

derived from the high school records produced unusually low

correlations with first semester grade-point average. The

high school science grade average, on the other hand, was

substantially correlated with the criterion. The results

demonstrated that academic competence in E.C.P.D. (Engineers

Council for Professional Development) accredited engineering

technician curricula can be significantly improved by means

of a pre-technology semester for applicants whose initial

academic qualifications for admission are either minimally

acceptable or marginally unacceptable.

Rowe, in 1957, developed and evaluated a course in

non-transferable (remedial) general mathematics for term-

inal, non-technical junior college students. A comprehen-

sive questionnaire was used to select the content and

objectives of the course which was then developed into a

syllabus and used as the basic text for the course. The

course was taught for one semester to two experimental

groups of terminal students in a California junior college.

The control group was a similar group of matched students

not taking the course.[26]

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[26]Jack L. Rowe, "General Mathematics for Terminal
Students in California Junior Colleges," University of
Colorado, 1957, _Dissertation Abstracts_, 19, 2:255, 1958.

Significant gains in achievement for the experimental group in comparison with the two control groups, were reported by Rowe. He found that terminal, non-technical students in junior colleges were able to learn considerable mathematics in spite of allegations of many to the contrary.[27]

Programmed Instruction and Remedial Mathematics

There have been many studies comparing the effectiveness of two or even three methods of teaching mathematics. Relatively few studies, however, have been reported on the effectiveness of teaching remedial mathematics by the programmed method as compared with the traditional or lecture method.

Alton, at Michigan State University in 1965, developed programmed material for the remedial mathematics course and compared its effectiveness with that of a self-help and tutor method. Based on the statistical results of her study she found that the adjusted mean scores for the experimental groups were all higher than for the control group. As used in her study, the programmed materials were more effective than a combination workbook-text method for

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[27]Ibid.

teaching a non-credit algebra course at the college level. Students who used the programmed materials felt that some tutorial help would have been helpful.[28]

A year later, Yesselman compared the gains made by students using programmed materials under three conditions of supervision. Her subjects were seventy-six college students in three successive sections of a remedial, non-credit mathematics course. The amount of supervision and control varied from almost none, to a great amount during later semesters. She found that varying the amount of supervision does not significantly affect learning from a program. A secondary, but important, finding was that the number of dropouts increases significantly when the program is presented in a non-supervised setting. And finally, students of different combinations of F-scale score and high school mathematics grade-point average learn equally well from a program under all conditions of supervision.[29]

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[28]Elaine V. Alton, "An Experiment Using Programmed Material in Teaching & Non-credit Algebra Course at the College Level (with) Supplement," Michigan State University, 1965, Dissertation Abstracts, 26, 8:4488, 1965.

[29]Charlotte B. Yesselman, "Programmed Instruction in College Mathematics," The University of New Mexico, 1966, Dissertation Abstracts, 27, 6A:1568, 1966.

Another study in 1966 was conducted by Goodman in an attempt to determine the remedial effectiveness of algebra and English grammar programmed for a group of college freshmen. The immediate effects were that the experimental groups showed a slight, but significant, superiority on the measure of achievement in comparison to the control groups. High school average was significantly related to grades in relevant courses for control group subjects, but not for the experimental group subjects. Goodman cited this as evidence that the programmed instruction had some effect in changing predicted performance in school. Programmed instruction was hypothesized to have induced resistance to learning in those students most in need of help.[30]

An experimental study by Dukeshire, in 1966, was conducted to demonstrate that the traditional lecture method of teaching college mathematics combined with a self-teaching workbook resulted in greater comprehension of the course than the lecture method alone. Using the classical randomly selected experimental and control groups taught by two

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[30]Jesse S. Goodman, "Programmed Instruction, Remedial Treatment, and Resistance to Learning: An Experimental and Exploratory Study of Facilitating and Hindering Factors in Remedial Programmed Instruction," New York University, Dissertation Abstracts, 28, 2A:496, 1966.

professors, the mean scores of daily tests were signifi-
cantly higher for the experimental class than for the
control group. Confirming the findings of this study,
Dukeshire reported a correlation between the content or
number of items in the workbook and the mean point differ-
ence of each test item. The greater the number of items in
the workbook about a topic the higher the number of points
earned on the test question concerning that topic.[31]

Summary of Remedial Mathematics Research

Among most educators, there is general agreement,
and much evidence, that the average college freshman comes
poorly prepared in mathematics and is in need of remedial
instruction. There is also a consensus that the problem of
wide variation in ability and background exists, but there
is little agreement on what to do about it. Some educators
would discontinue all remedial work, not only mathematics,
from the colleges on the basis that such work is the sole
responsibility of the high school. These people advocate
strict selective admission policies as the solution.

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[31]Mabel E. Dukeshire, "An Experimental Study of the
Relative Effectiveness of the Lecture Method of Teaching
and the Lecture Method Supplemented by a Self-Teaching
Workbook," Rutgers--The State University, 1966, Dissertation
Abstracts, 27, 6:1539-40, 1966.

From the various studies reviewed, high school grades, class rank, and achievement tests were the best criteria for predicting college success or for grouping students for differentiated instruction. Several studies indicated the need for each institution to establish its own local norms in the use of predictive types of tests.

Reports indicated that many institutions were satisfied with the results of their remedial programs in mathematics. Others expressed doubt about the use of predictive instruments in the selection of students who need remedial work, citing poor correlations between the predictors and course grades.

CHAPTER III

DESIGN OF THE STUDY

The main purpose of this chapter is to describe in considerable detail the design or methodology of the experimental procedures used to determine (1) the academic effectiveness of the developmental mathematics course at the Lehigh County Community College, and (2) to analyze and determine the adequacy of the selection process which identifies certain entering students as needing remedial work in mathematics.

Selection of the Sample

During their senior year in high school, in the spring of 1970, over 1100 potential Lehigh County Community College students took the ACT Battery designed by the American College Testing Program. On the basis of their scores on the ACT Mathematics sub-section and their most recent high school grades in mathematics, 237 students were selected as potential candidates for the developmental mathematics course, MAT-099. Any student whose ACT Mathematics score was 15 or lower was felt to be in need of some kind of remedial work before enrolling in one of the initial

college mathematics courses and was strongly urged by his counselor and academic advisor to take the developmental mathematics course. Students whose most recently recorded grade in high school mathematics was a "D" or lower were also strongly recommended to register for developmental work even though their ACT Mathermatics scores might have been above the cut-off level of 15. When both conditions, low ACT Math score and low high school math grade were coupled together, the student became a prime candidate for developmental mathematics and usually needed no further reminding, either by his counselor or himself.

Several exceptions were observed for returning veterans and mature students who were not required or even urged to take the course regardless of their ACT scores and high school grades. This slight deviation from the already arbitrary standards proved to be justified in most cases. Completely exempted from the sample selection were students repeating their freshman year and/or those who had previous college experience elsewhere. Finally, for purely statistical reasons, foreign students and those auditing courses were not part of this study.

The developmental mathematics course, MAT-099, was offered during the 1971 Summer Session, and was open to all

students planning on entering the Lehigh County Community
College as freshmen in August for the Fall term. From the
group of 237 potential remedial students, only 114 were con-
sidered as likely candidates because of their intended major.
Out of this reduced group, thirty-two enrolled in the summer
course and became the experimental group. Because of the
stated philosophy of the open-door policy, common to all
Pennsylvania Community colleges, compulsory enrollment was
neither required nor, in fact, possible. Since random
assignment of subjects to treatments is the proper, and if
possible, preferred experimental procedure, the effect of
self-selection or voluntary participation in the study was
examined.

It is well known that research results are important
only if they can be replicated by others. A statistically
significant difference between two groups is of no partic-
ular value if a similar difference cannot be found between
two other similar groups by another investigator at some
other time and/or place.

What, then, can be said about the non-random sample?
Tate has observed, ". . . that the majority of the samples
used in educational research are nonrandom; and it is
likely, because of administrative and other practical diffi-
culties, that the practice cannot be avoided, at least in

many instances." He further stated:

>   Since it cannot be considered to be represen-
> tative of any know population, the information
> it yields, strictly speaking, does not permit
> generalization. However, it would be incorrect
> to conclude that the study of a nonrandom sample
> is without significance. The investigation may
> be worthwhile, both because the sample evidence
> may be important in itself and because the in-
> vestigation may suggest significant problems and
> hypotheses for more extended and general study.
> Furthermore, there is always the possibility
> that a nonrandom sample is adequately represen-
> tative of other groups, so that what has been
> observed will have some generality.[1]

Most authors of textbooks on the subject of statistics

for education and psychological research refer to samples

which result from other than random methods of selection as

accidental or incidental samples.

O'Regan, in his study of remedial mathematics, en-

countered the same voluntary, self-selection situation and

concluded that, ". . . the presence of self-selection would

not seriously interfere with the purposes of this study."[2]

He observed, quite correctly, that the results of the study

would be biased in favor of the remedial group if volunteers

are more industrious and would probably do better than the

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[1]Merle W. Tate, <u>Statistics in Education</u> (New York: The Macmillan Company, 1955), p.13.

[2]O'Regan, <u>op. cit.</u>, p. 39.

control group, increasing the likelihood of a Type I error. There is general agreement, however, among most educators that the difficulties usually encountered in first-year college mathematics cannot be overcome by the industriousness of students needing remedial work. In fact, O'Regan's study suggests that if a student works diligently at trying to fit the new course into the pattern of his limited expectations, he may simply make matters worse.

The control group was selected from the remaining eighty-two students who were previously identified as potential remedial students, but who decided to enroll in their first-year college mathematics course without taking the developmental mathematics course. An attempt was made to randomly select this group, but difficulty arose as before with the experimental group when only 36 of the 82 students registered for freshman mathematics. These 36 students became the control group. Since the experimental group and the control group were actually subsets of the set of all potential students entering the Lehigh County Community College with known deficiences in mathematics, both groups were analyzed to determine if they could be considered as coming from the same population. They matched surprisingly well on all of the variables selected and were

considered as equivalent for the purpose of this study.

TABLE 1

EQUIVALENCE OF THE EXPERIMENTAL AND CONTROL GROUPS

Variables	Experimental n=30		Control n=28	
	Mean	S.D.	Mean	S.D.
High School Math Average	1.7	0.88	1.5	0.85
I.Q.	107.6	8.80	106.4	9.60
ACT Mathematics Score	14.3	2.50	14.1	3.10
ACT Composite Score	17.5	2.60	17.0	2.70

The student population from which the samples were taken is presented in Table 2 along with the various categories and reasons for elimination or participation in the study. The mathematics performance of Lehigh County Community College students as listed in the ACT Class Profile of Test Data for the years 1967 thru 1971 has not changed significantly, and there is reason to believe that these samples are representative of the population of students who enter the college each year with mathematical deficiences.[3]

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[3]ACT Class Profile Test Data, 1967-1971, Compiled by Jack A. Burger, (in the files of the Guidance Division, Lehigh County Community College), 1971.

TABLE 2

SUMMARY OF THE STUDENT POPULATION FROM
WHICH THE SAMPLE WAS SELECTED

Number of Potential Freshmen whose ACT Scores were examined	1124
Number whose ACT Mathematics subtest score was 15 or lower and whose high school mathematics grade average was either a D or an F.	237
Number of students likely to take remedial mathematics because of college major	114
Number of students who actually enrolled for MAT-099 Developmental Mathematics during the Summer Session	32
Total number of students who entered as Freshmen in August, 1971	740
Number of students who took Freshman mathematics	426
Number of students who took no mathematics courses	228
Number of students who withdrew	10
Number who failed the developmental course	1
Final number of students in the Experimental Group	30
Final number of students in the Control Group	28

Collection of the Data

The data used in this study came from a variety of sources and included many different types of information on each of the subjects. All of the students' past and current

records that were used were taken from the official files in the Admissions Office of the college. These included the following types of information:

1. Student's name

2. High school graduated from

3. Rank in graduating class

4. Student's I.Q.

5. Mathematics courses taken in high school

6. Mathematics grades received in high school

7. ACT Mathematics subtest score

8. ACT Composite score

9. College program - Career or Transfer

10. College major

From the official files in the Division of Guidance and Counseling, information was available on the American College Testing Program tests, test results and interpretations, manuals, and research reports. In cooperation with the Office of the Director of Admissions, this source provided much valuable data and helpful information.

The office and files of the Dean of Students also proved to be a useful source of academic, as well as personal, information concerning the students that were part of the study.

The Mathematics Division was the final source of information on the class rosters, semester grades in the various mathematics courses involved in the study, course outlines and basic texts used in the courses, and the Cooperative Mathematics Test Forms, answer sheets and test results. The course grades in the first-year college mathematics courses were, for statistical purposes, converted to numerical scores by assigning: A=4, B=3, C=2, D=1, and F=0.

The Experimental Procedure

Controlled experimentation is nothing new in the field of education. Early experimental schools in America attempted to evaluate teaching methods and principles under actual classroom conditions. Francis W. Parker's experimental school in Chicago in 1883, and the Laboratory School of the University of Chicago founded by John Dewey in 1896 were early examples of significant attempts to find the answers to many educational problems. For Dewey, the experimental or scientific method represented the only way to logically arrive at some worthwhile conclusion. In 1929, he wrote that, "in contrast to experience gained through trial-and-error, unguided by any conscious insight, an experiment represents directed observation guided by the purpose of the

study and by an understanding of the conditions."[4]

The experimental group was given the Cooperative Mathematics Test, Form A, as a pre-test during the first meeting of the developmental mathematics course in July of 1971. The control group was given the same test during the first week of classes of the regular Fall term in September of 1971. The students in both groups were not given the post-test until the week before they completed their first semester of college mathematics. Because of withdrawals or failure to appear for the post-test in December, the experimental group was reduced to thirty students and the control group was reduced to a total of twenty-eight students.

Description of the Instruments Used

The instruments utilized in this study were the American College Testing Program battery (ACT tests), and the Cooperative Mathematics Tests, Algebra II, Form A.

American College Testing Program. The ACT battery consists of four tests that measure academic potential in areas of English usage, mathematics usage, social studies, reading, and natural sciences reading. These tests contain

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[4]John Dewey, The Quest for Certainty (New York: Minton, Balch and Company, 1929), p. 84.

a large proportion of complex problem-solving exercises and proportionately few measures of narrow skills. The ACT tests are oriented toward major areas of college and high school instructional programs rather than toward a factorial definition of various aspects of intelligence. The tests measure as directly as possible the abilities the student will have to apply in his college work. In addition to a composite score, scores are reported in each of the four sub-test areas. Past tests have yielded mean reliability coefficients of 0.85 on the sub-tests and 0.94 on the composite score.

This study was primarily concerned with the scores in the mathematics sub-test area.

The mathematics usage test is a 40-item, 50-minute examination that measures the student's mathematical reasoning ability. This test emphasizes the solution of practical quantitative problems which are encountered in many college curricula. It also includes a sampling of mathematical techniques covered in high school courses. The test emphasizes reasoning in a quantitative context, rather than memorization of formulas, knowledge of techniques, or computational skill. There are two general types of items: the first, verbal problems, presents quantitative problems

in practical situations; the second consists of formal exercises in arithmetic, algebra, and geometry. The format of the item is a question with five alternative answers, the last of which may be "not given."

In general, the mathematical skills required do not exceed those included in high school plane geometry and first and second year algebra. In addition, approximately one-half the items are verbal descriptions of quantitative problems arising in realistic situations. The following areas of mathematics are included:

Advanced arithmetic.--Topics include proportion, averages, interpretation of quantitative statements, linear interpolations, indirect measurement, and implicit relationships in data.

Algebra.--This includes operations with signed numbers, operations with polynomials, manipulation of algebraic fractions, factoring algebraic expressions, dependence and variation of quantities related by given formulas, arithmetic and geometric series, derivation and application of equations and formulas, binomial theorem, solution of equations in one unknown, solution of simultaneous equations, inequalities, logarithmic principles, and exponents and radicals.

Geometry.--Topics include mensuration of lines and
plane surfaces, properties of polygons, angular relation-
ships involving parallel lines and polygons, relationships
involving circles and properties of circles, loci, solid
geometry, trigonometric principles, and the Pythagorean
theorem.[5]

The Cooperative Mathematics Tests; Algebra II, Form
A. This test is designed to measure achievement in algebra
at the intermediate level which corresponds approximately
to the completion of two years of high school algebra.
Achievement is assessed in terms of the student's compre-
hension of the basic concepts, techniques, and unifying
principles in each content area. Where possible, many of
the newer trends and emphases in mathematics are represented
in the tests, but content has been selected carefully to
insure the appropriateness of the tests for most students.
Ability to apply understanding of mathematical ideas to new
situations and to reason with insight are emphasized.
Factual recall and computation are minimized. The test con-
sists of forty multiple-choice items and the time allowed is

[5]The American College Testing Program Technical
Report, 1968-69 Edition, ACT Publications, Iowa City, Iowa,
pp. 12-13.

forty minutes.[6] National and urban norms have been developed by the publisher and information on developing local norms is contained in the test Handbook. The publisher recommends that each user make an individual judgment of content validity with respect to his own course content and educational aims. Reliabilities, computed using the Kuder-Richardson Formula 20, are reported in the .84 through .89 range.[7]

Statistical Methods Used

The purpose of this section is to present a general description of the statistical procedures used in this study to test the hypotheses stated in the problem and in analyzing the various factors associated with the effectiveness of the developmental mathematics course.

The first step was to compute the means and standard deviations for all scores of the experimental and control groups. The Cooperative Mathematics Tests were given early in the study and near the end of the study in order to

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[6]Oscar K. Buros, Editor, The Sixth Mental Measurements Yearbook (Highland Park, N. J.: The Gryphon Press, 1965), pp. 886-88.

[7]Cooperative Mathematics Tests Handbook, Educational Testing Service, Cooperative Test Division, Princeton, N. J., 1964, p. 62.

determine if the comparative gain scores were significant.
The pre-test scores were analyzed and "t" tests used to
determine whether or not there were significant differences
in achievement of the experimental group students and con-
trol group students before the treatment. The "t" tests
were then run on the post-test scores of both groups to
determine if the significance, if any, existed after the
treatment. The significance of the differences were tested
at the .05 level.

To account for the intervening variables that might
exist between the groups and any bias because the selection
of the sample was not random, it was decided to use the
analysis of covariance procedure as described by Edwards[8]
and McNemar.[9] Availability of pre-test data from both the
ACT and Cooperative Mathematics Tests was also a factor in
the decision to use the analysis of covariance method.

Multiple correlation techniques were used to deter-
mine which of the two pre-tests, the ACT or the Cooperative
Mathematics Tests, served as the better predictor of success
in first-year college mathematics courses, or if the

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[8]Allen L. Edwards, Experimental Design in Psycho-
logical Research, Revised Edition (New York: Rinehart and
Company, Inc., 1960), pp. 281-95.

[9]Quinn McNemar, Psychological Statistics (New York:
John Wiley and Sons, Inc., Fourth Edition, 1969), pp. 413-29.

combined scores distinguish between or predict the student's success.

Item and content analyses were conducted on the ACT and Cooperative Mathematics Tests to determine the specific areas of the students' weaknesses in mathematics. The correct responses made to the test items by both groups were tabulated under various content classifications to compare their performances and identify their difficult problem types.

Finally, a Chi square analysis was conducted on the frequency distribution of the final grades made by the Experimental and Control groups in their initial college mathematics courses to determine how they differ from a normal distribution.

CHAPTER IV

RESULTS AND INTERPRETATIONS OF THE STUDY

This chapter presents the findings that were obtained
by using the statistical methods described in Chapter III,
and the interpretations of these findings in terms of the
hypotheses stated in Chapter I. Each hypothesis will be re-
stated, then the statistical analysis of the data used to
test the hypothesis will be presented immediately followed
by an interpretation of the findings.

The First Hypothesis

The first experimental null hypothesis predicted that
there would be no significant difference in the gain scores
of elementary algebra, as measured by the pre- and post-test
results of the Cooperative Mathematics Tests, between the
students who took the developmental mathematics course and
those students who did not.

Both groups took Form A of the Cooperative Mathema-
tics Test in Algebra II. This was the pre-test. Approxi-
mately fifteen weeks later, both groups took the same test
again after completing their first semester of college
mathematics. This became the post-test. All references to

pre- and post-test scores are related to these two tests. The results of both tests for both groups are found in Appendix B. Presented here, are the results of the statistical analysis of the data; first for each group separately, and then in tabular form for both groups in order to clarify the findings.

For the experimental group, a summary of the descriptive statistics obtained from the results of the Cooperative Mathematics Tests follows:

<u>EXPERIMENTAL GROUP</u>

n = 30

	<u>Pre-test</u>	<u>Post-test</u>
Mean	9.70	13.80
Standard deviation	2.98	4.96

Mean difference = 4.10

Standard Error of the difference = 1.07

Degrees of freedom = 29

These data allow us to compute the "t" ratio and test for the significant difference between the means.

$$t = \frac{\text{Mean difference}}{\text{Standard error of the mean difference}}$$

$$t = \frac{4.10}{1.07} = 3.84 > 2.045 = P \ (.05)$$

This **first** finding was an indication that the difference between the means was highly significant at the .05 level. We could now state that the experimental group scored significantly higher on their post-test than on their pre-test. An identical analysis was then performed using the data obtained for the control group.

CONTROL GROUP

n = 28

	Pre-test	Post-test
Mean	10.20	12.10
Standard deviation	3.20	3.24

Mean difference = 1.90

Standard Error of the difference = 0.88

Degrees of freedom = 27

The "t" ratio is now computed from the above data.

$$t = \frac{\text{Mean difference}}{\text{Standard error of the mean difference}}$$

$$t = \frac{1.90}{0.88} = 2.16 > 2.052 = P (.05)$$

The **second** finding was that the control group also performed significantly better on the post-test than on the pre-test. Now it was concluded that both groups scored significantly higher on the Cooperative Mathematics post-test than on the pre-test.

The fact that the mean gain scores were found to differ significantly from the pre-test values did not directly establish the first hypothesis. The mean gain of 4.10 made by the experimental group was considerably higher than the mean gain of 1.90 made by the control group, and an analysis of variance revealed that the difference was indeed significant at the five per cent level.

The results of the analysis of variance procedure allowed us to reject the null hypothesis that there is no significant difference between the mean gain scores of the experimental group and the control group. This was the third finding. The computed value of the F ratio, as shown in the following summary, Table 3, was significant at the five per cent level.

TABLE 3

SUMMARY OF ANALYSIS OF VARIANCE

Source	S.S.	d.f.	M.S.	F
Between Group Means	527.25	3	97.50	6.90
Within Group Means	1677.00	118	14.1	
Total	2204.25	121		

$$F = \frac{97.50}{14.1} = 6.90 \qquad F_{95} \ (3,118) = 2.69$$

The first two findings were also strongly significant at the .05 level, but we were comparing the mean gain scores of two correlated samples and were interested only in determining if the students in each group performed better on the pre-test than on the post-test. We found that they did. It was not known, however, whether the test scores of the two groups differed because of the effectiveness of the treatment--the developmental mathematics course--or because the groups were different to begin with, or whether there was an interaction of several unknown variables, or if it was perhaps due to some chance phenomenon. Early in the study, it had been found that the two groups were statistically equivalent in many areas. It must now be assumed they were not completely comparable or else the test scores would have been almost identical. They were not. Chance errors caused some of the variation; individual student variables like ability, motivation, interest, etc. caused some; and test conditions and errors of measurement probably accounted for other variations. The analysis of variances allowed for the known, identifiable variables and enabled us to conclude that both samples came from the same population and, therefore, the difference between the mean gain scores was the direct result of the treatment and not due to chance.

Interpretation of initial findings

Both the experimental and control groups performed significantly higher on the post-test Cooperative Mathematics test than on the pre-test. The highly significant difference between the mean gain scores of the pre- and post-tests were interpreted as an indication that both groups improved their ability and proficiency in elementary algebra. The experimental group mean gain, however, was enough higher than the mean gain of the control group to attribute the main effect or cause to the developmental mathematics course. Support for the rejection of the null hypothesis that there would be no significant difference between the means came from the analysis of variances when the F ratio was found to be significant at the .05 level. The alternative hypothesis that there is a significant difference between the mean gain scores of the two groups was accepted.

Performance in First-Year College Mathematics

The second experimental hypothesis predicted that there would be no significant difference in performance in first-year college mathematics between the students who took the developmental mathematics course and those students who did not. The criteria were the final semester grades in

three first-year mathematics courses, MAT-101, MAT-103, and MAT-107.

The analysis of covariance method was used to test for significance primarily because the two groups were not randomly selected. Differences were known to exist between the groups being tested and measures of these variances were available prior to the application of the treatment. The procedure removes the effect of a potential disturbing variate by integrating the techniques of regression and analysis of variance. Fisher, in 1946, said, ". . . covariance combines the advantages and reconciles the requirements of regression and analysis of variance."[1]

During the process of tabulating the data required for the analysis of covariance, the means and standard deviations for the criterion variable, grade point average, and the covariates were computed. The values are shown in Table 4.

The results of the separate analysis of covariance procedures are given in tabular form for easy comparison in terms of performance in first-year college mathematics courses as measured by the criterion variable, final grade,

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[1]R. A. Fisher, <u>Statistical Methods for Research Workers</u>, (Edinburgh: Oliver and Boyd, Ltd., 1946), p. 289.

and the various covariates.

TABLE 4

MEANS AND STANDARD DEVIATIONS OF THE CRITERION
VARIABLE AND THE COVARIATES FOR MAT-101

N=22

Variable	Function	Group	Mean	Standard Deviation
Final Grade	Measure of Performance	Exper. Control	2.30 1.91	0.90 0.99
H. S. Grade	Covariate	Exper. Control	1.90 1.54	0.94 0.89
I.Q.	Covariate	Exper. Control	107.9 102.2	11.80 8.32
ACT Math Score	Covariate	Exper. Control	15.0 12.7	2.82 2.48
Cooperative Math Score	Covariate	Exper. Control	10.8 9.1	2.18 1.14
ACT Composite Score	Covariate	Exper. Control	18.3 17.0	2.28 1.59

All of the covariates were considered as intervening
or uncontrolled variables since they were measures that were
obtained prior to any treatment. For the purpose of this
study it was desirable to adjust or correct the means of the
differences between variables that for some reason or other
could not be controlled by matching or by random selection
procedures. Since we had to work with relatively small

intact existing groups the analysis of covariance allowed

for these distrubring differences between and within the

means.

TABLE 5

SUMMARY OF COVARIANCE ANALYSIS FOR MAT-101
WITH ACT MATH SCORE AS COVARIATE

Source of Variation	S.S.	d.f.	M.S.	F
S_5: Treatments	2.87	1	2.87	7.96
S_2: Error	13.95	39	0.36	
S_4: Total	16.82	40		
		$F_{95} = 4.10$		

Results for MAT-101

The fourth experimental finding was that the students

in the experimental group performed better in MAT-101 than

those in the control group when the covariate was their ACT

mathematics score and the criterion variable was their final

grade in the course. The differences were significant at

the .05 level. The null hypothesis of no difference in per-

formance in first-year college mathematics between the stu-

dents who took developmental mathematics and those who did

not was rejected, when the course was MAT-101 and the co-

variate was the ACT mathematics score. The alternate

hypothesis that the experimental group will perform better than the control group in their first-year college mathematics course was accepted for MAT-101.

In addition to the covariate of ACT Math, the composite ACT score and the students' I.Q. correlated highly with the criterion variable and were considered as possible sources of adjustments. Shown in Table 6 are the zero-order correlations between the criterion variable and the independent variables for MAT-101.

TABLE 6

CORRELATIONS BETWEEN THE CRITERION VARIABLE
AND THE INDEPENDENT VARIABLES FOR MAT-101

Independent Variables	Zero-Order Correlation
ACT Mathematics Score	0.39
ACT Composite Score	0.52
I.Q.	0.61

All of the correlations were found to be significant at the .05 level. Using the ACT Composite Score as a covariate, the results of the covariance analysis are in Table 7.

The _fifth_ significant finding was that the performance of the students in the experimental group was better than

TABLE 7

SUMMARY OF COVARIANCE ANALYSIS FOR MAT-101
WITH ACT COMPOSITE SCORE AS COVARIATE

Source of Variation	S.S.	d.f.	M.S.	F
S_5: Treatments	2.86	1	2.86	5.63
S_2: Error	19.80	39	0.51	
S_4: Total	22.66	40		

$$F_{95} = 4.10$$

the control group in MAT-101 when the criterion variable was

the course final grade and the covariate was their ACT mathe-

matics score. The difference between the means was signifi-

cant at the .05 level, and the null hypothesis of no

difference was rejected. The alternate hypothesis that the

students who took developmental mathematics would perform

better in their first-year college mathematics course than

those who did not was accepted.

When I.Q. was used as the covariate, the analysis of

covariance yielded the results shown in Table 8.

Again, the difference was found to be significant at

the 0.5 level, and the performance of the group that took

the developmental mathematics course was significantly

better in MAT-101 than the students who did not take the

course. The null hypothesis of no difference in performance

was rejected for MAT-101 when the covariate was the students'

I.Q. This was the sixth separate finding and in conjunction

with the two other findings for MAT-101 we rejected the

second null hypothesis of no difference in performance in

first-year college mathematics between the students who took

developmental mathematics and those students who did not

take the course.

TABLE 8

SUMMARY OF COVARIANCE ANALYSIS FOR MAT-101
WITH I.Q. AS COVARIATE

Source of Variation	S.S.	d.f.	M.S.	F
S_5: Treatments	1.38	1	1.38	4.76
S_2: Error	11.32	39	0.29	
S_4: Total	12.70	40		

$$F_{95} = 4.10$$

Results for MAT-103

Examined next were the experimental results of the

students whose first-year college mathematics course was

MAT-103. The means and standard deviations of the criterion

variable and possible covariates are presented in Table 9.

TABLE 9

MEANS AND STANDARD DEVIATIONS OF THE CRITERION
VARIABLE AND THE COVARIATES FOR MAT-103

N=16

Variable	Function	Group	Mean	S.D.
Final Grade	Measure of Performance	Exper. Control	1.62 1.25	0.86 0.97
H.S. Math Grade	Covariate	Exper. Control	1.12 1.12	0.78 0.60
I.Q.	Covariate	Exper. Control	110.0 106.0	7.92 13.93
ACT Math Score	Covariate	Exper. Control	16.1 15.9	2.58 5.15
Cooperative Math Score	Covariate	Exper. Control	10.8 11.6	3.00 3.32
ACT Composite Score	Covariate	Exper. Control	17.4 16.4	3.60 3.26

For MAT-103 the correlations between the final course
grades and independent variables were, in most cases, not
high enough to justify their use as potential disturbing
differences between the two sample means. Individual tests
for significance of the correlations resulted in the Coop-
erative Mathematics Test scores and the students' I.Q. as
the only covariates worthwhile controlling for effects.
Table 10 lists the zero order correlations between the

course grades and the independent variables.

TABLE 10

CORRELATIONS BETWEEN THE CRITERION VARIABLE
AND THE INDEPENDENT VARIABLES FOR MAT-103

Independent Variable	Zero Order Correlation
ACT Math Score	0.25 N.S.
Cooperative Math Score	0.41 *
H.S. Math Grade	-0.27 N.S.
ACT Composite Score	None
I.Q.	0.47 *

*Significant at the .05 level

It should be explained that the high school mathematics grades used in this study were the self-reported grades found on the ACT profiles. The students' individual official records in the Admissions Office revealed that these grades were, in most cases, conservative and on the low rather than on the high side of the actual average mathematics grade. This was considered to be a possible explanation for the low and negative correlations between high school and college mathematics grades for both MAT-101 and MAT-103.

Using the Cooperative Mathematics Test Score as a

covariate, the results of the analysis of covariance are shown in Table 11.

TABLE 11

SUMMARY OF COVARIANCE ANALYSIS FOR MAT-103
WITH COOPERATIVE MATHEMATICS SCORE AS COVARIATE

Source of Variation	S.S.	d.f.	M.S.	F
S_5: Treatments	0.70	1	0.70	1.59
S_2: Error	12.90	29	0.44	
S_4: Total	13.60	30		

$$F_{95} = 4.17$$

The <u>seventh</u> finding was that there was no significant difference in performance in MAT-103 between the experimental group and the control group when the covariate was the Cooperative Mathematics Test scores. The null hypothesis of no difference in performance was accepted.

There was an appreciable correlation between the final course grade and the students' I.Q. although the numerical difference was only four points in favor of the experimental group students. The results of the analysis of covariance for MAT-103 with I.Q. as the covariate are presented in Table 12.

TABLE 12

SUMMARY OF COVARIANCE ANALYSIS FOR MAT-103
WITH I.Q. AS COVARIATE

Source of Variation	S.S.	d.f.	M.S.	F
S_5: Treatments	0.27	1	0.27	0.65
S_2: Error	12.33	29	0.42	
S_4: Total	12.60	30		

$$F_{95} = 4.18$$

The eighth finding was that there was no significant difference in performance in MAT-103 between the students who took the developmental mathematics course and those who did not when the criterion variable was the final grade and the covariate was the students' I.Q. The null hypothesis of no significant difference in performance was accepted.

The experimental group performed slightly, but not significantly, better than the control group in this first-year algebra and trigonometry course. Had the mean I.Q.'s of the two groups been reversed, i.e., if the control group mean had been four points higher than the experimental group mean I.Q. the difference between the means would have been significant. The group with the lower initial I.Q. would then have performed more effectively and surpassed or

overtaken the other group. The analysis of covariance

allowed for this effect and, unlike the "t" test, did not

result in a significant difference value when in fact there

was none.

Results for MAT-107

Of the three first-year college mathematics courses

involved in this study, MAT-107, College Algebra was con-

sidered to be the most difficult. It was also the course

that was least likely to be affected in terms of academic

performance by students who either took or did not take the

developmental mathematics course. Past records indicated

that students who did not receive an A or B in MAT-099 were

not likely to perform successfully in MAT-107.

The means and standard deviations of the criterion

variable and the independent variables are shown in Table 13.

Not all of the independent variables listed corre-

lated to the degree necessary for justification as a

covariate. The high school mathematics grade, long con-

sidered as one of the most reliable factors for predicting

performance in college mathematics, correlated near zero

and was not used as a possible uncontrolled variable. As

TABLE 13

MEANS AND STANDARD DEVIATIONS OF THE CRITERION
VARIABLE AND THE COVARIATES FOR MAT-107

N-21

Variable	Function	Group	Mean	S.D.
Final Grade	Measure of Performance	Exper. Control	1.40 1.33	0.96 0.95
H.S. Math Grade	Covariate	Exper. Control	2.25 1.11	0.93 0.99
I.Q.	Covariate	Exper. Control	105.0 109.4	6.83 8.02
ACT Math Score	Covariate	Exper. Control	12.80 13.6	3.22 2.22
Cooperative Math Score	Covariate	Exper. Control	8.1 10.8	2.40 3.05
ACT Composite Score	Covariate	Exper. Control	17.2 16.6	2.08 3.13

shown in Table 14, the Cooperative Mathematics Test score

and the students' I.Q. were chosen as the covariates even

though they were moderately low.

Both the Cooperative Mathematics Test score and the

student's I.Q. when used as covariates had the effect of

equalizing the two groups prior to any treatment. Since the

students in the control group had a mean I.Q. of 4.4 points

higher than the experimental group, and a mean Cooperative

TABLE 14

CORRELATIONS BETWEEN THE CRITERION VARIABLE
AND THE INDEPENDENT VARIABLES FOR MAT-107

Independent Variable	Zero Order Correlation
ACT Math Score	0.23 N.S.
Cooperative Math Score	0.35 *
H.S. Math Grade	0.18 N.S.
ACT Composite Score	0.08 N.S.
I.Q.	0.35 *

*Significant at the .05 level

Math Score of 2.7 points higher than the experimental group,
it would be expected that the mean final grade of the con-
trol group might also be several points higher than the mean
final grade of the experimental group even though the
correlations were not appreciable. In fact, however, the
mean of the experimental group was slightly higher than the
mean of the control group. Table 15 summarizes the results
of the covariance analysis when the Cooperative Mathematics
Test score is the covariate.

This value was obviously not significant at the .05
level, and the ninth finding was that there was no differ-
ence in performance in MAT-107 between the students who took

TABLE 15

SUMMARY OF COVARIANCE ANALYSIS FOR MAT-107
WITH COOPERATIVE MATHEMATICS SCORE AS COVARIATE

Source of Variation	S.S.	d.f.	M.S.	F
S_5: Treatments	0.80	1	0.80	1.95
S_2: Error	16.10	39	0.41	
S_4: Total	16.90	40		

$$F_{95} = 4.10$$

the developmental mathematics course and those who did not
when the covariate was the Cooperative Mathematics Test
score. Therefore, the null hypothesis of no difference in
performance was accepted.

Using the students' I.Q. as the covariate, the re-
sults of the covariance analysis are presented in Table 16.

TABLE 16
SUMMARY OF COVARIANCE ANALYSIS FOR MAT-107
WITH I.Q. AS COVARIATE

Source of Variation	S.S.	d.f.	M.S.	F
S_5: Treatments	0.60	1	0.60	1.33
S_2: Error	17.50	39	0.45	
S_4: Total	18.10	40		

$$F_{95} = 4.10$$

This value was not significant at the .05 level.
The tenth finding was that there was no difference in per-
formance in MAT-107 between the students who took the devel-
opmental mathematics course and those who did not when the
covariate was the students' I.Q. The null hypothesis of no
difference in performance between the groups was accepted.

Interpretation of the Findings for the Second Hypothesis

The second hypothesis predicted that there would be
no significant difference in performance in first-year col-
lege mathematics between the students who took the develop-
mental mathematics course and those who did not. The study
tested the hypothesis for each of three mathematics courses
in order to differentiate between the course content and the
degree of difficulty.

For MAT-101, the experimental group students per-
formed significantly better than the control group students
and the null hypothesis was rejected. The correlations be-
tween the criterion variable, final course grade, and the
selected independent variables were positive and signifi-
cant at the .05 level. There was evidence to support the
alternate hypothesis that the students who take the develop-
mental mathematics course will perform better in their first-
year college mathematics course--MAT-101--than those

students who did not take the developmental course.

In MAT-103, there was no significant difference in performance between the experimental and control groups and the null hypothesis was accepted. The mean final grade of the students who took developmental mathematics was 0.40 points higher than the control group mean final grade, but the correlations between the final grades and the covariates were not high enough to effectively adjust the mean differences. This study supported the findings of an earlier study by Blyler in 1970 where he reported that the students in MAT-099 and MAT-103, ". . . earned just about the same grade in both courses."[2] There was no indication that students performed better in MAT-103 after taking MAT-099. There was evidence, however, that students who did well in developmental mathematics also did well in MAT-103.

MAT-107 is recognized as the most difficult of the first-year mathematics courses involved in this study and it was not expected that the developmental students would make any significant gains in performance over those students who did not take the developmental mathematics course. Again, as in MAT-103, the mean final grade of the experimental

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[2]Blyler, op.cit., p. 6.

group was slightly higher than that of the control group,

but not enough to report a significant difference at the .05

level. The mean differences between the two groups on the

factors of I.Q. and the Cooperative Mathematics Test score

almost allowed the statistically uncontrolled variables to

compensate for the small mean difference between the final

grades. An analysis of the final grades in MAT-107 and

MAT-099 indicated that students who receive a grade of "C"

or lower in the developmental course did not perform well in

MAT-107. There is also evidence, however, in the form of

the results of a student questionnaire that the develop-

mental mathematics course was helpful to the degree that had

they not taken it they might not have performed as well as

they did. Refer to Table 28 in the Appendix.

The Third Experimental Hypothesis

The third hypothesis stated that there is no rela-

tionship between the students' ACT Mathematics sub-section

score and ACT Composite score and their success in first-

year college mathematics. Success was defined earlier as

being synonomous with satisfactory performance or having

received a final course grade of either an A, B, or C.

The computed point biserial correlations between the

ACT scores and the dichotomous "successful - unsuccessful"

status of the first-year mathematics students are presented

in Table 17. It was decided to also compare the scores of

the Cooperative Mathematics Test with the ACT scores to

determine which was a better predictor of success in college

mathematics.

TABLE 17

POINT BISERIAL CORRELATIONS BETWEEN ACT AND
COOPERATIVE MATHEMATICS TEST SCORES
AND SUCCESS IN FIRST YEAR
COLLEGE MATHEMATICS

N=58

Test	Point Biserial Correlation
ACT Mathematics Score	0.22 N.S.
ACT Composite Score	0.34 *
Cooperative Mathematics Score	0.08 N.S.
*Significant at the .05 level	

The results indicated that the ACT Composite Score

was a better predictor of success in first-year college

mathematics than either the ACT Mathematics Score or the

Cooperative Mathematics Test scores. The null hypothesis

was rejected for the ACT Composite Score as a predictor and

accepted for the ACT Mathematics Score and Cooperative

Mathematics Test Score as predictors. The eleventh finding

was that the ACT Composite Score was the best single pre-
dictor of success in first-year college mathematics. A
related finding was that the Cooperative Mathematics Test
Score was a poor predictor of success in first-year college
mathematics.

Multiple correlations between the final course
grades, ACT Mathematics Scores and Cooperative Mathematics
Scores yielded some useful information concerning the com-
bined effects of the two mathematics test scores on the
final grades. The results are shown in Table 18.

TABLE 18

CORRELATIONS BETWEEN THE FINAL GRADE AND THE
ACT MATHEMATICS AND COOPERATIVE
MATHEMATICS TEST SCORES

N=58

Variable	Correlation
r_{12} Final Grade/ACT Math	0.29
r_{13} Final Grade/Cooperative Math	0.38
r_{23} ACT Math/Cooperative Math	0.52
$R_{1.23}$	0.49

The results were interpreted as supporting the null
hypothesis of no relationship between the ACT Mathematics

and Cooperative Mathematics Test Scores and success in first-
year college mathematics. The moderate but significant
correlations indicated only a relationship between test
scores and grades, not between test scores and success as
defined in this study. The multiple correlation, $R_{1.23}$ of
0.49 was computed on the basis of the final course grades of
both the experimental and control groups on all three first-
year mathematics courses. Had they been analyzed separately,
as before, the wide range of test scores and the unequal
gains made by both groups would have yielded lower corre-
lations, except in MAT-101 where the ACT scores were
significant at the .05 level.

Results Pertaining to the Fourth Hypothesis

The fourth hypothesis was concerned with the rela-
tionship, if any, between the areas of the students' mathe-
matical weakness and the content of the developmental
mathematics course. The posed null hypothesis stated that
there was no such relationship. The findings used to test
the first hypothesis were also helpful in supporting the
findings reported for this fourth hypothesis.

An analysis was made of the Cooperative Mathematics
Test results to identify the areas of mathematical weakness
of both the experimental and control group students. The

results of a student questionnaire and an examination of

the admissions records revealed that these students were,

with few exceptions, graduated in the upper-half of their

high school class. Approximately sixty-five per cent had

completed one year of high school algebra and about ten per

cent had taken two years of algebra and/or trig. It must

also be remembered that these students scored low on their

ACT tests and, in most cases, were aware of their weakness

in mathematics. The following observations served to estab-

lish the need for a program to strengthen these weaknesses.

1. Seventy per cent of the Control group and fifty-

eight per cent of the Experimental group could not solve the

quadratic equation $x^2 - 7x + 12 = 0$ for x.

2. Eighty-three per cent of the Experimental group

and eighty per cent of the Control group were unaware that

$5^o = 1$.

3. Given that $x = \frac{3}{4}$, ninety-two per cent of the

Control group could not find the value of x^{-2}. Eighty-five

per cent of the Experimental group had the same difficulty.

4. Eighty-nine per cent of the Control group and

seventy per cent of the Experimental group had trouble find-

ing the slope of the line whose equation was $3y - 6x = 4$.

5. No one in the Control group and only ten per cent

of the Experimental group could simplify $\dfrac{x^6}{x^4 + x^2}$

6. Given that $\log_{10} x = 2$, only twelve per cent of the Control group and fifteen per cent of the Experimental group could solve for x.

7. Eighty-five per cent of the Control group and eighty-three per cent of the Experimental group could not identify the equation of a line whose X and Y intercepts were given.

8. Seventy-nine per cent of the Control group and seventy-four per cent of the Experimental group failed to solve a quadratic equation that was easily factored.

9. Only thirty-one per cent of the Experimental group and thirty-three per cent of the Control group could successfully solve a first degree inequality.

10. Ninety-two per cent of the Control group and eighty-seven per cent of the Experimental group could not solve $3^{2x} = 81$ for x.

11. Seventy-four per cent of the Control group and eighty-five per cent of the Experimental group were unable to determine the coefficient of x^2 in the product of two polynomials.

12. Eighty per cent of both groups could not solve a system of linear equations for x and y.

13. Seventy-seven per cent of the Experimental group and eighty-one per cent of the Control group could not identify the quadratic equation whose roots were given.

14. Only seven per cent of the Experimental group and eleven per cent of the Control group could correctly find the product of two complex numbers, (2i) (- 3i). The most common answer was the algebraic sum.

15. Eighty-four per cent of both groups were unable to determine the "b" coefficient of a quadratic equation which would make the roots equal.

16. Eighty-six per cent of the Control group and eighty per cent of the Experimental group could not find the sum of three complex numbers even when the value of "i" was given.

17. Seventy-four per cent of the Experimental group and eighty-five per cent of the Control group had difficulty finding the sum of numbers with negative exponents.

18. Given: $f(x) = 3x^2 - 4x + 1$, only forty per cent of the Experimental group and fifteen per cent of the Control group could find $f(2)$.

19. Seventy-eight per cent of the Control group and fifty-eight per cent of the Experimental group could not determine the 13th term of a fractional arithmetic progression.

20. Sixty-one per cent of the Experimental group and seventy-four per cent of the Control group could not find the sum of three radicals.

The results of the item analysis presented in Table 19 were used to determine the specific areas of the students' weaknesses. Both groups took the post-test, but only the results for the experimental group were shown in the table for comparison with the pre-test results.

The criterion used to identify the areas of mathematical weakness was when over 50 per cent of the students could not answer an item correctly. An exception to this single criterion was when the per cent correct on a national basis was lower than fifty per cent. There were several items that were answered correctly by from fifteen to thirty-seven per cent of the students in the 224 high schools that were selected for determining the national norms. One item, for example, was answered correctly by thirty per cent of the students on a national basis and twenty per cent of the experimental group in this study. That item was not considered as being in one of the areas of the students' mathematical weaknesses.

TABLE 19
ITEM ANALYSIS OF THE COOPERATIVE MATHEMATICS
TEST RESULTS – FORM A

	Per Cent Correct Control Group	Per Cent Correct Experimental Group	
Item	Pre-test	Pre-test	Post-test
1	48	52	62
2	63	80	80
3	56	55	58
4	30	42	55
5	41	29	30
6	26	39	51
7	45	48	50
8	20	17	35
9	30	52	58
10	66	58	60
11	8	15	28
12	11	30	30
13	19	23	30
14	19	23	25
15	0	10	23
16	12	15	14
17	15	17	29
18	21	26	47
19	37	52	55
20	33	31	35
21	8	13	30
22	26	15	28
23	41	42	44
24	22	42	45
25	20	20	25
26	37	42	48
27	19	23	32
28	15	39	42
29	11	7	10
30	19	26	33
31	16	16	25
32	14	20	20
33	48	43	42
34	15	26	32
35	15	40	44
36	30	13	18
37	11	7	12
38	8	20	20
39	19	20	28
40	11	15	18

The following specific areas were selected as those representing the students' weaknesses and most in need of development.

1. Algebraic expressions

2. Factoring

3. Exponents and Radicals

4. Quadratic equations

5. Complex numbers

6. Logarithms

For the developmental mathematics course, MAT-99, to effectively improve or strengthen the weak areas as determined by the analysis of the Cooperative Mathematics Test items, it seems reasonable to conclude that the course content must include instruction in those areas. The first four specific areas--Algebraic expressions, Factoring, Exponents, Roots and Radicals, and the Solution of Quadratic Equations--are fully covered in the developmental course as presently offered. The last two areas listed--Complex Numbers and Properties of Logarithms--are not a part of the course syllabus. For a complete description of the MAT-99, Developmental Mathematics course refer to the Course Outline in the Appendix.

Two out of six weak areas not being covered would

partially explain the relatively poor performance of both groups in MAT-103 and MAT-107, their first-year college mathematics courses. It also explained their relatively good performance in the MAT-101 course which does not require a complete mastery of the students' demonstrated weak areas for comprehension and success in the course. Reference to the Course Outline for MAT-101, Foundations of Mathematics 1, in the Appendix will confirm this observation.

The six areas of the students' weaknesses represent twenty-six out of forty problems or sixty-five per cent of the Cooperative Mathematics Test items. Figures 1 and 2 graphically present the relative performance of the Control and Experimental groups on each content area of the Cooperative Mathematics Test items.

The performance of both groups was relatively poor as compared to national and local norms with the Experimental group answering thirty-seven per cent of the items correctly and the Control group answering only thirty-one per cent correctly. This indicated that the students' preparation in mathematics was poor and that there is an urgent need for some kind of remedial mathematics program before they take their first-year college mathematics course.
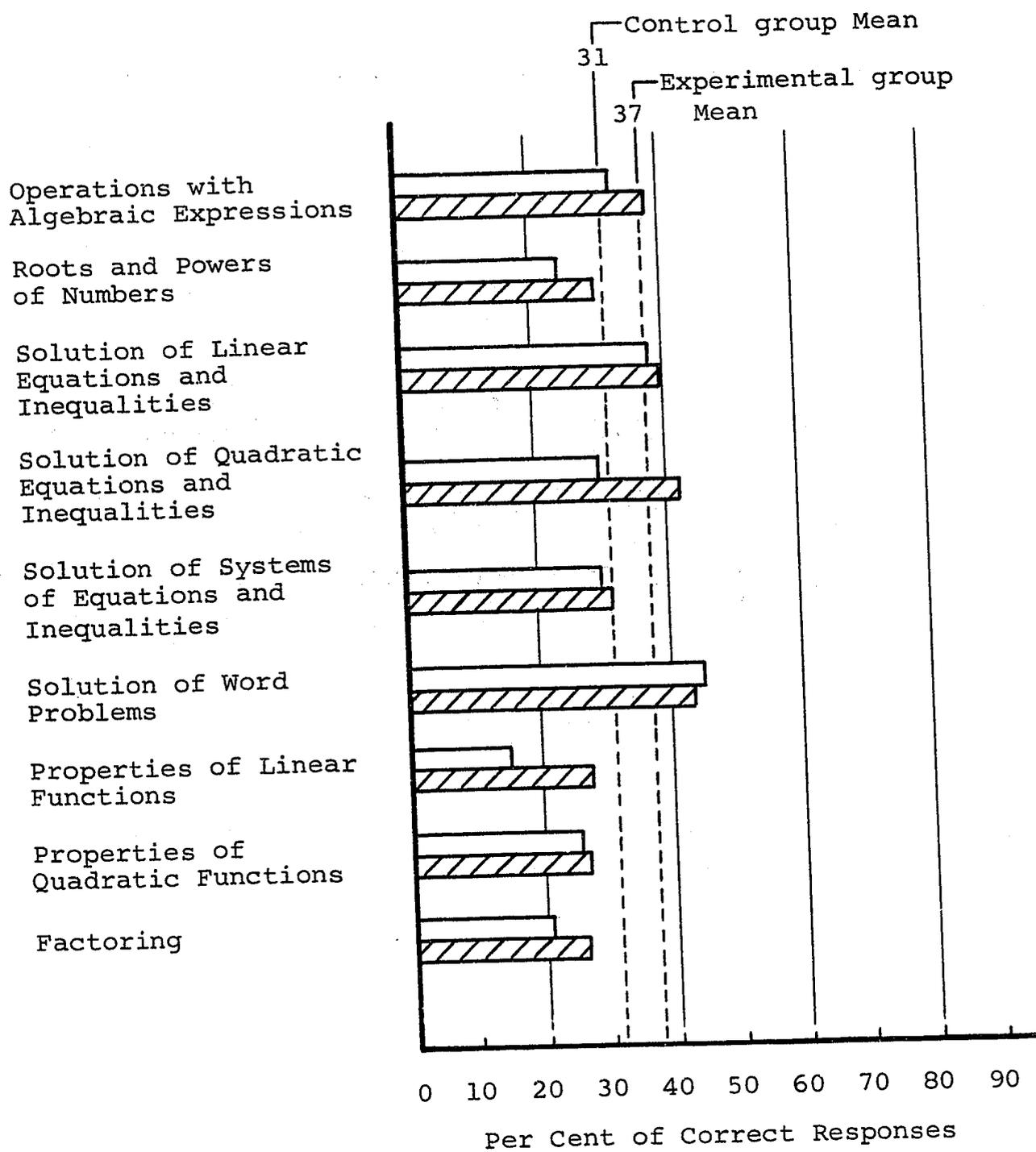
FIGURE 1

EXPERIMENTAL AND CONTROL GROUP PERFORMANCES
ON THE COOPERATIVE MATHEMATICS TEST – FORM A

☐ Control   ▨ Experimental

N = 28   N = 30
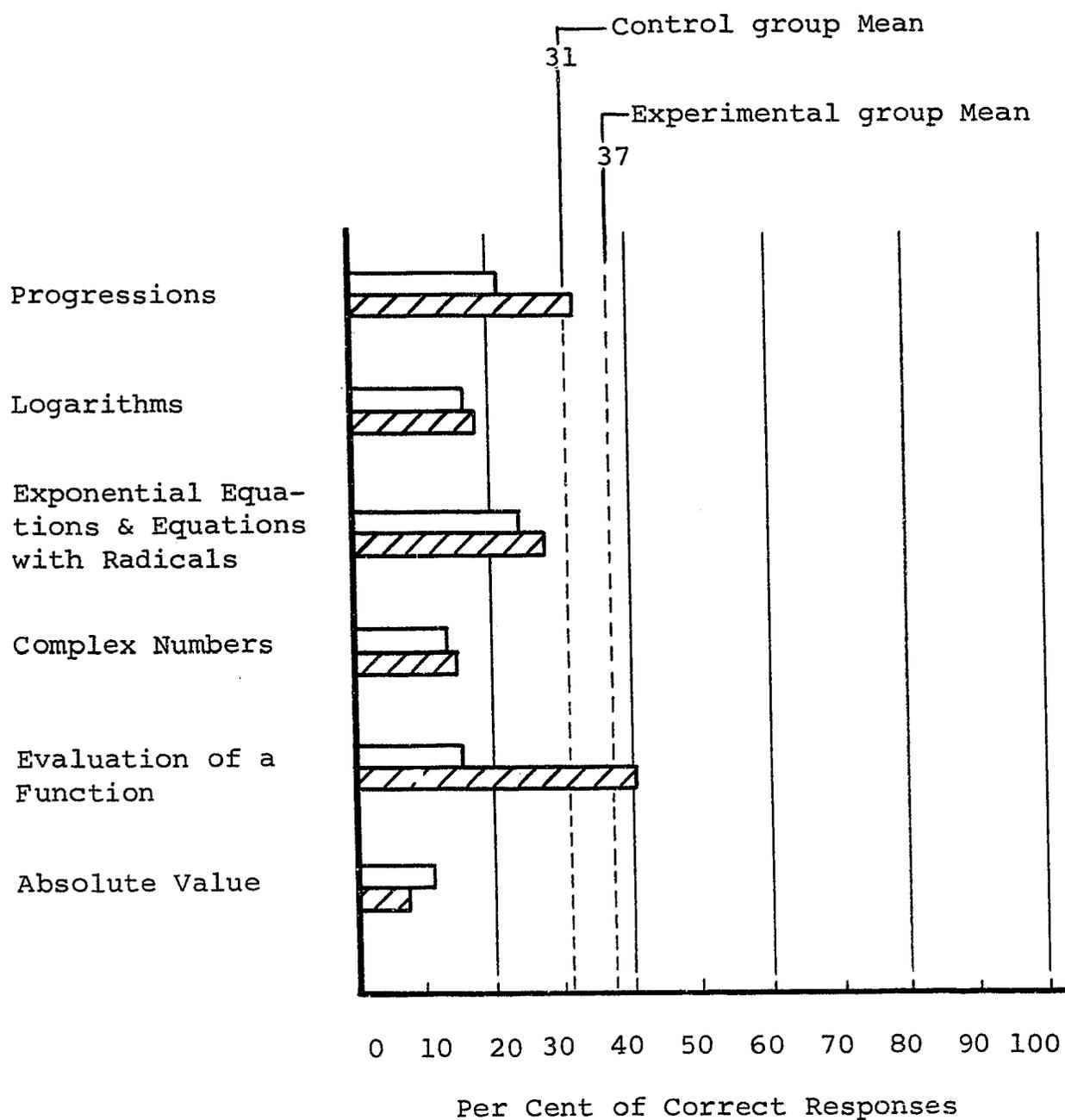
FIGURE 2

EXPERIMENTAL AND CONTROL GROUP PERFORMANCES
ON THE COOPERATIVE MATHEMATICS TEST — FORM A

Figure 3 shows the performance of the Experimental

group on the Cooperative Mathematics Pre-test and Post-test

on selected items in the areas of their mathematical weak-

nesses. There was an appreciable improvement in each of

the areas covered in the developmental course. The slight

improvement in the three areas not covered might have been

due to chance or by remembering the item from the pre-test.

In either case, it was significant that the immediate effect

of the developmental course was to improve the students'

performance in their weak areas.

It was not practicable to analyze the ACT Mathematics

test items to the same degree, but a comparison between se-

lected items revealed, as expected, the same areas of weak-

nesses. This explained why the students in this study

scored so low on their placement tests and also why the ACT

and Cooperative Mathematics Test scores correlated reasonably

high. This was further evidence that from thirty to forty

per cent of the entering freshmen are deficient in mathe-

matics and in need of some kind of remedial or developmental
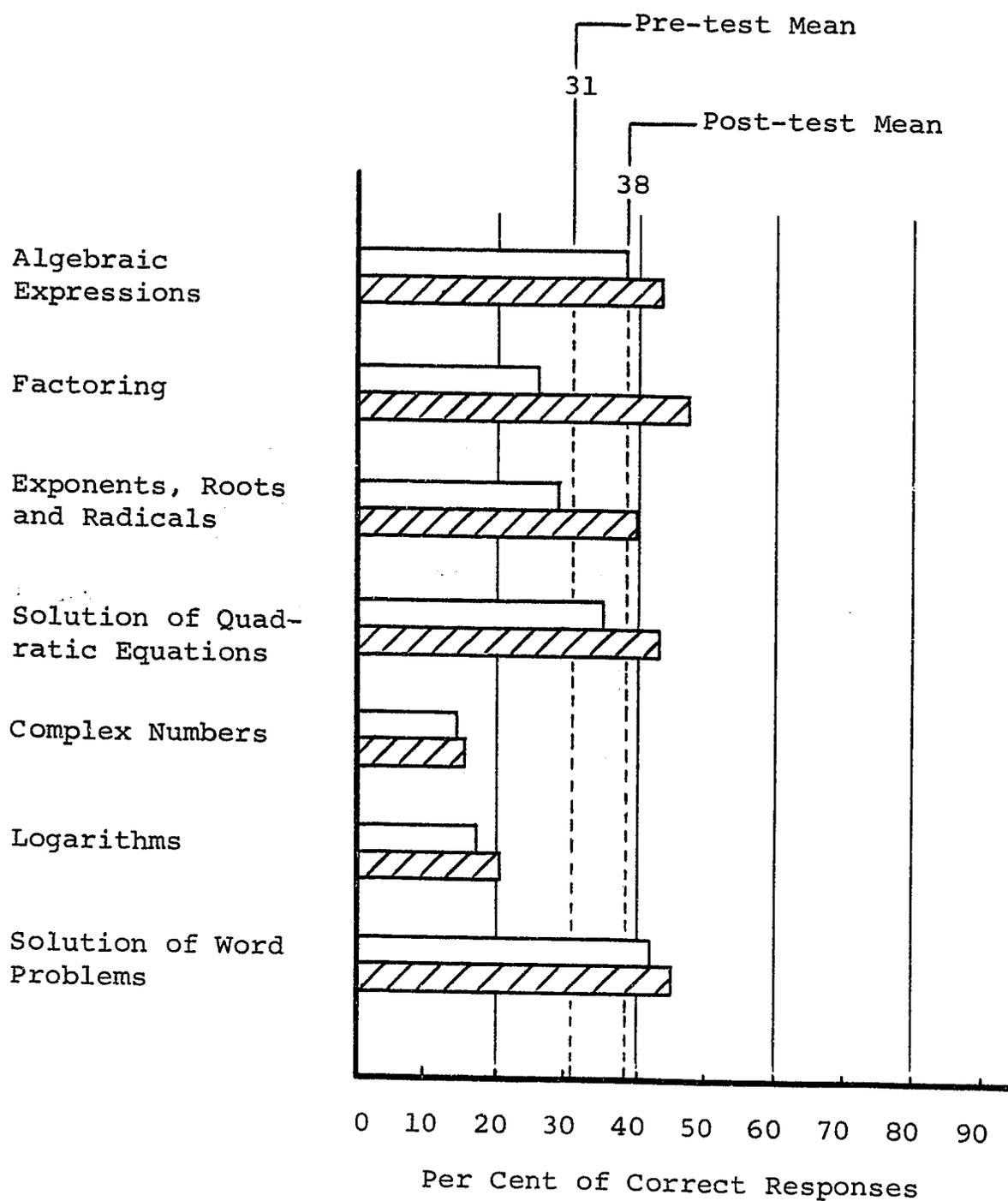
mathematics.

FIGURE 3

EXPERIMENTAL GROUP PERFORMANCE ON COOPERATIVE
MATHEMATICAL PRE- AND POST-TEST ITEMS IN THE AREAS OF
THEIR MATHEMATICAL WEAKNESSES

Operations with
Algebraic Expressions   xxxxxxxxxxxxxxxxxxxxxxxxxxxx

Roots and Powers
of Numbers   xxxxxxxxxx

Solution of Linear
Equations   xxxxxxxxxxxxxxxx

Solution of Quadratic
Equations   xxxxxxxxxxxxxxxxxxxxxx

Solution of Systems
of Equations   xxxxxxxxxxx

Properties of Linear
Functions   xxxxxxxxxxx

Properties of
Quadratic Functions   xxxxx

Factoring   xxxxxxxxxxxxxxxxx

Exponential Equations
and Equations with
Radicals   xxxxxxxxxxxxxxxxxxxxx

Absolute Value   xxxxx

```
   0     2     4     6     8     10
```

Hours

FIGURE 4

APPROXIMATE TIME DEVOTED TO EACH TOPIC IN
ONE SEMESTER OF DEVELOPMENTAL MATHEMATICS, MAT-99
INCLUDING TESTS

CHAPTER V

SUMMARY, CONCLUSIONS AND IMPLICATIONS

Summary

This study was primarily concerned with the problem
of determining the effectiveness of the developmental
mathematics program at the Lehigh County Community College.
The "open-door" admissions policy of Community Colleges in
Pennsylvania extended the opportunity for higher education
to all eligible high school graduates, with the result that
many students are being admitted who are poorly prepared in
mathematics. Remedial programs are offered, but not re-
quired by most colleges. On the basis of the students' ACT
scores and high school grades, the guidance counselors
identify those in need of remedial work and recommend that
they enroll in the developmental mathematics course. An
arbitrary score of 15 in the ACT mathematics sub-section
test is presently used as the cut-off point. For comparison,
this is equivalent to a standardized test score on the
national level in the 28-30 percentile range. The local
and national means of the ACT mathematics tests are 17.2
and 19.0 respectively.

For the developmental mathematics course to be effective, it must improve the students' proficiency in elementary algebra and enable them to perform successfully in their initial college mathematics courses. Once the student is allowed to enter, the college has the responsibility of not permitting him to register in courses for which he is not prepared. The wide range of educational backgrounds of entering freshmen has made the problem of placement in mathematics classes a matter of concern for the colleges. They have been forced to offer remedial courses, because the secondary schools have failed to adequately prepare the students for college level work in English and mathematics.

The Lehigh County Community College does not use any particular placement test, but relies heavily on the individual one-to-one counseling and advisement process. The student who scores 15 or lower on the mathematics sub-section of the ACT tests is not considered as having the background to be successful in his first-year mathematics course, and is advised to take the developmental course. There is always the risk that the voluntary or self-selection process will result in some students not taking the course who might have benefited more from it than those who subsequently

took the course only because their counselors strongly
advised it.

There is no agreement among educators concerning the
content or methods of teaching remedial courses except that
the net result should be a measurable improvement of the
students' proficiency.

The trend towards accountability has required that
educators be held responsible for the expenditure of funds,
both private and public, for programs that do not produce
effective measurable results. Remedial mathematics pro-
grams are no exception. In an effort to better identify the
students' areas of mathematical weaknesses, the Mathematics
Division of the Lehigh County Community College revised its
developmental program and changed from the traditional
text book lecture method to a programmed workbook method
hoping this would allow the student to progress at his own
rate.

The null hypotheses tested in this study were all
concerned with determining how effective the developmental
course is in meeting its objectives.

1. There is no significant difference in the gain
scores in elementary algebra between the students who took
the developmental mathematics course and those who did not

take it, as measured by the pre- and post-test scores of the
Cooperative Mathematics Test. The hypothesis was rejected.

2. There is no significant difference in performance
in first-year college mathematics between the students who
had taken the developmental mathematics course and those stu-
dents who had not taken it, as measured by their final
course grades. This hypothesis was rejected for MAT-101 and
accepted for MAT-103 and MAT-107.

3. There is no relationship between the students'
ACT and Cooperative Mathematics Test scores and success in
first-year college mathematics. The hypothesis was accepted
for both tests.

4. There is no relationship between the students'
areas of mathematical weaknesses and the content of the
developmental mathematics course. This hypothesis was re-
jected.

In addition to the test results from the ACT and
Cooperative Mathematics Tests, the final grades of three
first year college mathematics courses were used to statis-
tically test the above experimental hypotheses.

Because the college requires no admissions test or
administers no placement test, the experimental group con-
sisted of 30 students who registered for remedial mathe-
matics on the basis of their low ACT scores and the advice

of their counselor. The control group consisted of 28 students who, on the same basis, should have taken the remedial course, but decided not to. Voluntary participation in the study, rather than random selection, made it advisable to examine the possible effects of the self-selection factor. Both groups were analyzed on several variables to determine if they could be considered as coming from the same population. They matched well on all factors and were considered as equivalent for the purpose of this study.

The pre- and post-test scores were analyzed and "t" tests used to determine the significance of the difference in gain scores made by the experimental and control groups. The .05 level was selected as the value of significance to be used as the criterion.

To control for the effects of any differences between the groups prior to the treatment, the analysis of covariance procedure was used to test the null hypothesis of no difference in performance in freshman mathematics between the two groups. F tests were used to test for significance.

Multiple correlation techniques were used to test the null hypothesis of no relationship between the ACT and Cooperative Mathematics Test scores and success in freshman mathematics, and to determine which of the two tests is the

better predictor of success in first-year college mathematics courses.

For investigating the relationship between the developmental students' mathematical weaknesses and the content of the MAT-099 course, item analyses were conducted on the ACT and Cooperative Mathematics Tests items and responses to determine the specific areas of the students' weaknesses.

The frequency distributions of the final grades of the experimental and control group students in their first-year college mathematics courses were analyzed by a chi square test to compare them with the normal distribution, and the effects of several different instructors.

Conclusions

The results of the study indicated that the developmental mathematics course, MAT-099, is doing a reasonably good job of improving the students' proficiency in elementary algebra and increasing their chances of success in MAT-101, MAT-103, and MAT-107, the initial mathematics courses analyzed in this study. The significant gains made by the students in the experimental group on their post-test, particularly in their specific areas of weaknesses, allowed them to compete favorably with students who entered college not needing remedial work in mathematics. This was

especially true in MAT-101, where the previously identified areas of mathematical weaknesses were not emphasized in the course.

The moderate difference in performance between the experimental and control groups in both the MAT-103 and MAT-107 courses was not found to be statistically significant, but the results favored the experimental group. The results of the content analyses of the Cooperative Mathematics Test items demonstrated that the students who took the developmental mathematics course improved their performance in their weak areas to a degree that brought them success in a course they may have failed.

Both the ACT and Cooperative Mathematics Tests were relatively poor predictors of success in first-year college mathematics courses. Each test, however, served as useful diagnostic instruments when the test items were analyzed to identify and classify areas of strengths and weaknesses.

There was a positive relationship between the students' known areas of mathematical weaknesses and the topical content of the present MAT-99 course. Although both the experimental and control groups performed poorly on the pre- and post-Cooperative Mathematics Tests, the detailed item analysis revealed that the experimental group performed

significantly better on over 50 per cent of the items.

Implications

Remedial programs have become a major issue in most colleges and universities because there is little or no conclusive evidence of their actual benefit to the poorly prepared student. Ad hoc committees are appointed to study the situation, but their recommendations are seldom, if ever, acted upon. There are enough research studies and sufficient evidence to conclude that there are certain benefits to be derived from almost any well designed remedial program in almost any subject. There are also certain risks that may exist in some situations, and educators cannot seem to agree on whether the accrued benefits of the program are worth the accompanying possible risks to the student who needs the remedial work and does not take it as well as the student who, in fact, does not need the remedial work but is required to take it. A study by O'Regan concluded that in some cases remedial work may actually do more harm than good.[1] Based on several major theories of learning, this controversial conclusion seemed to support the premise that if the content of the remedial course was not what the student expected it

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[1]O'Regan, op. cit., p. 73.

to be, it might actually impede rather than improve proficiency. The results of this study do not support this theory in any respect. They instead, imply that the more practice the student receives in the areas of mathematics that he has demonstrated a weakness in the better he will perform. This is to say that the risk of boring or causing some students to lose interest or motivation is worth the benefits to be gained by most students who take remedial work. Until this is recognized by educators, a mandatory remedial program for all students failing to meet certain reasonable standards based on objective research and statistical data should be offered to these students prior to their initial college mathematics courses.

The failure of the ACT and Cooperative Mathematics Tests to serve as predictors of success in first-year mathematics courses for the students needing remedial work implies that the arbitrary cut-off score of 15 is perhaps too low. For the entering students with ACT scores of 18 and higher, the test does seem to have value as a predictive instrument.

A further implication of the study, based on the findings of the test item analyses, was that the content of the developmental course should be redesigned to include complex numbers, the properties of logarithms and the solution of

word problems. The results indicated that these three areas were not covered at all and combined with the students' weakness in fundamental arithmetic skills and elementary algebraic operations contributed to their poor overall performance in the Cooperative Mathematics Tests.

The moderate correlation (.48) between the per cent of items correct in each of the 15 content areas and the amount of time the students spent on these areas in their workbook supports the effectiveness of the developmental course in meeting their objective.

Recommendations

The conclusion that the present remedial course is measurably effective and doing a good job, and the implication that it could be even more effective and do an outstanding job, suggest several areas for further study and consideration.

1. One recommendation, suggested by an early limitation of the study, would be to take a good look at the present voluntary procedure for the determination of who should take the developmental mathematics course. The findings indicate that all entering students who score below 18 on the mathematics sub-section of the ACT test, should be required to take the developmental mathematics course,

MAT-099, before being allowed to register for their initial college mathematics course. This would result in the ACT test becoming a mathematics placement test with the cut-off score being increased from an arbitrary 15 to an experimentally determined 18. Not only would this greatly increase the students' chances for success in subsequent mathematics courses, it would increase enrollment in the developmental course to the level it should be according to the findings of this study.

2. Another suggestion would be to grant at least one college credit for the successful completion of the developmental mathematics course. There is much evidence to indicate that the student who needs remedial work simply cannot be relied upon to voluntarily register for it even after his counselor strongly recommends remedial work be taken. Receiving college credit would also help to convince the student that the college is interested in his success and feels that the course is beneficial.

3. A recommendation is that the college administration try to confirm the results of this study and work with the high schools personnel to find the real source of the remedial problem. Why should over 30 per cent of entering freshmen be unable to perform well in a mathematics placement

test that covers only the basic essentials of high school mathematics? Should not the successful completion of high school mathematics imply that the student is prepared for the initial freshman mathematics courses?

4. Based upon the findings, particularly those that resulted from testing the second null hypothesis, the content of the developmental mathematics course should be more closely coordinated with the contents of the first-year college mathematics courses. The MAT-099 course prepared the students for MAT-101 more effectively than for either MAT-103 or MAT-107. It is recommended that a text be adopted that includes as many as possible topics—preferably all—that are common to all three initial college mathematics courses. Only in this manner can the college assure the student that every effort is being made to adequately prepare him for his next mathematics course.

5. Another suggestion would be to offer more than one remedial mathematics course with graduated sequences covering several different topics and serving as prerequisites for different courses. The college could then allow a student to work with self-study materials in his weak areas before taking his first and, in many cases, his only college mathematics course.

6. One final recommendation, would be to find a way to encourage the students to overcome their poor preparation. This research is evidence enough that the student allowed to enter college through the "open door" policy having deficiencies in mathematics needs extra work for adequate preparation. An encouraging part of the study was that so many of these poorly prepared students can be helped to perform successfully in college mathematics. Students who scored as low as 6 and 7 correct out of 40 test items were later able to compete successfully with others in first-year college mathematics courses. This was evidence that the developmental mathematics course at the Lehigh County Community College is effectively improving the understanding of the basic mathematical skills of these students and increasing their chances for success in their initial college mathematics course.

There is reason to believe that the findings and conclusions of this research study are applicable to other Community Colleges and that the average college freshman who is deficient in mathematics can be expected to benefit significantly from remedial work.

APPENDIX A

List of Formulas Used in this Study

LIST OF FORMULAS USED IN THIS STUDY

1. Mean

$$\overline{X} = \frac{\Sigma X}{N}$$

2. Standard Deviation

$$S = \sqrt{\frac{\Sigma x^2}{N}}$$

3. Variance

$$S^2 = \frac{\Sigma x^2}{N}$$

4. Sum of the Squares

$$\Sigma x^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{N}$$

5. Standard error of the mean

$$S_{\overline{X}} = \frac{S}{\sqrt{N-1}}$$

6. Standard error of the difference between two means, uncorrelated data

$$S_{D\overline{X}} = \sqrt{S_{\overline{X}_1}^2 + S_{\overline{X}_2}^2}$$

      when: $N_1 = N_2$

$$S_{D\overline{X}} = \sqrt{\frac{\Sigma x_1^2 + \Sigma x_2^2}{N(N-1)}}$$

7. "t" test

$$t = \frac{\overline{X}_1 - \overline{X}_2}{S_{D\overline{X}}}$$

8. F test

$$F = \frac{S_1^2}{S_2^2}$$

9. Pearson Product-moment correlation coefficient, raw score

$$r_{xy} = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{[N\Sigma X^2 - (\Sigma X)^2][N\Sigma Y^2 - (\Sigma Y)^2]}}$$

10. Point-biserial correlation coefficient

$$r_{pb} = \frac{\overline{X}_P - \overline{X}_t}{S_t} \sqrt{\frac{p}{q}}$$

11. "t" test for point-biserial r

$$t = \frac{r_{pb} \sqrt{N-2}}{\sqrt{1 - r_{pb}^2}}$$

12. Multiple correlation R three variables

$$R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - (2 r_{12} r_{13} r_{23})}{1 - r_{23}^2}}$$

13. "Between" Sum of Squares

$$\Sigma x^2 = \Sigma (\overline{X} - \overline{X}_t)^2 n$$

14. "Within" Sum of Squares

$$\Sigma x^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n}$$

15. F test

$$F = \frac{\text{Mean square "between" groups}}{\text{Mean square "within" groups}}$$

16. Chi square

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

APPENDIX B

Performance Data for Experimental & Control Groups

TABLE 20

PERFORMANCE DATA FOR THE EXPERIMENTAL GROUP

N=28

Student No.	Cooperative Math		ACT		Final Grade
	Pre	Post	Math	Composite	
1	6	8	15	18	2
2	12	17	19	22	1
3	*	–	–	–	–
4	10	14	14	17	3
5	15	23	15	17	3
6	6	8	17	20	0
7	*	–	–	–	–
8	11	16	15	17	3
9	7	10	12	16	0
10	11	16	11	15	1
11	16	26	18	18	2
12	8	11	15	17	1
13	7	10	15	18	0
14	7	10	15	16	2
15	12	18	15	18	1
16	8	12	14	17	2
17	6	8	12	17	2
18	11	16	15	18	3
19	5	7	9	15	2
20	12	17	13	18	1
21	5	6	5	12	2
22	11	15	12	15	1
23	7	9	12	14	2
24	14	22	15	18	2
25	9	13	12	15	1
26	13	19	22	23	4
27	7	9	15	17	2
28	9	12	14	16	1
29	10	14	15	17	2
30	13	19	12	17	3
31	12	16	12	18	3
32	11	14	12	9	1
Sums:	291	414	417	505	53
Means:	9.70	13.80	13.9	16.8	1.76

*Students who did not enroll for a freshman math course.

COOPERATIVE MATHEMATICS TEST – EXPERIMENTAL GROUP

Pre-test	Post-test

Mean:

$$\bar{X} = \frac{\sum X}{N} = \frac{291}{30} \qquad\qquad \bar{Y} = \frac{\sum Y}{N} = \frac{414}{30}$$

$$\bar{X} = 9.70 \qquad\qquad\qquad \bar{Y} = 13.80$$

Mean Difference: $\qquad \bar{Y} - \bar{X} = 13.80 - 9.70 = 4.10$

Sum of the Squares:

$$\sum x^2 = \sum X^2 - \frac{(\sum X)^2}{N} \qquad\qquad \sum y^2 = \sum Y^2 - \frac{(\sum Y)^2}{N}$$

$$= 3089 - \frac{(291)^2}{30} \qquad\qquad = 6452 - \frac{(414)^2}{30}$$

$$= 266 \qquad\qquad\qquad = 739$$

Standard Deviation:

$$S_X = \sqrt{\frac{\sum x^2}{N}} \qquad\qquad S_Y = \sqrt{\frac{\sum y^2}{N}}$$

$$= \sqrt{\frac{266}{30}} \qquad\qquad\qquad = \sqrt{\frac{739}{30}}$$

$$= 2.98 \qquad\qquad\qquad = 4.96$$

Standard Error of the Mean Difference:

$$S_{D\bar{X}} = \sqrt{\frac{\sum x^2 + \sum y^2}{N(N-1)}}$$

$$= \sqrt{\frac{266 + 739}{30(29)}}$$

$$= 1.07$$

TABLE 21   112

PERFORMANCE DATA FOR THE CONTROL GROUP

N=28

| Student No. | Cooperative Math | | ACT | | Final Grade |
	Pre	Post	Math	Composite	
1	10	13	14	16	1
2	9	12	14	19	3
3	10	12	14	14	0
4	*	–	–	–	–
5	8	10	14	16	2
6	10	12	10	18	3
7	*	–	–	–	–
8	14	14	17	18	3
9	*	–	–	–	–
10	*	–	–	–	–
11	17	21	19	20	3
12	12	14	15	16	1
13	*	–	–	–	–
14	8	10	13	18	1
15	12	13	13	17	1
16	13	14	20	19	2
17	9	12	15	19	3
18	9	11	12	18	2
19	11	13	15	17	1
20	15	18	14	24	1
21	15	17	15	16	3
22	*	–	–	–	–
23	17	18	12	15	2
24	7	9	13	15	3
25	*	–	–	–	–
26	9	11	14	18	2
27	8	9	6	14	0
28	13	14	15	17	0
29	8	9	10	15	1
30	7	8	13	16	1
31	6	8	10	12	1
32	*	–	–	–	–
33	6	7	5	10	0
34	8	10	21	19	1
35	5	7	14	14	1
36	10	12	21	18	2
Sums:	286	338	389	467	43
Means:	10.20	12.10	13.90	16.70	1.54

*Students who withdrew from school or did not complete the course.

COOPERATIVE MATHEMATICS TEST - CONTROL GROUP

Pre-test	Post-test

Mean:

$$\overline{X} = \frac{\Sigma X}{N} = \frac{286}{28}$$

$$= 10.20$$

$$\overline{Y} = \frac{\Sigma Y}{N} = \frac{338}{28}$$

$$\overline{Y} = 12.10$$

Mean Difference: $\overline{Y} - \overline{X} = 12.10 - 10.20 = 1.90$

Sum of the Squares:

$$\Sigma x^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{N}$$

$$= 3210 - \frac{(286)^2}{28}$$

$$= 290$$

$$\Sigma y^2 = \Sigma Y^2 - \frac{(\Sigma Y)^2}{N}$$

$$= 4392 - \frac{(338)^2}{28}$$

$$= 292$$

Standard Deviation:

$$S_x = \sqrt{\frac{\Sigma x^2}{N}}$$

$$= \sqrt{\frac{290}{28}}$$

$$= 3.20$$

$$S_y = \sqrt{\frac{\Sigma y^2}{N}}$$

$$= \sqrt{\frac{292}{28}}$$

$$= 3.24$$

Standard Error of the Mean Difference:

$$S_{D\overline{X}} = \sqrt{\frac{\Sigma x^2 + \Sigma y^2}{N(N-1)}}$$

$$= \sqrt{\frac{290 + 292}{28(27)}}$$

$$= 0.88$$

APPENDIX C

The Lehigh County Community College
Course Outlines

<u>COURSE OUTLINE</u>

for

<u>MAT 99: DEVELOPMENTAL MATHEMATICS</u>

Date Submitted: August 26, 1970
Clock Hours: (3)
Semester Hours: (3)

<u>Course Description</u>

Since this course is to provide the necessary back-
ground for mathematics courses in both the college transfer
and college career programs it must have a wide range of
applicability. It must emphasize both concepts and tech-
niques. For these reasons MAT-99 will be comprised of
topics from elementary and intermediate algebra and ge-
ometry. The emphasis will be on algebra.

<u>Course Objectives</u>

In common with most community colleges any high
school graduate may be admitted to the Lehigh County
Community College. However, for various reasons, many of
these students do not have the mathematical background
necessary for success in college mathematics. It is the
aim of this course to give the student that necessary back-
ground.

<u>Topic Outline</u>

<u>PART ONE</u>

    I. Sets

   II. Counting Numbers

  III. Integers

  IV. Rational Numbers

<u>PART TWO</u>

    I. Equations Involving Two Variables

II. Algebraic Polynomials, Factoring and Fractions

III. Solving Fractional and Quadratic Equations

IV. Quadratic Equations with Irrational Solutions

Teaching and Grading Procedures

Students taking this course show a wide variety of individual backgrounds in mathematics courses previously taken and in length of time since last taking a formal course in mathematics.

With the wide variety of student backgrounds some students are going to find that they need additional help over and above the normal classroom and office hours help.

Quizzes, tests and a final examination; with respective weights of approximately 20%- 50%- 30%; determine the grading in this course.

Bibliography

Basic Text: Alwin, R. H. and Hackworth, R. D. Algebra Programmed, Parts One and Two; Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1969.

Supplementary Texts: Wade, T. L. and Taylor, H. E. Fundamental Mathematics,

Hemmerling, E. M. Elementary Mathematics.

COURSE OUTLINE

for

MAT 101 FOUNDATIONS OF MATHEMATICS I

Date Submitted: September 1, 1971
Clock Hours: (3)
Semester Hours: (3)

Course Description

In order to better understand the advantages and disadvantages of our base 10 place value system of numeration, other systems of numeration and systems using other bases are studied.

To study number systems from a contemporary point of view, material on sets, relations, and their properties are presented. The real numbers are then built up by successive extension of the whole numbers, the integers, and the rational numbers.

Objectives

The main objective of this course is to develop in the students an understanding of the real number system. A second objective derived from the first is that the student must become aware of the differences between a system of numeration and a number system. He should also discover the advantages of our place value system over other systems of numerations.

Topic Outline

I. Sets
    A. Description of sets
    B. Set Notation
    C. Subsets
    D. Operations involving sets
        1. Union
        2. Intersection
        3. Complement
        4. Cartesian Product
    E. Membership tables

II. Relations and their Properties
    A. Illustrations of relations
    B. Properties of relations
    C. Equivalence relations
    D. One to one correspondence
    E. The cardinal of a set
    F. Relations as sets

III. The Real Numbers
    A. The system of whole numbers
      1. Counting sets
      2. Whole numbers
      3. Ordinal and cardinal use of numbers
      4. Systems of numeration and number systems
      5. The equals relation
      6. Binary operations
      7. Properties of binary operations
      8. Addition and multiplication of whole numbers
      9. Properties of the binary operations, addition and multiplication in W.
     10. The system of whole numbers
     11. Order relations for whole numbers
     12. Finger counting
     13. Place value systems with bases other than 10
     14. The algorithms
     15. Computations in bases other than 10
     16. Computer arithmetic

    B. The System of Integers
      1. The set of integers
      2. Properties of the set of integers
      3. The system of integers
      4. The cancellation laws
      5. Prime numbers and composite numbers
      6. Prime factorization
      7. The division algorithm
      8. The greatest common divisor
      9. The least common multiple
     10. Order relations for the integers
     11. Absolute value
     12. Clock arithmetic
     13. The congruence relation

C. The System of Rational numbers
    1. Interpretation of number pairs
    2. The set of rational numbers
    3. Equivalent relation for ordered pairs of integers
    4. Equivalence classes of ordered pairs of integers
    5. Rational numbers as equivalence classes
    6. Addition of rational numbers
    7. Multiplication of rational numbers
    8. Naming of classes (reducing fractions)
    9. The system of rational numbers
    10. Order in the rational numbers
    11. Interpretations of rational numbers
    12. Decimal fractions

D. The System of Real Numbers
    1. Introduction to irrational numbers
    2. The number line
    3. The set of real numbers
    4. Order relations in the reals
    5. The system of real numbers
    6. Real numbers as infinite decimals
    7. Repeating decimals
    8. Approximations
    9. Decimal approximations of rational numbers
    10. Rounding off decimal approximations
    11. Decimal approximations of irrational numbers
    12. Square roots

Teaching Procedures

In order to teach the structure of the real number system this course starts with the whole number system and builds up system by system to the real number system. This upward movement allows the student to see the development of a new system as being necessary to answer questions having no answer in the old system. He also finds that in developing the new system the old system is retained as a sub-system of the new.

Grading Procedures

Quizzes, tests, and a final examination; with

respective weights of approximately 20%- 50%- 30%; determine the grading in the course.

Bibliography

Basic Text: Peterson, J. A. and Hashisaki, J. Theory of Arithmetic.

Supplementary Texts: Fehr, H. F. and Hill, T. J. Contemporary Mathematics for Elementary Teachers.

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COURSE OUTLINE

for

MAT 103: ALGEBRA AND TRIGONOMETRY I

Date Submitted: August 27, 1971
Clock Hours: (3)
Semester Hours: (3)

Course Description

This course is designed for students interested in pursuing a technical program stressing applications of basic mathematical concepts. Topics studied include fundamental concerpts and operations, linear functions and graphs, trigonometric functions, linear equations, determinants, and vectors. The prerequisite is MAT-099 or one year of High School Algebra.

Objectives

This course is intended primarily for students in the technology division and is given concurrently with their technical courses; such as electronics and chemical technology. These allied courses normally require that students have a certain mathematical maturity; and it is one of the objectives of this course to provide that maturity as the courses progress from topic to topic; therefore, this is an integrated course giving topics in both algebraic and trigonometric terms concurrently; not separating the two. The primary objective of this semester's work is to lay a firm foundation in algebra and trigonometry and to impress upon the student the practicality of the mathematics being taken. The course is also intended to help the student develop a feeling for mathematical methods, and not simply to have a collection of formulas when he has completed his work in this course.

Topic Outline

I. Fundamental Concepts and Operations
   A. Fundamental laws of algebra
   B. Exponents and Radicals
   C. Addition, subtraction, multiplication and division of algebraic expressions
   D. Equations and formulas

II. Functions and Graphs
    A. Functions
    B. Graphs of functions

III. The Trigonometric Functions
    A. Values of the trigonometric functions
    B. The Right Triangle

IV. Linear Equations and Determinants
    A. Graphical and algebraic solution of systems of equations
    B. Solutions of systems by determinants

V. Factoring and Fractions
    A. Factoring
    B. Simplifying fractions
    C. Multiplication, division, addition and subtraction of fractions

VI. Quadratic Equations
    A. Solution by factoring
    B. Completing the square and the quadratic formula

VII. Trigonometric Functions of any Angle or Number
    A. Signs of the trigonometric functions
    B. Radians and their applications
    C. Functions of any angle

VIII. Vectors and Triangles
    A. Applications of vectors
    B. The Law of Sines
    C. The Law of Cosines

Teaching Procedures

This course is taught as an integrated course to give a sound mathematical background to the future technician. Numerous applications are presented from many fields of technology; however, few are developed in detail. The applications in this first semester are primarily to indicate where and how the mathematical techniques are used. The approach used is not a rigorous one, although all appropriate terms and concepts are introduced as needed and given an intuitive or algebraic foundation. An extensive use is made of examples and graphs to introduce as well as clarify and illustrate points made in the text.

Grading

Sufficient quizzes and tests are given before a final examination to allow the student to analyze his progress through the course and adjust his study habits accordingly. The overhead projector is heavily used as a visual aid to compliment lectures and question-answer sessions.

Bibliography

Basic Text: Washington, A. J. Basic Technical Mathematics with Calculus, Menlo Park, California: Cummings Publishing Co., 1970.

Supplementary Text: Cairns, E. S. Mathematics for Applied Engineering, Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1967.

COURSE OUTLINE

for

MAT 107: COLLEGE ALGEBRA

Date Submitted: February 1970
Clock Hours: (3)
Semester Hours: (3)

Course Description

This course initially reviews high school algebra with topics on the real numbers, polynomials, rational exponents, and open sentences in one variable. The rest of the topics generally center around the function concept. Polynomial functions are covered in detail; variation is treated from the function standpoint; and sequences are treated through functions having positive integers as domain. Techniques for sketching the graphs of functions are emphasized. Complex numbers are introduced with emphasis placed on equation solving.

Objectives

There are three main objectives for this course:

1. To review and extend the algebraic concepts studied in previous courses.

2. To provide an adequate background for those students who intend to continue their studies in mathematics at least through calculus.

3. To provide an adequate background for those students taking technical programs.

Topic Outline

I. Properties of Real numbers
      A. Definitions and Symbols
      B. Operations on Sets
      C. Classification of Numbers

II. Polynomials
   A. Definitions
   B. Sums
   C. Products
   D. Factoring
   E. Quotients
   F. Equivalent Fractions
   G. Sums of Rational Expressions
   H. Products and Quotients of Rational Expressions

III. Rational Exponents
   A. Roots and Exponents
   B. Powers with Rational Exponents
   C. Radical Expressions
   D. Approximation of Irrational Numbers

IV. Open Sentences in one Variable
   A. Equivalent Equations
   B. First-Degree Equations
   C. Second-Degree Equations
   D. Substitution in Solving Equations
   E. Solution of Linear Inequalities
   F. Solution of Quadratic Inequalities
   G. Open Sentences involving Absolute-Value Notation

V. Relations and Functions
   A. Cartesian Products
   B. Subsets of Cartesian Sets
   C. Linear Functions
   D. Forms for Linear Functions
   E. Special Functions
   F. Graphs of First-Degree Relations
   G. Quadratic Functions
   H. Quadratic Inequalities
   I. Polynomial Functions
   J. Rational Functions
   K. Conic Sections
   L. Variation as a Functional Relationship

VI. Systems of Equations
   A. Systems of Linear Equations in 2 Variables
   B. Systems of Linear Equations in 3 Variables
   C. Systems of Nonlinear Equations
   D. Systems of Inequalities

VII.   Complex Number System
     A.   Definitions
     B.   Absolute Value
     C.   Quadratic Equations

VIII.   Sequences and Series
     A.   Mathematical Induction
     B.   Sequences
     C.   Series
     D.   Arithmetic Progressions
     E.   Geometric Progressions
     F.   Limit of a Sequence
     G.   Infinite Geometric Progressions
     H.   The Binomial Theorem

Teaching and Grading Procedures

There is a wide variety in the backgrounds, abilities and objectives of the students taking this course.   In order to try to meet the needs of all types of students, manipulative as well as theoretical aspects will be emphasized.   Where possible, applications of topics will be indicated. The method of presentation will be primarily lecture with as much student participation as time permits.   Grades will be based on quizzes, tests, and a final examination.

Bibliography

Basic Text: Beckenbach, Drooyan, Wooton. College Algebra, Second Edition, Wadsworth Publishing Company.

Supplementary Text: Rosenbach, Whitman, Meserve, Whitman. College Algebra, Fourth Edition. Ginn and Company.

APPENDIX D

Grade Distribution

TABLE 22

FREQUENCY DISTRIBUTION OF FINAL GRADES
MADE BY THE EXPERIMENTAL GROUP
IN FIRST YEAR COLLEGE MATHEMATICS

N=30

Developmental * Mathematics Grade	First-year College Mathematics Grades				
	A	B	C	D	F
A	1	3	4	0	0
B	0	1	4	3	1
C	0	2	2	6	2
D	0	0	0	0	0
F	0	0	1	0	0
	1	6	11	9	3

* MAT-099 Course

TABLE 23

FREQUENCY DISTRIBUTION OF FINAL GRADES
MADE BY THE CONTROL GROUP
IN FIRST YEAR COLLEGE MATHEMATICS

N=28

Control Group * Cooperative Math Post-test Score	First-year College Mathematics Grades				
	A	B	C	D	F
16-18	0	1	0	1	0
13-15	0	3	4	2	0
10-12	0	1	1	5	2
7-9	0	1	0	4	0
4-6	0	1	0	0	2
	0	7	5	12	4

* This group did not take the Developmental Mathematics
  Course

APPENDIX E

Performance of Freshman Class

TABLE 24

PERFORMANCE OF FRESHMAN CLASS
IN THREE FIRST YEAR MATHEMATICS COURSES
FIRST SEMESTER 1971

N = 426

Mathematics Course		Mean	Standard Deviation
MAT-101	n=149	2.21	0.86
MAT-103	n=101	2.73	0.94
MAT-107	n=176	2.68	0.96

* This N does not include the 58 members of the
control and experimental groups.

TABLE 25

COMPARISON OF EXPERIMENTAL AND CONTROL
GROUPS PERFORMANCE WITH THAT OF TOTAL FRESHMAN CLASS

N = 484

Mathematics Course	Experimental n = 30	Control n = 28	Total Class n = 426
MAT-101	2.30 *	1.91	2.21
MAT-103	1.62	1.25	2.73
MAT-107	1.40	1.33	2.68

* Developmental Mathematics especially effective for
MAT-101.

APPENDIX F

ACT and Cooperative Mathematics Test
Item Content Classification

TABLE 26

ACT MATHEMATICS TEST

Item Content Classification

Content Classification	No. of Items
Terminology	1
Combining Terms	3
Colution of Linear Equations	4
Translation from Verbal to Algebraic Expressions	4
Substitution in Algebraic Expressions and Equations	4
Solution of Literal Equations	1
Exponents and Roots	3
Algebraic Multiplication and Division	5
Averages	1
Systems of Linear Equations	2
Graphs of Linear Functions	2
Linear Inequalities and Order	4
Factoring and Quadratic Equations	4
Division by Zero	1
Variation	1
	40

TABLE 27

COOPERATIVE MATHEMATICS TEST, ALGEBRA II

Form A

Item Content Classification

Content Classification	No. of Items
Operations with Algebraic Expressions	5
Roots and Powers of Numbers	5
Solution of Linear Equations and Inequalities	3
Solution of Systems of Equations and Inequalities	4
Solution of Quadratic Equations and Inequalities	1
Solution of Word Problems	2
Properties of Linear Functions	3
Properties of Quadratic Functions	6
Factoring	1
Progressions	2
Logarithms	2
Exponential Equations and Equations Involving Radicals	2
Complex Numbers	2
Evaluation of a Function	1
Absolute Value	1
	40

APPENDIX G

Results of Student Questionnaires

TABLE 28

RESULTS OF THE QUESTIONNAIRE
GIVEN TO THE FIRST-YEAR MATHEMATICS STUDENTS

N = 53

	Frequency of Responses					
Item	5	4	3	2	1	Mean
1	2	31	20			3.66
2	7	31	15			3.92
3		20	26	7		3.25
4a	12	23	15	3		3.92
4b	10	25	18			3.57
4c	5	28	20			3.62
4d	2	17	26	8		3.25
4e	4	13	25	10		3.21
4f	3	8	22	19		2.90

QUESTIONNAIRE – MAT–

Name _____ Instructor_____

1. To what extent do you think that your preparation for this course was adequate? Circle one.

   Adequate   5   4   3   2   1   Inadequate

2. Do you feel that MAT 099 helped you in this course? (Circle only one)

   Helped   5   4   3   2   1   Did not help

3. Do you think the material covered in MAT 099 was: (Circle only one)

   Too Difficult   5   4   3   2   1   Too Easy

4. Try to identify your weak areas and circle the number that states whether MAT 099 was of any help in your present math course.

Arithmetic processes	Helpful	5	4	3	2	1	Not
Fractions	Helpful	5	4	3	2	1	Not
Simple Equations	Helpful	5	4	3	2	1	Not
Factoring	Helpful	5	4	3	2	1	Not
Radicals and Exponents	Helpful	5	4	3	2	1	Not
Sets and Inequalities	Helpful	5	4	3	2	1	Not

   Thank you

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