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Integrating the Pedagogy of United States' Fourth-Grade Math Teachers with International Approaches

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This is to certify that the doctoral study by

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has been found to be complete and satisfactory in all respects,
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Walden University
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Abstract

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by

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Dissertation submitted in Partial Fulfillment

of the Requirements for the Degree of

Doctor of Education

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May 2023

Abstract

Standardized testing results in fourth-grade math have shown that students in the United States continually score below students in many other nations, despite funding education at levels well above the global average. The problem addressed in this study was that fourth-grade math students in the United States were not performing as well on standardized tests as students of other nations and international cities. Guided by Vygotsky's social learning theory, the purpose of this qualitative, exploratory case study was to investigate the perceptions of teachers from U.S. and international settings about different approaches to effective fourth-grade math instruction, inherent obstacles to sound instruction, their college training, and professional development (PD). Purposeful sampling was used to identify 4 teachers in New York State and 8 teachers in Singapore and Shanghai who completed semistructured interviews for this study. Interview data were analyzed across teachers' responses using inductive, open coding to identify themes. Key findings indicated that New York State participants reported a lack of adequate timeframes for teaching collaborative math activities, a prevalence of low expectations connected to ability grouping, the absence of specialized math instructors, a dearth of math courses in pre-service education, as well as the unavailability of ongoing, relevant PD as compared to teachers in Singapore and Shanghai. In light of these findings, any successful effort to create positive social change and bring U.S. students' math performance into line with international students' performance should include enhancements in these practices, resources, and supports across the U.S. education sector.

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Chapter 1: Introduction to the Study

The United States Department of Education's stated mission is to promote student achievement and enhance their global competitiveness by cultivating educational excellence (U.S. Department of Education, 2022). The National Assessment of Educational Progress (NAEP) has associated itself with this goal by prioritizing student proficiency or advanced levels in mathematics (NAEP, 2019). Proficient or advanced knowledge of mathematics will enable students in the United States to successfully compete on international tests as well as acquire the necessary mathematical skills to obtain careers in science, technology, engineering, and mathematical (STEM) (Boaler & Sengupta-Irving, 2016; Peng & Lin, 2019; Vakil & Ayers, 2019). To meet these expectations, school administrators must confirm that fourth-grade teachers master an array of eclectic and effective instructional skills and approaches.

Research has revealed a strong connection between the capacities of teachers and the academic achievement of students (Rodriguez-Lopez et al., 2019). Many studies have found that using efficacious instructional approaches is critical to ensuring the academic and professional success of our students (Boaler & Sengupta-Irving, 2016; Pew Research Center, 2018; Swanson & Williams, 2014; Trinter et al., 2015). However, fourth-grade students in the United States are not amassing mathematical skills at the same level as many of their international peers. The problem that prompted this qualitative case study, which was identified in recent research literature and supported by standardized tests results, was that many fourth-grade math students in the United States were not meeting proficiency benchmarks, nor performing as well on standardized tests as students of other nations and international cities (NAEP, 2019; Pew Research Center, 2018; TIMSS, 2019; Woessmann, 2016). Low benchmark test results by students in the United States have occurred despite the fact that teachers are utilizing many instructional approaches, especially student-centered teaching (NAEP, 2019; Pew Research Center, 2018; TIMSS, 2019; Woessmann, 2016). The NAEP indicated that average math scores for fourth-grade students in the United States have consistently fallen below proficiency benchmark levels between the years 2007 and 2019 (The Nation's Report Card, 2019). Proficiency ratings in the Ithaca City School District in New York, which provided participants for my study, reflected an analogue of national trends. Results indicated that nearly half of all fourth-grade math students failed to meet proficient levels in 2019 (NYSED Data Site, 2019). The NAEP stated that students who reach "proficiency" have demonstrated competency over challenging subject-matter knowledge, can apply this knowledge to real-world scenarios, and will be able to use analytical skills relevant to the subject matter (NAEP, 2018).

The malaise experienced by America's fourth-grade students in math has extended to the upper grades as well (NAEP, 2018). National proficiency test results from 2017 revealed that 67% of eight-grade students in America were not proficient in math (NAEP, 2018). According to the NAEP, only 25% of American high-school seniors were proficient in math in 2017 (NAEP, 2018). These low levels of math achievement have provided the impetus for the creation of the study's purpose. The purpose of this qualitative exploratory case study was to develop and analyze thick rich descriptions of an international array of teacher perceptions about different approaches to effective fourth-grade grade math instruction, inherent obstacles to sound instruction, their college training, and PD. The study, therefore, focused on collecting and analyzing the perceptions of four fourth-grade teachers in schools in New York as well as four teachers in both Singapore and Shanghai. Teachers in the United States may use the results and findings of the study to integrate successful Singaporean and Shanghainese instructional strategies, as well as fine-tune their own current approaches. Teachers may also use the outcomes to expand their teaching capacity and, subsequently, to escalate the math achievement of fourth-grade math students.

The study was significant as a conduit for positive social change for many interrelated reasons. Studies have shown that early success in mathematics often enables students to subsequently experience achievement in advanced math courses (Boaler & Sengupta-Irving, 2016; Peng & Lin, 2019). These personal academic accomplishments may allow students later entrée into STEM-related careers (Boaler & Sengupta-Irving, 2016; Peng & Lin, 2019). This professional advancement, in turn, often serves as a gateway to economic upward mobility and increased civic enfranchisement which is fundamental to positive social change (Peng & Lin, 2019)

The remainder of Chapter 1 will include several subsections. The background section will highlight the necessity of the study which is to better understand how the poor performance of fourth-grade math students in the United States may be related to the teachers' preservice training and PD as well as to the implemented instructional approaches. Later sections will explain how studying the perceptions of teachers about math instruction, their training experiences, as well as attending influences may reveal the strengths and weaknesses of current approaches. Other sections will further expand on how these insights may be used to ameliorate the attending failure of teachers to increase student achievement (see NAEP, 2019; Swanson & Williams, 2014; Trinter et al., 2015). The conceptual framework section will center on Vygotsky's SLT which provided the study's contextual lens and served as a common nexus for all integral parts of the dissertation. Vygotsky's theory articulates the collaborative social aspects of learning (Lasmawan & Budiarta, 2020*;* Nguyen, 2017; Vygotsky, 1979). This philosophy is constitutive in understanding the nature of learner-centered approaches. I will also demonstrate the applicability of the qualitative exploratory case study design and explain how it will promote the collection and interpretation of deep-seated teacher mindsets regarding instruction as well as their prior and current training. I will then show how the qualitative design is aligned with the purpose, problem, and data collection process. The definition segment, in turn, will unveil the meaning of salient terms that are germane to the dissertation. These clarifications include such descriptors as student-centered and teacher-centered instruction. This section will also define 21st century skills and problem- and project-based learning, among others. The assumption section will contain a delineation of my presumptions. These preconceptions include those pertaining to the veracity of participant responses and researcher bias, as well as beliefs about the influence that the small, purposeful sample exerts on the study. Penultimately, I will expound on how improved classroom practices and training, as well as positive social change regarding STEM-related employment may constitute desirable outcomes of my study. The summary section, which will end the chapter, will be comprised of a synopsis of the chapter's notable points.

Background

International test results from the last 10 years, emanating from TIMSS, have suggested that many fourth-grade math students in the United States are not performing as well as their international peers (Pew Research Center, 2018; TIMSS, 2019; Woessmann, 2016). Many of these fourth-graders have also failed to attain proficient ratings in math for the years 2007-2019 (The Nation's Report Card, 2019). The disappointing test results reported by TIMSS as well as by the NAEP, have occurred even though teachers are utilizing many instructional approaches, especially student-centered teaching (Aslan & Reigeluth, 2015; The Nation's Report Card, 2019; TIMSS, 2019). Therefore, the failure of math teachers in the United States to prepare many of their students to perform competitively with their international peers, as well as achieve proficient ratings in math, represented a gap in the practice. To address this gap, the purpose of this qualitative exploratory case study was to develop and analyze thick rich descriptions of an international array of teacher perceptions about different approaches to effective fourth-grade grade math instruction, inherent obstacles to sound instruction, their college training, and PD. These elucidations included the beliefs of teachers who represented educational systems in New York State, Singapore, and Shanghai. In accordance, the study's scope encompassed the collection and analysis of the perceptions of four fourth-grade teachers in New York State schools, as well as four teachers in Singapore and four teachers in Shanghai.

A perusal of research literature supported the notion that fourth-grade math students in the United States were not performing at the same level as other international students (Boaler & Sengupta-Irving, 2016; Hanushek et al., 2014; The Nation's Report Card, 2019). To illustrate, one study documented that students in the United States consistently performed below students from Singapore and Shanghai on tests that were controlled for economic and cultural determinants (She et al., 2018). These results highlighted the serious ramifications of American students' underperformance in mathematics given the increasing importance of quantitative literacy in the United States (Boaler & Sengupta-Irving, 2016, Peng & Lin, 2019).

Teachers in the United States often emphasize student-centered math instruction (Adeleye, 2021). Studies have documented some vulnerabilities related to student-centered instruction. For example, Hui-Chuan and Stylianides (2018) argued that teachers are often unable to provide equitable facilitation in large classes which feature student-centered approaches. Retna and Pak (2016) and Koh-Chua et al. (2021) produced research-based evidence that student-centered instruction was not compatible with the timeframes allotted to math instruction in many schools. Stockard et al. (2018), in contrast, evidenced the efficacy of teacher-centered instruction, currently prevalent in

Singapore and Shanghai. These researchers reported that DI, when integrated with intermittent group work often produced salutary outcomes. My study was therefore significant in understanding the connection between the poor performance of many fourth-grade math students in the United States and the teachers' instructional approaches, preservice training, and ongoing PD. The study also explored other factors that may influence math instruction: (timeframes, class size, teacher-parent relationships, ability grouping, et cetera). To mitigate against these deficiencies, the study identified successful instructional strategies as well as a host of ancillary factors that affect preservice and ongoing teacher training. To that end, teachers may use the study's findings to integrate and supplement their own capacity with an array of approaches and supportive measures gleaned from their peers. In this way, teachers in the United States may implement more effective instructional strategies, as well as undertake initiatives to improved teacher training. These palliatives, in turn, may act as catalysts in increasing the math achievement of fourth-grade students.

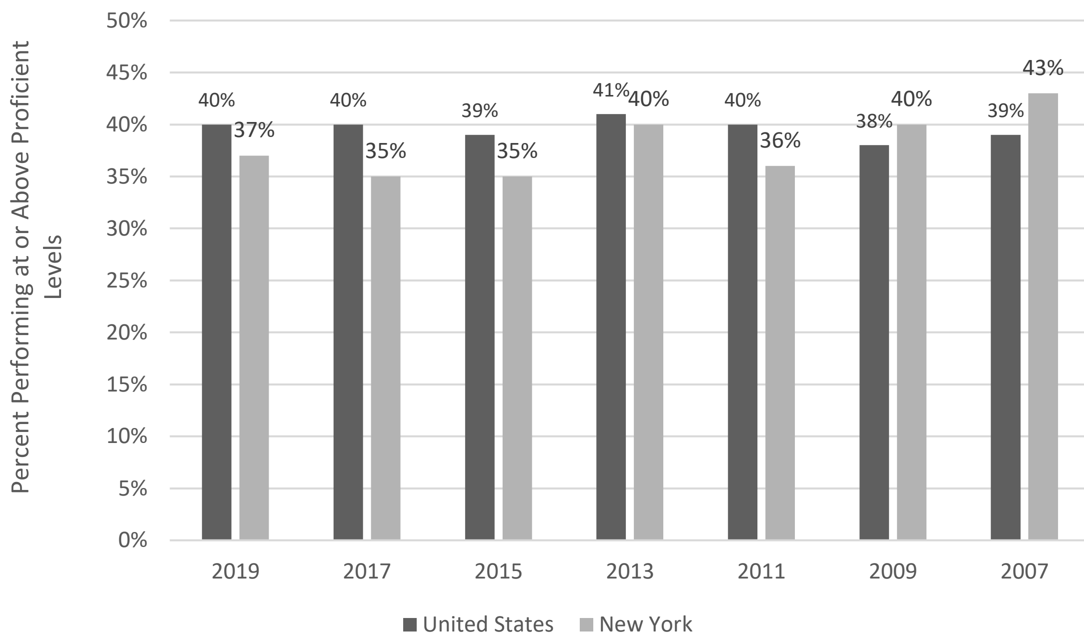
Problem Statement

The problem of this study, identified in current research literature, and supported by standardized tests results, was that fourth-grade math students in the United States are not performing as well on standardized tests as students of other nations and international cities (Pew Research Center, 2018; TIMSS, 2019; Woessmann, 2016). To compound the problem, a large percentage of the nation's fourth-grade math students have consistently failed to reach proficient levels in national tests in the last 10 years (The Nation's Report

Card, 2019). Specifically, just 40% of fourth-grade math students were rated as proficient according to the 2019 standardized test results (The Nation's Report Card, 2019). This is analogous to the proficiency level at which students performed in 2007 (The Nation's Report Card, 2018). New York State reported proficiency ratings for fourth-grade math students at 37% (The Nation's Report Card, 2019).

Figure 1

Proficiency Ratings: Fourth-grade Math

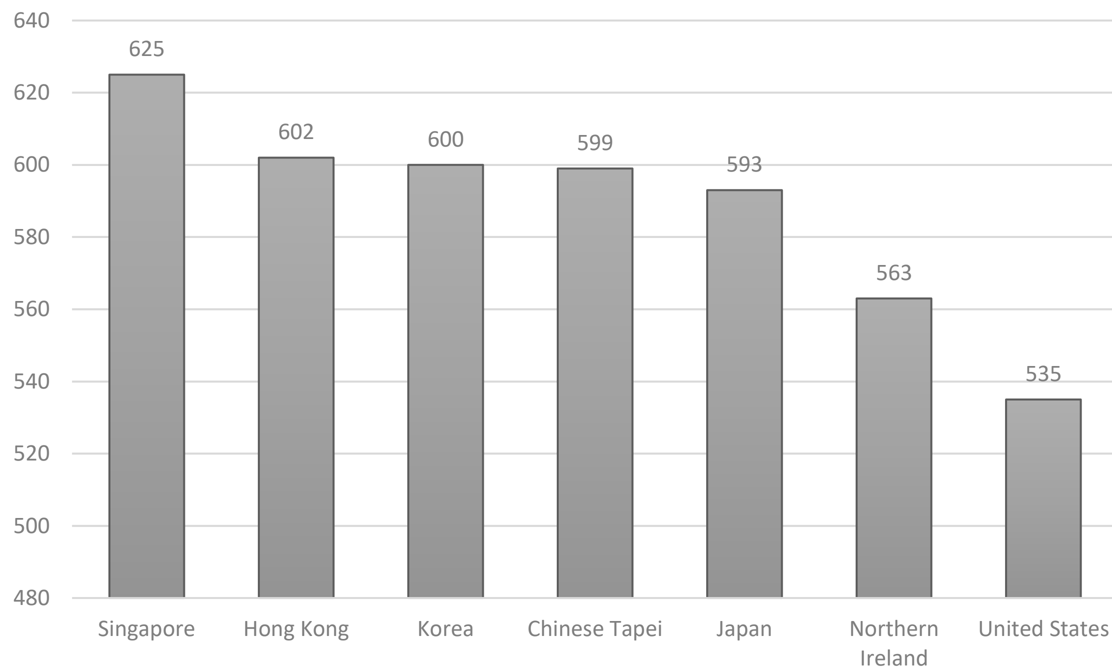


Note. From The Nation’s Report Card (2019) & NYSED Data Site (2019).

This dilemma was supported by a plethora of data related to longitudinal international tests results as well as national math proficiency scores (Pew Research Center, 2018; The Nation’s Report Card, 2019; TIMSS, 2019; Woessmann, 2016). These test results by United States’ students in general and New York State in particular, continue to trend in spite of the fact that teachers in America have utilized a number of instructional approaches, including student-centered teaching.

Figure 2

TIMSS (2019) Test Results: Fourth-grade Math



Note. From National Center for Educational Statistics (NCES) (2019).

As the figure depicts, the problem concerning the failure of fourth-grade math students in the United States to compete with international peers is current. The chart above illustrates exactly the profound nature of the problem. (Shanghai did not participate in the 2019 TIMSS tests.) The problem is also relevant and significant for the following reasons. For example, low math achievement not only affects classroom math performance but also exerts a deleterious impact on the global ascendency of the United States (DeJarnette, 2018;). Mathematics is one of the core disciplines which constitutes a critical element of elementary school academics (DeJarnette, 2018). Early success in math is essential in ensuring America's global competitiveness (Boaler & Sengupta-

Irving, 2016; DeJarnette, 2018). Success in math during these incipient years of development is vital both for college readiness and thereupon, the ability of students to compete globally in STEM careers (Boaler & Sengupta-Irving, 2016; DeJarnette, 2018; (Vakil & Ayers, 2019). Other research substantiates the relevance of instructional practices and teacher capacity in addressing the study's problem (Minarni et al., 2018; Swanson & Williams, 2014; Trinter et al., 2015). One study revealed that difficulties in learning mathematics may be ameliorated in the primary grades by improved teacher instruction (DeJarnette, 2018; Park et al., 2016). In the early elementary grades, teacher-reported instructional practices were linked to the development of students' motivational frameworks, which, in turn, were connected to students' mathematics achievement (Koh-Chua et al., 2021; Park et al., 2016). Success in mathematics in the primary grades can also provide a point of entry to algebra, geometry, trigonometry, and calculus in high school (DeJarnette, 2018). These academic accomplishments, in turn, provide a gateway to economic upward mobility and increased civic enfranchisement (DeJarnette, 2018).To ensure success in the higher grades, as well as in STEM careers, students in the United States must display competitive competency in mathematics apropos their international peers or reach advanced proficiency benchmarks (Alharbi et al., 2020). Resultantly, my doctoral study analyzed teacher perceptions to build upon data obtained through the perusal of prior articles and studies to address the gap in the practice. This gap was identified as the failure of math teachers in the United States to prepare many of their students to meet national and state proficiency ratings and exhibit the same level of math expertise as their international peers.

Purpose of the Study

The purpose of this qualitative exploratory case study was to develop and analyze thick rich descriptions of an international array of teacher perceptions about different approaches to effective fourth-grade grade math instruction, inherent obstacles to sound instruction, their college training, and PD. Alignment between the key components is paramount in a qualitative research study (Burkholder et al., 2019). In compliance with Burkholder et al. (2019), the purpose statement described here shows how the current study addressed the problem: many fourth-grade students in the United States are not meeting proficiency benchmarks nor are they performing as well as their peers on international math tests. Hence, the purpose is in alignment with the problem.

Research Questions (Qualitative)

The four research questions were informed and crystallized by a review of current literature. The questions addressed effective math instructional practices, challenges which served as obstacles to effective instruction, the role that SLT played in the instruction, and teacher preservice education and ongoing PD. The four queries were intrinsically aligned with the significant problem of why many fourth-grade students in the United States are not reaching proficiency benchmarks or performing as well as their peers on international math tests. The questions were also aligned with the purpose of developing thick rich descriptions of the perceptions of teachers on effective math instruction, inherent obstacles to sound instruction, college training, and PD. The interrogatives stem from and were aligned with the SLT conceptual framework. Moreover, these questions served as a fountainhead from which I derived interview questions for teachers.

Research Question 1 (RQ 1): According to fourth-grade teachers in the United States, Shanghai, and Singapore, which teaching practices best support math instruction?

Research Question 2 (RQ2): What challenges do fourth-grade teachers from the sample cite as obstacles to effective math instruction?

Research Question 3 (RQ 3): How do other determinant influences, such as teachers' preservice education and PD, affect math instruction in the United States, Singapore, and Shanghai?

Research Question 4 (RQ 4): How does SLT influence fourth-grade math instruction in the United States, Singapore, and Shanghai?

Conceptual Framework

I used Vygotsky's SLT as the contextual lens to explain the phenomenon of this dissertation (see Lasmawan & Budiarta, 2020*;* Vygotsky, 1978). Vygotsky proposed that students construct knowledge actively and interactively. Vygotsky's beliefs are thereupon foundational to constructivism. Constructivism, with its roots firmly ensconced in the precepts of Vygotsky's learning theory, emphasizes the importance of language, prior knowledge, authentic tasks, readiness levels, scaffolding, and the building of knowledge in a social setting (Lasmawan & Budiarta, 2020; Phillips, 2000; Leon & Castro, 2017; Piaget & Inhelder, 1970).

I will now expand on these constructivist practices by relating the following research-based information. Hardman (2021), as well as Leon and Castro (2017), attested that in a constructivist learning environment, students actively create their own meanings and conceptual understandings as they link prior knowledge with new information. Since learners similarly cultivate their own knowledge from real-world experiences, constructivist paradigms must include authentic tasks. Piaget and Inhelder (1970) and Hardman (2021) surmised that constructivist teaching should center on the intrinsic qualities of the student including their stage of development, mental processes, and cultural identities. These principles adumbrated the current practices related to student readiness and differentiated instruction. Leon and Castro (2017), as well as Lasmawan and Budiarta (2020*)* stressed the importance of social interaction and language in knowledge building. Steffe (2016), Serin (2018), and Pie-Ling Tan et al. (2017) proffered the idea that constructivism promotes active learning, as the teacher decentralizes himself or herself, and adopts the role of a facilitator of learning. Constructivism therefore serves, along with SLT, as a touchstone for student-centered instruction (Bruner, 1996; Jacobs & Renandya, 2019; Krahenbuhl, 2016; Pie-Ling Tan et al., 2017; Vygotsky, 1978). Constructivism also helps inform other student-centered practices. These include the formation of small groups for maximum communication, student collaboration and self-direction, and the role of teacher as a facilitator (Krahenbuhl, 2016; Lasmawan & Budiarta, 2020; Pie-Ling Tan et al., 2017; Rogers, 1969).

Student-centered instruction is edified by the Socratic method, which is based on the philosophy of Socrates (Wilberding, 2021; Yudcovitch & Hayes, 2014). Socrates was a Greek philosopher who sought to reveal the basis for his students' points of view by asking a series of questions (Ludovici & Hayes, 2014; Wilberding, 2021). These teacher-activated discussions sought to encourage student dialogue and reveal the participants' mindsets (Adeleye, 2021; Halina, 2014). Hence, the Socratic method emphasized the active learner, who acquires knowledge and cultivates critical thinking and creativity, while interacting with others in a social situation (Adeleye, 2021). The student-centered approach in education had its modern realization in the Progressive movement which began in 1917 (Nardo, 2018). John Dewey was one of its first advocates (Blasco, 2017; Nardo, 2018). Dewey viewed the developing child, and his or her curiosity, skills, and interests, as the starting point for all curriculum development (Blasco, 2017; Nardo, 2018). In congruence with Vygotsky's SLT Dewey averred that knowledge is constructed through interactions with others (Dewey, 1916; Nardo, 2018; Vygotsky, 1978). Shabani (2016) and Hardman (2021) contended that SLT is a process mediated by symbolic tools such as language. The concept of mediation suggests that human relations with the world are not direct but mediated, or interceded, by symbolic tools such as language (Hardman, 2021; Shabani, 2016). As a result, through the mediation of language, which supports dialogic negotiation and interaction among learners, a higher form of mental functioning is created (Hardman, 2021: Shabani, 2016).). In this cooperative and communicative ambient, students apply past learning to the current lesson. Thereupon, new knowledge is acquired through interactions with classmates and an informed adult mentor (Lasmawan & Budiarta, 2020; Vygotsky, 1978). Higher order thinking may result, in due course, through these social engagements (Lasmawan & Budiarta, 2020; Pie-Ling Tan et al, 2017). In this social learning milieu, knowledge is shared, diverse viewpoints are expressed, conflicts are resolved, creativity is ignited, and new knowledge is synthesized into existing schemas (Lasmawan & Budiarta, 2020; Pie-Ling Tan et al., 2017). This learning-centered environment is of critical importance in cultivating many 21st century skills (Leon & Castro, 2017; Serin, 2018). These skills include critical thinking, problem solving, cooperation, communication, self-efficacy, self-direction, and creativity (Leon & Castro, 2017; Serin, 2018).

The “zone of proximal development” (ZPD) is another maxim of Vygotsky’s SLT (Lasmawan & Budiarta, 2020; Vygotsky, 1978). This zone describes a student on the cusp of advancing from prior, mastered, and independently applied knowledge to new understanding. To achieve this higher level, students must elicit support from more capable others (Lasmawan & Budiarta, 2020; Nguyen, 2017). The word “proximal” indicates the closeness, proximity, and readiness of the student to achieve this cognitive progression. The advance in knowledge crucially occurs through social intermingling. Vygotsky (1978) further avowed that a student’s level of knowledge will progress when it is developed in an environment of assisted learning that incorporates problem solving in collaboration with more capable peers and is under the facilitating guidance of a knowledgeable teacher (Lasmawan & Budiarta, 2020; Nguyen, 2017). Enhanced cognition, therefore, is the final product of the above-mentioned series of social intercommunications (Hardman, 2021; Nguyen, 2017).

Correspondingly, by using Vygotsky's SLT as a conceptual lens, I could better understand the underlying principles which inform student-centered instruction as practiced by American teachers. As a result, I was able to better address the study's problem. The logic for this assertion is thus: SLT informs student-centered instruction; both SLT and student-centered instruction share attributes such as learning in a social environment and the development of social skills (Keiler, 2018; Dole et al., 2016; Vygotsky, 1978). Moreover, both SLT and student-centered instruction support an atmosphere that cultivates collaboration, communication, critical thinking, and shared problem solving (Pie-Ling Tan et al., 2017; Serin, 2018; Vygotsky, 1978). SLT and student-centered instruction also promote the goals of student self-direction, self-management, and self-efficacy (Churcher et al., 2014; Pie-Ling Tan et al., 2017; Serin, 2018). Therefore, Vygotsky's SLT is an appropriate lens since student-centered instruction is utilized by an increasing number of math teachers in the United States (Dansis, 2014; Dolmans et al., 2016; Serin, 2018). Concomitantly, there is a logical link between the student-centered approach, with all of the documented affordances of SLT, and the problem of the dissertation. The problem, identified in current research literature and supported by standardized tests results, was that many fourth-grade grade math students in the United States, despite relying on student-centered instruction, are not meeting proficiency standards, nor performing as well on standardized tests, as students of other nations and international cities (Pew Research Center, 2018; The Nation's Report Card, 2019; TIMSS, 2019; Woessmann, 2016).

Subsequently, the conceptual lens established foundational knowledge which helped frame many of the interview questions. These questions explored the strengths and weaknesses of SLT as it is implemented in student-centered classrooms. In mutual fashion, challenges inherent in SLT and student-centered approaches, such as classroom management, constricted timeframes, over-burdened teachers unable to attend to all students in equal fashion, as well as students' lack of focus, provided an entry point for dissecting the tenets and assets of teacher-centered classrooms which are prevalent in Asian schools (Hui-Chuan & Stylianides, 2018; Keiler, 2019; Kumar, 2016). Finally, the conceptual framework helped guide the data collection by generating appropriate interview questions. The reason for using this data collection tool was emphatic. It was to better understand the perceptions of teachers concerning the strengths and weaknesses of student-centered strategies, the attending value of teacher-centered instruction as well as the impact on instruction of other factors including teacher training. I will undertake a more thorough and detailed analysis of the conceptual framework and the logical connection among its elements in Chapter Two.

Nature of the Study

This qualitative study, which examined the perceptions of a purposeful sampling of 12 fourth-grade math teachers, utilized a case study design. This approach was taken to promote in-depth interviews in a naturalistic setting (Mishra & Dey, 2021; Yin, 1984). As a result, I was able to collect thick rich descriptions of teacher practices and viewpoints regarding math instruction as well as other determinant factors that influence that instruction such as preservice education and PD. During my study, I used open coding and thematic analysis regarding the interview transcripts. From these perspectives, I analyzed and interpreted the collected data related to the perceptions of four teachers in New York State, four teachers in Singapore, and four teachers in Shanghai. The exploratory case study design supported the qualitative ordonnance. Case study methods, through in-depth scrutiny of a very limited number of individuals, enables the examination of data within a specific complex context (Yin, 2018). Fundamentally, by using a case study design, I was able to describe data accrued in an empirical environment, as well as reveal the intricacies of real-life perceptions and beliefs (Yin, 2018).

Definitions

I included the following literature-based definitions which help describe concepts requisite to this study.

Collaborative learning: collaborative learning describes the practice of students working together in small groups, as well as in consultation with the teacher, to produce knowledge. Collaborative learning often changes the nature of authority in the classroom (Adeleye, 2021; Davidson & Major, 2014; Retnowati et al., 2017).

Student-centered learning: a collaborative and cooperative classroom environment, in which students and teacher share power (Hanewicz et al., 2017; Kaput, 2018). Moreover, SCL centers on authentic tasks, peer-to-peer learning, formative assessments, and the development of life skills (Hanewicz et al., 2017; Kaput, 2018). The teacher, in the role of facilitator, encourages students to adopt an active role in developing life skills such as critical thinking, problem solving, communication, and creativity (Hanewicz et al., 2017; Kaput, 2018).

Problem-based learning: problem-based math lessons offer compelling, real-life examples which require students to apply their current understanding and skills to new contexts (Davidson & Major, 2014; Dolmans et al., 2016; Siagan et al., 2019). In the problem-based milieu students work in teams and use critical thinking, communication, and problem-solving skills to understand the point at issue, identify necessary resources, utilize many strategies to find the correct solution, and ultimately, disseminate their findings (Davidson & Major, 2014; Dolmans et al., 2016; Siagan et al., 2019).

Proficiency: proficiency is described as the level of learning attained by students who have demonstrated competency over challenging subject-matter knowledge and can analyze and apply this knowledge to real-world scenarios (NAEP, 2018).

Project-based learning: project-based learning (PBL) is a standards-based instructional strategy, lasting days or weeks, where teachers, as facilitators of learning, encourage students, working in small groups, to exercise voice and choice as they collaborate on engaging real-life tasks (Chen & Yang, 2019).

Purposeful sampling: purposeful sampling denotes the idea that individuals are deliberately chosen to participate in the research for particular reasons (Ravitch & Carl, 2020). The rationale behind their selection includes their specific experiences, their knowledge of a certain phenomenon, or their employment in a particular school or location (Ravitch & Carl, 2020).

Teacher-centered learning: teacher-centered learning (TCL) environments promote the ascendency of the instructor where the teacher is considered the primary source of knowledge while the student is regarded as the receiver of that information (Baeten et al., 2016; Butler, 2020).

Twenty-first century skills: twenty-first century skills pertain to those abilities and life skills, especially promulgated in learner-centered classrooms, that are applicable to the work center (Chalkiadaki, 2018). These skill sets include, among others, critical thinking, problem-solving, inter personal and intra personal communication, collaboration, risk taking, inquiry, and creativity (Chalkiadaki, 2018, Ozcan et al., 2017).

Assumptions

Assumptions in qualitative studies are governed by the idea that knowledge is attained through the subjective experiences of people (Holley & Harris, 2019). There were three assumptions germane to this doctoral study. These assumptions pertained to the methodology, the accuracy of the data, and the implicit goal of improving teachers' capacity as well as their instructional approaches to fourth-grade math.

First, I assumed that the qualitative exploratory case study methodology, through the use of in-depth interviews derived from two data sources: New York State and Asian participants, was the most appropriate method to collect each participant's perceptions. As previously divulged, my purpose, in response to the problem related to American students' low test scores, was to develop thick, rich descriptions of the perceptions of an international array of 12 teachers concerning different approaches to fourth-grade math instruction, obstacles to sound instruction, as well as exploring the effect that preservice education and PD has on this instruction. Pertinently, research has shown that deep understanding of lived experiences is optimally collected using a qualitative case study (Creswell, 2018).

Secondly, I presupposed that the participants' responses to my questions accurately reflected their beliefs about instructional approaches, preservice education, and PD. To support this presumption, I prepared and rendered inclusive confidentiality agreements to secure the participants' privacy. The resulting privacy helped ensure the accuracy of the participants' responses. Member checking, where participants reviewed transcripts for verisimilitude, also helped ensure the accuracy of responses.

The final assumption of this study was that the dissemination of results will improve fourth-grade teacher instructional practices in math. This assumption was based on the following logic. The knowledge pool of fourth-grade math teachers should increase as a result of collecting, analyzing, synthesizing, and disseminating the instructional strategies, preservice education criteria, and PD experiences of an international array of teachers. The majority of these teachers emanate from Singapore and Shanghai which have achieved excellent results in international math tests (Pew Research Center, 2018; TIMSS, 2019; Woessmann, 2016). The quality of Singaporean and Shanghainese math instruction is bolstered through selective hiring practices combined with generous pay scales. High-scoring nations, such as Singapore and Shanghai, recruit their teachers exclusively from the top of their academic cohorts in college (Hanushek et al., 2019). Studies have postulated that teachers' cognitive skills are related to teacher quality and have a significant impact on student learning (Hanushek et al., 2019). In contrast, in the United States, just 23 percent of new teachers come from the top third of their graduating class with the majority emanating from lower end of the college skill distribution (Hanushek et al., 2019). Studies have therefore suggested that differences in United States teachers' cognitive skills and those of Singapore, may exert an impact on students' performance in school (Hanushek et al., 2019). Consistent with this logic, research divulged that teacher numeracy skills also have a strong association with student math performance. Compellingly, other studies have determined that teachers who are confident in their math abilities consider time constraints as less of a challenge than those teachers with low self-efficacy (Depaepe & Konig, 2018). Moreover, according to scholarly investigations, an increase of one standard deviation in teacher cognitive math skills was associated with an increase of 10 to 15 percent of a standard deviation in student performance (Hanushek et al., 2019). This implies that one quarter of the gap in average student performance between the United States and high scoring nations would be closed if America was to raise its teachers' cognitive skills from its current 47th percentile in math to the 74th percentile accrued by teachers in other nations (Hanushek et al., 2019). Scholarly scrutiny has posited two reasons for the cognitive differences among teachers in the United States and those in Singapore. First, women now have greater job opportunities outside the field of teaching in the United States (Hanushek et al., 2019). Secondly, teacher salaries are 22% lower in the United States in relation to those earned by other four-year college-degree holders (Hanushek et al., 2019).

In addition to high cognitive skills, teachers in Singapore undergo rigorous ongoing PD to improve their content knowledge and instructional capacity (Koh-Chua et al, 2021; Retna & Pak, 2020). Therefore, I theorized that, due to the dissemination of pedagogical knowledge from these international entities, teachers will be able to synthesize new information and strategies into their instructional repertoire. This may, as a consequence, increase their instructional capacity as well as enhance student achievement.

Scope and Delimitation

This study addressed the problem concerning the inability of many fourth-grade math students in the United States to meet proficiency benchmarks nor performing as well on standardized tests as other nations and international cities (Pew Research Center, 2018; TIMSS, 2019; Woessmann, 2016). To amend this dilemma, the study's focus was to collect and analyze data related to the perceptions of an international array of teachers about fourth-grade math instruction, obstacles to sound instruction, preservice education, and PD.

The ambit of the study, which included the delimitations, or the boundaries of the study, was limited to a purposeful sampling of 12 experienced fourth-grade math teachers and will be confined to three elementary schools in the United States, four elementary schools in Singapore, and four elementary schools in Shanghai. As required, each teacher in the study was fluent in English. The qualitative case study design for this doctoral study precipitated the collection of rich data from in-depth interviews with the 12 teachers, from New York State and Asian data sources.

I chose Vygotsky's SLT as my conceptual lens, but two other perspectives may also be related to the focus of my study. These include Engelmann's theory of DI, as well as hermeneutics. The cornerstone of the study was to collect and analyze data related to the phenomenon of interest, (i.e.), the perceptions of an international array of 12 teachers regarding fourth-grade math instruction. I therefore alluded to theories related to DI, which is extensively practiced in Asian schools, as well as interpretive beliefs, which helped clarify the teachers' perceptions. Engelmann's theory of DI was pertinent to the study (Engelmann, 2007; Spenser, 2021). Engelmann professed that explicit instruction places a high priority on teacher autonomy (Engelmann, 2007; Spenser, 2021). In consonance with Engelmann (2007) and Heward et al. (2021) teacher autonomy reduces the chances of students' misinterpreting ideas. As part of their advocacy for teaching for mastery, practitioners of DI emphasize the review and application of previously learned knowledge and skills (Engelmann, 2007; Spenser, 2021). Engelmann's theory was relevant to this study since its tenets strongly influence math instructional approaches currently used in Singapore and Shanghai (DeSouza, 2018; Ye & Cheng, 2017; Zhao et al., 2014).

Hermeneutics was another conceptual lens that has application to this study. Hermeneutics has been used as a method in educational research for many years (Can et al., 2018). Hermeneutics can be defined as a philosophy of interpretation through language or dialogue (Bleicher, 1982: Gadimer, 2007; Suddick et al., 2020: Yeo, 2017). Hermeneutics may also be described as using a subjective viewpoint to report, inform, understand, and analyze a phenomenon (Can et al., 2018; Kleinberg-Levin, 2021). Hence, this set of beliefs is concerned with making meaning of a person's lived experience (Can et al., 2018). To capture lived experiences, researchers conduct interviews that employ probing, open-ended questions (Yeo, 2017; Zeivots, 2018). The overarching goal is to create meaning and achieve a deeper understanding of these past experiences (Zeivots, 2018).

In congruence with the hermeneutic credo, understanding is deeply entwined in human experience Kim, 2018). Proponents believe that assumptions and biases, on the part of the researcher, are authentic components of the interpretive process (Galehbakhtiari & Hasangholi-pouryasouri, 2015; Kim, 2018). Therefore, a person's ability to understand does not necessarily involve neutrality (Kim, 2018). Rather, the researcher must be aware of one's own inherent biases so that they can invoke ameliorating practices such as dialogue with others and self-reflection (Kim, 2018). Collaterally, the interpretive process accentuates prior self-knowledge and subjective understanding (Kim, 2018; Yeo, 2017). Consequently, the interpretive method involves cyclical movement from the particular (subjective) to the general (objective) and then back to the particular (Kim, 2018). Complete understanding ultimately takes place when all the differing subjective and objective viewpoints are synthesized into a cohesive whole (Kim, 2018). Lastly, since this study involved a nonrandom purposeful selection of a small number of participants and sites, the emphasis of the study was not on its transferability; rather the thrust was to gain a deeper understanding of the particularities and complexities of a limited number of cases regarding teacher perceptions of fourth-grade math instruction, obstacles to sound instruction, preservice education, and PD (Ravitch & Carl, 2020; Yazan, 2015).

Limitations

There were limitations related to this doctoral study. The 12 fourth-grade teachers who participated in this exploratory case study were purposely chosen and represented three schools in the United States, four schools in Singapore, and four schools in Shanghai. As a result, the transferability of the findings may not be fully realized. However, there were advantages to having a purposeful sampling. Research has espoused that a deep, careful, and complete exploration of the cases can illuminate substantive findings relevant to the discipline (Creswell, 2018). Advocates of qualitative studies have also avowed that multiple, experienced-based interpretations of reality exist (Creswell, 2018). As a result, each person's perception may exhibit a unique understanding of reality, as opposed to subscribing to universal characteristics (Creswell, 2018).

Another limitation involved the veracity of participants' responses, the lack of which might have exerted a deleterious effect on the studies credibility. To address issues related to credibility, the same interview questions were repeated with the queries rephrased. Member checking was also used to bolster credibility. Member checking involved the practice of participants reviewing a copy of the transcriptions to certify that I had accurately captured their responses (Castillo-Montoya, 2016; Liao & Hitchcock, 2018). I also enlisted the support of my colleagues when reviewing the proposed questions. Here, my colleagues were asked to closely examine and determine whether the questions were clearly posed, nonjudgmental, and contained neutral language.

Moreover, an understanding between interviewer and interviewee that the accrued responses will remain private is integral to ensuring truthful responses (Ravitch & Carl, 2020). Requisitely, I utilized unique identifiers, such as Participant 1, to mask the real names of the participants as well as school sites. This layer of privacy encouraged participants to be more forthright in their responses. Finally, to help ensure the authenticity and credibility of the conclusions, the data were triangulated using both New York State and Asian sources to support my interpretations (Chen, 2015; Natow, 2020; Watts et al., 2017).

A third limitation, often associated with qualitative studies, was researcher bias (Creswell, 2014; Ravitch & Carl, 2020). Studies have indicated that, in qualitative explorations, the researcher adopts the role of the principal collector, as well as analyst of information (Ravitch & Carl, 2020). In this role, I therefore became part of the research. For that reason, I utilized a researcher reflective log to sustain an unbiased position. Considering this, I was able to maintain the exactitude of the participant's responses as well as my subsequent interpretations.

Significance

My doctoral study was significant since the collection, analysis, and dissemination of data related to fourth-grade math instruction in the United States, Singapore, and Shanghai may enhance teachers' knowledge and skills and increase student achievement in math (Aljaberi & Gheith, 2018). This sharing of data was an intrinsic component of the effort to improve the below-par achievement of many fourth-grade math students in the United States. Boaler and Sengupta-Irving (2016), Keiler (2019), and The Nation's Report Card (2019) described studies which found that the underperformance of math students in the United States has profound implications given the importance of numeracy on the global stage.

In accordance, researchers have demonstrated the urgency for investigations which focus on teachers' perceptions of their instructional methodologies (Aljaberi & Gheith, 2018). Aljaberi and Gheith (2018) attested that understanding teacher perceptions related to instruction and factors influencing instruction may lead to improved teaching practices, increased student achievement, as well as advanced knowledge in the discipline. Aljaberi and Gheith's study supported the intent and purpose of my dissertation which was to better understand the perceptions of an international array of teachers on the different approaches to fourth-grade math instruction and obstacles to sound instruction, as well as their points of view on how preservice education and PD impacted instruction (Aljaberi & Gheith, 2018). Therefore, my purpose fundamentally addressed the problem of the study, the seriousness of which was evidenced by standardized test results, as well as underscored by the previously-mentioned literature. The problem was that many fourth-grade math students in the United States are not meeting proficiency benchmarks, nor performing as well on standardized tests as other nations and international cities on standardized tests (Pew Research Center, 2018; The Nation's Report Card, 2019; TIMSS, 2019; Woessmann, 2016). Ergo, the study had additional significance since the subsequent dissemination of results may prove germinal and advance the synthetization of successful East Asian teacher training and instructional strategies into America's pedagogical repertoire. At this inflection point, the increase of teachers' knowledge and skills may help United States' students acquire the necessary math expertise to advance in STEM-related fields (Boaler & Sengupta-Irving, 2016; Keiler, 2019)

This advancement in students' knowledge and skills has serious implications for positive social change (Boaler & Sengupta-Irving, 2016; Vakil & Ayers, 2018). Boaler and Sengupta (2016) and Vakil and Ayers (2018) concluded that poor math performance in the early grades has a negative impact on the students' future academic and professional trajectory. These findings are underscored by studies that show the achievement gap in math, strongly related to income and race, has failed to close in the last 50 years (Hanushek et al., 2019). Therefore, student success, at this early point of entry, may emphatically serve as an avenue to economic upward mobility and increased civic enfranchisement by underserved populations (Vakil & Ayers, 2018).

In a germane manner, my study particularly focused on the ability groupings in differentiated instruction as practiced in New York State classrooms. Many of these formulations subsumed an inherent and often overt implication of lower expectations for some of the students. Research has revealed that low expectations may serve as an obstacle to the goal of achieving equity. Louie (2019) and the National Council of Teachers of Math (NCTM) (2019) divulged that excellence in mathematics education requires equity. Equity has been defined as having high expectations and strong support for all students (Louie, 2019). Cultivating students' sense of their own efficacy and agency is often viewed as a vital means to advance equity (Brinkmann, 2019; Snell & Lefstein, 2018). Therefore, ability grouping can sometimes work against equity as teachers address certain groups with basic problem solving and close-ended and unchallenging questions while reserving complex problem solving and open-ended and more sophisticated questions for higher-performing groups (Darling-Hammond, 2010; Louie, 2019; Snell & Lefstein, 2018).

Furthermore, Brown and Bates (2017) and Vakil and Ayers (2018) underlined the importance of equity in the classroom, citing it as a transformational force for the upward mobility of the underserved. Lastly, Wright (2016) as well as Vakil and Ayers (2018) avowed that mathematics education should play a role in addressing difficulties faced by our society, including growing inequality and human rights abuses.

Summary

The following precis encapsulates the main points of the first chapter of my dissertation study, “Integrating the Pedagogy of United States’ Fourth-Grade Math Teachers with International Approaches.” The introduction described the vital role mathematics plays in our society (Brown & Baltes, 2017; Vakil & Ayers, 2018). The background component of the chapter delineated the study’s significance to both the discipline as well as the progression of positive social change. It also provided a summary of related research literature. The problem section included evidence that the problem of the study was current and significant and exposed a gap in the practice. The purpose section revealed the intent of the study, highlighted the linkage of the purpose to the problem and focus of the investigation, and described the research paradigm. Other chapter constituents contained definitions of important terms, framed the research questions, and characterized SLT as the conceptual lens. Subsequent elements showcased the qualitative nature of the study and alluded to the methodology which included the collection and analysis of data. Finally, I identified my assumptions, elaborated on the scope and limitations of the study, and expanded on the significance of the doctoral dissertation.

Chapter 2 will be comprised of a review and synthesis of current literature related to fourth-grade teachers' instructional practices. The literature review will include an examination of studies related to the research questions. This perusal will also help establish the relevance of the problem which is the failure of students in the United States to meet proficiency benchmarks or perform as well as many international students on standardized tests. I will also describe studies that are associated with my phenomena of interest and identify methodologies that are congruous with my own topic and scope. Additionally, I will use evidence from scholarly articles to justify my approach. Supplementary studies will be analyzed and synthesized into my current understanding to broaden and deepen my comprehension of the attending concepts. Next, a summary of what is known and not known in the discipline, as it relates to my topic, will be provided. Finally, I will summarize the major themes emanating from the literature review and describe how my current study fills the gap in the literature

Chapter 2: Literature Review

The problem that served as an impetus for this qualitative exploratory case study, which was identified in current research literature and supported by standardized tests results, was that many fourth-grade math students in the United States were not meeting proficiency benchmarks, nor performing as well on standardized tests as students of other nations and international cities (Pew Research Center, 2018; The Nation's Report Card, 2019; TIMSS, 2019; Woessmann, 2016). This problem was aligned with the purpose of the study which was to collect thick rich descriptions of fourth-grade teachers' perceptions about effective math instruction, inherent obstacles to sound instruction, college training, and PD. The study's qualitative methodology efficiently linked all of the study's components to each other, that is, the problem, purpose, research questions, and conceptual framework, as well as data collection, analysis, and findings. Accordingly, the qualitative method was used as a platform to address the problem of United States students' failure to attain benchmarks in fourth-grade math proficiency or perform as well as many international students on standardized tests. The qualitative method facilitated the purpose of collecting rich thick data from a purposeful sample of 12 participants through semi structured, in-depth interviews. Research has shown that deep understanding of lived experiences is most optimally collected using a qualitative, exploratory case study (Creswell, 2018). The research questions complemented the purpose and addressed the problem by eliciting information about instructional practices and teacher training. The research questions also served as the basis for collecting data relevant to SLT which served as the conceptual lens and strongly influenced math instruction in the United States. The data was then analyzed for patterns and themes across teacher responses utilizing open coding and thematic analysis. The study findings revealed the need for increased timeframes, specialized teachers for math instruction, relevant and ongoing PD, robust preservice education, and maintaining high expectations during instructional practices related to differentiation.

A perusal of current literature unveiled the relevance of the study's problem. Trends in Mathematics and Science Studies released test results from 2019. Fourteen countries (out of 58) had statistically higher average fourth-grade math scores than the United States (TIMSS & PIRLS, 2019). This represented a decline from the 2015 scores (Pew Research Center, 2017; TIMSS & PIRLS, 2019). Moreover, 53% of Singapore students reached fourth-grade benchmarks in math compared to 15% of United States' students. The national proficiency scores, unfortunately, have also failed to improve during the last decade in the United States (The Nation's Report Card, 2019). Just 40% of fourth-grade math students were rated as proficient according to the 2019 standardized test results (The Nation's Report Card, 2019). This is basically the same proficiency level at which students performed in 2007 (The Nation's Report Card, 2019).

These scores by fourth-graders are portentous since a lack of success in math in the primary grades often exerts a deleterious effect on a student's later performance (Boaler & Sengupta-Irving, 2016; DeJarnette, 2018). Hanashek et al. (2014) and She et al. (2018) found that 15-year-old math students in the United States continued to post lower scores than students from many international countries and cities when samples were controlled for economic, cultural, and social factors.

There are many determinants which influence math instruction in the United States, Singapore, and Shanghai. Many scholars have noted the increasing proliferation of ineffective teacher preparation programs in the United States (Cochran et al., 2015; McDiarmid, 2019). McDiarmid (2019) reported that Singaporean teachers are selected from the upper tier of their graduating class. Intelligence or cognitive ability can be defined as the ability to reason logically, solve problems, think abstractly, understand complex ideas, and learn from experience (Bardach & Klassen, 2020). However, research on teacher effectiveness has largely ignored intelligence as a potential predictor of how well teachers enable student learning (Bardach & Klassen, 2020). Research is also scarce concerning the inability of American universities to attract top tier candidates to educational fields (Kafir, 2021). Both of these deficiencies represent part of the gap in the literature.

The remaining sections of this chapter will include the literature search strategy, a description of the conceptual framework, and an exhaustive review of the literature. The literature review strategy will include a listing of all utilized data bases and search engines, a delineation of key search terms, and an overall description of the search process. I will afterwards describe the conceptual framework. Here, I will identify the central phenomenon and thereafter include primary writings by significant theorists. This research demonstrated how the current study benefits from the past articulations found in seminal writings. The definition segment will unveil the meaning of salient terms that are apposite to the phenomenon. In the review itself, I examined over 75 current and seminal scholarly articles that are related to my constructs of interest, research questions, and selected methodology. Current literature and seminal studies were used to justify the rationale for the method. This review summarized the studies, identified common themes, and described how previous researchers have approached the concepts.

Literature Search Strategy

The following databases were searched to identify the literature for this study: Educational Resources Information Center (ERIC), Education Source, Academic Search Complete, Sage Journals, Dissertations & Theses @ Walden University, ProQuest Education Database, and Taylor and Francis Online. Search engines were confined to Google and I discovered that the database Google Scholar is an effective tool for Citation Chaining. To identify the relevant literature, I used the following search terms either singularly or in combinations: *SCL, learner-centered education, teacher perceptions, mathematics, elementary education, academic achievement, teacher-centered instruction, TCL, international or foreign countries, Shanghai education, Shanghai teachers, Shanghai teachers' PD, Shanghai preservice training of teachers, Singapore: Preservice training of teachers, Singapore education, Singapore teachers, Singapore teachers' PD, Singapore: Quality of teacher-candidates, United States: Quality of teacher-candidates, PD in the United States, preservice training of teachers in the United States, fourth-grade* *math, mathematics education, SLT, Vygotsky, math instruction, teacher instruction, qualitative studies, case studies, and mathematical literacy.*

Conceptual Framework

I utilized Vygotsky's SLT to view the phenomenon of this dissertation study (Hardman, 2021; Vygotsky, 1978). Vygotsky's theory asseverated that social interaction, mediated through language, served as the basis of all learning and development. As alleged by Vygotskian perspectives, enhanced cognition and increased development are the final outcomes of interactive learning (Heward et al., 2021; Nguyen, 2017). Vygotsky also professed that learning is a process of apprenticeship and internalization in which skills and knowledge are transformed from the social strata into the cognitive plane (Barohny, 2019; Nguyen, 2017; Shabani, 2016). Related to this, Vygotsky observed that intellectual development is socially acquired and postulated the idea that learners absorb, synthesize, and develop the practices, attitudes, and expressed thoughts of those in their social milieu (Hardman, 2021; Vygotsky, 1978). Vygotsky's theory, which professed that students actively construct knowledge in a collaborative learning environment, is foundational to constructivism. Constructivism, in turn, articulates the importance or prior knowledge, authentic tasks, and the building of knowledge in a social setting (Krahenbuhl, 2016; Matthews, 2020; Pie-Ling Tan et al., 2017). Therefore, constructivism informs many other facets of SCL including experiential learning, collaboration, hands-on or heuristic activities, scaffolding, and the role of teacher as facilitator of learning (Albanese-Benevento, 2016; Hardman, 2021; Morcom, 2014; Nguyen, 2017; Vygotsky, 1978).

As one might deduce, the above-mentioned commonalities demonstrate a nexus between SLT, constructivism and SCL. Ergo, SLT is an appropriate conceptual lens since the student-centered approach is the strategy of choice for many math teachers in the United States (Krahenbuhl, 2016; Matthews, 2020; Pie-Ling Tan et al., 2017).

Although Vygotsky lived and worked in the early 20th century, his philosophy of learning remains relevant to teaching and learning in the new millennium (Balakrishnan & Naraez, 2016; Hewerd et al, 2021; Pie-Ling Tan et al., 2017). Vygotsky embraced the idea that social learning antecedes mental development and is the key determinative for the optimal functioning of the child's cognitive processes (Hardman, 2021; Nguyen, 2017). Albanese-Benevento (2016), Lasmawan and Budiarta (2020), and Pie-Ling Tan et al. (2017) further explained Vygotsky's theory. According to these researchers, as students develop, they progress through novel learning tasks supported by scaffolding. Parenthetically, scaffolding entails the following set of actions. As the teacher introduces new concepts to the pupils, they support those who need assistance. This aid will be provided until students are able to perform the tasks independently (Shvarts & Bakker, 2019).

Other aspects of Vygotsky's philosophy as well as student-centered practices postulate that children, as social creatures, are curious, active learners who are intuitively involved in their own knowledge construction (Barohny, 2019; Nguyen, 2017). McLeod

(2014), Baeten et al. (2016), Lasmawan and Budiarta (2020), and Nguyen (2017) found that students progress from the social and cognitive contributions made by teachers and more advanced peers. Shabani (2016) and (Ediger (2018) maintained that SLT is a process mediated by symbolic tools such as language. The concept of mediation suggests that human relations with the world are not direct but rather interceded by symbolic tools. The use of language among individuals in a synergistic environment is, for that reason, central to the learning process (Hardman, 2021; Vygotsky, 1978). Language is also fundamental to student-centered instruction. Through the interposition of language, which supports dialogic negotiation and interaction among learners, a higher form of mental functioning is created. This elevated state includes more advanced linguistic and communicative skills, enhanced problem solving, and improved memory schemas (Hardman, 2021; Vygotsky, 1978). The resulting new knowledge is then internalized, stored in memory, and can be retrieved in subsequent, varying situations (Lasmawan & Budiarta, 2020; Vygotsky, 1978). Under those circumstances, the advanced knowledge becomes part of the students' repertoire (Lasmawan & Budiarta, 2020; Vygotsky, 1978). Proponents of SLT educed that without these social interactions, the cognitive growth of a child may be limited (Butler, 2020; Churcher et al., 2014; Nguyen, 2017; Shabani, 2016).

Barohny (2019) and Vygotsky (1978) postulated that it is crucial to distinguish between knowledge and learning. As attested to by social constructivists, knowledge is co-constructed in a social, collaborative environment (Altaftazani et al., 2020; Barohny, 2019; Vygotsky, 1978). Learning occurs when this knowledge is internalized (Heward et al, 2020; Vygotsky, 1978). Learning, as a consequence, materializes at the individual level and is a product of the interiorizing of information. Hence, the internalization of information, or learning, is regarded as both an individual and social process (Hardman, 2021; Vygotsky, 1978).

Furthermore, in these collaborative environments, which may also describe facets of a student-centered classroom, active learners link prior learning to the current task and acquire new skills and knowledge through communicative interaction with their peers and a knowledgeable other (teacher, mentor, or tutor) (Heward et al, 2020; Nguyen, 2017; Vygotsky, 1978). In light of this accentuation, adults are viewed as an important source of cognitive development for children. As a result, enhanced critical thinking skills may be inclulcated through these social engagements as knowledge is shared and analyzed, diverse viewpoints are expressed, conflicts are resolved, consensus is built through the evaluation of ideas, creativity and self-efficacy are ignited, and new knowledge is synthesized into existing schemas (Barohny, 2019; Butler, 2020; Kang & Liem, 2017; Mathews, 2020; Pie-Ling Tan, Choo). This learning culture also helps to cultivate many 21st century skills currently promulgated through student-centered instruction (Mathews, 2020; Pie-Ling Tan et al., 2017). These skills include the ability to frame, explore, and solve problems. These proficiencies also subsume heightened capabilities in collaboration, communication, critical thinking, and creativity (Butler, 2020; Dole et al., 2016; Pie-Ling Tan et al., 2017).

The ZPD is another tenet of Vygotsky's theory of social learning (Lasmawan & Budiarta, 2020; Vygotsky, 1978). Zones of proximal development are the distances between students' ability to complete a task with adult or peer scaffolding and their aptitude to perform the work independently (Barohny, 2019; Vygotsky, 1978). According to Vygotsky, ZPD is the primary space in which learning occurs (Lasmawan & Budiarta, 2020; Shabani, 2016). Advocates of SLT claim that a student's maximum amount of learning occurs in the ZPD, at a level exclusive to each child (Barohny, 2019; Churcher et al., 2014; Vygotsky, 1978). Zones of proximal development also inform the practices of instructional differentiation and readiness which are associated with student-centered classrooms (Barohny, 2019; Geelan et al., 2015). Differentiated instruction theory can be described as an adaptive practice in which teachers modify the curriculum, instructional methods, resources, learning activities, and student products to address the various needs of students and maximize student achievement. (Deunk et al., 2018). In addition, differentiation practices encourage teachers to evaluate the needs of each student based on ability, interest, readiness, and learning preferences, and to subsequently provide high quality instruction grounded in those prerequisites (Barohny, 2019; Geelan et al., 2015).

The word "proximal" in ZPD indicates the closeness and readiness of the student to achieve a cognitive progression (Lasmawan & Budiarta, 2020; Vygotsky, 1978). Therefore, ZPD represents a timeframe where knowledge, while still in the nascent stage, is on the cusp of advancement. Thusly, Vygotsky alternately defined learning as evolving from the ZPD to the "zone of actual development" (ZAD). Vygotsky (1988) and

Lasmawan and Budiarta (2020) attested that a student who currently needed adult support to complete a task should be capable of mastering it independently at a later date. Ineluctably, effective instruction anticipates future growth in a “zone of actual development.” The teacher, for this reason, guides and instructs the student by focusing not only on the child’s current level of knowledge and readiness but also on their potential advancement (Hardman, 2021; Vygotsky, 1988). The instructor, axiomatically, concentrates on what the child may possibly achieve independently (Heward et al, 2021; Vygotsky, 1978). In accord with Vygotskian perspectives, enhanced cognition is the final product of the above-mentioned series of socialized learning interactions undertaken through successive zones (Lasmawan & Budiarta, 2020; Nguyen, 2017).

Vygotsky’s theoretical framework has been an important catalyst for shifting the focus from teacher-centered DI to student-centered instruction (Dole et al., 2016; Hardman, 2021; Pie-Ling Tan et al., 2017). Vygotsky’s theory has many implications for contemporary educational paradigms. As a result, my study accrued many benefits through utilizing Vygotsky’s SLT as a conceptual lens. Social learning theory fostered a deeper understanding of the reasoning that sustains the student-centered based instructional practices of teachers in the United States. As a result, I was able to better address the study’s problem. The social learning conceptual lens also informed the purpose of the study by contributing essential knowledge concerning the teaching practices of many of the participants. Vygotsky’s philosophy framed the research questions as well. These requisite queries explored the strengths and weaknesses of SLT as it was implemented in student-centered classrooms. Furthermore, challenges inherent in SLT and learner-centered approaches, such as classroom and time management as well as ambiguity and student focus, provided the rationale for dissecting the postulates and assets of TCL. Congruently, TCL approaches were correlative to current instructional practices in many Asian schools (Kumar, 2016; Serin, 2018). The conceptual framework also guided the collection of data and its subsequent analysis by generating appropriate interview questions relevant to student-centered teaching practices. The data were then used to gauge the attending value of teacher-centered instruction.

Social learning theory has often been applied and articulated in prior studies. Ediger (2018) showed how differentiated instruction increased students' math performance in inner-city schools. The key, according to Ediger (2018), was linking teacher instruction to the students' ability, learning preferences, and readiness levels. Inquiry-based math activities also instance Vygotsky's theory (Maker & Fielding-Wells, 2018). The authors determined that these pedagogical practices have roots in social learning as prescribed by Vygotsky. The authors explained that inquiry learning involves a process where students communicate and collaborate in small groups as they create research questions, cooperatively conduct ensuing explorations, collect and analyze data, and derive possible solutions. The teacher, in this environment, acts as a facilitator over long-range projects. With the help of teacher guidance, students work together as they concentrate on real-life problems, identify resources, conduct research, reflect on their progress, and discover solutions for authentic audiences (Maker & Fielding-Wells, 2018).

Literature Review Related to Key Concepts

International Test Results

The problem of this study, identified in current research literature, and supported by recent standardized tests results, was that fourth-grade math students in the United States are not performing as well on standardized tests as students of other nations and international cities (Pew Research Center, 2018; TIMSS, 2019; Woessmann, 2016). To compound the problem a large percentage of the nation’s fourth-grade math students have consistently failed to reach proficient levels in national tests in the last ten years (The Nation’s Report Card, 2019)**.** The problem of the study is in strong alignment with the purpose (to develop thick rich descriptions of the perceptions of an international array of 12 teachers concerning effective 4th grade math instruction, inherent obstacles to sound instruction, college training, and PD). The problem was also intertwined with the qualitative exploratory case study methodology (which featured in-depth interviews), the research questions (which served to elicit information about instructional practices as well as other determinant factors), and the conceptual lens of SLT (which strongly influences math instructional practices in the United States).

Student achievement, as measured by standardized tests, may be considered as an indicator of the quality of education (Eriksson et al., 2019; Karakoc et al., 2016). A review of recent test results found in TIMSS provided the propelling investigative force for this qualitative exploratory case study (TIMSS & PIRLS, 2019). TIMSS is a global assessment of international student achievement in fourth- and eighth-grade mathematics and science (Helenius & Ryve, 2019; Hsiang, 2016). The assessments have been conducted in four-year cycles since 1995 and provide achievement results for a myriad of participating countries, regions, and cities (Chen, 2014; Eriksson, Helenius & Ryve, 2019). TIMSS uses a rigorous sample and assessment framework to collect educational information at the student and teacher levels (Chen, 2014; Eriksson et al., 2019). TIMSS also uses similar sampling designs in all nations, cities, and regions (TIMSS & PIRLS, 2019). Student-level factors include socioeconomic components, levels of home environmental support, students' perceptions of school experiences, as well as learner motivation (TIMSS & PIRLS, 2019). The teacher-level elements encompass instructor preparation and instructional methods (Chen, 2014; Eriksson et al., 2019; Hanushek et al., 2014). The significant information in the TIMSS database has allowed researchers to conduct national or global comparative studies of educational contexts and, more specifically, to examine constituents related to student performance in mathematics (Helenius & Ryve, 2019; Woessmann, 2016). Researchers may utilize these disaggregated data to explore the connection between student success in mathematics and classroom instructional strategies (Helenius & Ryve, 2019; Woessmann, 2016). The success of the Singapore and Shanghai models, as reflected by the results of these standardized tests, and the difference in instructional approaches between these two entities and those used in the United States, have raised compelling questions. I intend to provide answers to these questions when I interpret the findings of the study and offer recommendations in Chapter Five.

The purpose of this qualitative exploratory case study was to develop and analyze thick rich descriptions of an international array of teacher perceptions about different approaches to effective fourth-grade grade math instruction, inherent obstacles to sound instruction, their college training, and PD. For this reason, I used the Literature Review to create a narrative that explored such instructional approaches as student-centered practices (prevalent in American schools) and teacher-centered practices (pervasive in Asian schools) (Darling-Hammond, 2017; DeSouza, 2018). I also perused and incorporated peer-reviewed scholarly articles that attended to the research questions and identified other determinant influences that may impact the efficacy of instructional practices. Later in the section, I will discuss the similarities and differences between the research findings as well as address controversial aspects of the studies. These emotive issues include the caliber of teacher applicants in the United States, specialization among math teachers, and the efficacy of ability groupings. I will also highlight any pertinent issues that have remained obscure.

It is noteworthy to draw attention to the viable connection between pedagogic research and TIMSS-related data. As background, and in the context of this study, Singaporean and Shanghainese fourth-grade students have consistently outperformed their United States' peers according to the TIMSS and Program for International Student Assessment (PISA) test results (Pew Research Center, 2018; Sun-keung Pang & Zhuang, 2017; TIMSS, 2019; TIMSS & PIRLS, 2019; Woessmann, 2016). Singapore is an economically-developed country whose students generally effectuate high achievement on international tests of mathematical competencies (Furhan, 2016; Lee, 2017; Tan & Ng, 2018). Data from 2019 international test results revealed that the percentage of Singaporean students reaching ascendant benchmarks in TIMSS fourth- and eighth-grade tests were far greater than their American counterparts (TIMSS & PIRLS, 2019). Incidentally, Shanghai students did not participate in these tests. Singapore's progress was particularly distinguished between 2003 and 2019. For example, in 2003 38% of Singapore students were classified as attaining advanced stratus on tests compared to 7% of American students. In 2019 the percentage of Singapore students attaining advanced status had risen to 54% while only 14% of United States' students reached that level (TIMSS & PIRLS, 2019). Parenthetically, according to TIMSS and PIRLS (2019), advanced fourth-grade students can apply their understanding and knowledge when undertaking various relatively complex problems and explain their reasoning. Students can also solve a variety of multistep word problems which incorporate whole numbers. Students must demonstrate an understanding of fractions and decimals as well. They must also show that they can apply understanding of two- and three-dimensional shapes in various situations and represent data to solve multistep problems.

Shanghai students, during recent years, have also ranked near the top in mathematics on international assessments during recent years (OECD, 2019; Yao et al., 2018). The 2018 PISA results showed that Shanghai students helped to propel China to the top of the PISA mathematical scores (OECD, 2019). Singapore held second position, while the United States finished 25th. The United States has maintained this relatively low level of math achievement since the tests' inception in 1967 (OECD, 2019; Sun-keung Pang & Zhuang, 2017).

Mathematics Instruction in Shanghai, Singapore, and the United States

Teaching is one of the cardinal practices related to student achievement (Darling-Hammond, 2017; Mathews, 2020). By extension, students' achievement will directly affect societies' level of education (Tonga et al., 2019). Singapore's educational cognoscenti have therefore striven to attract the most academically qualified candidates to the field (Darling-Hammond, 2021; Zhang et al., 2017). Moreover, Singapore and Shanghai have centralized, cohesive educational systems that feature a common curriculum (Darling-Hammond, 2021). The educational landscape of the United States, however, remains largely fragmented despite the efforts of Common Core and the NCTM to establish a congruous curriculum linked to robust learning standards (Asempapa et al., 2017; Hamlin & Peterson, 2018). Moreover, school leaders from various states in America have recently retreated from their prior commitments to adopt stringent Common Core Standards (Hamlin & Peterson, 2018). As a result, American educational leaders have failed to commit to a high-quality national curriculum for all fifty states. This omission has obviated the opportunity for students, in some states, to acquire a high quality education (Hamlin & Peterson, 2018; Shoenfeld, 2007). This foundering has also allowed many of the nation's school districts to pursue locally derived agendas. Often, in these local districts, there is wide variance in the caliber and rigor of instruction (Hamlin & Peterson, 2018). To add to this dilemma, many school districts in the United States camouflage this lack of rigor by lowering benchmarks for academic proficiency (Hamlin & Peterson, 2018).

A succinct review of the Singapore model of teaching mathematics was especially instructive in understanding one of the reasons for its excellent results. Singapore's national mathematics framework can be instantiated in a pentagonal design with mathematical problem-solving at the center (Jaciw et al., 2016; TIMSS & PIRLS, 2019). The five sides of the pentagon consist of attitudes (beliefs, interests, confidence, and perseverance), metacognition (monitoring one's own thinking), procedures, numerical concepts, and skills (numerical calculations) (Jaciw et al., 2016; TIMSS & PIRLS, 2019). This quintet of functions in tandem to support problem-solving. This framework is buttressed by textbook density as well as in-depth, whole class explorations of the anchor problems (Boyd & Ash, 2018; Ginsburg et al., 2005; Yang et al., 2017). Further, the Singapore approach to math instruction encompasses visualization and model drawing strategies, the aforementioned problem-solving skills, and promotes deep understanding of mathematical concepts as well as procedural knowledge (Jerrima & Vignoles, 2016; Purwadi et al., 2019; Yang et al., 2017). According to the Singaporean approach, value is placed on working through a lesson or concept in a sequential manner until students attain mastery (Hardman, 2021). Engelmann (2014) and Hardman (2021) also believed that mastery is crucial if the student is to sustain the new learning, apply it in different contexts, and store it in his or her cognitive repertoires for future application.

At the epicenter of the Singaporean model is the Concrete to Pictorial to Abstract (CPA) approach to instruction (Abdoulaye, 2020; Jerrima & Vignoles, 2016). With the CPA technique, learning is gradual and sequential. Students, under CPA guidance, initially use hands-on manipulatives. Afterwards, they learn to visually represent math concepts through pictorials (Abdoulaye, 2020; Ginsburg et al., 2005; Jerrima & Vignoles, 2016). These initial activities reflect an Aristotelian approach to learning which emphasized that mathematical concepts are developed through logical reasoning and empirical investigations (Yeping. & Schoenfeld, 2019). On this account, concrete experiences prepare students to later apply abstract symbols such as numbers (Abdoulaye, 2020; Ginsburg et al., 2005; Jerrima & Vignoles, 2016). This process also affords learners the opportunity to create their own generalizations about the concept being learned, as opposed to memorizing seemingly discrete facts in isolation (Abdoulaye, 2020; Jerrima & Vignoles, 2016). By gradually transitioning from objects to pictures to symbols, CPA provides variegated circumstances for the pupils to learn basic concepts. This incremental approach may particularly benefit students who struggle with mathematics (Purwadi et al., 2019). The CPA strategy also advances the teaching of problem-solving strategies (Purwadi et al., 2019; Rittle-Johnson & Schneider, 2015). During the problem-solving process, consecutive applications of activities, ranging from hands-on to pictorial to abstract delineations, can offer an array of different problem-solving approaches (Purwadi et al., 2019; Rittle-Johnson & Schneider, 2015).

DeJarnette (2018) proffered the axiomatic premise that mathematics knowledge and skills are essential to math- and technology-based societies such as those found in the United States, Singapore, and Shanghai. The TIMSS and PIRLS (2019) results showed that Singaporean students had an appreciably higher mean achievement score in math than their counterparts in the United States. PISA results from 2018 showed that 98% of Shanghai students attained Level 2 or higher in mathematics compared to the OECD average of 76% (OECD, 2019).

Studies have demonstrated that self-confidence is one of the most salient trait related to math success (Ciftci & Yildiz, 2019). Other studies have presented similar results regarding student self-efficacy. These investigations disclosed that students' mathematics achievement was related to students' self-concepts about their math abilities (Bandura, 1982; Ciftci & Yildiz, 2019; Cvencek et al., 2015; Furner, 2017; Mullis, Martin et al., 2004). These researchers purported that instruction should incorporate facets which increase students' confidence by also addressing readiness levels and implementing ongoing scaffolding (Matthews, 2020; Retnowati et al., 2017). Other determining factors for math achievement included student motivation and engagement (Ciftci & Yildiz, 2019; Hsiang-Wei, 2016). To enhance motivation, teachers should create lessons around authentic, relevant tasks which appeal to the student's personal interests (Caldor, 2015; Fredericks et al., 2017; Matthews, 2020). These lesson characteristics will motivate and engage learners as they engender positive responses in the task-at-hand (Caldor, 2015; Fredericks et al., 2017; Matthews, 2020; Singh et al., 2002).

Tan (2019) and Retna and Pak (2016) vitalized the idea that Singapore's educators are currently placing less emphasis on test preparation and evolving towards more balanced instructional approaches. On the authority of the authors, the reform movement often incorporated cooperative problem solving to inculcate critical thinking and creativity on the part of the students. Tan (2019) and Retna and Pak (2016) also espoused the idea that Singapore educators are evolving towards a "teach less, learn more" paradigm. The topical regnant reform movement, at this inchoate stage, is promoting engaged learning, collaborative problem solving, autonomy, and creativity on the part of the students. In concurrence with the educational reorientations, many Singaporean teachers have currently begun to incorporate critical creative thinking, global awareness, civic literacy, cross-cultural skills, and the cultivation of communication, collaboration, and information retrieval abilities (Tan, 2019).

Jensen et al. (2016) as well as Park et al. (2020) professed that elementary school teachers need a strong, coherent foundation in mathematics as well as pedagogical knowledge. However, in many initial teacher education programs in the United States, as well as in PD programs, these skills, including a deep understanding of the attending content, were not being taught (Jensen et al., 2016; Ren & Smith, 2018; Tuncel & Cobanoglu, 2018). Many American elementary school teachers, therefore, are not confident about their content knowledge nor their instructional prowess in mathematics

(Ren & Smith, 2018). This uncertainty negatively affects their delivery and may deleteriously influence the self-efficacy of many of their struggling students (Jensen et al., 2016; Ren & Smith, 2018). To compound the problem, the United States has also experienced a decline in the quality of candidates being accepted into teacher education programs due to low application requirements and the abundance of more lucrative professional opportunities (Alisov et al., 2020; Jensen et al., 2016). Correspondently, in the United States, education majors have historically registered lower SAT scores than students entering other domains (Alisov et al., 2020)

Specialization is one way to help teachers develop deep expertise in math. Unlike elementary teachers in the United States, who generally teach all subjects in the elementary grades, teachers in Shanghai and Singapore usually concentrate on one subject (Kuennen & Beam, 2020). As a result of this specialization, as well as their intensive preservice educational programs, elementary school teachers in Shanghai and Singapore are renowned for having strong subject expertise, particularly in mathematics (Fennell, 2018; Jensen et al., 2016; Kuennen & Beam, 2020).

Teacher-Centered Instructional Approaches in Shanghai and Singapore

The persistent success of East Asian students in international assessments has engendered great curiosity about their instructors' academic background and classroom practices (Dimmock & Tan, 2016; TIMSS & PIRLS, 2019). Teacher-centered instruction has been the traditional conduit for imparting knowledge in Singapore and Shanghai (DeSouza, 2018; Zhao et al., 2014; Serin, 2018). Teacher-centered instruction is distinguished from student-centered instruction by the degree of teacher control and the level of student participation in classroom activities (Yoonjeon, 2018). Serin (2018) contended that explicit instruction, as practiced in teacher-centered classrooms, places a high priority on teacher autonomy. The goal of teacher-directed instruction, thereupon, is for the instructor to don the mantle of authority and transmit information, skills, and concepts to students in a didactic manner (Serin, 2018). However, there is a degree of differentiated instruction in these classrooms as well, as the teacher develops a framed and controlled academic experience in consideration of the students' readiness levels, learning preferences, and work predilections (Yoonjeon, 2018). The resulting differentiation may be seen in scaffolding, intermittent group work, and mini lessons. Mastery learning, an essential component of the teacher-centered paradigm, includes structured practices, which allow students to progress at their own rate (Engelmann, 2014; Magliaro et al., 2005; Serin, 2018; Stockard et al., 2018). This calculated pace affords slower students the opportunity to acquire mastery of the concepts while higher performing students undertake enrichment activities (Engelmann, 2014; Magliaro et al., 2005; Serin, 2018; Stockard et al., 2018). Tasks selected for group work, in this learning culture, should be rich enough to elicit a variety of problem-solving strategies and sufficiently multifarious to offer entry points to students at different levels of understanding (Munter & Stein, 2015; Stockard et al., 2018). Further, as part of their advocacy for teaching for mastery, practitioners of DI revisit previously learned knowledge and skills in recursive cycles (Rosenshine, 1979; Stockard et al., 2018). In consonance with Engelmann (2014), teacher autonomy, an important facet of DI, also reduces the chances of students' misinterpreting ideas (Heward et al., 2021). This is largely due to the unambiguous nature of the instruction, as well as frequent questioning and feedback by the teacher (Stockard et al., 2018). To bolster this explicitness, the direct instructive approach promotes the use of clear, specific examples which are sequenced to encourage correct inferences from the students (Stockard et al., 2018).

Teacher-centered instruction, in Singapore and Shanghai, is often associated with strong math achievement by students (DeSouza, 2018). Studies have identified explicit, systematic math instruction as an effective instructional approach which supports the efforts of struggling students as well (Butler, 2020; National Mathematics Advisory Panel, 2008). According to Butler (2020), these practices often place less demand on attention, working memory, language, and general cognitive resources. Explicit instruction may also include the sequencing of problems, the highlighting of crucial aspects of each problem, and offering students multiple application opportunities abetted by teacher guidance (Butler, 2020; National Mathematics Advisory Panel, 2008).

The main purpose of math education in Singapore is to cultivate a balanced picture of mathematics (Heng & Lynn, 2021). This intent subsumes both classroom learning and real-world practicality. As a result, "Thinking Schools, Learning Nation," promoted a method of preparing citizens to meet future challenges with an education system designed to address the demands of the 21st century (MOE, 2019). In addition to mastering mathematical concepts and skills through hands-on activities, the Singapore and Shanghai mathematics curriculums also promote the understanding of mathematical processes (Heng & Lynn, 2021). Conspicuously, the dual purpose of combining the "why" with the "how" is supported by a well-educated and highly trained teaching staff in both Singapore and Shanghai (Tan, 2019). In this dual modality, which includes process and procedure, students use manipulatives to unveil the reasoning behind the formulas. Teachers then demonstrate, through modeling, both specific and various procedures for solving problems (Heng & Lynn, 2021). Afterwards, students are provided with repeated opportunities, accompanied by the teacher's feedback, to independently practice these methods with pictures and abstract symbols (Magliaro et al., 2005; Munter & Stein, 2015; Stein, Silbert, & Carnine, 2004; Wong, 2020). This process can be succinctly stated as modeling with reinforced guided performance (Joyce et al., 2000; Munter & Stein, 2015; Keiler, 2018).

Memorization, which is often associated with teacher-centered classrooms, is viewed by many Singaporean and Shanghainese educators as complementing deeper understanding (Yoonjeon, 2018). Many of these instructors believe that higher order thinking requires content knowledge recall to solve content-related problems (Greif et al., 2015; Hogue, 2017: Koh & Chong, 2014; Wu et al., 2020). These ideas are generally antithetical to the American notion that memorization and "direct-facts" approaches should be avoided in favor of applying reason-based strategies (Baroody et al., 2016; Butler, 2020). However, according to Wu et al. (2020) the memorization tool is not used by East Asian students in an exclusive fashion; rather, it is utilized in conjunction with elaboration and meta cognitive strategies. Finally, Darling-Hammond et al. (2017) and Butler (2020) reinforced the point of view that Asian teachers often cultivate critical thinking through explicit instruction. Blakey et al. (2014) and Adeleye (2021) defined critical thinking as the ability to reason logically while analyzing, synthesizing, and evaluating. To activate students' critical thinking, Asian teachers use such verbal and written prompts as "why," "how," and "what if." Instructors in Singapore and Shanghai supplement these approaches by modeling and conducting "think aloud" strategies where they explain the criteria for each reasonable procedural decision and welcome a plethora of student questions (Darling-Hammond et al., 2017; Serin, 2018).

The Ministry of Education of the People's Republic of China (MOE) has, for the past decade, promoted reform efforts to replace passive learning and rote-memorization with students' active participation, hands-on activities, critical thinking, and inquiry-based learning abetted by greater interaction and collaboration (Lee, 2017; Tan 2019; Tan, 2020, MOE, 2019). However, Shanghai's conception of knowledge acquisition, personal inquiry, and critical thinking is somewhat paradoxical (Tan, 2020). Traditional Chinese philosophy embraces an objectivist posture which contends that knowledge exists as an external reality (Tan, 2020; Ye & Cheng, 2017). Hence, the emphasis on independent thought under the current reform movement has not resulted in the production of new lines of inquiry by the student. Instead, the current conception of acquiring knowledge focuses on the reproduction of traditional knowledge (Tan, 2020). The students' responsibility, therefore, is not to challenge objective knowledge as presented in textbooks; rather it is to learn and apply objectified knowledge to solve real-life problems (Tan, 2020).

All Shanghai schools use the same math textbooks (Lee, 2017; Tan & Ng 2018) . Shanghai teachers are generally highly respected and tend to be authoritative (Lee, 2017; Tan & Ng 2018). Students are typically well-disciplined. Shanghai teachers can therefore employ whole-class teaching more effectively. Shanghai mathematics instruction is characterized by topics taught from a DI standpoint, progressing from basic to advanced levels through systematic exercises. The Shanghai mathematics syllabus is consequently linear and deductive (teacher-controlled) with fewer topics taught in greater depth (Lee, 2017; Tan & Ng 2018).

Still, both teacher-centered and student-centered methods exist simultaneously in Shanghai with teachers relying on the transmission of knowledge as they employ student engaged learning methods (Tan, 2020). However, the traditional influences are evident in an exam-oriented system. Despite the intermittent inclusion of open-ended questions in high-stakes standardized tests, the majority of exam questions are still closed-ended with objectively prescribed answers (Tan, 2020). Summarily, Shanghai, despite reform efforts, still relies on an exam meritocracy, didactic teaching, common textbooks, a non confrontational view of knowledge acquisition, as well as a hierarchical relationship between students and teachers (Tan, 2020).

Finally, Ning et al. (2016) and Tan (2019) disclosed that Shanghai's teachers and students enjoy a close relationship with one another. However, a competitive culture exists among students (Ning et al., 2016; Tan, 2019). The resulting contentiousness has created a great degree of academic pressure which may decrease students' self-efficacy and personal appreciation for school (Ning et al., 2016; Tan 2019).

Student-Centered Instructional Approaches

Transitioning from the traditional, didactic, and teacher-centered instructional approaches to SCL entails redefining both the teachers' and students' responsibilities (Burns et al. 2014; Serin, 2018). This transformation also causes a metamorphosis in the teacher and student relationship and alters the instructors' organizational, instructional, and assessment practices (Burns et al., 2014; Eriksson et al., 2019). Social learning theory, and the notion that language is the mediator in mathematics knowledge building, has strongly influenced student-centered instruction (Hardman, 2021; Sibanda, 2017; Vygotsky, 1978). Student-centered instruction also promotes the belief that each student is unique (Gardner, 1983; Jacobs & Renandya, 2019; Manić & Ranđelović, 2017; Piaget, 1958; Vygotsky, 1978). Similarly, every learner has distinct methods of absorbing and processing information, interacting with resources, and constructing knowledge (Burns et al., 2014; Hardman, 2021; Manić & Ranđelović, 2017). Gardner's theory of multiple intelligences further purported that the cognitive process of constructing knowledge is discrete and idiosyncratic for each learner (Gardner, 1983; Manić & Ranđelović, 2017; Rotnitsky & Yavich, 2020). Gardner, in his multiple intelligence theory, identified eight distinct types of intelligences (Bordei, 2017; Rotnitsky & Yavich, 2020). These are: Linguistic, Logical-Mathematical, Spatial, Bodily-Kinesthetic, Musical, Interpersonal, Intrapersonal, and Naturalistic (the ability to distinguish among different types of plants, animals, minerals, and meteorological formations found in the natural world). Appositely, according to this theory, students may display disparate learning styles, aptitudes, and intelligences (Rotnitsky & Yavich, 2020). Consequently, many student-centered proponents contend that the effect of students' individual cognitive characteristics, as recognized in the multiple intelligence theory, has important ramifications for the learning process (Ahvan & Pour, 2015; Aydemir & Karalib, 2014; Kartikasari & Widjajanti, 2016; Rotnitsky & Yavich, 2020).

Advocates of SCL further argued that students possess unique working styles such as learning independently or interacting collaboratively in either small groups or in pairs (Ediger, 2018). The awareness of these divergences in learning and working styles has vitalized the idea that instruction must be differentiated (Burns et al., 2014; Ediger, 2018). Differentiated instruction may entail the following actions to account for the heterogeneous proclivities of students. The instructor may place students who are prone to making specific types of mathematical errors in small groups composed of four or five individuals (Ediger, 2018). Another cluster may be comprised of gifted learners who require more challenging work. Pupils who require more background information or assistance staying on task can form a third assemblage. Students who wish to pursue individual experiences can choose to work independently (Ediger, 2018).

Student-centered activities furnish students with opportunities to be actively involved in the process of generating mathematical knowledge through problem solving and inquiry-based learning (Ediger, 2018; Lattimer, 2015). Formative assessments and self- and peer assessments are a crucial part of this learning process. These evaluations help teachers not only monitor student progress but help educators individualize instruction to meet current student needs (Eronen & Karna, 2017; Lattimer, 2015; Serin, 2018). Blumberg's (2016) study, as well as research by Jacobs and Renandya (2019), reported that instructors, in student-centered classrooms, required pupils to take responsibility for learning. As a result, students become designers of their own learning as they consult a variety of content and contexts (Adeleye, 2021; Lee & Hannafin, 2016).

However, according to my review of current literature, one aspect of SCL remains controversial. Proponents of learning styles theory (LST) contend that students have inborn preferences for learning such as using an auditory, visual, or kinesthetic mode. According to this theory, these preferences predispose students to experience enhanced learning if information is presented in their idiosyncratic style (Antoniuk, 2020; Brown & Kaminski, 2018; Furey, 2020). However, many cognitive scientists and neurosurgeons have found LST to be otiose (Antoniuk, 2020; Brown & Kaminski, 2018; Furey, 2020; Wammes & Jonker et al., 2019). Researchers cite a lack of empirical evidence supporting LST, as well as identifying diagnostic difficulties and other confounding factors (Antoniuk, 2020; Brown & Kaminski, 2018). Furthermore, scientific explorations of memory issues have identified contraindications to LST. Studies have found that memory is improved by adding information from a variety of modalities rather than focusing on a single source (Wammes & Jonker et al., 2019).

However, in the academic community, there continues to be widespread misunderstanding concerning the importance of learning styles. To elucidate, 67% of teacher-preparation programs in the United States required students to incorporate learning styles into lesson-planning assignments and 59 percent of textbooks supported this idea (Furey, 2020).

Operationalizing the practices inherent in SCL classrooms finds students discussing the problems in small groups, sharing a number of strategies in solving the problem, and reflecting on their efforts (Butler, 2020; Leon & Castro, 2017). During this process, students also compare and contrast strategies, resolve conflicts, negotiate agreements, synthesize new ideas, and reach a consensus (Butler, 2020; Leon & Castro, 2017). These series of student interactions once more support the proposition that learning is a socially and linguistically arbitrated construct (Amineh & Asl, 2015; Bell & Kozlowski, 2008; Mathews, 2020; Vygotsky, 1978).

Problem solving is an essential characteristic in the student-centered instructional culture (Dolmans et al., 2016, 2016; Jacobs & Renandya 2019; Toh et al., 2014). The NCTM, in the United States, averred that problem solving should be one of the central foci of the mathematics curriculum (Finken, 2016; Hobri & Naja, 2018; Toh et al., 2014). Dewey also proposed problem solving as a model for thinking in the field of mathematics (Dewey, 1916; Nardo, 2018; Nurdyansyah & Bachtiar, 2017). Polya (1945) contributed to this strand by developing a multi-step process for solving math problems (Tohir & Hobri, 2018). Polya’s strategy helps students develop effective problem-solving approaches when confronted with novel examples (Irvine, 2018; Toh et al., 2014). The four stages, depicted by Polya, involve the following. The first step is to understand the non routine problem. Next, the student should devise a plan, carry out the plan, and lastly reflect on his or her efforts (Irvine, 2018; Polya, 1945).

Hummell (2017) and Butler (2020) highlighted the importance of SCL assets in the cultivation of math expertise. These advantages may include collaborative problem solving, formative assessments, and personalized learning as well as implementing DI at appropriate intervals. In many interactive SCL milieux, computation is first taught through DI (Butler, 2020; Polly et al., 2014; Rittle-Johnson & Schneider, 2015). Problem-solving strategies may then be activated among groups of students to solve computational problems (Jacobs & Renandya, 2019; Polly et al., 2014; Rittle-Johnson & Schneider, 2015). Certain problem-solving strategies, such as using manipulatives or drawings to comprehend the inherent concepts, allow students the opportunity to first acquire a more complete understanding of the essential ideas before moving on to traditional algorithms (Mathews, 2020).

Student-centered practices also subordinate, to a lesser level, the importance of procedural fluency (Chen et al., 2018; Leon & Castro, 2017). Instead, SCL approaches place a greater emphasis on understanding the underlying mathematical content and concepts (Chen et al., 2018). Student-centered instruction also highlights group reflections on the completed activity (Yoonjeon, 2018). Complex problems are well-represented in SCL activities (Yoonjeon, 2018). Parenthetically, complex problems are defined as those problems, based on real-life situations, which lack an obvious solution (Yoonjeon, 2018). Complex instruction involves mathematical tasks that require students to reactivate previously learned concepts, as well as exercise critical thinking and reasoning when applying concepts in a novel context. This process is diametrically opposed to procedural instruction which involves tasks that entail memorized or routine methods (Yoonjeon, 2018). Complex instruction should, in turn, help students realize the usefulness of particular mathematical skills and knowledge as they apply these to real-life situations. Complex instruction also helps students discern the contexts in which these understandings are acquired, as well as fathom how this new knowledge relates to other, previously-learned information (National Research Council, 2001; Yoonjeon, 2018). Complex instruction should, as well, result in greater skill retention and fewer problem-solving errors, and, over time, increase students' mathematics achievement (National Research Council, 2001; Yoonjeon, 2018). Likewise, another SCL facet, the opportunity to communicate and reflect upon mathematical understanding may strengthen children's metacognitive reasoning (Chen et al, 2018; Munter & Stein, 2015; Weimer, 2013). Further, students may also benefit from student-centered instructional practices due to the locus of internal control, as well as the greater organizational, social, verbal, and task demands inherent in this approach (Chen et al, 2018; Lee & Hannafin, 2016).

Nonetheless, SCL does present challenges. Hui-Chuan and Styliandes (2018) cited studies which showed student-centered classrooms, involving many pupils, led to unequal distribution of effective instruction. There are other challenges immanent in student-centered classrooms. These points of contention include difficulties in classroom management, extended timeframes for effective collaborative learning, and the non alignment between student understanding and standardized assessments (Buchs et al., 2017; Luitel & Pant, 2019; Polly et al., 2014).

Collaborative learning, as previously noted, plays an indispensable role in the student-centered instructional modality (Eronen & Karna, 2018; Smajic et al., 2014). Math instruction in many American classrooms is appurtenant to collaboration. Collaborative learning alludes to the cooperative interactions among a constellation of learners and teachers (Eronen & Karna, 2018). Collaborative learning is reported to be an effective teaching method that can promote student interactions in small groups to achieve a common goal (Chen & Kuo, 2019). Concurrent with SLC practices, students cooperatively establish self-direction in making decisions regarding their own learning (Eronen & Karna, 2018). Eronen and Karna (2018) purported that the collaborative sharing of knowledge, as highlighted in the constructivist paradigm, has implications for teaching and learning mathematics. When learning material is apportioned among group members, each student's working memory processes less information. Hence, the information load is reduced for every pupil (Mathews, 2020; Retnowati et al., 2017).

Collaborative learning invariably involves small groups or teams of four or five students (Leon & Castro, 2017: Serin, 2018). Each team member should be assigned a specific role and is responsible for a certain segment of the task. Reflection, a requisite activity associated with collaborative exercises, is considered crucial to deep understanding and meta-cognition (Leon & Castro, 2017; Serin 2018; Weimer, 2013). Parenthetically, during reflection students focus on their own thinking processes and attempt to understand how they learn while cultivating an awareness of their particular strengths and weaknesses (Erdogan, 2018).

Other best practices associated with these synergetic classrooms include offering pupils the opportunity to use critical thinking skills to solve real-world problem, encouraging students to utilize their learning preferences in performing tasks and creating deliverables, and utilizing multi-sensory, integrative, and interdisciplinary modalities to improve their transfer skills (Adeleye, 2021; Leon & Castro, 2017; Wiles & Bondi, 2014). All these affordances will help promote students' future success in the 21st century workplace as well (Dilek et al., 2016; Mathews, 2020). In addition, studies have indicated that collaborative learning has a salutary effect on cognition, motivation, as well as intrapersonal and interpersonal goals (Mathews, 2020). Chen et al. (2018) and Hsiao et al. (2014) further asservated that successful collaborative learning consists of the following elements: small group formulations, interactions among pupils, group processing, and individual responsibility. Each member of a communal learning group must depend on and help each other and should assume responsibility for success or failure. As a result, cooperative learning benefits students in terms of achievement, motivation, and social skills (Chen & Kuo, 2019).

Embedded assessments are also part of the collaborative communal experience (Blanco et al., 2015; Burns et al., 2014; Grasser et al., 2017; Kaput, 2018). Here, students demonstrate their learning through a variety of means and activities that not only match their learning preferences but also enhance their real-world, 21st century skills (Kaput, 2018). In addition to written and oral tests, these assessments may also take the form of student portfolios and performance exhibitions (Kaput, 2018).

Other Determinant Influences that Affect Students' Math Achievement

Other determinant elements that may influence student achievement in mathematics involve teachers' preservice education and their subsequent PD (Ölçü Dinçer & Seferoğlu, 2018; Rodriguez-Lopez et al., 2019; Van den Bergh & Ros, 2014). The quality of a teacher's preservice education has been found to be a critical factor in teaching and learning (Maphoso & Mahlo, 2015; Rodriguez-Lopez et al, 2019). Studies by Maphoso and Mahlo (2015) and Rodriguez-Lopez et al (2019) evidenced that a teacher's content and pedagogical knowledge, as well as understanding the role of assessments, are associated with higher student test performance. Other studies have shown that students' mathematics achievement is directly related to the teachers' level of content knowledge (Buddin & Zamarro, 2009; Newsome et al., 2019).

Professional development has been described as playing a crucial role in elevating teachers' knowledge and skills (Ölçü Dinçer & Seferoğlu, 2018; Van den Bergh & Ros, 2014). Previous research has averred that the ongoing training of educators often results in an increase in theoretical knowledge, enhanced classroom applications, and the accretion of shared ideas (Ölçü Dinçer & Seferoğlu, 2018). The newly acquired PD-derived expertise may also activate, in teachers, a deeper conceptualization of content knowledge, as well as an enhanced pedagogical understanding (Rodriguez-Lopez et al., 2019; Wang et al., 2014).

Teacher quality, derived from preservice education and sustained through ongoing PD, plays an important role, not only in the instruction of students, but also in supporting the United States' global ascendency (Newsome et al., 2019, DeJarnette, 2019). Proximately, the following syllogistic reasoning may be implied from the research of DeJarnette (2019), presented in her article attesting to the importance of Science, Technology, Engineering, Arts, and Mathematics (STEAM-based education). The development of our society is dependent upon skilled STEAM-derived skills and knowledge. This expertise is possible only through quality education, which, in turn, is dependent on highly qualified teachers.

Teaching

Darling-Hammond (2021) as well as Çer and Solak (2018) postulated that teaching is one of the preeminent school-related factors related to student achievement. Congruently, ongoing PD and preservice education has been shown to exert a direct effect on the quality and capacity of the teaching staff (Qian & Walker, 2022; Tonga et al., 2019). Furthermore, Darling-Hammond (2011), as cited in Zhang & Zheng, 2020), indicated that schools which provide teachers with professional learning opportunities are capable of continuously enhancing instructors' teaching practices that promote student learning.

In particular, since educational-related knowledge is a constantly evolving and expanding dynamic, effective PD is climacteric in enhancing teacher capacity (Bentley & Cason, 2019; Matherson & Windle, 2017). To be effective, PD learning opportunities must be intensive, ongoing, and connected to practice (Huijboom et al., 2021; Matherson & Windle, 2017). PD should also be interactive, engaging, and relevant (Matherson & Windle, 2017; Nelson & Bohanon, 2019). The most useful PD emphasizes active participation and a hands-on experience rather than abstract discussions (Jiang et al., 2018). Suitably, PD is more efficacious when it is explicitly tied to classroom lessons and contains appropriate andragogical principles (Desimone & Garet, 2015; Tonga, et al., 2019).

Recent research further suggested that a high-quality teaching staff and an ongoing teacher training system energized Singapore and Shanghai's educational achievement (Çer & Solak, 2018; Goss & Sonnemann, 2020; NCEEa, 2021; NCEEb, 2021; Tonga et al., 2019; Zhang et al., 2017). Professional learning communities in both Singapore and Shanghai focus on student learning and outcomes through improvements in teaching (Tonga et al., 2019; Zhang et al., 2017). Shanghai's PD, in particular, emphasizes improving collaboration with parents (NCTM, 2022). Professional learning communities in Singapore and Shanghai are generally embedded in teachers' work sites and complement their schedules (Tonga et al., 2019; Ye & Zhou, 2022; Zhang et al., 2017). Professional development, as practiced in Singapore and Shanghai, focuses on student learning and facilitates the development of specific teaching skills (Qian & Walker, 2021; Tonga et al., 2019; Zhang et al., 2017).

United States' schools, despite significant and continued efforts, have not seen a consistent and sustainable improvement in test scores (The Nation's Report Card, 2019; TIMSS & PIRLS, 2019). The lack of effective PD may contribute to this malaise (Bentley & Cason, 2019). Matherson and Windle (2017) and Bentley and Cason (2019) found that many United States' teachers who participated in PD sessions felt the training was unrelated to classroom problems. Therefore, the PD sessions had little positive impact on their pedagogical practice or on student achievement.

According to Qian and Walker (2021) and Tonga et al. (2019), teacher-related attributes were central factors in Shanghai students' continued stellar performances in PISA tests. Rigorous PD, as practiced in Shanghai, includes collaboration and cooperation among teachers and schools, learning about the educational practices of other countries, and focusing on student learning (NCEEb, 2021; Qian & Walker, 2021; Tonga et al., 2019). From this perspective, school leaders in Shanghai sustained and supported the express goal of increasing teacher capacity through ongoing PD (Tonga et al., 2019). Shanghai's educational cognoscenti viewed teaching as a common responsibility with collective accountability (Hairon & Tan, 2017; Tonga et al., 2019). Sharing information about daily practices is the norm among teachers in Shanghai (Hairon & Tan, 2017; Qian & Walker, 2021; Tonga et al., 2019). School leaders also promoted the intermingling of teachers' experiences among different schools. This allows teachers from high-quality schools to volunteer to teach in at-risk schools (Qian & Walker, 2021; Tonga et al., 2019). Darling-Hammond et al. (2017) as cited in Darling-Hammond (2021) also addressed the issue of PD. The author conjectured that teacher education and ongoing PD must prepare and sustain teachers who are self-motivated in improving their own pedagogical skills, reflective about their practices, and proficient in utilizing theories and research to innovate their instruction and increase their students' learning. Additionally, Goldhaber and Brewer (1997), Dimmock and Tan (2016), Rodriguez-Lopez et al. (2019), as well as Burns et al. (2014), indicated that the level and intensity of a teacher's PD and preservice training may be directly associated with the students' mathematics achievement. The authors concluded that a teacher's mastery of subject-specific content has a significant impact on student success in mathematics. Teachers who are well-trained and have expanded PD opportunities to increase content knowledge are found to render more effective classroom instruction, and subsequently, exert a greater influence on student learning (Burns et al., 2014; Dimmock & Tan, 2016; Goldhaber & Brewer, 1997; Rodriguez-Lopez et al., 2019).

As previously mentioned, PD was especially beneficial when professional training was linked with student outcomes (Burns et al., 2014; Darling-Hammond, Hyler, & Gardner, 2017; Huijboom et al., 2021). These findings dovetailed well with the conclusions of another study by Jiang et al. (2018). These authors avowed that Singapore's educational system offers vigorous and continuous enhancement of math teachers' instructional knowledge and skills. Other authors also instanced the positive influence that teacher training has on effective math instruction (Dimmock & Tan, 2016; Kit, 2020). Singapore's policy of "teach less, learn more" provides additional time in the school day for PD as well as for planning and working with students outside the classroom (Darling-Hammond, 2010; Hairon, 2019; Hairon & Tan, 2017). One particular strand of PD in Singapore focuses on lesson study (Jiang et al., 2018). Here, teachers collaborate on the development of a unit of work. This included a detailed research lesson plan, followed by an observation of the lesson being taught, which, in turn, was sequentially accompanied by an assessment of student learning (Jiang et al., 2018)

Continuous PD is essential for effective teaching in Shanghai as well (Qian & Walker, 2021; Zhang et al., 2017). Shanghai has established stringent requirements, as well as carved-out opportunities for PD (Jensen et al., 2016; Tonga et al., 2019). Firstly, novice teachers are assigned mentors during their first three years as instructors (Ye & Zhou, 2022; Tonga et al., 2019). Shanghai then provides time, space, resources, planning, and other structural supports to teachers. These instructors are obligated to take additional professional training over the next five years (Çer & Solak, 2018; Jensen et al., 2016; Zhang & Zheng, 2020). To augment this enterprise, many schools in Shanghai embody a caring, trustful, and respectful school atmosphere (Zhang & Zheng, 2020).

Collectivist practices, deeply engrained in the Chinese culture, promote the value of professional learning communities in Shanghai (Tan & Ng, 2018; Zhang & Sun-Keung Pang, 2016). Shanghai is one of many Chinese cities which has systematic teacher research groups which focus on classroom improvement (Ye & Zhou, 2022; Zhang et al., 2017; Zhang & Zheng, 2020). Teachers in these venues discuss their classroom experiences, exchange ideas about new theories, create exam questions, and conduct research (Zhang et al., 2017; Zhang & Zheng, 2020). Moreover, China, unlike some Western countries, has always situated teachers' learning at their workplace (Zhang & Zheng, 2020).

Instructors in Shanghai, in comparison to teachers in the United States, spend more time developing their pedagogical expertise (Lee, 2017; Stewart, 2011; Zhang & Zheng, 2020). Further, PD, in the United States, is not always linked to the instructional agenda (Bentley & Cason, 2019; Matherson & Windle, 2017; Stewart, 2011). Professional development, in the United States, is therefore often perceived as incoherent and non essential. As a consequence, many teachers avoid participation (Bentley & Cason, 2019; Matherson & Windle, 2017; Stewart, 2011).

As previously noted, Singapore and Shanghai, according to international testing results, have systematically produced students who are among the world's highest achievers in math and science (Jensen et al., 2016; National Center on Education, 2018; TIMSS, 2019). The Singapore education system, which is grounded in exam meritocracy, has a centralized agenda managed by the government (Ro, 2020). Singapore's MOE sets national academic priorities, establishes the curriculum, generates national examinations, creates criteria for teacher development, assessments, and promotion, and hires most education officers (Ro, 2020). Singapore's MOE requires teacher education institutions to establish initial teacher competencies that directly relate to the national standards

(Darling-Hammond et al., 2017; Tonga et al, 2019). Preservice education not only includes theory but must be school-based and relevant to school operations (Cochran-Smith, 2015; Darling-Hammond et al., 2017; Tonga et al, 2019). Teacher preparation is carried out at the National Institute of Education (NIE), which is ranked among the best education universities in the world (Revai, 2018). The NIE is the nation's only teacher-education institution. Teacher preparation at NIE is rigorous, grounded in a coherent curriculum, and offers teacher-candidates abundant opportunities for extended practicums in local schools (Revai, 2018). The NIE selection process is highly competitive. Acceptance rates for education courses are based on the estimated number of teachers required to fill open positions (MOE, 2023). Therefore, only the most capable applicants are chosen to attend the NIE (Revai, 2018). Candidates are recruited from the top third of secondary graduates and less than 20% of those who apply are accepted (Revai, 2018). Appositely, Jaciw et al. (2016) and Tonga et al. (2019) reported that Singaporean teacher-candidates must show evidence of high levels of mathematics skills and a strong desire to teach before embarking upon their teacher education programs. Student teachers receive tuition grants and a monthly stipend during preservice training (MOE, 2019). Under MOE's auspices, students are guaranteed teaching positions after graduation but are required, in return, to teach for three to four years (MOE, 2023). Newly hired teachers in Singapore are restricted to 17 hours of classroom teaching each week. For the remainder of the week they consult with mentors, meet with parents, conduct research, and participate in PD programs (Çer & Solak, 2018; Ministry of Education, 2018). In addition to their mentor teacher, new teachers are assigned a buddy (Darling-Hammond, 2017; Tonga et al, 2019). This educator is an experienced peer, who has expertise in the same subject-area. A supervisor is also part of the orientation team. These supplemental resources help support the acclimatization process for newly-hired educators (Darling-Hammond, 2017; Tonga et al, 2019). In addition, the MOE provides support for teachers' professional growth through collaboration with the NIE, the Academy of Singapore Teachers (AST), and district schools (MOE, 2023). Conspicuously, Toropova et al. (2019) found that schools where teachers were offered administrative support and mentoring programs at early career stages had higher job satisfaction and lower attrition rates than schools without these appurtenances. However, in the United States, a much smaller percentage of novice teachers receive these ranges of support, (i.e.) regular mentoring, shared planning time, a teaching buddy, and a reduced teaching load (Bentley & Cason, 2019; Darling-Hammond, 2017).

Tonga et al. (2019), purported that in diversified economies, such as those found in the United States, Singapore, and Shanghai, education must compete with other sectors to acquire the most competent candidates. Singapore, in accordance with these dynamics, offers high salaries and good working conditions. The NIE, as well as district leaders, may therefore choose those teacher-candidates who have excelled academically or professionally (Tonga et al., 2019). The United States, in contrast, does not limit the numbers of people who train to become teachers (Darling-Hammond, 2021; Stewart, 2011). However, there are subsidies for older, work-experienced candidates in United

States' schools (Darling-Hammond, 2017; Thomas & Mockler, 2018). These individuals enter teaching through alternative programs, such as Teach for America (Thomas & Mockler, 2018). Such non standard avenues often entail only a few weeks of preservice training and have precipitated the lowering of teacher requirements. This is especially impactful in many inner-city schools in the United States (Darling-Hammond, 2017; Thomas & Mockler, 2018).

Restructuring teacher education in the United States will require a large increase in school funding. Federal, state, and local governments should encourage the recruitment of high-caliber math students with student loan forgiveness for those who commit to teaching (SREB Teacher Preparation Commission, 2019). Government should also provide stipends for extended year-long practicums. Moreover, the coursework in math methods courses, which accompany these practicums, should subsume systematic analysis of practice via videotaped lessons involving the student-teachers (Santagata et al., 2019). The subsequent observations and co-constructed interpretations of videotaped episodes may result in the cultivation of learning-from-teaching competencies thereby linking theory and practice (Santagata et al., 2019). Lastly, governments must find the financial resources to provide math teachers with the same level of compensation that is on a par with other math professions (SREB Teacher Preparation Commission, 2019).

Çer and Solak (2018), Darling-Hammond et al. (2017), and Tonga et al. (2019) reported that there is an abundance of educator-candidates applying for every teaching position in Shanghai. Collaterally, East China Normal University and Shanghai Normal

University, the two learning centers charged with preparing teachers, select only the best qualified applicants (Tonga et al., 2019). Preparation programs in Shanghai emphasize foundational instruction in academic content, pedagogical knowledge, and professional standards (Darling-Hammond et al., 2017; Tonga et al., 2019). The universities' curricula also highlight research and preparing teachers to conduct ongoing research. After graduation, students must pass national examinations in psychology, pedagogy, and teaching methods as well as a district level test of content mastery (Darling-Hammond et al., 2017; Tonga et al., 2019). Although teachers in Shanghai have lower average salaries than in most nations, they are compensated by a higher social status. This incentive may help attract more qualified candidates (Darling-Hammond et al., 2017; Tonga et al., 2019).

Summary and Conclusions

My study addressed a current problem, which has been identified in the literature, concerning the inability of many fourth-grade math students in the United States to meet proficiency benchmarks and to perform as well on standardized tests as students of other nations and international cities (Pew Research Center, 2018; The Nation's Report Card, 2019; TIMSS, 2019; Woessmann, 2016). The failure of math teachers in the United States to prepare many of their students to perform competitively with their international peers, as well as achieve proficient ratings in math, represented a gap in the practice. To address this gap, the purpose of this qualitative exploratory case study was to develop and analyze thick rich descriptions of an international array of teacher perceptions about different approaches to effective fourth-grade grade math instruction, inherent obstacles to sound instruction, their college training, and PD. An exhaustive review of the literature provided insights into what are confirmed as best practices in math instruction, obstacles to sound instruction, as well as other determinant factors that influence the teaching of math such as preservice education and PD. However, there appeared to be a gap in the literature concerning the efficacy of the specialized teaching of math and the effects of limited timeframes on the Common Core curriculum. Then too, I detected a gap in the literature concerning the caliber of teacher-candidates in the United States. Probing these and other issues with my 12 participants should enable teachers to integrate this acquired information into their professional practice. Policy makes and members of academia in the United States may also use the study's contributions to precipitate the implementation of more effective use of class time, specialized teachers for math instruction, as well as enhancing teacher-candidate selection process, teacher preparation programs, and ongoing PD.

I identified the following patterns and commonalities after an extensive and exhaustive review of the literature. These patterns included the difference in math instructional approaches that exists between many Asian and American schools and the challenges and advantages inherent in student-centered and teacher-centered practices. Other identified patterns included the high caliber of Asian teacher-candidates, the specialized credentials of Asian instructors, and the distinctions between the quality of Asian and American preservice education as well as PD.

In Chapter 3, I will connect the chosen qualitative methodology, which includes gathering and analyzing the beliefs and practices of a purposeful sampling of teachers, with the attending gap in the practice. This gap in the literature can be addressed by an enhancement of knowledge about certain best practices connected to fourth-grade math instructions, as well as the increasing ameliorating information regarding other determinant factors influencing instruction.

Chapter 3: Research Method

The purpose of this qualitative exploratory case study, which was aligned with the problem, research questions, methodology, and conceptual lens, was to develop and analyze thick rich descriptions of an international array of teacher perceptions about different approaches to effective fourth-grade grade math instruction, inherent obstacles to sound instruction, their college training, and PD. The coterie of 12 educators represented in the sample included teachers who practiced in Asian school systems as well as in New York State. In this chapter, I will restate the research questions and define the central phenomenon of the study. The qualitative case research design will be identified and justification for its selection will be explained in detail. I will also define my role as a researcher and participant. Moreover, Chapter 3 will include the methodology, with a focus on the participant selection and instrumentation, as these relate to data collection. I will also elucidate upon the data analysis plan and incorporate steps taken to ensure the trustworthiness of the study. Finally, issues related to the exploration's ethicality will be addressed.

Research Design and Rationale

The research questions are as follows:

Research Question 1 (RQ 1): According to fourth-grade teachers in the United States, Shanghai, and Singapore, which teaching practices best support math instruction?

Research Question 2 (RQ 2): What challenges do fourth-grade teachers, represented in the sample, cite as obstacles to effective math instruction?

Research Question 3 (RQ 3): How do other determinant influences, such as teachers' preservice education and PD affect math instruction in the United States, Singapore, and Shanghai?

Research Question 4 (RQ 4): How does SLT influence fourth-grade math instruction in the United States, Singapore, and Shanghai?

Qualitative research is a complex, recursive, and methodological endeavor (Creswell, 2018; Creswell & Poth, 2016). The researcher, when adopting this design, seeks, in a naturalistic setting, to understand people's perspectives and beliefs (Creswell, 2018; Merriam & Tisdell, 2015). Qualitative studies concordantly permit researchers to intensely investigate how people interpret their life experiences (Creswell, 2018). In congruence with these characteristics, I utilized an exploratory case study approach to collect rich descriptions of the 12 participants' experiences within the bounded setting of 11 schools. Incidentally, the sample size of 12 was deemed appropriate since it facilitated the collection of rich thick data albeit at the expense of attaining statistical transferability (Mishra & Dey, 2021; Şahin et al., 2016).

One of the salient characteristics of a case study is its holistic approach (Mishra & Dey, 2021; Sewell et al., 2017). Therefore, an exploratory case study may be employed to fully garner essential details through multiple data points of particular groups in a contextualized setting (Sewell et al., 2017; Yin, 2014; Yin, 2018). This approach was judged germane to the study due to its ability to provide a platform for assiduous investigations of unexplored phenomena (Cetenkaya, 2019; Flavell et al., 2019). In accordance, the exploratory case study format complemented the intent of the study which was to examine the perceptions of a purposeful sampling of 12 fourth-grade math teachers.

The case study design offered other benefits as well. A case study promoted a full description of the data accrued from a variety of sources, specifically in-depth interviews from two distinct sources. Markedly, these provenances provided information relevant to the purpose of the study. In this way, I was able to triangulate my research as I revealed the intricacies of American, Singaporean, and Shanghainese teachers' educational experiences. As a result, this research design dovetailed well with the focus of my dissertation. This focus, which represented the nucleus of the investigation, was to collect and analyze data related to the phenomenon of interest, (i.e., the perceptions of an international array of teachers regarding fourth-grade math instruction and other determinant factors related to that instruction).

Role of the Researcher

I was the sole researcher of this study. That being the case, I can affirm that I had no professional or personal relationships with any of the participants. My responsibilities, as an active participant interfacing with the respondents, included collecting data through interviews, maintaining a personal journal of contemporaneous reactions and insights, transcribing the accumulated information, and analyzing the data. In this role, I became part of the research (Creswell, 2014; Creswell & Poth, 2018; Cypress, 2017). Therefore, it was vital that I maintain an unbiased stance. Appropriately, I utilized my interactions with the respondents to better understand and accurately transmit unencumbered teacher beliefs concerning their instructional practices, preservice education, and ongoing PD.

One of the most important data collection instruments in qualitative studies is the in-depth interview. Qualitative interviews contain the following key characteristics. Qualitative interviews are relational since they feature a relationship based on mutual trust and reciprocity between the interviewer and the respondents (Ravitch & Carl, 2020). Qualitative interviews are also highly contextualized since every interview is operationalized within multiple, intersecting, and idiosyncratic situations (Ravitch & Carl, 2020). Therefore, the interviewer attempts to understand how these various factors shape an individual's experiences and perspectives in relation to the study's research questions (Creswell & Poth, 2018). Qualitative interviews are also person-centered (Ravitch & Carl, 2020). In this ambient, the participant remains the omphalos of the interview and is considered the expert of their own experience. Similarly, careful attention must be paid to the individual's opinions, feelings, and ideas. Finally, qualitative interviews are subjective (Ravitch & Carl, 2020). In other words, the interviewer and participant are both engaged in interpretive endeavors. These interpretations have been filtered through both individual's set of beliefs, experiences, biases, and values. This dynamic leads to a reciprocal awareness, or how the researcher and participant's subjectivities develop in relation to each other (Ravitch & Carl, 2020).

The cultivation of a trusting and comfortable relationship between the interviewer and interviewee is an important aspect of the qualitative interview process (Creswell &

Poth, 2018). To cultivate this atmosphere, the researcher should explain the process and convey their expectations for the interview (Creswell & Poth, 2018; Merriam & Tisdell, 2015). This information includes the timeframe, the presence of a recording device, conditions of confidentiality, the risks and benefits of the study, and how the results will be disseminated. Once the interview commences, the interviewer should remain a careful and empathetic listener and formulate probing and follow-up questions but never ones that cause discomfort to the interviewee (Creswell & Poth, 2018).

Interviews may include different types of questions that guide the data collection. The first type of question alludes to the participant's experience and behavior. These queries focus on actions the person has taken, is currently taking, or will take in the future (Ravitch & Carl, 2020). Another type of interview question explores opinion and values (Ravitch & Carl, 2020). These interrogatives are designed to investigate how a person discerns an experience and the merit they attach to it. Then too, there are questions that identify the participant's pool of knowledge and facts concerning the phenomena (Ravitch & Carl, 2020). Lastly, background questions center on the respondent's self-reported social location, identity, and positionality (Ravitch & Carl, 2020). I consequently included each of these interrogative types during the course of my interviews.

As previously stated, as sole investigator in a qualitative study I became part of the study. I therefore assiduously identified and controlled my personal feelings to ensure the trustworthiness of my qualitative study (Creswell, 2018). Berger (2015) and McGrath, Palmgren, and Liljedahl (2019) posited that qualitative researchers should attempt to suspend their own perspectives. They must also be aware of their positionality. Positionality refers to the researcher's role and social identity in relation to the contexture of the study (Berger, 2015; McGrath et al., 2019). Positionality also delineates how external and internal aspects of the researcher's values, experiences, and biases influence the meaning-making process (Berger, 2015; McGrath et al., 2019). Incontrovertibly, it will be of critical importance to sustain an unbiased position to maintain the accuracy of the participant's responses as well as my subsequent interpretations. My interview questions consequently included neutral language (Creswell, 2018). Moreover, I constantly scrutinized my preconceived feelings and opinions. I suspended these predispositions by bracketing and holding in abeyance my preconceptions, cultural factors, and past experiences (Creswell, 2018; Dforfler & Stierand, 2020). I also refrained from using audible intonations during the interview that revealed any form of bias. Creswell (2018) contended that self-reflection helps creates a credible narrative. Additionally, I continually monitored my actions and reactions through reflexive metacognition. These thoughts, related to self-learning and the identification of researcher bias, were recorded in a personal journal and were extended through the analysis phase (Creswell, 2018). Member checking may also minimize bias (Shaked et al., 2019). Member checking occurs when the researcher presents transcriptions of responses to the participants so they can evaluate their replies and make any necessary additions or modifications (Shaked et al., 2019). Consequently, I sent a copy of the appropriate transcript to each participant for their validation.

Methodology

The methodology section of a study includes components utilized to address the investigation's research questions (Creswell & Baez, 2020; Taylor et al., 2016). Therefore**,** methodology incorporates descriptions of the setting and the sample population and illustrates how participants were selected. Equally important, methodology describes the instrumentation for data collection, as well as reports on how data will be analyzed and disseminated (Creswell & Baez, 2020; Taylor et al., 2016).

Participant Selection

A climacteric task of my study was to determine who would be represented in the field of participants and how I would gain access to those individuals. Prior to the participant selection process, and in the interest of ensuring the safety of my respondents, I perused and remained compliant with the compilation of Office of Human Research Protections' (OHRP) laws, regulations, and guidelines on human subjects' protections (OHRP, n.d.). Afterwards, I dispatched emails to teachers and administrators at their respective schools. Following a fallow period of non responses due to the Covid pandemic, the Institutional Review Board (IRB) granted permission to offer each participating teacher a fifty-dollar gratuity. This initial outreach following this offering attracted four participants from New York State. I also gained access to the Walden Participant Pool. The Participant Pool is described as essentially a virtual bulletin board where researchers post studies and individuals participate in studies for which they qualify. As part of the process, the IRB reached out to the Participant Pool on my behalf. After my study was accepted, I was able to recruit four participants from Singapore and four participants from Shanghai. As previously noted, my exploratory case study incorporated a purposeful sampling of 12 fourth-grade math teachers who represented a total eleven elementary schools in these nations, states, or international cities: Singapore, Shanghai, and New York State. Each subject in the sample revealed they had four or more years of teaching experience. The important point, in this regard, was that each participant was familiar with the essential characteristics of fourth-grade math instruction, including but not limited to teacher- and student-centered instruction, discovery- and project-based learning, inductive reasoning, and conceptual development. In addition, all Singaporean and Shanghainese instructors were fluent in English.

My reasoning for selecting 12 participants is thus. Qualitative inquiry has no rigid criteria regarding the sample size (Ellis, 2020). However, I selected 12 participants based on the calculus of acquiring an equal number of subjects from Singapore, Shanghai, and New York. The objective was to obtain thick deep descriptions of teachers' perspectives concerning fourth-grade math instruction, preservice education, and PD. The limited sample size was ideal in that it counterpoised sufficient breadth with a depth of inquiry. Moreover, to enable this depth, samples for qualitative studies are usually smaller than those used in quantitative studies (Creswell & Poth, 2018; Merriam & Tisdell, 2015). According to Creswell (2018), the smaller the sample size, the richer the accrued data.

Similarly, the purposeful sample included 12 experienced fourth-grade teachers from the represented schools. Furthermore, purposeful sampling, in exploratory case studies, affords the researcher the opportunity to choose individuals and settings that can further the understanding of the problem of the study as well as address the primary phenomenon of the investigation (Creswell, 2018; Shaheen, 2019). Notably, all schools subscribed to the state or national curriculum of their respective regions.

Instrumentation and Data Collection

My study incorporated researcher-produced semi structured interviews. These data collection instruments subsumed telephone interviews as well as those using Zoom technology. A panel of three experts in the field reviewed the interview questions before the data collection took place. In addition, I maintained a personalized contemporaneously-derived running log of my reactions and insights regarding each interview. The researcher who uses reflexive engagement, via a log, while conducting research cultivates an ongoing, recursive appraisal of their actions and reactions regarding the mitigation of bias (Phillippi & Lauderdale, 2018; Probst, 2015). Triangulation was achieved through the incorporation of multiple disparate populations derived from the Asian and New York participant pools. This triangulation helped bolster the trustworthiness of the endeavor (Dooly et al., 2017; Korstjens & Moser, 2018).

The interviews were conducted in one of two ways. The first method consisted of telephone interviews. The second option was Zoom-enabled interviews. The study's research questions served as a basis for derivative interview queries. The research questions encompassed effective instructional strategies, challenges inherent in math instruction, how SLT influences instruction, as well as how other determinant factors might also impact the quality of fourth-grade math instruction. Each Zoom- or telephone-enabled interview session was recorded using audio equipment.

The in-depth interview is a fundamental integrant in qualitative studies. Interviews can be structured, unstructured, and semi structured (Goodell et al., 2016; King et al., 2019). Structured interviews are formatted to pose identical questions to each individual (Goodell et al., 2016; King et al., 2019). Unstructured interviews do not use prepared questions; rather the researcher and participant explore broad areas which create extempore questions and produce data based on the individual's experiences (King et al., 2019). I utilized semi structured interviews, which embodied prepared questions to guide the discussions as well as specific, pertinent follow-up questions dependent on the idiosyncratic nature of the initial response.

The questions included in the semi structured format were probing and open-ended (King et al., 2019). This configuration afforded the opportunity to not only include considered questions; it also offered the flexibility to react spontaneously to participants' responses with additional probing queries. In other words, key questions were preplanned, but the interviews also produced questions flowing from previous responses. These follow-up queries helped stimulate the creation of holistic reasoned responses on the part of the participants. The questions progressed from general inquiries to more specific interrogatives. Iterative questioning was also operationalized (Goodell et al., 2016; King et al., 2019). Iterative questions assisted in the detection of misleading information supplied by the participant (Goodell et al., 2016; King et al., 2019). Respect for each participant's time schedule and availability was an important component of data collection. Therefore, each interview session ranged from 45-minutes to 75-minutes in duration.

Ricci et al. (2019) cited the importance of inviting input from the population of interest when creating interview questions (Ricci et al., 2019). I did this to ensure that the interview questions fully reflected the respondents' perspective and that the inquiries were contextually relevant (Ricci et al., 2019). This method enabled the population of interest, rather than the expectations of the researcher, to generate some of the interview queries (Ricci et al., 2019).

The open-ended and non ambiguous interview questions were constructed to provide answers for the four research questions which guided the study. Incidentally, a list of the interview questions, which served as the data collection instruments, can be found the Appendix. In the course of the interviews, participants divulged information pertaining to best teaching practices, obstacles to teaching and learning, the quality of preservice training and PD, as well as the practical application of SLT in the classroom. I also encouraged the subjects to offer additional insights into issues that were not covered by the interview questions.

Procedures for Recruitment and Participation

The criteria for participant selection focused on the professional experience of the teachers as well as their fluency in English. The recruitment process began by contacting the principal of each school and providing each with an invitation and a letter of cooperation. The purpose of this letter was to request permission to conduct the study as well as issue other details about the process. The invitation missive included an introduction to the study as well as elaborated on the purpose and procedures germane to the research. It also described any risks as well as unveiled the potential benefits my study might yield to the field of education. The principal was then asked to forward a third missive, the invitation to the teacher, to all fourth-grade instructors who were fluent in English and had four or more years of teaching experience. In an effort to obviate the possibility of coercion, the potential participants then responded directly to me. Consequently, the principal was not apprised as to which teacher had decided to participate and who had declined. However, due to the initial lack of response from the likely pool of teachers at the beginning of the data collection process, primarily due to the coronavirus pandemic, I found it necessary to expand my search for candidates. The IRB, in response to my adjustments regarding assessment samples, allowed me to forego the invitation letter to the principal as well as the letter of cooperation. This resulted in an enhanced pursuance which ultimately resulted in contacting over 200 schools in Singapore and Shanghai. I also reinvigorated my search in local districts to include more schools.

Subsequent to receiving the first positive response from a candidate, which materialized in the fall of 2020, I emailed a consent form. The consent form clarified, with potential respondents, the purpose of the study and the role of the participants. Moreover, in this missive, I informed teachers of precautions taken to ensure their confidentiality and safety. Participants were also apprised of my intention to audiotape the interviews. All participants were asked to read this form. The letter explained that by returning the consent form they were acknowledging that they were willing to take part in the study. The first teacher from each school, who agreed to take part in this research and met the requirements, was confirmed. Singularly, no teacher withdrew from the study once they established their acceptance.

I also notified local teachers, who practiced teaching in western New York, that the interviews may be conducted via telephone or Zoom technologies. All four New York participants selected a telephone interview option. All interviewees were notified of my intention to audiotape the session. Each participated in a single interview from her home The interview timeframes for the New York State participants ranged from 45- minutes to 75-minutes. However, following the interview, I corresponded, via email, with all the New York participants requesting further clarification or additional information. I concluded each interview by thanking the participant and asking if they had any additional information to share. Teachers who practiced in Singapore and Shanghai were sent similar notices. The time schedules were negotiated with each teacher. All participants utilized Zoom technology from their homes and agreed to a single audiotaped interview. The timeframes for the interviews averaged 50-minutes. At the conclusion of every interview I thanked the teacher for their participation and asked if they had any added comments. As with the New York participants, I reached out to each participant shortly after the interview, via email, requesting additional clarifications. All 12 participants from New York State, Singapore, and Shanghai were also asked if they were willing to take part in a member checking activity. Each accepted the offer and, in this context, read the interview transcriptions and gauged the accuracy of their input. Notably, none requested any changes.

Data Analysis Plan

Qualitative data analysis is an essential procedure that requires intensive and meticulous attention. My coding procedures commenced with the use of a Microsoft Word document. The results, which emanated from the analysis of data, provided answers to the research questions. These interrogatives sparked inquiry about the most effective math instructional practices, the greatest challenges to successful instruction, how SLT influences math instruction, as well as how other determinants influences, such as preservice education and PD, affect didactic practices in math.

Data analysis, in qualitative studies, consists of organizing the data, coding, creating categories and themes, representing data, and interpreting the larger meaning of the results (Creswell, 2018; Miles et al., 2016; Stuckey, 2015). According to Saldaña (2021), the quality of the research is dependent on the rigor of the coding. A code in qualitative research is a diminutive word or phrase that distills the essence of a part of the collected narrative (Saldaña, 2016; Saldaña, 2021; Timberlake, 2014). I used open coding on the transcribed data that emerged from audiotaped recordings. I then reduced the data and combined similar responses to form categories. Categories were analogues of code clusters (Cetenkaya, 2019; Glegg, 2019; Şahin et al., 2016; Stuckey, 2015).

A theme, which evolves from the categories, is the underlying unifying idea that links the categories to each other (Glegg, 2019; Şahin et al., 2016; Stuckey, 2015). Thematic analysis transcends the counting of specific words and phrases (Blair, 2015; Saldaña, 2018). Thematic analysis centers on identifying and describing implicit and explicit ideas from the data (Blair, 2015; Saldaña, 2018). Probing teacher perceptions through qualitative methods allowed these themes to emerge based on the respondents' feelings. After confirming the trustworthiness of a study, themes may be used to fill gaps in the literature, inform the practices of professionals in the field, as well as serve as a catalyst for future studies (Blair, 2015; Miles et al., 2021).

Discrepant cases or contradictions in the data can give rise to unexpected findings (Nowell et al., 2017; Rose & Johnson, 2020). Discrepant cases occur when a code does not fit into any of the established categories (Nowell et al., 2017; Rose et al., 2020). This indicates that a participant's perception or experiences differ from the mainstream evidence. The use of disparities can bolster the overall conclusions of the study if they address the research questions (Nowell et al., 2017; Rose et al., 2020). My study identified a single discrepant case. The teacher, Participant 2, from New York, not only differed from her American colleagues on the issue of student-centered instruction but also eschewed many of the best teaching practices as delineated in scholarly research. I will further explain my findings related to this case in Chapter 5.

Trustworthiness

Trustworthiness in qualitative research refers to the degree of confidence the reader has in the presented data (Creswell & Miller, 2020; Hussein, Jakubec, & Osuji, 2015). The trustworthiness of a qualitative study is based on its accuracy, replicability, lack of bias, and transferability (Creswell & Miller, 2020). The issue of trustworthiness is critical to all studies involving qualitative research. Equivalently, the value of a qualitative study is often directly proportional to its degree of trustworthiness (Creswell & Miller, 2020). Qualitative researchers consider trustworthiness analogous to being thorough and accurate when conducting a study (Cypress, 2017; Stahl & King, 2020). Trustworthiness was established in my study through the clear materialization and alignment of the research problem, purpose, conceptual framework, research questions, instruments for data collection and methods of analysis (Goodell et al., 2016; Rose & Johnson, 2020). This precise methodology will allow the reader to reach determinations about the quality of a study's research process

Certain criteria must be met to attain trustworthiness. The criteria that support trustworthiness are credibility, confirmability, transferability, and dependability (Nowell et al., 2017; Rose & Johnson, 2020). Credibility is essential in substantiating trustworthiness. Credibility refers to the accuracy of the study's findings (Amankwaa, 2016; Stahl & King 2020). In other words, a credible study measures what it is intended to measure. Credibility is augmented through truthful descriptions of the teachers' lived experiences (Cypress, 2017). Strategies that helped ensure the credibility of my study incorporated the following research-based practices. First, I offered iterative or rephrased questions throughout each interview. Iterative questions were implemented to detect misleading information supplied by the participant (Cypress, 2017; Merriam & Tisdell, 2015; Saldaña, 2021). Credibility was also strengthened through reflective activities, such as incorporating a research log, as well as systematic and thorough data analysis (which included coding, categorization, and thematic scrutiny) (Dorfler & Stierand, 2021; Cypress, 2017). Studies have affirmed that the use of various data such as multi-source interviews as vehicles for triangulation, can be used to support evidence, validate conclusions, and add to the credibility of case studies (Arriaza et al., 2015; Korstjens & Moser, 2018; Yin, 2014). Saturation detection is another strategy which supports the credibility of a study (Hennink & Kaiser, 2019; Hussein et al., 2016). Saturation takes place when redundancies appear in the collected data *(*Hennink & Kaiser, 2019; Hussein et al., 2016). Therefore, at this juncture, the researcher may decide that additional information would not promote further understanding and, for that reason, appending more data would not contribute to the analysis (Hennink & Kaiser, 2019; Hussein et al., 2016). Saturation occurred in my study as I detected redundancies in data related to student- and teacher-centered approaches as well as teacher-parent relationships, homework assignments, preservice education, and PD appraisals. I also identified a surfeit of similar responses concerning teachers specializing in math, timeframes, teacher collaboration, and the role of technology.

In the interest of enhancing credibility, an examination of past scholarly investigations authenticated the degree of congruence between prior studies and the current one (Korstjens & Moser, 2018; Noble & Smith, 2015; Nowell et al., 2017). Finally, credibility was intensified through my prospective member checks where the participants perused interview transcripts to authenticate their responses (Kornbluh, 2015; Korstjens & Moser, 2018).

Dependability may be defined as the extent to which the study can be replicated with consistent results (Lemon & Hayes, 2020; Forero et al., 2018). The degree of dependability is closely aligned with the study's measure of credibility (Forero et al., 2018). Analogously, in the interest of dependability, I triangulated findings from two distinct sources: Asian and New York State participants, sent transcripts of the interviews to each participant for validation, and maintained a reflexive log.

Confirmability denotes that the study's findings are the result of a participant's responses which are unfiltered by the personal biases and predispositions of the researcher (Amankwaa, 2016; Huttunen & Kakkori, 2020). Confirmability also signifies the extent that the results can be corroborated by other investigators (Forero et al., 2018). Confirmability may be augmented through the reflexivity (Forero et al., 2018). Reflective activity on my part, and the subsequent inscription of these metacognitions in a journal, helped the investigator identify the extent of personal biases (Cypress, 2017; Dorfler &

Stierand, 2021). I used these journals to critically asses my reactions, inner beliefs, and responses. I also helped ensure confirmability through member checking (Arriaza et al., 2015; Kornbluh, 2015; Lemon & Hayes, 2020; Nowell et al., 2017). Member checks are often referred to as the gold standard for establishing a study's trustworthiness (Arriaza et al., 2015; Kornbluh, 2015; Lemon & Hayes, 2020; Nowell et al., 2017). In this regard, I sent copies of the interview transcripts to the participants so they might confirm that the document truly reflected their intended meanings.

Transferability may be described as the ability to apply the current study's results to similar situations or contexts (Hussein et al., 2015; Lemon & Hayes, 2020; Nowell et al., 2017). One strategy to advance transferability is the acquisition of thick descriptions concerning the participants' perceptions (Creswell & Miller, 2020). By describing a phenomenon in sufficient detail I began to evaluate the extent to which the conclusions drawn were transferable to other contexts. A second strategy that furthered transferability was journaling. Systematic journaling provided contextual information which will assist other researchers determine the universality of the conclusions (Kornbluh, 2015; Noble & Smith, 2015; Phillippi & Lauderdale, 2018).

Ethical Procedures

The dynamic and reciprocal nature of qualitative studies involve complex ethical responsibilities (Iphofen & Tolich, 2018). Thus, many codes of ethics were created to regulate the relationship between qualitative researchers and participants (Flick, 2018). Qualitative researchers must be proactive in certifying the ethical components of their study (Iphofen & Tolich, 2018). For instance, researchers must thoughtfully delineate their roles and responsibilities. Furthermore, the investigator must decide on the potential risks and benefits for the participants as well as incorporate privacy safeguards (Iphofen & Tolich, 2018; Von Unger et al., 2016). Many ethical prescriptions accentuate respect for the individual as a central tenet (Iphofen & Tolich, 2018; Von Unger et al., 2016). Respect for the respondents is particularly crucial in cultivating an inviting and safe atmosphere for those studies which include interviews. As a result, the researcher should disavow an expert-learner modality (Ravtch & Carl, 2020). This stance, which emphasizes the ascendency of the interviewer, assumes that the questioner has more knowledge than the participant (Ravtch & Carl, 2020). Instead, the researcher should embrace the idea that the respondents are experts of their own experiences (Ravitch & Carl, 2020). The interviewer should also avoid deficit orientations (Flick, 2018). A deficit orientation takes place when the researcher believes that people from various cultural or social groups lack a certain degree of knowledge, skills, or values (Arriaza et al., 2015; Flick, 2018). Equally important, I demonstrated respect for the interviewees by explaining the purpose of the research study, the timeframe for interviews**,** the presence of audiotape equipment**,** participation requirements and rights, and who will be privy to the data during the data collection, transcription, analyses, and dissemination processes (Creswell, 2018; Patton, 2002; Yin, 2014). I also elucidated on the process of member checking where, after the interview was completed, participants perused the transcripts and authenticated their responses. Moreover, before the commencement of the interviews, I made the respondents aware that all coding procedures, as well as the final approved version of the dissertation, would include a unique identifier rather than their name to assure that their privacy was maintained. That being the case, in lieu of their real name, each participant was styled as such: (Participant 1, Participant 2, et cetera).

Privacy is a paramount issue in qualitative research (Office of Research and Doctoral Support). As a consequence, confidentiality and anonymity are considered critical precautions. Confidentiality is closely aligned with privacy and encompasses decisions such as how the data will be gathered and disseminated (Roth & Von Unger, 2018). However, the terms anonymity and confidentiality are often incorrectly conflated (Roth & Von Unger, 2018). Anonymity is achieved when the identities of the research participants remain unknown (Roth & Von Unger, 2018). In contrast, confidentiality refers to the fact that the researcher knows, but chooses not to reveal the identity of the respondent (Roth & Von Unger, 2018). Participants must therefore be assured that, due to the confidentiality agreements, the research study will protect their identities to avoid any adverse risks to them such as estrangement from colleagues or reprisals by leadership (Bree et al., 2018; Jessica & Anders, 2018; Patton, 2002).

Furthermore, confidentiality can be safeguarded by downloading, transferring, and storing data obtained from the participants' e-mail responses, including information derived from interviews, in a secure electronic device (Leland et al., 2019). Once collected, the information was transferred to a hard drive and flash drive. Later, all data, including electronic hardware, will be archived in a locked file and will be stored for a minimum of five years and then destroyed (Office of Research and Doctoral Support, 2019). Password protection will help secure electronic files during this interim (Office of Research and Doctoral Support, 2019).

A consent document is an important facet of qualitative research (Office of Research and Doctoral Support, 2019). Therefore, after dispatching the agreement, I suggested that each teacher read, understand, and acknowledge agreement with the informed consent document prior to taking part in my study. This assured that their privacy would be respected. Parenthetically, informed consent was a process of explaining the intricacies of the study to the participants and responding clearly and accurately to their questions (Office of Research and Doctoral Support, 2019). The consent forms should be inclusive, transparent, and clearly worded **(**Office of Research and Doctoral Support, 2019; Twining et al., 2017). The criteria for valid informed consent embraces three elements: information, comprehension, and voluntariness (Office of Research and Doctoral Support, 2019). The informational component required the researcher to clearly describe the research procedure. This methodology included the analysis of perceptions and practices related to information collected and collated from the interviews. The informed consent document also expanded upon the purpose of the format, the attending risks and benefits, privacy issues, and the timeframe. Additionally, the informed consent agreement contained sample interview questions and a proviso in which respondents were offered the opportunity to withdraw at any time from the research (Office of Research and Doctoral Support, 2019). The second element, comprehension, indicated that it was the responsibility of the researcher to certify that the participants were able to understand the contents of the informed consent agreement. Therefore, I ascertained that the presentation of information in the agreement was aligned with the subject's intellectual capacities (Office of Research and Doctoral Support, 2019). Informed consent also subsumed the third concept of volunteerism. Volunteerism means that the subject freely chooses to participate in the study. This element of informed consent further required conditions free of coercion (overt threat of harm) and undue influence (excessive compensation) (Office of Research and Doctoral Support, 2019). Conspicuously, my study was free of coercion and undue influence for the following reasons. I did not conduct the study within my own work environment. There were also no power differentials between the researcher and the participants. However, a small incentive, approved by the IRB, was offered.

As an ethical researcher, I also allotted adequate time for the participants to review the information contained in the informed consent pact as well as formulate and pose questions prior to giving assent (Office of Research and Doctoral Support, 2019). The respondents were made aware that consent forms did not require signatures if the participant indicated approbation by some action, such as simply returning the completed form (Office of Research and Doctoral Support, 2019). I also included a reminder that the participants print a copy of the consent agreement (Office of Research and Doctoral Support, 2019) . The consent form also provided contact information explaining how the participant can reach the researcher and the university’s Research Participant Advocate.

The principles of justice and beneficence, as they relate to ethics, are two other essential aspects in qualitative research (Flick, 2018). Justice stipulates that the benefits of taking part in a study will be maximized and distributed in an equitable manner. Beneficence lends prominence to the concept that researchers should always forefend the participants' welfare (Flick, 2018). In other words, as a researcher I confirmed to the respondents that they would suffer no harm as a result of engaging in the study.

There are many types of malfeasance that qualitative researchers must avoid. To clarify, the researcher should not mislead the participant about the purpose or timeframe of the research, nor present poorly worded or incomplete consent forms, compromise any confidentiality or anonymity agreements, or write reports that portray participants in ways that are inaccurate or deficit oriented (Ravitch & Carl, 2020).

In a supplementary manner, the IRB creates ethical guidelines for all research prior to, during, and following the study. The IRB precepts confirm that principled procedures are being followed (Creswell & Poth, 2018). Every researcher starts the approval process by completing IRB-issued Form A (Office of Research and Doctoral Services, 2019). They then receive further guidance from the IRB regarding other documentation, such as Form C, as needed. These forms, as well as other templates, safeguard the integrity of the study as well as the safety of the participants.

Summary

I began Chapter 3 by restating the research questions and describing the central phenomenon of the study. Following this, the qualitative case research design was identified as an exploratory case study and the rationale for its selection was explained. Afterwards, I defined my role as a researcher and participant. I delineated the attending methodology and placed especial emphasis on participant selection and instrumentation as each related to data collection. I also described the data analysis plan in detail. In other sections, I explained the various steps taken to ensure the trustworthiness of the study. Next, I elucidated upon a plethora of ethical aspects related to the study.

Chapter 4 will commence with a review of the purpose and research questions immanent to the study. I will then explain the setting and include a description of the participants' demographics. I will also expound on the steps taken in collecting the data and describe the process of data analysis. Next, I will indicate how the data lead to the study's results, conclusions, and recommendations. Afterwards, evidence of trustworthy qualities, indigenous to the study, will be proffered. Chapter 4 will then be summarized and I will describe the transition to Chapter 5.

Chapter 4: Results

The purpose of my qualitative exploratory case study was to develop thick, rich descriptions of teacher perceptions about math instruction, inherent obstacles to good instruction, college training, and PD. For that reason, a purposeful sampling of 12 fourth-grade math teachers was selected. The sample was comprised of four teachers from Singapore, four teachers from Shanghai, and four teachers from the United States. All 12 participants were female teachers with four or more years of teaching experience.

The study's research questions, which were informed by current literature, advanced my exploration of the strengths and weaknesses of student-centered instruction and teacher-centered instruction as these approaches pertained to selected classrooms in Singapore, Shanghai, and New York State. Additional research questions undergirded my examinations of how SLT influenced math instruction and how other determinants, such as teachers' preservice education and PD, influenced the quality of classroom instruction. These questions were aligned with the purpose of collecting and analyzing thick rich descriptions of teacher perceptions. The study's four primary interrogatives, listed below, also served as the touchstone for my interview queries**.**

Research Question 1 (RQ 1): According to fourth-grade teachers in the United States, Shanghai, and Singapore, which teaching practices best support math instruction?

Research Question 2 (RQ 2): What challenges do fourth-grade teachers from the sample cite as obstacles to effective math instruction?

Research Question 3 (RQ 3): How do other determinant influences, such as teachers' preservice education and PD affect math instruction in the United States, Singapore, and Shanghai?

Research Question 4 (RQ 4): How does SLT influence fourth-grade math instruction in the United States, Singapore, and Shanghai?

Chapter 4 will be organized in the following manner. I will initially describe the setting and will include a synopsis of the participants' professional background. Later, steps taken during data collection will be recounted. I will then render a detailed account of the data analysis process. Next, I will indicate how the data adumbrated the study's results, conclusions, and recommendations. Afterwards, evidence of trustworthy qualities, immanent to the study, will be offered. Chapter 4 will then be summarized and I will provide a segue to Chapter 5.

Setting

Twelve teachers, representing New York State, Singapore, and Shanghai volunteered to take part in my study. The total sample consisted of four teachers from each of those three locations. All participants were women who were fluent in English. The 12 teachers in the sample taught in 11 individual schools. The New York teachers taught for an average of 16 years while the Asian teachers' medium professional experience was 7 years. All Asian schools subscribed to the national curriculum. All eight Asian classrooms, additionally, contained a mixed socioeconomic group of learners. All four New York schools had a mixed socioeconomic group of students. However, three of the four New York classrooms contained a sizable number of students on free lunch programs. Lastly, the New York schools were under the auspices of the Common Core curriculum. The interviews took place in the fall and winter of 2020-2021. Zoom technology was used to conduct the Asian interviews while the New York teachers spoke by telephone. I conducted the interviews from my home office. All educators responded from their own homes and each was currently teaching the fourth-grade. Lastly, I can confirm I had no knowledge of any personal or organizational conditions which influenced the teachers nor the data collection.

Data Collection

This qualitative study, which examined the perceptions of a purposeful sampling of 12 fourth-grade math teachers, utilized a case study design. Walden University's IRB originally granted approval for the collection of data in January 2020. This approval (#01- 03-20-0089862) was renewed on January 7, 2021. I commenced the data collection outreach process by sending letters and e-mails to potential schools and participants throughout the spring and summer of 2020. However, due to the COVID pandemic there was limited response to my outreach. Subsequently, I made numerous telephone calls throughout the spring and summer of 2020. Still, by October, no potential participants had replied. In the late fall and winter of 2020-2021, four eligible teachers from New York State replied to my emails in a positive manner. After joining the Walden Participant Pool in October 2020, I was able to recruit four eligible Singaporean teachers and four eligible Shanghainese teachers as well.

The data collection was comprised of interviews from two distinct sources, that is, Asian and New York State teachers. Twelve semi structured interviews were scheduled on the basis of mutual convenience. The interviews lasted an average of 50-minutes. Each participant was assigned a number from one to 12 to ensure their confidentiality and privacy. Since my collection of data began during the global pandemic, social distancing requirements dictated that communication with the teachers be conducted either by Zoom software, telephone, or email. Zoom provided the opportunity for audio, video, and digital recordings. I also used an independent voice recorder as backup. In addition, I utilized emails for member checking as well as for addressing additional questions.

I encouraged the participants to select their own media preference. All chose either Zoom or telephone interviews. I emailed a secure link to the Zoom video chat to each teacher who preferred that medium. Once each chat was initiated, I reminded the participant that I would be recording the interview. Although I reiterated my willingness to allow anyone to withdraw from the study at any time, none of the participants demurred. Each participant took part in one interview. Subsequently, after the interviews concluded, I submitted additional questions to each participant via email.

During the Zoom session (password protected) I utilized a Sony voice recorder to audio record the conversations. During the interview process, as well as throughout the data collection and analysis stage, I used a log to monitor my personal experiences, beliefs, and biases. In this manner, I was able to isolate my viewpoints from those of the participants.

The New York State teachers chose to be interviewed by telephone. These exchanges were recorded on two Sony voice recorders. Later, password protected Ebby software was used to transcribe each interview recording. Afterwards, I perused the transcript while simultaneously listening to the audio recording of each interview. In this way I was able to interpret any indistinct responses. These transcriptions were then transferred to Word documents and saved to both a thumb drive as well as to my laptop. After verifying the accuracy of the transcripts I emailed the transcript to the participants for review. All of the teachers confirmed the fidelity of the transcriptions. Furtner, there were no unusual circumstances or variations in the data collection. Lastly, the notebook, along with the laptop, thumb drive, and voice recorders were secured in a locked cabinet in my home office.

Data Analysis

Codes, Categories, and Themes

The following section describes the process of explication where the data evolved inductively from coded/ category units to the larger representations of themes. According to Saldaña (2018) a code is a short word or phrase that the researcher assigns to a portion of the text to capture its essence. Innately, codes are descriptive rather than interpretive and address a small portion of the text. Codes, clustered together according to similarity, nurtured the cultivation of categories. Categories are, therefore, collections of codes which share similar meanings. A theme, which develops from these categories, is the underlying or palpable unifying idea that binds the categories together. Themes represent patterns across data sets and includes information that addresses the research questions. Hence, the consolidation of codes/categories moves the researcher from a particular reality toward more abstract and encompassing themes (Saldaña, 2018).

My first step in the post-interview process of data analysis was to run off hard copies of each of the transcripts. I then read through all responses line by line. I underlined relevant phrases, sentences, or paragraphs. Next, I jotted down an appropriate code or label, related to the applicable interview question, in the right-hand margin. This label or code captured, in a succinct manner, the meaning of the corresponding section of the transcript. Afterwards, I listed all the meaningful units, or codes, connected to each participant’s response in a Word document (Belotto, 2018). These codes were color-coded to reflect the individual participant and all codes were organized in a manner which indicated the appropriate geographical region (New York, Singapore, or Shanghai). As I organized the codes in the first round I kept a tally of each participant as well as by each geographical region. This resulted in a total of 1466 codes. I also catalogued these data by research question. I accomplished this by separating each participant’s responses and highlighting chunks of the transcriptions (codes) with four different colors to represent each of the four research questions. In this way, I was able to cross-reference the data by individual responses, research questions, and regions. I utilized a similar method to cross-reference my notes and comments. I then perused the aggregated data in the second cycle of coding and eliminated repetitive, closely related, synonymous, and less relevant codes. The codes were then synthesized with appropriate categories to form the hybrid codes/categories. This reduced total of 111 codes/categories was depicted on the nine tables. Later, I analyzed how these codes/categories were connected. The similar codes/categories were then elided into relevant patterns or themes (Belotto, 2018).

The data accrued from Singapore and Shanghai teachers were combined since saturation appeared early due to the commonality of responses. Therefore, these aggregated data from Singaporean and Shanghainese teachers were placed under the heading of Asian teachers. In addition to the two geographical entities' responses to the four research questions, there remained the issue of one discrepant case. Data for the discrepant case can be found in Table 9. All 111 codes/categories, and 30 themes that materialized from the totality of data are listed below in Tables 1 through 9.

Table 1

New York Teachers' Descriptions of Effective Instruction

Codes/Categories	Themes
• Learning standards. • Teacher as facilitator. • Concrete manipulatives. • Concept development. • Higher order thinking. • Reflections. • Cross curricular subject matter. • Real world applications. • Spiraling. • Multiple approaches to problem-solving. • Justify thinking.	• Rigorous research-based approaches to instruction with teacher as a facilitator in NY classrooms.
• Differentiate. • Ability Grouping. • Scaffolding. • Study aides. • Mini lessons. • All students have chrome books, access to interactive software exercises for review.	• Personalized learning with tiered ability grouped instruction among New York students.
• Formative tests. • Ongoing feedback. • No student input in exams. • Self-and peer-assessments.	• NY teachers conduct primarily. informal monitoring of student progress.
• Small class sizes. • Safe, non competitive atmosphere. • Encourage interaction between pupils. • No alphabetic grades.	• NY teachers cultivate a safe non competitive classroom climate conducive to learning.
• Weekly outreach to parents.	• Parental involvement encouraged through multiple avenues in NY.

Table 2

Asian Teachers' Descriptions of Effective Instruction

Codes/Categories	Themes
• High expectations for all students.	
• Teaching for mastery.	• Rigorous research-based approaches to math instruction with Asian teachers as the source of knowledge and authority.
• Teacher is specialized in the teaching of math.	
• Learning standards.	
• Concrete manipulatives.	
• Concept development.	
• Memorization.	
• Spiraling.	
• Teacher leads unambiguous discussions.	
• Encourage interaction/questioning between students and teacher.	
• Technology for research.	
• Opportunities for gifted students.	
• Cross curricular themes.	
• Real world applications.	
• Multiple approaches to problem solving.	
• Justify answers.	
• Student reflection.	
• One-hour daily homework.	
• Scaffolding.	
• Mini lessons.	• Targeted personalized learning, as appropriate, in Asian classroom.
• Frequent formative tests.	
• Ongoing feedback	• Ongoing and frequent formal monitoring of student progress in Asian classrooms.
• Self and peer-assessments.	
• Student input into assessments.	
• Competitive atmosphere.	• Asian teachers utilize students' competitive instincts for the value of increased learning.
• Grades emphasized as extrinsic rewards.	
• Math competitions for gifted students.	
• Frequent communication with parents.	• Ongoing outreach to Asian parents.

Table 3

Asian Teachers' Challenges to Effective Instruction

Codes/Categories	Themes
• 7-9 ½ hour school day. • 193-day school year. • 2-3-hour daily math. • Large class size.	• Rigorous workload in Asian schools combined with a long school day.
• Classroom lack material resources including chrome books and software programs.	• Lack of readily available technology and other material resources.
• Students demonstrate a lack of interest.	• Negative attitudes towards math among many Asian students.

Table 4

New York Teachers' Challenges to Effective Instruction

Codes/Categories	Themes
• 6-hour school day. • Daily math: 45-minutes to 1½ hours. • 180-day school year. • Lack of student requisite skills. • Student attitudes. • Lack of specialized teachers of math. • Parents express unfamiliarity with algorithms. • State test results tied to teacher evaluations. • Teacher expectations. • Lack of gifted programs. • Mainstream student with unique challenges	• Inadequate time frames for NY teachers. • Students' lack of readiness from earlier grades as well as negative attitudes challenge NY teachers. • Teachers lament the paucity of opportunities to specialize in math. • Lack of parental support, especially on the issue of homework. • State tests, connected to teacher evaluations, often compel NY teachers to "teach to the test." • Gifted students, in NY classrooms, often lack challenging material. • Special Ed students in NY teacher classrooms.

Table 5

Asian Teachers' Determinant Influences that Affect Instruction

Codes/Categories	**Themes**
• Teacher training rated on average four out of five stars. • PD once or twice a month, systematic and ongoing. • PD often linked to classroom activities. • Monthly sharing with 4th grade teachers.	• Asian teacher college training in math was deemed effective. • Ongoing, systematic classroom-related PD among Asian teachers leads to improvement in the capacity and attitudes of teachers. • Systematic collaboration by Asian teachers among grade-level colleagues leads to increase in pedagogical skills and collegiality.

Table 6

New York Teachers' Determinant Influences that Affect Instruction

Codes/Categories	Themes
• Poor to good teacher training in college. • PD: hit-or-miss. • PD not systematic or ongoing. • PD rarely linked to classroom activities. • Ongoing informal PD discussions. • Little PD in math. • PD has not increased teacher capacity.	• NY participants, in college, generally experienced poor preparation for teaching math. NY participants found PD to be ineffective due to abstract material which was infrequently rendered.
• Sharing often with grade-level teachers.	• Grade-level collaboration among NY teachers leads to increased capacity and collegiality.

Table 7

Asian Teachers Use of SLT

Codes/Categories	**Themes**
• Primarily teacher-centered. • Explicit instruction. • Intermittent small group instruction. • Individual roles in groups. • Accountability in groups. • Group captains. • Reflections as assessments.	• Asian teachers use direct and whole class instruction and assessments supplemented by occasional collaborative activities

Table 8

New York Teachers Use of SLT

Codes/Categories	**Themes**
• Interaction between students. • Routine collaboration given heavy emphasis. • Group captains. • Life skills. • Individual accountability in group work. • Students answer most questions. • Students coach each other. • Heterogeneous or homogenous pairs. • Establishing expectations for groups.	• NY teachers use collaborative small group tiered activities with an emphasis on student autonomy during math lessons.
• Less frequent DI.	• NY teachers use targeted DI on appropriate occasion.

Table 9

Discrepant Case of New York Teacher P2

Codes/Categories	Themes
• Little collaboration. • Teacher responds to most questions. • No inductive reasoning • No cross curricular themes • No modeling behavior for groups. • Limited differentiation. • No group captains. • No self- and peer- assessments. • No spiraling. • Infrequent feedback. • No student coaching. • No life skills	• One discrepant case involving a NY participant exhibited many pedagogical approaches which were not supported by the research-based findings in the literature review.

Outcomes

The problem of this study, identified in current research literature and supported by standardized tests results, was that many fourth-grade math students in the United States were not meeting proficiency standards, nor performing as well on standardized tests, as students of other nations and international cities (Pew Research Center, 2018; The Nation's Report Card, 2019; TIMSS, 2019; Woessmann, 2016). The purpose of this qualitative exploratory case study was to develop and analyze thick rich descriptions of an international array of teacher perceptions about different approaches to effective fourth-grade grade math instruction, inherent obstacles to sound instruction, their college training, and PD. When analyzing the thick descriptions, I not only noted the participants' perceptions, but was also mindful of their voices, feelings, and actions. Then too, I was also privy to my own concurrent thoughts since I had created a log of my reactions as I conducted the interviews. After analyzing the data, I was able to discern many characteristics which distinguished the New York teachers from their Asian counterparts. I noted a general embracement as well by most New York and all Asian participants of such practices as encouraging critical thinking, justifying answers, student reflections, and outreach to parents. Moreover, there was a prevalent belief, by both international groupings, absent my discrepant case in some instances, in the merits of establishing a safe environment, adhering to learning standards, utilizing concrete manipulatives to reify abstract ideas, providing ongoing feedback, and inculcating multiple approaches when solving problems. Lastly, all teachers from both international cohorts reported that many students displayed negative attitudes toward math.

Research Question 1

RQ 1: According to fourth-grade teachers in the United States, Shanghai, and Singapore, which teaching practices best support math instruction?

I will begin this section by describing how the aggregated data, derived from four semi structured interviews with the New York State subjects, addressed my first research question. The first research question, as well as the remaining three, served as the fountainhead for the interview questions. As a result of the open coding and thematic analysis process, 30 codes/categories and five themes ultimately emerged from the New York State participants' responses.

As a point of interest all four New York State teachers in the sample (styled P1 through P4), subscribed to the concept of differentiation. All agreed they utilized the same state-wide learning standards for all students but individualized the students' lesson plans and assigned each student to one of three homogenous groups.

P1 would then form groups based on individual needs.

P1 added:

> Right now I have a fourth-grader who's doing high school algebra. He does the regular exercises because I feel he's valuable part of the fourth-grade group and that discussion is important. But he also uses a program that the district recommends called IXL.

Incidentally, I Excel or IXL is a personalized interactive program, accompanied by diagnostic tests, that allows students master many advanced math skills ranging from algebra to calculus. The needs of struggling students in P1's classroom were also addressed.

> The standards are the same. Everyone is definitely exposed to the same standards and the same content so I would be challenging some students with extra steps and then average students with less steps and stressing basic skills with others.

The teacher also felt that concrete manipulatives helped her to differentiate lessons for these students.

> They are usually working with unifix cubes or pennies, something they can touch and count and develop that understanding of one to one relationship. And then once they get that concept they can move to something like a disc that has a ten on it. Instead of having to count out ten cubes they can move to something more abstract with numbers and symbols.

P1 added:

> We try to use a lot of visuals such as cardboard pieces of pizza. They play with these and handling the materials helps them to develop an understanding plus it promotes the interaction between students and leads to a discussion of the concept in the subject's vocabulary.

Memorization of facts was also an issue with struggling learners. P1 offered her thoughts on this matter.

> Initially, I give them a chart that they can use as a tool. That is so they eventually can internalize the information. It's like in reading if you have to keep going back and looking at the alphabet it really slows you down, right?

P2 disclosed, “I typically differentiate within my own classroom by forming homogenous groups and I teach all of the groups at different levels. For example, in in fourth-grade they want us to be able to solve multi-step word problems. I would challenge some students with extra steps and then average students with less steps and stressing basic skills with others.” P2 advocated the use of tape diagrams for the neediest pupils. Tape diagrams are simply three identically shaped rectangles, one of which is larger than the other two. The larger piece represents the whole while the two smaller rectangles symbolize parts of the whole. When dealing with addition or subtraction problems the children understand that when they already know both parts of the whole they must add and when they know the whole and one part they must subtract. Tape diagrams, consisting of one long rectangle and two smaller ones, can be used as well for multiplication, word problems, and number bonding. Using the three rectangular pieces of tape or paper adds a visual representation of the problem as well as a tactile dimension. I-Ready is another program that P2 used to differentiate the instruction. I-Ready is a diagnostic program that is given three times a year. It covers a large range of math skills and identifies areas of needs and offers levels of difficulty. Additionally, P2 used technology to address the matter of memorization. Reflex Math offers an online game format where students practice math facts. The scores are analyzed to indicate mastery or to signify the need for more practice. Academic Intervention Services (AIC) provides a 30-minute time frame where certain students leave the room for help in different subject matter, including math. This time frame is also utilized for remediation purposes in the homeroom as well. Here, P2 offered targeted instruction and mini lessons. Lastly, on the topic of gifted students, P2 recounted, “We don't have a large number of students that are working above grade-level. So we don't have a gifted and talented program or anything like an accelerated program.”.

P3 also promoted differentiation as a best practice. P3 has employed a panoply of differential strategies. One such approach was conducting mini lessons with heterogeneous groups.

P3 explained:

> Sometimes I pair strugglers with kids at a higher level so that peers can help peers I will also reduce expectations in terms of the sheer number of problems or the difficulty level of the problems presented to lower performing kids. One example, I would I give lower performing kids dice that only go up to six so that multiplication facts are so much easier to recall. My higher kids are given dice that go to 12.

P3 believed that concrete manipulatives play a vital role in fourth-grade math.

> You start off with concrete first and then move into the abstract. I find that because of fourth-grade students' different levels and abilities, you definitely need all those different things in the classroom. So some kids are still definitely at that concrete level. They need to be able to touch and manipulate and move things around and visually be able to see it where some kids are already at that higher-level of abstract thinking. I don't think, from my perspective as a teacher, that it's bad to introduce concepts at the concrete level for all kids because it just reinforces what they already knew.

P3 also effected bar models for operations ranging from addition to multiplications, word problems, and fractions. The student, utilizing bar models, breaks the problem into chunks, uses appropriately sized rectangles to represent numbers, and writes numbers, in brackets, outside the rectangles. Memorization is also a concern. P3 stated:

> I can tell you that the more that kids memorize facts or the more efficient kids are at knowing at how to identify a basic fact, the easier the process is. Being a Special Ed teacher for many years I know some kids are never going to memorize their facts and so the memorization piece of it, though it's an ideal thing and some kids can get there, some kids are not going to get there either in fourth-grade or ever. So I don't try to push the sheer memorization of it necessarily. My job is to give them tools and strategies to get to it even if they don't have it memorized and to try to make those tools and strategies (arrays, skip counting) available as fast as possible.

P3 lamented the lack of gifted programs.

> I also know, as a classroom teacher, you can't forget about those higher kids because I think they get bored and often times because they are decent students and things come easier to them. I don't see us giving as many opportunities in math. We don't find ways to challenge them enough. So that's definitely something that I think we could strongly consider.

P4 offered her views on differentiation:

> As a fourth-grade math teacher, I had two groups out of my three that were significantly below grade level (homogeneously grouped). I would work with students individually or give them tools or strategies that they could use to understand and work with concepts better. My biggest focus is making sure that those (struggling) kids get as much hands-on concrete practice as they can. Eventually, they're going to move away from them and be able to draw representations and then move into the mathematical part of it. Anytime you're teaching a new concept at grade level or a new concept in the younger grades, you should always be starting with concrete manipulatives.

Scaffolding for struggling students included place value charts, multiplication tables, templates for long division, and graph paper to help align the numbers.

> We use a lot of number lines for a lot of different concepts and different models for a fraction. So maybe we'll use an area model or a circular model

> or the number line or a set of things rather than one whole. So I use as many different representations that I can to make sure students are encouraged to find one that they comfortable with. I would work with students individually or give them tools or strategies that they could use to understand and work with concepts better (place value charts for those with visual/tracking difficulties, multiplication charts for those with memory issues, manipulatives for those who are not thinking abstractly yet.

P4 further explained:

> We've gotten away from gifted kids as a focus. I think teachers are struggling so much to just get their average and lower average kids to be where they need to be. The last thing they're worried about is those higher-end kids. They're going to pass the math test no matter what. I hate to say that we're not teaching to a test but every teacher is going to care about that because that's how we're judged. That's the way it is. Generally, with gifted kids, I focus on different strategies and not just on one right answer. I ask them to show me two other ways to do the same thing or don't just tell me that one fraction is equivalent to 3/4. How many others can you find? Also, creating open-ended questions, finding more challenges and more ways that they can represent their thinking.

P4 weighed in on technology and memorization and how these topics affected differentiation.

> Technology is used mostly for practice. There's a lot of activities and games they can use to learn math facts. We obviously utilize those facts every day. I encourage them to practice their facts on a daily basis. We have online and hands-on games and activities to help them practice. However, a lot of my kids cannot memorize. I know that the state wants memorization to play a bigger part but I have students using strategies for figuring out the facts more than I do memorization. I believe the process is more important than memorizing their facts.

The Asian assemblage of participants produced 27 codes/categories and five themes related to the first research question: “According to fourth-grade teachers in the United States, Shanghai, and Singapore, which teaching practices best support math instruction?” (Please refer to Table 2). All eight Asian respondents eschewed the use of differentiated instruction. The teachers from Shanghai and Singapore revealed that they taught the same material to the entire class and gave identical tests to all students. However, they individualized their approach by continually addressing the student’s ability and readiness levels and offering ongoing scaffolding and remediation in the form of mini lessons and individualized attention. Firstly, students were encouraged to present evidence of learning in diverse ways that aligned with their learning preferences. Pupils could show their work on “one size fits all” math problems using pictures, numbers, or words. This approach also made the work more manageable for the less advanced students. The instructors also used continuous scaffolding to lend additional support to needier children. One teacher, P5, articulated:

> I give more attention to those who have difficulty grasping the content. I give students the opportunity to ask questions. I use groups so that the fast learners can help the slow learners. I also use cubes for slow learners especially with teamwork. It encourages inductive reasoning and helps them further understand the content and give the correct answers. It helps speak to their minds.

Readiness levels and mastery are esteemed in P5's classroom. "I give frequent quizzes to assess their current skills. Then I create mini lessons." P6 professed her support for scaffolding:

> After repeating the directions multiple times, I then focus on the slow learners and give them additional help. When students come to me and express concerns I organize a mini lesson. There is no sense in re-teaching the entire class so I concentrate on the small group. We also use manipulatives in our class so that they can know the "why" before the "how".

P7 asservated, "I give special attention to the slow learners and am very patient with them. When they have free time I encourage them to do math exercises." P8 surmised, "I know that each and every student is different. I encourage the fast learners to help the slow learners in groups when they have questions. Concrete manipulatives help as well.

They enable them to visualize the concept in their heads." "I teach the whole class and circulate asking them are there any questions. I attempt to address them fully and not ignore anyone" explained P9. "Memorization is very important too. When they memorize basic facts it makes it much easier to work with the formulas." P10 agreed. "Memorization lets them concentrate more on the immediate problem rather than looking up facts." P11 chimed, "They have to memorize because these formulas keep on recurring." P12 attested, "I give the instructions generally to the whole class but I concentrate on the slow learners. You have to allow room for a lot of questions, corrections, patience, and clarifications.".

P12 further offered:

> Gifted students receive the same content as the rest of the class but they are allowed to take part in competitive events in different schools. Often the students receive, as a result, full scholarships. That motivates all the students and helps build math skills.

Asian teachers were in agreement that grades should be used as extrinsic rewards. P5 succinctly stated, "Grades are rewards" and P7 believed that using grades as extrinsic rewards led to greater motivation by the pupils.

The use of technology was an important characteristic of effective instruction for both New York and Asian teachers. Distinctly, New York and Asian teachers view technology's role in the classroom through unambiguously distinctly different lenses. P1, from New York, considered online learning to be among the best classroom practices. P1 found technology useful for students who need practice with math facts and for those who wanted a challenge. P1 also viewed technology as a means to supplement the classroom instruction. P2 acknowledged “I don't use it (technology) for main instruction. I just use it as a support for practice of skills or for learning math games and learning math facts. P3 is a strong believer in the role that technology plays in math instruction.

> Well, the world is technology-based at this point and so it's becoming more and more important and there's actually a lot of more really cool tools and apps and games and resources out there to use. So I've used them more than ever in the last couple of years.

P3 continued:

> It (technology) can be a good change-of-pace. They can present things in a different way and reinforce skills and make things more engaging for the students. But I don't think we should be solely relying on that even though technology is extremely important. I think that the kids still need to have a little bit of this old school side of me when it comes to math.

P4 agreed. “I think that (the use of technology) depends on the ability of the kids. For me, it's mostly been a lot of practice. There's a lot of multiplication facts or whatever activities and games they can do for that.”

When the New York participants were asked whether all students had their own chrome book or tablet, each one answered in the affirmative. Conversely, in Singapore and Shanghai, none of the participants’ students had access to a personal computer. All visited a computer room. Here they could log on to the internet which was primarily used for research. No interactive programs or games were generally available.

The New York and Asian participants also differed on the subject of homework. My interviews with the eight Asian teachers revealed that all considered the assignment of homework as a best practice. Each stressed the importance of assigning a half hour or more math homework on a daily basis. Concomitantly, the teachers felt, when assigning homework, that parental involvement was a key component. They disclosed that the communication piece between the educator and the caregiver was crucial. Therefore, they encouraged parents to be closely involved with the child's assignments. P8 disclosed:

> The parents have to have an interest in mathematics. The parents should pay attention to the children's needs so they do not feel alone with math outside of class. There is a large number of students in class and it is not easy to give attention to each one. So I rely on parents to keep motivating the students.

P7 rejoined, "When they (parents) buy the textbook it is a very valuable resource." P5 divulged, "I encourage parents to become involved in the students' success. Even if they do not have a lot of time I still encourage them to become involved with whatever amount of time they can spare." P12 summed it up:

> Parents should be supporting their children with math. They have the responsibility of wanting to know what the children are learning and how their children are performing in school. Because if it is only left upon the teacher how the student is faring it becomes a very big challenge because

> the parents have a lot of time they spend with the students. So they have to be there, present for the student as they progress through mathematics.

Conversely, my interviews with the four New York teachers showed that three participants assigned no math homework whatsoever. The fourth, P4, assigned 15-20 minutes of math homework each night. P4 stated:

> I also try to make sure that when I send home assignments, there's some problems already done showing them how to do it in terms of the parents trying to jump in right from the get-go and try to teach them from scratch. So, this way they have an understanding of what they've done in class.

The three New York teachers who assigned no homework cited either district or school policies as a reason for their decision. P1 explained, “No homework is the policy of the school. At our school, prior to Covid, fourth-grade did not generally assign math homework unless a student had missed school and needed to catch up.” Explicitly, some of the New York participants explained that part of the dilemma was the number of new problem-solving algorithms, such as partial products and sums, mandated by the Common Core curriculum. Some parents complained that they did not understand these new methods. P2 disclosed:

> The communication piece is extremely important, especially when using alternative methods. We are sending home information explaining how and I'll be honest and say, with a lot of the Common Core activity, we just don't include homework because parents say, “Well, I just taught them how I

> knew how to do it.” Sometimes it involves leaving parents out of it because it sometimes leads to confusion. There is a lot of pushback from parents when you are using multiple methods. Parents are on board for the standard algorithm across the board. I would say majority of parents want their students to learn the way that they learned.

P3 concurred.

> I think it's a little more challenging for parents than it was ten years ago. Common Core wasn't around. Parents tell me ‘I never learned it this way. I don't understand. It doesn't make sense.’ So they often revert to wanting to teach the kids the way they learned. For this reason, many parents have experienced frustration when helping their child with math homework.

P3, who is head of the mathematics department at her school, has been proactive in the face of the multiple methods conundrum.

> I have weekly newsletters that updates parents on what we're doing in every area, math being one of them. I try to inform them of what methods we are using and I also actually give them letters explaining multiplication and division methods and I send home examples of the different strategies that we're using. They can then try to help their child in a way that's going to make sense to their kid rather than try to frustrate them.

P3 also pinpointed to the importance of parents having a positive attitude towards math. “I think that in general parents and home environment plays a big factor. The value that the family places on education determines how important school is to the student as a whole." P4, the only New York teacher who assigned homework, did not evince high expectations when it came to the child completing the math assignment. "Just encouraging parents to make sure their child sits down and at least attempts the assignment and lets me know if there's problems in doing that."

Assessments were the next code found under best practices. Assessments can be presented in summative and formative iterations. Summative exams are assessments *of* learning (Baht, 2020; Rakoczya et al., 2019). Formative tests are assessments *for* learning. P1, from New York, declared that everyday tasks that students undertake is part of an ongoing formative assessment. P1 mentioned, "Every day is a formative assessment because when I'm looking at their work or even walking around the room I see that this group of kids needs to do more in this area. I feel like everything is sort of a formative assessment."

P1 offered explicit feedback following written summative exams.

> For example, I might write "Good thinking you understand the problem but you made a subtraction error here". Or, "This is great. Your diagram is accurate but you forgot to label it so when you got to the end you got a little mixed up." I might even have a student who can do that very well present to the class because sometimes they can show it or explain it in a different way so that kids think, "Okay, now I get it."

P1 further addressed the issue of summative assessments.

> We do a formal mid-module assessment to guide the rest of the module. Then a formal assessment at the end. On the formal assessments I write on the last page what their strengths are and what their goals are and what to review. Or I write, “You're doing really well at this but you need to learn this.”.I tell the kids this test is not just a test of what you can do, but it's also a test of how well did I teach this to you.

P1 used small groups to review the test and explore student weaknesses. “I would meet with a group the day after and group them by areas they are having difficulty with. Then we would go over it.” Higher order thinking is also included on the summative exams, P1 explained:

> The questions at the start of the test are just informed for the numeric answers and then on each page they get more difficult. The last few are word problems where they need to explain their thinking and that is the most challenging.

P1 also advocated for self- and peer-assessments.

> I think it’s really good for the kids to do self-reflection and also to see samples of other students’ work because they think, “if they can do it I can do it”, and it also sort of raises the bar.

Lastly, P1 demurred on the idea of students offering input when creating exams.

> In math, it's mostly not even teacher created. It goes with the program that we use. The drawback of children offering input is that it is harder for that assessment to meet the standards. Especially in math since it is so specific.

P2 confirmed that they created assessments with a team. Therefore, there is zero input from students. P2 treated practice as a formative exam. P2 also assigned a formal test at the end of each Common Core topic. “That way we can see who needs further instruction,” this participant explained. “We might form an AIS group solely on the data from those. When the children in the AIS group leave class in the afternoon, I arrange mini lessons for targeted math instruction.” P2 added:

> We definitely look at the data from summative assessments including state exams. I spend a lot of time looking at and using it as a reflection tool as a teacher. I use the data to drive instruction based on the number of people mastering the skill and to find areas where the students are needy. I try to find different ways to teach based on the data from assessments.

P2 did not offer students the use of self- and peer-assessments. P2 encouraged a thorough analysis of the problem through the use of the “read, draw, write” strategy. P2 stated, “First they read the problem three times, then they draw a picture that represents the information. Lastly, they write a number sentence or statement based on the drawings.” P2 did not feel there is much time for revisiting previously taught material, or spiraling, on tests. Instead, they concentrate on the current topic. P2 also refrained from offering substantial feedback.

> Sometimes I will go over the exam and talk about the answers with the whole class. Sometimes I do require kids to correct and return and that's how parents come in on it. I will write comments which helps parents see what they're not able to do so they can help them out at home.

P3 divulged that she offers quick checks, or standard-based assessments, about every two weeks. Summative unit tests are given every six weeks. This participant also monitored students on a daily basis and issued exit tickets at the end of the lesson. Exit tickets normally take about five minutes to complete and require students to respond to specific prompts as they apply what they have learned during that day's lesson (Fowler et al., 2019). Proximately, feedback from exit tickets, leads to targeted mini lessons where topics can be revisited. P3 clarified:

> Finding kids with similar needs and pulling them together in small group time helps us differentiate the level a little bit more. After the kids take a quick check or test and it's been graded, I give them back their tests or whatever to look at and then I give them a reflection sheet. It gives them a spot to put down whatever standard or topic or the unit we were doing. Then they assess themselves like something they were proud of doing or something that they recognized they struggled with or still need work and they can circle what they did and then at the bottom write, "I still need to work on this and I'm proud of myself for accomplishing this. So this was a

> problem that I really was stuck with but this is a strategy that I used to help me."

P3, who advocated for the use of peer- and self-assessments, remarked:

> You have to kind of model and practice the thinking behind it and the purpose of it and teach kids how to talk to each other. I think if that's done properly it can be helpful and impactful to hear things from your peers.

Although P3 did not elicit student input when creating exams, she did endorse the use of rubrics and models to which learners can aspire. P3 also explained that higher order thinking is an important component in teaching math. "Critical thinking has more to do with showing the work or justifying your answers or explaining your thinking." P3 promoted the use of spiraling when teaching math. "I feel that math is a subject that needs constant spiral and review so I would say I do that daily. Half the skills we work on each day are older skills." P3's district, along with P2's, disavowed the use of traditional grading.

> We use a one, two, three, four kind of grading rubric scale where three is grade level expectation. Four is exceeding, two is approaching grade level and one is you're below grade level. They get an overall score for math. They also get an effort score: excellent, satisfactory, inconsistent, but then below that it's broken down into some different sub skills. Problem solving, math fact efficiency, things of that nature so a kid could be a three on grade level for concepts and skills and how they've done on tests, but they could

> be a two in problem-solving or a two in their math fact awareness or whatever. So there's a little bit of variation on the report cards.

P3 added:

> After a quiz they have a chance to go over and self-reflect on a reflection sheet. Then I typically will go over the test. If it was one student who would really struggle with something I may pull them individually. Generally, there's trends in the assessment that can apply to more than one student. So I may choose a few questions to specifically highlight with the whole group. Or pull a small group of kids if there were some commonalities in their errors.

P4 concurred with two of her New York colleagues concerning the beneficial effects related to feedback, mini lessons, critical thinking, spiraling, and the lack of student involvement when creating assessments. P4 particularly relied on exit tickets as a formative appraisal.

> I will use exit tickets just to see where each kid is at and who may have some misconceptions. So that helps me identify those kids a little bit more. I'll do an occasional quiz, especially with a longer unit just to kind of break it up a little bit and make sure that they're up to where they need to be before we go further with that concept.

P4 was adamant about the use of spiraling.

> Later in the year, I'm testing what they learned earlier this year because even though it's been three months, we still have to know how to do this. I think they're trying to do some constant review and everything that we've learned and building on that throughout the year is important as well.

P4 also used screening tools three times a year to indicate the progress students have made relevant to the standards. However, P4 refrained from using self- and peer-assessments. This participant also used formative assessments less frequently (every two weeks) than her colleagues. P4 often followed these tests with learning centers. “I use more of a learning center format. For example, my kids who maybe were struggling with place value I might offer them a learning center that will help you practice that skill.” The eight Asian participants offered a panoply of opinions on the role that assessments play in math instruction. All subscribed to the value of testing frequently. Two offered twice weekly written formative exams while six preferred weekly tests. All participants stressed the efficacy of using the results to calibrate their lessons. P5 imparted, “They tell us how much the students understand. If the students are having challenges, we will know how to address the weaknesses.” P7 responded, “I give mini lessons and written feedback so that they are moving in the same direction.” P7 also underscored the importance of using formative tests to tweak their lessons. “I am very flexible about changing the next lesson.” All endorsed the idea that grades on summative exams were significant as a motivating and accountability tool. The Asian participants were in sync with their New York colleagues as they accentuated the importance of including higher order thinking on their assessments. P6 declared “They have to explain their answer step by step. Not just give an answer but provide the reasoning behind it.” The Asian teachers all promoted the practice of allowing the students to offer input when creating exams. P8 commented, “I encourage them to have input into tests because then it helps them to be motivated. P12 divulged, “I allow them input. It gives them ownership.” P9 allowed students to help create rubrics as well. Furthermore, all the Asian participants were in favor of self- and peer-assessments. P10 replied, “They help them know how they are doing in comparison with other students. P9 added, “It helps them solve problems by themselves.” P11 mentioned, “It will enable them to be friendly with each other and be able to assist each other and work well in groups as well as independently.” Additionally, the Asian teachers considered spiraling as a vital component in math instruction. P6 said, “It has a very important role because some of this content is related to the next lesson.” P12 responded, “We do that often. It is usually relevant to a lot of different concepts.”

As previously noted, Shanghai and Singapore schools cultivated a competitive atmosphere as many fourth-grade math students vie for coveted slots in the inter school math competitions. P11 from Shanghai revealed that a competitive culture exists among the faculty as well. P11 disclosed:

> Yes, there is a competitive environment, but it allows you to perform at your best. I am friendly with the other teachers though. The school

> administration supports all of us and gives us room for improvement and even the competitive atmosphere encourages one to be a good teacher.

Specialization was another best practice. Following a review of the responses from the 12 participants in my study, I can report that seven of the eight Asian teachers were content specialists who concentrated on math. The remaining participant taught one subject besides math. In contrast, all four New York teachers taught in traditional self-contained classrooms and were responsible for teaching all the core subjects. Asian teacher P5 voiced, “It is better that I specialize in math rather than teaching a number of subjects. P8 reflected, “I prefer to teach math alone so that I can get the best outcomes among the learners. If you teach other subjects, like some of my colleagues, they get over exhausted. They fall into high stress levels.” P9 added, “Specializing in math is important because at the end of the day, they won't to feel overwhelmed. They also become more familiar with the concepts.” P11 offered, “I think that specializing is good because it allows us to give our best in one area rather than dealing with many areas.” P12 concluded, “It is very important because it enables you to specialize, and it leads to the division of labor among the staff.” Although the New York participants in my study all described themselves as generalists, each noted the merits of specialization. P1 reported:

> I have never tried specializing but it could be worth a try because if you were specializing in math you would be more effective. You would have more time to devote to developing themes and activities and more hands-on things.

New York teacher P2 elaborated:

> I would love specialization. I'm more of a language learner. I built my best understanding of fractions when I used the Common Core fraction module for fourth-grade for the first time. When I looked around at the other people in my advanced math class (in college) I felt they just get those concepts. It was tougher for me to understand. My brain is not like that. So if you did have master teachers of math, right at the elementary, I could see that is definitely being beneficial. Just being able to have people that have minds that work like that. They have passion about it. Then they can explain things to the kids in different ways or have different understanding that I don't.

P3 said:

> I think specialization is a very fundamental value. I know the principal at my school has a very good idea about what age kids should departmentalize. So we don't do that in my building until fifth or sixth grade where teachers specialize in two subjects instead of just all of them. I think if you have a faculty or staff member who maybe excels in an area it's not a bad idea to consider it just because then more students could benefit from that person's experience. I also think that when kids see someone else modeling excitement over math and talking about it constantly it opens their eyes to it.

P4 stated:

> Having taught in a middle school setting and now working in an elementary school setting I have heard administrators say, “You're an elementary teacher so you can teach anything. You don't have to be a specialist in this.” But I almost feel like for some things in math you do. As an interventionist I used to push in the classrooms to help teachers out and it was amazing to me how much re-teaching I need to do to teachers. They would say a number is one-hundred and thirteen. I would tell them to get rid of the "and" because once they get to fifth grade that "and" means you have a decimal point. Simple things that teachers should know but are making errors.

On the topic of teacher self-confidence when teaching math, P4 responded:

> Those that are not competent know that they are not competent. If we have a receptive administrator, he or she will say “Why don't we put you as a reading teacher and you can teach two sections of reading and this other teacher, who's more comfortable, can do the two sections of math.” I wish more administrators would do that. Parents come to you and they say, “Well I was never good at math, so I can't expect my kid to be.” Teachers say the same thing like I wasn't good at math in school either and now they're teachers and maybe they're not comfortable teaching it. So, we

> either need to find a way to make them comfortable or we need to make sure that we have the right people in that slot.

Research Question 2

RQ:2 “What challenges do fourth-grade teachers from the sample cite as obstacles to effective math instruction?” The New York teacher responses produced eleven codes/categories and seven themes (Please see Table 4). There were six codes/categories and three themes connected to the Asian responses to the second research question Please see Table 3).

Time constraints was designated as a major challenge by the New York compeers. Responding to the query of whether there was time to cover the math curriculum each year, P1 said, “I think the key word in that question is cover. You can get through all the topics but can everyone learn and understand it? No, because the pace is too fast for many students.” P2 was in consonance with P1.

> There is absolutely not enough time. You have to push during the course and then you have the New York State deadlines on top of it for the testing. So you're in this push, push, push to get the majority of concepts that are heavily tested on the New York State test. An example is our students are terrible at geometry because geometry is always shelved to the end. I have students in fourth-grade with very little basic shape knowledge because we're always trying to cover the big things so geometry and measurement are always pushed to the end and we have deficiencies in those.

P2 continued, “I would say fifty to sixty percent of the test questions are fractions and then 20% are multiplying and dividing. The rest is just a hodgepodge of everything else like a little bit of perimeter and geometry.” P2 concluded, “I always feel I'm always in this biggest time crunch. I don't always have time for the students to even share.”

P3 commented:

> It's been a struggle. I'll be honest, you know, we found some creative ways to try to make sure that we cover everything, but I wouldn't say that we are able to cover all of the units well in a school year. We have six math units that were modules we were expected to teach, and I would say we can get through four to five of them. Sometimes like one of the six left out to me is the geometry unit in fourth-grade. We often have to find creative ways to cover that because of time constraints so we might do it in small group or RTI time or something like that. But yes, I think time is always an issue that comes up in teaching because of a number of things that we have to teach and then just the level where kids are at. They're not coming to us ready to learn some of the skills we're trying to teach.

Lastly, P4 subscribed to the idea that there was not enough time to cover the curriculum and that, as a result, geometry was generally ignored. Antithetically, all Asian participants agreed that they had adequate time to cover the curriculum. However, they cited the long school day as an obstacle to effective instruction. P10 said, “We have to work very hard and get a lot of cooperation from the students and from the teachers. It is important that the teachers work hard together." P7 stated, "The teachers are very dedicated so we can cover the curriculum. We do this because the teachers work so hard." The Asian respondents did cite the paucity of technological hardware and software as a problem. None of the students were assigned personal computers. All were required to visit a computer lab to gain access to the Internet. Then too, interactive computer programs that enhance math skills were not generally available to the Asian participants.

Class size was another issue which arose in the interviews. A perusal of interview responses from the participants in my study indicated that New York respondents reported much smaller class sizes than the Asian compeers. The New York teachers' average class size of 17 compared favorably with the Asian group who reported an average class size of 32. These larger class sizes were seen by the Asian participants as factors which added to their rigorous workload.

Student attitudes towards math arose as a challenge for both New York and Asian participants. New Yorker P1 conveyed:

> When I introduce a new concept some kids look overwhelmed. They think 'I can't do this.' I remind them that there are 22 kids in the class and that they can teach each other. I know five of those kids really get it. When they partner during groups and start to teach each other and explain to each other and reinforce that teaching in a different way then they're working together and I think that's what makes it successful.

P2 acknowledged, “I feel like a lot of them do have the "I can't" attitude. They have the mentality of giving up very easily. So we definitely use perseverance.” P3 imparted:

> My answer is two-fold. I would say some obstacles in teaching kids fourth-grade math is that kids often have a fixed mindset about themselves with math. Sometimes our students have a bad attitude about math and just don't like it. Other times it's just a lack of confidence that kids have because they haven't experienced success or they know it's hard for them.

P4 revealed that she often heard a common rejoinder from children, "This is too confusing.”

The Asian contingent also encountered negative attitudes regarding math from the students. P5 confirmed, “Students generally don't like mathematics so I encourage them.” P6 has noticed that some students avoid math. “Sometimes they are sick or have family issues. Some of the children are spoiled and so they are allowed to stay home.” P7 lamented, “It is very sad, for instance, students don't have that passion. They are not very interested. Most don’t love math.” P10 summed it up for the remaining participants, “Most students have a negative attitude towards math. When we put them in groups they tend to not want to go back a second time.”

Student readiness emerged as a challenge exclusively for the New York participants. P2 responded:

When Common Core was rolled out our elementary level (grades 3, 4, and 5) was all in on it right from the start but in the primary level school (grades K-2), everybody was using different materials. It wasn't until the last couple of years that our primary school is finally realizing they need to be all teaching the same thing. This led to filling in the gaps and not using the same common vocabulary across buildings which leads to other deficits.

P2 expanded:

> We feel we're superior to the lower grades, kindergarten, one, and two, which are located in the separate primary school. We never see them. We just know what we get from second grade and how far behind it is compared to what our expectations are. After so many years of it we're thinking, are they serious?

P2 then offered specific examples.

> Students get challenged at fourth-grade because they don't understand basic concepts of addition which is putting things together. And division is placing numbers into equal groups. So when they're trying to envision the problem as a whole the lack of understanding makes it difficult. They are always just grabbing at it. They want to grab two numbers and go with it. I will say that deficiencies in reading also play into word problems. So our reading problem also becomes our math problem solving word problems.

> Long division is introduced at the fourth-grade and without understanding of math facts it is very difficult to teach them the standard algorithm for long division. So we have to come up with different methods that don't rely on fact memorization.

P3 added:

> Kids don't often have a lot of the prerequisite skills needed. They don't come to fourth-grade with skills to meet some of the standards so having to try to backtrack and then fill in some of those gaps and holes is challenging. For example, word problems are always one of the most challenging to learn even for higher-level kids because it involves so many different skills. No two problems are the same and it integrates more than one type of operation. Just the sheer reading component is one of the more difficult things so students don't even have the ability to access the problem because of the reading part.

P1 also found student readiness to be a challenge.

> Sometimes I get kids that are on a first or second grade level and that is very difficult. I think the most important subject is reading because in math there is so much reading and students really need to be critical readers and understand one word can change the meaning of a problem. When they take the math tests there is not a single problem that is strictly computational. It really is dependent on being a solid reader.

The practice of mainstreaming children with disabilities has been long-standing in the United States but less so in other nations. Only one teacher, P4, among the New York State participants, reported having a student with disabilities in the class. P1addressed the issue of disabled students and state testing.

> It's true that Special Ed students take the test unless their parents choose to opt out. The student can also refuse. I think many parents of Special Ed. students choose to opt out. However, another concern that I have is that during testing, students who do not take the test do not receive the services they are supposed to get because the teachers are all busy providing testing and associated testing accommodations. It's “all hands on deck” for testing, so that is a problem with our system, in my opinion. Students who opt out end up basically just killing time.

P3 added, “Students with more severe disabilities do not take them (standardized tests). Those students are considered alternative assessment students and have a different process for showing or assessing progress by the state.” P4 confirmed that the test results from disabled students are given to the state. “The state then has their data, but from what I understand there is a complicated algorithm that they use to get their scores and it changes each year.”

Research Question 3

RQ: 3 “How do other determinant influences, such as teachers’ preservice education, and PD affect math instruction in the United States, Singapore, and Shanghai?”

The third research question, related to teacher quality, generated eight codes/categories and three themes among the New York group (please see Table 6).

New Yorker P1 reflected on her college training. “I think it was good. I had several teachers that created really interesting activities. One lesson was about making a bean salad to understand slope. Those kind of things stick in your mind.” P1 also underscored the need for effective teacher training in math. “There are often people who don't like math but if they had the right teacher they would understand better and learn how to make it more interesting and have it apply more to people's lives.” On the issue of PD P1 articulated, “I think formal PD is pretty hit-and-miss. Once in a while there is some good training and that would really be when it's ongoing.” P1 also related that grade-level teachers, due to contractual considerations, formally shared information once a month. “I think informally we meet much more often.” P1 felt that the success of grade-level collaboration depended on the quality of the peer relationships.

> For example, through “Kids Discover the Trail”, I'm partnered with a teacher who really likes math. So we often talk about what we are doing. That sort of a peer relationship is really important. Having colleagues that are interested in math where you can share your ideas and sort of develop your teaching through observing each other or discussing what you're doing is really important.

Incidentally, “Kids Discover the Trail” is a local program that provides curriculum-based field trips. Students visit venues that explore STEM, history, nature, and art (Discovery

Trail, 2021). P2 gave a poor grade to her college training, particularly in regards to math. “I only had one math methods class. We need to have more concentration on math in college.” P2 also gave poor marks to her PD programs.

> The focus for PD is never math. As far as someone coming in teaching new ideas, it's non existent for math. We are always into buying these big-name things like the Kagan method. It's a whole system of building your classroom on cooperative learning. I guess maybe that would tie into improving the math. Everybody ended up having to take two or three big training sessions on it. But it wasn't like you have to implement this, but obviously it was encouraged.

Moreover, P2 recommended having PD in math on a monthly basis. P2 also addressed the issue of teacher collaboration.

> We meet one to two times a week. We recognize we have a problem in math, among other areas, so we were working in conjunction with the Boards of Cooperative Educational Services (BOCES) math leader to try to look at our curriculum. These meetings take place once a month. It forces us to reflect on our teaching and to make change. I do think a lot of people can get stagnant and just do what they've always done. So it was forcing us to reflect and look at our practices and try to find where the gaps are and find strategies to fill them.

P3 explained that her college training was “good” due to a talented professor who taught the class. P3 added, “I think they did an okay job, but nothing compares to the real life experience of being in a classroom and teaching kids.” P3 related that she collaborated with her grade-level colleagues once a week.

> We also had meetings across grade levels the last few years. We had some math teams that met three times a year. One representative from each grade level would come and we would do some above and below standards and we were analyzing priorities and how things lined up.

P3 also shared her thoughts on PD.

> We have PD about four to five times in the school year. I believe that sessions that are part of a continuum are productive. I went to one about two years ago where they built on each previous one and they were connected and that was helpful.

Focusing on classroom strategies is the most productive, P3 opined.

> Some of the training is just understanding what works in the classroom. Some focused more on the dynamics of your students in a classroom, the mentality of your kids. Also, the experience of trying different things over the years is like your own PD, learning from classroom learning experiences.

P4 remarked that she would give her college training a “B.” “I had pretty good training”, she said. P4 wished she had more time in college working with the students. However, P4 felt that not much attention is given to PD in math.

> We've been trying for years to get some math PD. It is so hard to find any math training much less quality math training. So that's been a real challenge. Even in the last five or ten years I've noticed that's really gone downhill.

P4 related that she shared information and planning with her colleagues on a monthly basis.

> Last year we met about once a month for a math committee, which was a cross-grade level meeting. It was just nice to share ideas of what each grade level was doing and how we can move the students to the next grade level.

PD, however, is currently unavailable in P4’s district.

> It's probably been 3 or 4 years since I've even been to one.” P4 lamented, “I see new teachers coming in and they don't have the tools that I do and I wish there was more time to work with those younger teachers to help them head in the right direction.

The Asian cluster of teachers in my study produced four codes/categories and three themes related to the third research question (Please see Table 5). P5 affirmed that she received valuable instruction in college. “I did get good instruction. However, teaching skills should be emphasized because classroom experiences are very different from what you learn in college. I would like more classroom experiences.” P5 has regular monthly PD get-togethers. “They have helped develop other teaching skills: patience and creating interesting lessons. P5 emphasized that the best PD concentrates on classroom skills.” She described her weekly meetings with colleagues. “We share ideas and incorporate concepts that you didn't know and it encourages cooperation with your colleagues. We also coordinate lessons.” P6 agreed that her monthly PD sessions were beneficial. “Yes, it has enabled us to focus on the right teaching skills. We can also share the knowledge we are taught.” P6 rated her college training as “very good.” However, she would have liked more opportunities to do research. P6 met with grade-level colleagues twice a month. P6 mentioned, “It helps me think through my job performance. Also, it gives me skill-based training. You can learn something simple from other teachers.” P7 also rated her college training as beneficial. “I had good teacher training,” she stressed. P7 “exchanges ideas from different experiences” during her weekly collaboration with other teachers. Her monthly PD meetings were also advantageous in her estimation. “They help improve skills that are important for the classroom setting. They tend to make you a better teacher.” She preferred interaction with other teachers rather than listening to lectures. “Interacting with other teachers is important,” she declared. P8 has described her regular PD sessions in a favorable light. “It helps me love my career as a teacher and I can relate to other teachers' experiences. Interacting with others helps improve skills. Learning can take place in a formal or informal setting.” P8 participated in weekly collaboration with other grade-level teachers. “We exchange ideas. And it helps reduce competitiveness as we improve on how we relate to each other.” P8 recalled that she received a solid background in math instruction in college. “On a scale of one to five I would give it a four.” However, she additionally stated, “I would have liked more time for interactions with other teachers. Also, more interaction with a classroom experience working directly with children.” P9 took part in bi-monthly PD gatherings. She feels that it has increased her capacity as a teacher. She also declared that PD is especially helpful to neophyte instructors. “New teachers can use the knowledge to adjust quickly to the environment.” P9 gave good grades to her college training. “On a letter scale I give it a B.” P10 mentioned that she enjoys her monthly PD conferences. “I have had to do lots of research. It enabled me to improve my classroom performance and has let me leave my comfort zone and try new things. And it gives me the opportunity to talk with others.” P10 engaged with other grade-level instructors on a weekly basis. “I learn different approaches to learning. How to make a class lively and ensure that students are cooperating in group work. And how to come up with tests.” P10 recalled her college training in positive terms. “It was fine. It prepared me to be a teacher. But I think I would have more time in primary schools with a lot more interaction with the students. Also, the lecturers in college should be a lot friendlier and more open to the teachers.”

P11 looked back on her college training. “I think I got a good background that enabled me to succeed in what I am doing.” P11 believed that her twice-monthly PD assemblages increased her capacity as a teacher. “Things that help you learn from teachers who are doing things better than you to help you in the classroom setting. It has enabled me to love teaching. It is not just a job but teaching must be a passion." Her weekly meetings with grade-level colleagues has also proved helpful. "You exchange ideas so that your teaching skills can be improved. You can ask other teachers how to meet other challenges in the classroom." P12 declared that she shared experiences with her colleagues on a weekly basis. P12 engaged in PD every month.

> PD has given me more skills in teaching in the classroom setup. It has given me room for improvement. It has also enabled me to develop a continued cordial relationship between myself and the students. And between me and the parents and with the school itself. Both lectures and interaction with the teachers are quite helpful. I find that if PD sessions focused on classroom strategies in math that would be quite good.

P12 imparted that she was satisfied with her college training. She commented, "I had a good background." However, she felt there should be a more open climate at the college level. "I wish every teacher could express their point of view. And there should be more rapport between the professor and student. Also, more experience in the classroom with children."

Research Question 4

RQ:4 "How does SLT influence fourth-grade math instruction in the United States, Singapore, and Shanghai?"

The fourth research question elicited 10 codes/categories and two themes from the New York State participants (Please see Table 8). The Asian teachers generated seven codes/ categories and one theme (Please see Table 7).

A perusal of the totality of responses related to the fourth research question indicated that the Asian group unanimously favored DI as their primary conduit but utilized collaborative activities to complement their teaching. Conversely, three of the four New York State participants preferred collaborative approaches but used DI on certain occasions. The fourth New York State participant selected DI as her instructional mode and her experiences will be discussed in the Discrepant Case section.

P1, from New York State, explained the role of DI in her classroom.

> I almost always start off with some sort of DI. There is vocabulary and concept skills to model before the students start working on their own. So there's a gradual release of responsibility from modeling, actually talking about it, and then they get to work on it. I gradually delegate responsibility for the learning.

P1 felt that collaboration was indispensable. P1 described how students generally collaborated before they worked independently. "I circulate to see if there is misunderstanding. If I think I didn't explain it very well I might stop them and do some more DI to sort of redirect what they're doing."

P1 also used group captains in her collaborative activities. "If a handful of students understand a concept very well, they may be the leader in a breakout room." P1 believed that collaboration ignites creativity. “When students do group work, they witness a variety of methods and hear a variety of explanations. This encourages the students to be more flexible and creative, rather than thinking there is only one way to solve a problem.” P1 discounted the plausibility of utilizing discovery-oriented approaches. ‘Ideally, wouldn't it be great if they could do everything by discovery, building a project and all that sought of thing but with the amount of things we have to teach in a year doesn't really make it workable.

Moreover, P1 recounted that building a safe environment was crucial.

> I think the first thing you need to do is build a classroom community where the kids feel comfortable taking risks and they trust each other. I don't want them to collaborate if they feel insecure or if they feel like someone is just going to make fun of them or not help them.

P1 believed that collaboration helps develop life skills such as communication, cooperation, conflict resolution, consensus building, higher order thinking, and problem solving. P1 also revealed that she utilizes real-world problems in her instruction and requires her students to apply prior knowledge to new problems. In addition, P1 took into account the readiness levels, learning preferences, and personal interests of her students. P3 summarized her approach to SLT and DI.

> I do use DI, but my math classroom is very hands-on and practicing and interactive. I try to find a balance of both DI and collaborative learning. But in a typical year for me collaboration in math is essential to my classroom.

> But I think that because so many things in math are sequential or process-based kids need to see it be explicitly taught. During my DI I try to model a ton of multiple ways of doing things but also model my thinking because I think that's huge for kids at this level in terms of understanding.

P3 added authenticity to her lessons by connecting math problems to the real world. P3 believed long-term projects are also beneficial but not practical in her New York State classroom. “Sometimes what you think is best for students might not always be the way in which you can do things. I will say that the time factor for me is always there.” P3 also affirmed additional benefits to social learning. “I just think that different perspectives and different views always create better solutions than narrow single approaches to things.” Individual accountability is mandatory in P3’s class. P3 explained:

> Because if you don't do that (individual accountability) you have one kid that gets it who does all the work for everyone else and then you have the kids that sit there and don’t open their mouths. So that's a constant for me whether you're working in a partner pair or a small group. I need to see everybody participating and giving whatever it is that they can give. The Kagaan strategies that we use involve giving kids certain roles. These strategies involve making sure that kids get a say in things and so it also allows voice for those kids who are a little bit more socially shy or quiet or reserved. They also make the kids who are naturally more leaders and outspoken and louder sit back and listen and relax a little bit instead of

> constantly feeling like they do need to take over and take charge. So I think it works to balance out things.

Finally, P3, like her colleagues P1 and P2, found that there is simply not enough time to teach for mastery.

P4 divulged that DI was helpful for the struggling students when she stated

> I generally teach the lower end of the spectrum so DI is very important for those kids. Where you have your gifted or your higher kids you can just give them a problem and say, "Tell me about this, argue about it, figure out with your partner, talk about it. What do you think?" And it's really hard to do that with your lower kids because they just sit there and stare at each other.

P4 concurred with the other New York State participants who introduce new topics through DI.

> I think especially when you're introducing a concept, I do that and then the more the kids learn about it the more I can get away from that DI and get them thinking a little bit more deeply about it.

P4 relied on collaboration for most of the math instruction.

> I focus on collaboration. I mean Vygotsky, who was an educator who basically said, we learn best when we learn with someone else and eventually we're going to get to the point where we can learn on our own. So I rely on kids working together to help them understand it or model their

> thinking. A lot of kids won't do that with an adult but they will do it with each other.

The Asian aggregation generally relied on DI as their modus operandi. P5 described the reduced role that SLT plays in her teaching.

> The teacher being at the center and the students basically listening helps avoid confusion and everyone understands. I use DI most of the time. At various times I do use small group instruction but this is not very common. Students are not assigned roles. They come up with the common answer. So I find out how they managed to do it. They work mostly as a team. When time allows there is more individual accountability.

P5 recognized the value of collaboration. “Collaborating on a project enables students to have more interest in math and allows them further insights on any topic in math and allows them to be more comfortable when learning.” P5 was also in accord with the other seven Asian participants that her students acquired mastery of the concepts before moving on to other topics. P6 confirmed that she often taught the lesson while the students sat and listened. P6 also provides clear instructions with sequential steps related to math procedures. P6 does incorporate collaborative activities on occasion. “They gather in small groups at times and share information. I use groups so that the fast learners can help the slow learners. This helps them teach each other.” P6 divulged that there are no classroom management problems when the children participated in group work. “They are very cooperative and do not get agitated when in groups.” P6 assigned group leaders when collaborative activities occur. This participant also mandated individual accountability on many occasions.

P7, together with her other colleagues, prioritized DI when teaching math. However, P7 reported that she formed heterogeneous groups on occasion.

> There is individual responsibility to keep the students focused otherwise the children feel they are free to do anything. They should each have responsibility. Sometimes, at the end of the lesson, each of the students speaks out and expresses something related to the answer and sometimes there just a spokesperson for the group.

Uniquely, P7 did implement discovery-based projects “when there is time.” Here, students learn inductively, talk together, and come up with a solution or arrive at the understanding of a concept. “It’s important that they all get a chance to share.” P7 also stated that group work does not lead to class management problems. “The students are focused and the groups are patrolled so this is not a challenge.” P8 generally took an active role in presenting information to the entire class. P8 customarily models procedures. However, on occasion P8 utilized the affordances of SLT. “The students brainstorm ideas for solving problems in small groups.” This participant concurred with a few of her colleagues that the noise level does rise during group work. “You have to like control it and I think the group leaders help the students learn in a controlled manner.” P8 also believed that all students should participate in their groups. Individual accountability was required. “I watch them closely and at the end of the day I have them reflect.” P9 affirmed that she directly taught concepts, principles, cognitive strategies, and physical operations. “I provide instruction to the students while they listen.” P9 further asserted that students interacted with the instructor far more than they engaged with each other. She offered, “I respond to the majority of students’ questions.” However, P9 does complement DI with collaborative activities. “You give each child a role and make sure they are participating fully in the groups. They stay focused and they know there is individual accountability at the end of the lesson so they don't make much noise.” P10 relied on explicit instruction as her primary mode of instruction. P10 described how she introduced a new topic.

> Every student knows that it is a completely different topic. You have to explain clearly. You have to attract their attention by asking questions like what do you think this lesson is about. They raise their hand. The student explains. If they explain it incorrectly you correct them. If they explain it right you appreciate them. Make it interesting. Come up with models and go through the steps and encourage them to ask questions. At this point they are watching me demonstrate but they are very keyed in.

P10 also used “think aloud” strategies when modeling procedures. P10 found merit in collaborative activities as well.

> Group work helps students who may not be comfortable asking questions to the entire class. When they talk among themselves in small groups they come up with ideas about how to grapple the problem. They can

> communicate how they feel and share what they know. Each student adds to the other students' knowledge and that would help them to think creatively.

P11 explicitly taught concepts, principles, strategies, and operations. P11 described how she commenced a typical lesson.

> I make sure they know that this is something they are going to like. They should remain focused and they just write down their objectives. You have to convince them that the concept is very easy to learn. Just simplify it. Also, when I teach division, I put models on the board and encourage questions.

P11 relied on group captains when she implements collaborative activities. “The noise level does go up but you have to regulate it.” P11 spotlighted the value of group work. “Because they can apply what they know together. The students build on each other's knowledge and this starts them in new directions so they might create new ideas.” Each student in the group is assigned a role. Individual accountability is imperative. “I call on students at random and also have a group discussion.” P12 stated that in her classroom she provided a series of steps to follow when solving a math problem. Afterwards, students routinely worked alone when solving problems. P12, in a manner similar to the rest of the Asian colleagues, found value in collaborative activities. “When they are in groups they feel that they are at liberty to express themselves. They build on each other and this sets them in new directions.” P12 assigned roles to each member of the group.

She then assessed the learning in the following manner. “You'd ask questions in the form of a class discussion. Call on students to explain something.”

Discrepant Case

Discrepant cases or contradictions in the data can give rise to unexpected findings (Nowell et al., 2017). Discrepant cases occur when a code does not fit into any of the established themes (Nowell et al., 2017). This indicates that a participant’s perception or experiences differ from the mainstream evidence. Such was the case of my New York State participant, P2. After studying her responses to the interview questions I created the category “anomalies to best practices” to store codes which diverged from the main themes (Nowell et al., 2017). P2 differed from the other New York State teachers, not only in her denouncement of her math training in college, but also in her embracement of DI to the near exclusion of collaborative learning. P2 stated:

> I am a huge advocate for DI and when it comes to math, even though I know there's a better way. I always feel I'm always in this biggest time crunch and then I don't always have time for the students to share. I guess I just don't pay attention to the value of that. We do share as a group at times but it’s mainly discuss your answer with your partner and explain how they got the answer. But it's not a large part of my approach.

P2 believed that modeling group behavior would take up valuable time.

> You can't just throw kids together and say talk about it. You have to spend large amounts of time modeling that and showing them how to do that. I

> don't feel they come to me having those skills and knowing how to talk about math right off. So I would be starting fresh with how to have math conversations within a partnership or a group.

P2 does use collaborative activities during the 30-minute blocks of time allotted to AIS instruction.

> So the five or six kids that are left in the classroom with me might work on enrichment. We might take area and perimeter and work on a group project or an individual practice. That would be more like where project-based learning would play a role in math in my classroom as opposed to during that one hour.

P2 avoided the use of group leaders as well. “If I do a math class and I'm going to have a few people get together, I don't know if I would per se assign a leader in that scenario.” P2 did advocate for individual accountability but does not assign roles. “Requiring all the students to write the problem down. Things like that where you don't just have one person recording. Holding them each accountable for solving the problem.” P2 also believed there are benefits and drawbacks to collaborative groupings. “I feel kids that are prone to sharing and have a lot to say. It definitely can be really beneficial but I think it's sometimes painful for my lowest students. They may literally not have anything to share.” P2 used, in a supplementary manner, small homogenous groupings despite her belief that many of the lower performing students would have little to say. P2, unlike her New York State colleagues, relied primarily on DI. P2 reported that she took an active role in presenting information to the entire class while the students watch and listen. P2, rather than other students, responded to the majority of questions. “Yes, I pretty much use the ‘I Do We Do You Do’ method. First, I teach it through DI, then we do guided practice. We work on it together and then gradually they work into the independent practice part.” P2 then described her DI method of instruction in more precise terms.

> I explain what the objective is. What we need to be able to do so that they can see where we're going. At the beginning of a unit, what is our end game, what is our goal that we are trying to get to. Each day, at the beginning of the lesson, I tell them what we are trying to do by the end of the lesson. I am brutally honest with kids. I tell them, “Today, you're going to need to put on your perseverance pants. We're going in hard.” I'm not going to sugarcoat it like it's going to be so easy because perseverance is the main word I use in my classroom with kids, especially when it comes to math because I feel like a lot of them have the “I can't” attitude. They have the mentality of giving up very easily. So we definitely use perseverance. But yeah, I just give them the objectives, give them the goals of the lesson and then we dig in.

P2 responded to a question on introducing cross curricular themes in the following manner.

> I don’t use them too often. I would say more like real-world problem-solving, you know a situational kind of thing. Not like putting social studies in math.

P2 was asked her opinion of self- and peer-assessments. “I don't use them a lot actually. P2 also refrained from spiraling.

> We have a small amount of time for that but it all goes back to how much we’re expected to cover in one year's time. So there's not always a ton of time for reviewing. I basically concentrate on the current content on my exams.

P2 limited her feedback to students.

> I don't give them a lot of feedback. Sometimes I will for the whole class go over the whole exam and talk about the answers. Sometimes I do require kids to correct and return and that's how parents sometimes come in on it. For parents to see what they're not able to do and help them out at home. I will write comments and stuff but not heavily.

P2 also confirmed that students do not use inductive reasoning to reach generalizations about mathematical concepts and problem-solving strategies. “I do think concrete manipulatives are more important at the primary level, building the foundational.” Although P2 differentiated her instruction based on the difficulty of the offered problem, she does not take into account student learning preferences and their personal interests. P2 also disavowed the practice of students coaching each other. Finally, her math lessons do not incorporate life skills such as communication, cooperation, conflict resolution, and consensus building.

Evidence of Trustworthiness

The issue of trustworthiness is critical to all studies involving qualitative research. Trustworthiness refers to the degree of confidence the reader has in the presented data (Creswell & Miller, 2020; Hussein et al., 2015). The value of a qualitative study is often directly proportional to its degree of trustworthiness (Creswell & Miller, 2020). Qualitative researchers consider trustworthiness as being thorough and accurate when conducting a study (Cypress, 2017; Lemon & Hayes, 2020). Thusly, certain criteria must be met to attain trustworthiness in qualitative studies. These indicators are styled credibility, transferability, confirmability, and dependability (Lincoln & Guba, 1985; Nowell et al., 2017; Stahl & King, 2020).

Credibility is considered the most important factor in substantiating trustworthiness (Amankwaa, 2016; Rose & Johnson 2020). Credibility alludes to the accuracy and plausibility of the study’s findings (Amankwaa, 2016; Rose & Johnson 2020). Credibility is augmented through truthful descriptions of the teachers’ lived experiences (Cypress, 2017; Lemon & Hayes, 2020). The credibility of my study was amplified by the inclusion of thick detailed descriptions of the attending phenomena acquired through unambiguous, open-ended questioning (Lemon & Hayes, 2020; Noble & Smith, 2015; Nowell et al., 2017). Iterative questioning also played a role. Iterative, or rephrased questions, were operationalized throughout each interview to detect misleading information (Cypress, 2017; Merriam & Tisdell, 2015; Ravitch & Carl, 2020).

Credibility was also strengthened through reflective activities, such as inscribing my contemporaneous reactions in a research journal. Many of these inscriptions involved the process of bracketing. The purpose of bracketing is to hold in abeyance any preconceptions and assumptions which might adulterate the credibility of the study (Dörfler & Stierand, 2021). During bracketing, I identified, monitored and mitigated personal biases with the goal of interpreting data from a neutral standpoint. Moreover, I was particularly conscientious about obviating my own assumptions in my follow-up queries. To illustrate, when asking how participants felt about specializing in math, rather than teaching a number of subjects, I used neutral language and avoided leading questions. Requisitely, the responses to these questions reflected, clearly and verisimilarly, the teachers' perceptions about their lived experiences. Triangulation was another approach that bolstered the credibility of my study. Succinctly, triangulation refers to the use of multiple forms data when establishing findings (Korstjens & Moser, 2018). Research has affirmed that my use of triangulated data sources (interviews from two distinct sources) is vital in validly supporting evidence, substantiating conclusions, and enhancing the credibility of a study (Arriaza et al., 2015; Korstjens & Moser, 2018; Yin, 2014). Saturation detection is another technique I invoked to support credibility (Hennink & Kaiser, 2019; Hussein et al., 2016). Saturation takes place when redundancies appear in the collected data (Hennink & Kaiser, 2019; Hussein et al., 2016).

In other words, when the researcher realizes there is nothing new to be learned. Subsequently, when I reached this juncture with the Asian and New York State clusters, I decided that additional information would not promote further understanding (see Hennink & Kaiser, 2019; Hussein et al., 2016). My study's credibility was also intensified through member checks where the participants perused interview transcripts to authenticate their responses (Kornbluh, 2015; Korstjens & Moser, 2018). More specifically, I emailed the transcriptions of the interviews to my 12 participants to monitor for errors, clarify statements, or include supplementary information.

Transferability may be described as the ability to apply the current study's results to other contexts (El Hussein, Jakubee & Osuji, 2016; Hussein et al., 2015; Nowell et al., 2017; Stahl & King, 2020). Since qualitative studies generally do not include random sampling or a large number of participants one must concentrate on the depth rather than the breadth of the data. Thusly, one strategy to advance transferability is the acquisition of thick rich descriptions concerning the participants' perceptions (Creswell & Miller, 2020). By describing a phenomenon in sufficient detail other researchers can evaluate the extent to which the conclusions might be transferable to other contexts. I also established a research template by describing the setting, the participants' experiences, and their assigned teacher roles. This will offer future researchers the capability to coordinate their own efforts with my study. In addition, my sample included representatives from 11 schools across three nations or cities. This variation in participant selection also supported the transferability of my study. These participants offered an array of teaching experiences. Hence, when they expressed concurrence on a particular issue, the transferability of the findings gathered additional cache.

Dependability may be defined as the extent to which the study can be replicated with consistent results (Korstjens & Moser, 2018). According to Stahl and King (2020), dependability is authenticated by providing a description of a study with sufficient detail to enable another researcher, with a similar assemblage of participants and context, to reach comparable conclusions. I have addressed these criteria by presenting clear and sequential documentation of my research procedures from conceptualization and implementation to the reportage of results (Arriaza et al., 2015; Korstjens & Moser, 2018). Subsequently, I explained, in detail, how the data led to the interpretations, recommendations, and conclusions of the study.

Scholarly research as affirmed that the degree of dependability is closely aligned with the study's measure of credibility (Korstjens & Moser, 2018). Therefore, there is a strong connection between the dependability of my study and its degree of triangulation between the two groups of sources, member checking, and reflexivity. I further enhanced the dependability of my study by consistently addressing the same set of open-ended questions, albeit with appropriate follow-up queries, to all participants.

Confirmability denotes that I have maintained the objectivity of my study by mitigating the effects of any personal assumptions or biases (Korstjens & Moser, 2018). Researcher beliefs, values, and passions are seen in a positive light as an impetus for their engagement with the topic (Korstjens & Moser, 2018). However, I must monitor these inner credos on a continual basis so that the study's findings do not reflect my feelings; rather, they must be a true representation of the participant's responses (Amankwaa, 2016; Stahl & King, 2020). Confirmability may be augmented through the reflexivity (Forero et al., 2018). Reflective activity on my part, with the subsequent inscription of these metacognitions in a journal, helped me monitor the extent of any personal assumptions (Cypress, 2017; Dorfler & Stierand, 2021). These actions enabled me to lessen any personal bias during the data collection and analysis processes. To further validate the dependable nature of a study, the researcher can also integrate a technique known as an inquiry (Arriaza et al., 2015; Forero et al., 2018). To this point, I have offered, throughout this dissertation, explicit descriptions of each research step taken from the commencement of the project to the conclusion of the study (Arriaza et al., 2015; Forero et al., 2018). Fundamentally, I provided evidence that my study was systematic, objective, and worthy. First, I explained the problem and the purpose of the study. Moreover, I strove to ensure the transparency of the theoretical and methodological processes involved in my qualitative study. Accordingly, I disclosed the conceptual lens and detailed each pertinent element of the methodology including participant recruitment. I also clarified the data collection and analytic process and presented a well-reasoned and data-based interpretation of the results. In addition, I delineated the limitations of the study, described its implications, and offered my recommendations for further study. In addition, I enlisted the services of a peer debriefer who signed a confidentiality agreement. The peer debriefer scrutinized the codes/categories, themes, and the collected data, including my notes, to ensure the findings from my study were accurate, free of researcher influence, and authentically represented the experiences of the participants. The peer review affirmed my findings but suggested certain additions as well as refinements in the wording of the themes. Thereupon, I included these additions and made edits to the themes based on the peer debriefer's input. As a result, the particularized descriptions of the involved processes will enable others to evaluate the study's accuracy and determine whether the findings, interpretations, and conclusions were supported by the data (Forero et al., 2018). Finally, the itemized processes established the foundation for enabling other researchers to independently assess the viability of further research.

Summary

The purpose of this qualitative exploratory case study was to develop and analyze thick rich descriptions of an international array of teacher perceptions about different approaches to effective fourth-grade grade math instruction, inherent obstacles to sound instruction, their college training, and PD. Using open-coding and thematic analysis, I identified 111 codes/categories and 30 themes. The following is a summary of the themes as well as their pertinence to each research question.

RQ 1: According to fourth-grade teachers in the United States, Shanghai, and Singapore, which teaching practices best support math instruction?

Themes from the New York State teachers, emanating from RQ 1, confirmed their roles as facilitators of instruction in a collaborative setting. This approach offered students the opportunity to practice collaboration, communication, and consensus building as well as the heightening of work place skills. However, the collaborative format is time-consuming which resulted in instances where teachers could not complete the required curriculum by year's end. Collaborative formats may also lead students, after lengthy discussions, to reach ambiguous or incorrect conclusions. Personalized learning, with an emphasis on tiered instruction, was a second theme related to best teaching practices by the New York State participants. Adapting lessons to the individual interests and ability level of each student is productive and undergirded by research (Crutchfield & Inman, 2020). However, research has also revealed that students in each group may not be subjected to equally high expectations. This may have profound consequences concerning their learning curve. I also found, as a result of the interviews, that New York State teachers underlined their intentions to involve parents, as formidable partners, in their children's education. As a result, multiple avenues of communication and venues were offered to parents for their consideration. This practice aligned itself with, and is bolstered by, the predominance of research literature. Teachers from New York State also encouraged the cultivation of safe, non competitive classrooms, and generally utilized informal formative assessments based on observation rather than on written tests.

Responses from the Asian teachers regarding best instructional practices in the classroom unveiled the following. Direct instruction was the predominant method of purveying knowledge among the Asian participants. This method established the teacher in an authoritarian mode and created the groundwork for clear, unambiguous, time-saving instruction (Engelmann, 2014; Stockard et al., 2018). On the other hand, DI often left scant time for the practice of many life skills. The Asian participants also divulged that they selected whole class instruction for math generally eschewing tiered- or multi-group instruction. This option required high expectations for the entire class. Although an element of personalized learning was lost, such as addressing student skill levels, teachers felt they individualized learning by formulating mini lessons where slower students could be offered targeted scaffolding. In addition, Asian participants opted for frequent written formative tests to gauge the current progress of students. The Asian contingent also differed from the New Yorkers by honing the students' competitive instincts with strong attention to grades and scholarships as well as supporting competitive math contests among schools. Finally, regarding best practices, the Asian teachers endorsed the New York State teachers' viewpoints concerning the importance of frequent communication with the parents, concrete manipulatives, spiraling, reflections, self- and peer assessments, constructive feedback, real-world problem solving, and critical thinking.

RQ: 2 What challenges do fourth-grade teachers from the sample cite as obstacles to effective math instruction?

Many themes regarding challenges to effective instruction emerged as a result of interviewing the New York State teachers. Lack of adequate timeframes, perhaps due to the shorter school day and collaborative learning, materialized as the preeminent challenge. These compressed time schedules also affected, in a retro manner, the learning readiness of many students upon entering the fourth-grade. Moreover, the mainstreaming of special education students levied additional demands on the instructors' time. Finally, the lack of adequate temporal resources as well as the connection between state test results and teacher evaluations, compelled the New York State participants to often "teach to the test."

Although New York State teachers strove to enlist the support of the parents, confusion over Common Core math algorithms left many students without parental support on the subject of homework. Additionally, during the course of the interviews, participants disclosed that the New Yorker's classrooms were virtually devoid of gifted student programs. Lastly, all New York State teachers lamented the lack of opportunities to accrue specialization credentials to focus exclusively on math instruction.

The coterie of Asian participants reported fewer challenges. However, they were unanimous in describing the eight- or nine-hour school day as formidable. There was also a dearth of technology in the Asian classrooms. Students lacked access to personal computers as well as software programs for reviewing subject matter. Finally, teachers revealed frustration with the students' negative attitudes towards math.

RQ 3: How do other determinant influences, such as teachers' preservice education, and PD affect math instruction in the United States, Singapore, and Shanghai?

The New Yorkers identified three factors which influenced the quality of their math instruction. First, teachers concurred that they received fair to poor preparation in college regarding the teaching of math. Secondly, the participants found PD to be generally ineffective due to the abstract nature of the material as well as the infrequency of the sessions. Thirdly, the teachers recognized that grade-level collaboration was systematic and influenced their math instruction in a positive manner.

Alternatively, the Asian contingent deemed their college training in math as effective, found that ongoing and systematic classroom-related PD lead to improvement in their teaching capacity. However, the Asian teachers were consonant with their American colleagues on the positive aspects of systematic collaboration with other grade-level teachers.

RQ: 4 How does SLT influence fourth-grade math instruction in the United States, Singapore, and Shanghai?

The resulting data indicated that the New York State math teachers relied on many tenets of SLTby using collaborative, small group, tiered activities with an emphasis on student cooperation and autonomy. However, the New York State participants used targeted DI on appropriate occasions, usually followed by group interaction. In contrast, Asian teachers generally eschewed the principles of SLT, using direct and whole class instruction/assessments supplemented only occasionally by collaborative activities.

I identified one discrepant case among the New York State contingent. This participant exhibited many pedagogical approaches distinctly opposed to their colleagues' practices. However, these modalities were not supported by the research-based findings in the literature review.

In Chapter 5, I will complete this study with a compendium of the findings. I will also discuss my interpretation of the results, the limitations to the study, as well as recommendations and implications. Finally, in the conclusion, I will create a short narrative that captures the key essence of the study.

Chapter 5: Discussion, Conclusions, and Recommendations: Introduction

The dilemma of why many fourth-grade students in the United States were not reaching proficiency benchmarks or performing as well as their peers on international math tests provided the impetus for the study. The subpar performance of many math students in America has broad ramifications. Success in math during these early years of development is vital both for college readiness and thereupon, the ability of students to compete globally in STEM careers (Boaler & Sengupta-Irving, 2016; DeJarnette, 2018; Vakil & Ayers, 2019). Hence, low math achievement not only affects academic performance but also exerts a deleterious impact on the global ascendency of the United States (DeJarnette, 2018). Patently, educators may use the study’s findings to initiate changes that may lead to increased student achievement in math. In a congruent manner, the purpose of this qualitative exploratory case study was to develop and analyze thick rich descriptions of an international array of teacher perceptions about different approaches to effective fourth-grade grade math instruction, inherent obstacles to sound instruction, their college training, and PD. The nature of the study subsumed the characteristics of a qualitative exploration. That being the case, I examined the perceptions of a purposeful sampling of 12 fourth-grade math teachers, and utilized a case study design. The study’s key findings revealed the difference in many math instructional approaches among Asian and American schools, distinctions regarding specialization of instruction, discrepancies between American and Asian instructional timeframes, and contrasts between the quality of Asian and American preservice education as well as PD.

Interpretation of Findings

Findings related to the study included teacher viewpoints concerning best teaching practices in math, inherent obstacles to sound instruction, as well as college training and PD. In addition, the participants discussed the effects that SLT exerted on instruction. My qualitative approach was implemented to advance in-depth interviews in a naturalistic setting (Creswell, 2018; Yin, 1984). As a result, I was able to collect thick rich descriptions of teacher perceptions regarding math instruction as well as other determinant factors that influence instruction (Creswell, 2018). My study’s purposeful sample facilitated the exploration of the thoughts of four teachers in the United States, four teachers in Singapore, and four teachers in Shanghai. As the investigation progressed, I employed open coding and thematic analysis to organize, categorize, represent, and analyze the collected data.

The exploratory case study design supported the qualitative ordonnance. Case study methods, through in-depth scrutiny of a very limited number of individuals, enable the deep examination of data within a specific complex context (Gioia, 2020; Yin, 1984). Fundamentally, by using a case study design, I was able to describe data accrued in an empirical environment, as I revealed the intricacies of the teachers’ real-life experiences and beliefs (Gioia, 2020; Yin, 1984). Although the transferability of case studies is often viewed as a vulnerability, proponents contend that even a single observation can represent a tenet germane to numerous contexts (Lincoln & Gubba, 1985; Stahl & King, 2020).

The researcher is the instrument for the collection and analysis of data across all phases of a qualitative research project (Creswell, 2018; Yin, 1984). Therefore, since I was part of the process, bracketing became an essential element of my qualitative case study. Bracketing enables researchers, through on-going self-reflectivity, to identify and suspend pre existing cultural factors, biases, assumptions, preconceptions, or previous experiences (Dforfler & Stierand, 2020). These predispositions are then placed in brackets or held in abeyance during the data collection and analysis phases of the study (Dforfler & Stierand, 2020). As a result of bracketing, researchers can consider how their beliefs affect their understanding. This enables investigators to create interview questions and collect and analyze the data in a neutral manner (Eden & Ackermann, 2018). Consequently, by documenting and monitoring my reactions through note-taking, I was able to avoid asking leading questions while maintaining a neutral tone and language during the interviews. I was also prepared to detect nuances in the participants' responses to teacher-and student-centered practices as well as ability grouping. Resultantly, my coding became more explicit and I arrived at an accurate understanding of the interviewees' subjective accounts of their lived experiences.

My doctoral study was necessary since the collection, analysis, and dissemination of data related to fourth-grade math instruction in the United States, Singapore, and Shanghai may enhance teachers' knowledge and skills and increase student achievement in math (Aljaberi & Gheith, 2018). Therefore, the recommendations of this study will help amend the gap in the practice and may constitute an important component in the effort to improve the below-par achievement of many fourth-grade math students in the United States. Furthermore, my study may highlight the connection between students' achievement in fourth-grade math and the quality of preservice education and PD.

Data analysis revealed that my four research questions produced 30 themes. Please refer to the Tables 1-9 for the complete list of codes/categories and themes pertinent to each research question.

RQ 1: According to fourth-grade teachers in the United States, Shanghai, and Singapore, which teaching practices best support math instruction?

Scrutiny of the themes related to the first research question divulged the following. Participants from all geographical sections proffered many similar perspectives regarding a host of best instructional practices, including the use of concrete manipulatives, scaffolding, mini lessons, and ongoing feedback. Moreover, all participants endorsed critical thinking, reflections, and cross-curricular subject matter as well as noting the importance of parental support. However, there were significant differences between the New York State and Asian cohorts on how much emphasis should be placed on DI vis a vis collaborative learning. Each cohort also viewed the following topics through distinctly different lenses: differentiation, specialization in math teaching, homework assignments, teaching for mastery, frequency of formative testing, student input into assessments, grading, the role of technology, classroom size, mainstreaming, time constraints, and teacher training. Finally, there was one discrepant case, which emerged from the New York State group, who offered many unorthodox points of view.

Differentiated instruction evolved as an important code for effective instruction among the New York State participants. Differentiation is the process of personalized learning where the teacher correlates the students' learning characteristics and demonstrated abilities to a developmentally appropriate curriculum level and instructional strategy (Crutchfield & Inman, 2020). Differentiation subsumed a number of other codes according to the New York State teachers. These codes included ability grouping, concrete manipulatives, study aides, scaffolding, teacher expectations, technology, and mini lessons. All the New York State participants facilitated differentiated instruction by dividing the class, based on ability, into three homogenous groups. Forming ability groups was viewed as a logical strategy since fourth-grade school classrooms often feature a panoply of diverse learners in terms of their academic ability and achievement level (Prast et al., 2018).

In contrast, my study, supported by the research of Boyd and Ash (2018), found that teachers in Asia placed little emphasis on differentiation by task or content and largely avoided in-class grouping based on prior attainment. My study confirmed that all Asian participants abnegated differentiation, taught the same material to the entire class, and held high expectations for each student. Additively, the Asian teachers required all students to memorize facts in lieu of study aids while facilitating mastery of the content.

The Asian teachers also used extensive scaffolding and mini lessons to help struggling students.

As previously stated, all four New York State participants differentiated math instruction by forming three ability-based groups. Research has suggested that ability grouping is efficacious in classrooms where students exhibit a wide range of cognitive abilities. These groupings take into account the children's readiness levels, interests, and learning styles (Bodovski, 2020). Ability groups are assigned by the teacher within individual classrooms and last no longer than the term's end (Prast et al., 2018). Ability grouping offers instructors the opportunity to teach at a pace and difficulty level conducive for all learners. This will prevent struggling pupils from falling behind (Legette & Kurtz-Costes, 2020). However, studies have determined that differentiated instruction involves additional investments of time and effort (Crutchfield & Inman, 2020).

Teachers who practice ability grouping need to remain flexible by monitoring students' academic progress on a continuing basis to pinpoint learners' changing understanding and needs (Prast et al., 2018). Formative assessments are usually the tool of choice to monitor student progress on a timely basis. However, all New York State participants offered only intermittent written formative assessments, relying instead on informal observations. The Asian constellation, in contrast, gave formal formative assessments once or twice a week.

As background, assessments play a central role in the teaching and learning process. Le et al. (2021) postulated that good teaching is inseparable from good assessment. Formative assessments are assessments *for* learning which can be contrasted with summative assessments which are assessments *of* learning. (Baht, 2020; Rakoczya et al., 2019). Formative exams are utilized primarily to determine student understanding and teaching effectiveness at a specific point-in-time (Baht, 2020; Dixon & Worrell, 2016). The frequency of formative assessments exerts an essential role in explaining academic performances and are therefore integral in ensuring flexibility when moving students to a higher or lower ability group (Bochis et al., 2017; Le et al. 2021). Formative assessments have also been shown to exert a positive effect on student achievement and promote pupils' responsibility for their own learning results (Le et al. 2021). The resulting feedback offered by the teacher may be the central factor in the formative assessments' success (Lee et al., 2020).

Feedback on student work reflects information about students' performance. This helps to unveil learning processes, thus promoting student understanding which, in turn, encourages learners to take a more proactive role in their own acquisition of knowledge (Lee et al., 2020). Feedback, among my study's participants, with the exception of the discrepant case, included descriptions of the student's strengths, weaknesses, and goals.

Analysis of the data revealed that both New York State and Asian compeers relied on formative testing either in a formal or informal iteration. Teachers from both aggregations used the results of the assessments to tweak lessons and guide the content of mini lessons. The data from the tests were also used to guide the review of certain topics throughout the year. As previously mentioned, the Asian cohort offered frequent (twice weekly) formal formative tests and responded with ongoing written feedback to the students. Contrarily, the New York State teachers used informal formative testing based extensively on observations. Albeit, one New York State teacher created daily exit tickets which were comprised of one or two problems connected to the day's lesson.

In addition to summative and formative assessments there are also self- and peer-assessments. Self-and peer-assessments involve students making judgments about their own and other students' academic performances (Wanner & Palmer, 2018). One study indicated that students regard self- and peer-assessment as productive for improving their own work, thereby increasing their academic performance (Wanner & Palmer, 2018). Learners who self-assess or evaluate their peer's products were therefore active participants who share responsibility with the educator in the evaluation process (Atkinson et al, 2017; Wanner & Palmer, 2018). Both self- and peer-assessments epitomize assessments *for* learning since they require students to engage in critical thinking as they review, evaluate, and correct their own work or the work of others. Additionally, as learners immerse themselves in the evaluation of others, they develop deeper insights and cultivate a clearer understanding of the material (Atkinson et al., 2017; Hongli et al., 2021). As a result of the robust affirmation, in numerous studies, concerning the effectiveness of self- and peer-assessments, it was not surprising that 11 of the 12 participants in my study utilized these strategies as part of their teaching regimen. The only anomaly was the one discrepant case in the study (Atkinson et al., 2017; Hongli et al., 2021).

Returning to the differentiation data, I found that the New York State participants evinced reduced expectations for the lower groups. These students were presented with less demanding problems, were offered extensive use of study aides in lieu of memorization, and allotted additional time for task completion. The New York State teachers in my study professed that interactive software programs were used by the lower performing students for additional practice and review. However, studies have found, in some cases, that the academic failure of students has been ascribed to teachers' less demanding expectations (Francis et al., 2020; Kaymakamoğlu et al., 2017). These expectations become a self-fulfilling prophecy since the teacher's behaviors, classroom decisions, and activities are guided by those beliefs (Francis et al., 2020; Kaymakamoğlu et al., 2017). Furthermore, low expectations may serve as an obstacle to the goal of achieving equity. Louie (2019) stated that excellence in mathematics education requires equity. Equity has been partly defined as having high expectations and strong support for all students (Louie, 2019). Cultivating students' sense of their own efficacy and agency is often viewed as a vital means to advance equity (Snell & Lefstein, 2018). Therefore, ability grouping can sometimes work against equity as teachers address certain groups with basic problem solving and close-ended and unchallenging questions while reserving complex problem solving and open-ended and more sophisticated questions for higher groups (Darling-Hammond, 2010; Louie, 2019; Snell & Lefstein, 2018).

The Asian teachers, alternatively, taught the same content to the entire class. Axiomatically, these teachers communicated high expectations for all students. In addition, unlike the Asian teachers who required their students to memorize a wide array of math facts, the New York State instructors did not require students to commit math facts to memory. Instead, lower ability groups were afforded the use of study aides such as place value and multiplication charts. However, these tools were gradually minimized by some New York State participants as the students internalized math facts. Concrete enhancements were also available to struggling students. The manipulatives encompassed unifix cubes, as well as an array of other tactile objects. Abstract learning tasks were gradually introduced as lower-rung learners progressed.

The middle tier of students was given more challenging work by the New York State participants. This included abstract renditions and multi-step problems. Opportunities for the gifted students, however, were severely limited. All New York State teachers simply encouraged gifted students to solve multi-step problems or describe multiple ways to solve a problem. One New York State gifted student was allowed to use the IXL program. The New York State teachers reasoned that the gifted students would perform well on state-wide tests and therefore the teachers’ attention should be centered on the remainder of the class. This was considered a practical approach since the entire New York cohort stated that their professional teaching appraisals were strongly influenced by standardized test results. In contrast, as previously stated, all the Asian participants held high expectations for the entire class. Moreover, gifted students were afforded the opportunity to compete with students from other schools in math contests. Often, scholarships were rewarded to the winners.

Research concerning how higher or lower expectations (on differentiated groups) affect student achievement is scarce and somewhat ambiguous (Prast et al., 2018). However, a meta study has found that self-perception of academic competence is a strong predictor of students' success (Legette & Kurtz-Costes, 2020). There is also a concurrent argument as to whether self-perception can change in response to external stimuli (Valdes, 2021). Some research has emphasized that self-perceptions are malleable and adapt over the educational career in response to educational inputs (Valdes, 2021). Other authors have argued that self-perceptions are fixed at the start of academic life due to the influence of significant people (Strello et al., 2021).

Valdes (2020) professed that the level of stratification of an educational system has been known to affect educational outcomes. Another study identified a strong nexus between the teacher's understanding of the students' ability (based on standardized test scores and prior achievement), the students' consequent placement in tiered groups, and the students' perception of their own abilities (Legette & Kurtz-Costes, 2020). Over time, this would affect the self-trajectory of their educational goals (Holm et al., 2019).

There are additional dynamics at play when forming ability groups. Grouping students by level of achievement in different cohorts tends to increase the homogeneity of the resulting groups (Valdes, 2021; Van den Broeck et al., 2018). Therefore, the allocation of similar students in the same group exerts such a strong effect on students' expectations (Lorenz et al., 2020). More specifically, the assimilation effect suggests that students' high-group apportionment conveys the perception of enhanced ability which leads to increased academic self-concept (Legette & Kurtz-Costes, 2020). However, comparisons to other classmates in this higher group, rather than to the entire classroom, might lead high-tracked students to feel challenged and less competent, resulting in lower academic self-concept. At the same time, students in lower ability groups benefit from comparing themselves to their cohort peers rather than to the general student body, which would include high-performing students (Legette & Kurtz-Costes, 2020). However, one study found that ability grouping exerted a negative effect on the lower groups' achievement (Boyd & Ash, 2018). This may be due to the self-fulfilling prophesy or Pygmalion Effect among students involved in the lower-tiered instruction (Boyd & Ash, 2018).

Scaffolding, where a more capable adult or student supported the child's learning, was used extensively by both the New York State and Asian participants. Scaffolding math instruction for lower performing students is firmly authenticated in the primary school literature (Prast et al., 2018). Scaffolding also complements Vygotsky's ZPD and is well established in scholarly research (Prast et al., 2018). According to Vygotsky, the objective was to raise the child's actual achievement level to his or her potential level through interactions with a more knowledgeable person (Brower et al., 2017; Hardman, 2021). Mini lessons, or micro-adaptations (spontaneous adaptations in direct response to students' needs), which were embraced by all participants in my study, facilitated the scaffolding approach (Prast et al., 2018).

Mastery learning was another best practice found among my Asian teachers. By contrast, all New York State participants, due to time constrains, avoided the practice. Mastery learning has been described as an approach to instruction and assessment in which students are evaluated on a prescribed standard (Betts, 2019; Emery et al., 2017). Betts (2019) found that mastery-learning approaches have a positive impact on student achievement. Engelmann (2014) and Hardman (2021) also argued that mastery is crucial if the student is to sustain the new learning, apply it in different contexts, and store it in his or her cognitive repertoires for future application. Instructors who teach for mastery allot additional time, instruction, and learning opportunities to support students who failed to achieve the benchmark the first time (Betts, 2019; Emery et al., 2017). Hence, adequate time frames for teaching math must be scheduled for mastery learning to take place. Frequent formative assessments and corrective feedback, as practiced among the Asian participants, are part of the mastery learning approach. Formative tests determine whether the students have mastered particular course content (Bloom, 1968; Serin, 2018). Bloom (1968) as cited in Betts (2019) purported that mastery practices would precipitate mastery of the material for the vast majority of students.

However, none of the New York State participants taught math to mastery levels. Time constraints emerged as the primary reason for the reluctance of the New York State teachers to embrace mastery learning. The New York State cohort concurred that, considering the time strictures, they were principally concerned with covering topics found on standardized tests. To support this contention, the New York State participants revealed that, although the school day lasted approximately six hours, daily math lessons ranged from 45-minutes to one hour. In contrast, the Asian cohort reported that their school days ranged from seven to nine and one half hours while daily math lessons lasted from two to three-hours.

The effect of homework on student achievement is another topic where my participants failed to reach concurrence. New York State participants revealed a lack of parental involvement in the students' homework assignments. Singaporean and Shanghai students and parents, on the other hand, viewed learning as a family obligation, as well as a community and social responsibility (Liu et al., 2019). Thusly, there was strong parental support for Asian students undertaking their daily homework assignments. However, the efficacy of homework assignments continue to stir controversy in the academic community (Guven & Akcay, 2019). One study, which involved approximately 5000 students, found that standardized test results had no statistically significant correlation with assigned homework or students' allocated time for the homework (Guven & Akcay, 2019). The study also revealed students' achievement was significantly lower in countries in which homework contributed to marks, or when homework was frequently the basis for class discussion, or corrected in class. Another study found that homework only adds to the congested school curriculum, depriving students of valuable time to develop social skills (Trenholm & Chinnappan, 2019). Conversely, Trenholm and

Chinnappan (2019) cited other studies which found homework can have a salutary effect. Trenholm and Chinnappan (2019) revealed that homework is critical in complementing and consolidating students' understanding of content (Trenholm & Chinnappan, 2019) Homework can have a positive effect on academic achievement if certain criteria are met. These criteria are: time spent (positive effect disappears for homework assignments exceeding 70-minutes), the degree of parental involvement, the kinds of questions assigned, and how homework is assessed (Guven & Akcay, 2019). The study also concluded that Chinese parents appeared to play a more positive role in helping with homework than parents in the United States (Guven & Akcay, 2019). My own study concurred with this latter finding. The Asian assemblage reported no problems with obtaining parental support for homework. Therefore, Asian participants assigned math homework, ranging in time for a half-hour to an hour, each night. However, the New York State participants cited lack of parental expertise with current algorithms as a fundamental reason for not assigning math homework.

Teachers who specialize in math instruction was another Asian best practice found in my study. In the United States, elementary teachers are most often prepared as generalists in college and are qualified to teach an array of subjects (Myers et al., 2019). This customary preparation has led to a recognition that many elementary teachers need improved content knowledge and pedagogical capabilities for effectively teaching mathematics (Myers et al., 2019). Research has substantiated the idea that teacher preparation and teacher quality influence student achievement (Myers et al., 2019).

Several prestigious mathematics education organizations, including the Association of Mathematics Teacher Educators (AMTE) and the NCTM, surmised that every elementary school in the United States should have access to teachers who specialize in math (Myers et al., 2019). The NCTM recommends that math specialists should exhibit the following knowledge and skills. Firstly, content knowledge for teaching, including deep understanding of grades K-8 mathematics. Secondly, pedagogical knowledge for teaching, including a firm understanding of learners and learning, curriculum, and assessment (Kuennen & Beam, 2020; NCTM, 2022).

Most of elementary school students in the United States learn in self-contained classrooms where one instructor teaches all major subjects (Cohen et al., 2018). However, many elementary school teachers typically have more expertise in certain subjects (Bastian & Fortner, 2020). This indicates that assigning teachers to subjects based on their comparative abilities may enhance student learning (Bastian & Fortner, 2020; Campbell et al., 2014; Condie et al., 2014). Thus, math specialization may potentially enhance teacher effectiveness by developing a teacher's subject expertise (Bastian & Janda, 2018). Research has also illustrated that teaching the same grade repeatedly helps quicken the development of math proficiency in teachers (NaYoung & Kisida, 2021). Inversely, managing wider ranges of content decreases the generalists' effectiveness (Bastian & Janda, 2018). Teachers also revealed that preparing and teaching fewer subjects reduced stress and increased their job satisfaction (NaYoung & Kisida, 2021). The potential benefits for learners include expanded exposure to expert instruction, ability to benefit from various instructors' teaching styles, and preparation for departmentalization that will take place in middle school (Markworth et al., 2016) as cited in (Mills et al., 2020). However, math specialization does have a negative connotation as well. Subject-area specialization spreads teachers across more students thus weakening the student-teacher relationship (Bastian & Fortner, 2020). Hence, teacher specialization may have more detrimental effects on students from vulnerable populations who thrive on a closer relationship with their teachers (Ball et al., 2008) as cited in (NaYoung & Kisida, 2021).

Easy access to the Internet was deemed an effective practice by the New York constellation. All New York State participating teachers revealed that each student in their classrooms was assigned a personal chrome book. The chrome book was used extensively for math practice. This may be deemed as a best practice for New York State participants since research has shown that smart mobile devices are suitable in addressing the learning needs of students (Kaur et al., 2017; Leem & Sung, 2019). Smart mobile devices also afford students the opportunity for collaboration, engagement, motivation, and communication. Moreover, portability, versatility, and convenience of access to information provide additional benefits (Kaur et al., 2017; Leem & Sung, 2019). Furthermore, the use of smart mobile devices has increased the motivation to learn through challenges, curiosity, and control (Troussas et al., 2022). Students can manipulate the content, thereby enabling more repetition and practice. Finally, another study revealed that mobile devices have the potential to help students with learning disabilities by increasing learning opportunities, facilitating the ease of communication, and integrating learners into inclusive classroom settings (Chelkowski et al., 2019).

Finally, my Asian participants attached effective instruction status to the out-of-school math competitions offered to the gifted students. In contrast, all the New York State participants lamented the lack of any structured learning environment for their gifted students. Collaterally, they listed this gap as a challenge. Redding and Grissom (2021) concurred that gifted programs offer additional enrichment and challenge and help compensate for regular classroom settings which feature, for the gifted, lower expectations or weak academic rigor. However, some studies have asservated that gifted programs have little effect on math achievement. Researchers have suggested that the chief benefits of gifted programs may be attitudinal, that is, contributing to the student's self-concept and motivation (Redding & Grissom, 2021).

RQ: 2 What challenges do fourth-grade teachers from the sample cite as obstacles to effective math instruction?

The data from the New York State participants produced 11 codes/categories, and seven themes concerning the second research question. The data from the Asian assemblage produced six codes/categories and three themes relevant to the second research question. Please refer Tables 1-9 for a full listing. The Asian teachers cited students' lack of access to personal chrome books as a challenge to fourth-grade math instruction. A computer room provided alone provided access to the Internet during the school day. Additionally, the Internet was utilized primarily for research since interactive math programs were not available.

The New York State contingent referenced limited timeframes as an ongoing challenge. New York State participants unanimously agreed they were not allotted adequate time to teach the entire math curriculum as mandated by Common Core. Typically, New York State teachers in my study offered one hour of math instruction each day. As a result, the geometry strand was generally ignored and mastery of subject matter was rarely attempted. One of the specific problems contributing to this time crunch is reform-oriented teaching which is at the center of the Common Core Curriculum (Common Core State Standards Initiative, 2022). These approaches involve SCL, with the concomitant group work and discussions. SCL emphasizes interactive teaching where pupils play an active role by exploring problem-centered activities in small groups, contributing points to discussions, resolving conflicts, and explaining and demonstrating their methods and solutions to others in the class (Kaput, 2018). Compared to direct or traditional teaching, these activities, as well as mastering behaviors conducive to collaboration, require a commitment of additional class time (Mathews, 2020). However, curriculum planners, school boards, administrators, and teachers’ unions have failed to reconcile the dichotomy between the expanded curriculum demands and traditional daily time frames for math instruction (Leong & Kaur, 2019). Consequently, abbreviated time frames have presented significant obstacles to innovative practices and have constricted the opportunity for homework validation, spiraling (intermittent review of previous topics), and teaching for mastery (Leong & Kaur, 2019; Mathews, 2020). The allotted time for math in New York State classrooms has also compelled New York State participants in my study to concentrate on covering, rather than teaching, all areas of the curriculum. A final conundrum, as revealed by my participants, was the number of students who were not reading at grade-level or exhibiting readiness for fourth-grade math. This may have been partly the result of the cumulative time circumscriptions encountered by teachers and students during previous years. Since most authentic math problems involved reading as well as math, additional time was invested by math teachers for scaffolding these below-grade level performers. As a consequence of all these factors, teachers will require new ways of thinking about using class time. Policy makers and administrators, in turn, will need be supportive of changes in class scheduling to overcome these challenges.

The Asian participants, on the other hand, reported they were able to spend from two to three hours each day teaching math. As a result, all the Asian participants revealed they were able to cover all aspects of the curriculum each school year and teach for mastery, as well as intermittently review previously learned concepts. However, the Asian participants felt that the rigorous work load challenged their stamina. The Asian participants, whose school day lasted between seven and nine and one half hours (as opposed to the six-hour school day in New York State) did not perceive readiness issues as a challenge. Moreover, they taught classes which had 60% more students than the average New York State cohort’s. Additively, Confucian beliefs permeate East Asian societies (Kim et al., 2019). These beliefs emphasize education as a moral pursuit which is undergirded by a strong work ethic (Kim et al., 2019). These cultural values may carry over to the teaching and administrative segments of these societies. Therefore, the Asian teacher's dedication to long school hours may help explain student success despite larger class sizes.

Mainstreamed students have the potential to create challenges in an inclusive classroom. However, the effects of mainstreaming were minimal according to my participants. One New York State participating teacher reported having a mainstreamed child. However, the child did not require an aide. After perusing the interview data, I concluded that the effects on the aggregated standardized tests scores may be negligible. New York State allows parents of mainstreamed students to opt out (New York State Education Department, 2019). In addition, those students with severe disabilities are offered an alternative exam which includes many accommodations (New York State Education Department, 2019). Further, Individualized Educational Plans (IEP) are noted along with the test results for mainstreamed test-takers. Finally, New York State utilizes a complicated algorithm for factoring these additional data into their final test results. Conversely, Asian participants reported a total lack of mainstreamed students among their classrooms.

A close reading of my study's results indicated that all participants dealt with the challenge of students evincing negative attitudes towards mathematics. Studies have found that student attitudes are a key component of lower or higher performance in math

(Mazana et al., 2019). Scholarly investigations have also confirmed that student attitudes are affected by the teacher's instructional practices, the students' aptitude, and the school environment (Manzana et al., 2019). In addition, researchers discovered that parents' and teachers' attitudes toward math significantly influenced the viewpoints of students (Luttenberger et al., 2018). To engender a more positive student perception of math, the New York State instructors in my study stated that they cultivated a safe learning environment where students' question were welcomed. The New York State participants also strove to build self-esteem with feedback which emphasized the student's strengths and well as offering insights to improve their performance. Teachers from both international constellations provided relevant problems and hands-on material. Moreover, both cohorts stressed real-world practical applications of math. However, the overwhelming amount of content in the New York State curriculum, as well as the inherent readiness issues and limited timeframe, did not allow teachers to provide students with the proper foundation with which to appreciably alter negative attitudes. Teachers in this group also complained about their college training and the lack of PD in math. Both of these issues may have affected their confidence and expertise in teaching math. Ineluctably, negative attitudes by the New York State participants may have influenced the convictions of the students as well. Additionally, most of the parents of New York State pupils maintained that they did not understand New York State's Common Core strategies. This led to a loss of parental facilitation. The lack of parental assistance became so exigent that two of the three school districts represented in my study eliminated math homework altogether. However, the fourth participant assigned 20 minutes of math homework each night. Although students of the Asian participants also experienced negative attitudes towards math, the Asian parents, according to my participants, were invested in their children's success and lent support at home. Finally, in addition to the nightly homework assignments, Asian students were exposed to an additional one to two hours of math instruction each day.

Class size is one of the many factors thought to influence student learning (Blatchford, 2021). Class size is likely to affect the interactions and relationships that develop in a classroom as well as the quality of the learning experienced by pupils (Blatchford & Webster, 2018). My study revealed that class size, a code/category identified as supporting effective instruction among the NY group, was viewed as a challenge by the Asian teachers. I discovered, after perusing the results of my study, that the Asian participants' classrooms, which averaged 32 students, had 60% more students than the New Yorkers, whose classrooms contained an average of 17 pupils. Overcoming these inequities constituted a challenge for the Asian contingent. The Asian assemblage dealt with the challenge of large class sizes with numerous strategies. Firstly, they utilized the expanded school day and school year to full advantage by elongating math instruction. Secondly, they enlisted parental and collegial support. Thirdly, Asian teachers implemented intermittent group work among students. Lastly, they confirmed that their dedication to the profession, as well as their strong work ethic, were essential components in meeting the challenge of large class sizes.

Student Teacher Achievement Ratio (STAR) research is considered the most credible study related to class size reduction (Finn & Achilles, 1999) as cited in (Blatchford, 2021). In this investigation, students and teachers were randomly assigned to a small class, which averaged 15 students. Other teachers were placed in a class with 22 students. Results showed that the smaller classes (which represented a 31% reduction) improved student achievement in math by about three months over a four-year period (Chingos & Whitehurst, 2011) as cited in (Rushevsky, 2018). Secondly, the most compelling benefits from reduced class size were obtained when the size was reduced to 19 pupils or less. Moreover, the authors divulged that the study identified an array of other small class benefits including improved teaching conditions, enhanced student performance during and after the experimental years, augmented student learning behaviors, fewer classroom disruptions, and fewer student retentions (Finn & Achilles, 1999) as cited in (Blatchford, 2021). Supplementary evidence established, in both small and large class sizes, that the presence of a teacher aide had no significant effect on student achievement (Finn & Achilles, 1999) as cited in (Blatchford, 2021).

There have been additional studies related to class size in Texas and Israel. These studies verified that smaller classes led to better student achievement. However, the gains were smaller than those identified in the STAR study (Rushevsky, 2018). This latter study concluded that large class-size reductions, involving 7-10 fewer students per class, can have compelling long-term effects on student achievement. Correspondingly, small classes have the greatest impact when introduced in the earliest grades and students from lower socio-economic backgrounds benefitted the most (Blatchford & Webster, 2018; Rushevsky, 2018).

Other researchers affirmed that the disadvantages of large classes are often offset by contextual circumstances (Blatchford & Russell, 2020). To illustrate, many students are pulled out at various times during the day thereby reducing not only the homeroom's size but also creating small groups in the rooms to which they are migrating (Blatchford & Russell, 2020). Therefore, one can assume that class size is not a static number but one that should be evaluated in a larger context. Other research showed that large class sizes provide many educational advantages for students (Blatchford & Russell, 2020). To exemplify, more students in a class offer the learners an opportunity to independently master problem-solving skills since they rely on each other rather than the teacher. Moreover, additional pupils result in more ideas and insights among the student population. Finally, there is a political dimension to the topic as well. Most teacher unions, parents, and some researchers claim that small class sizes are beneficial for learning (Blatchford & Russell, 2020). In contrast, economists, think tanks, and policy makers maintain that class size is not a cost-effective method to improve education (Solheim & Opheim, 2019).

The limited amount of time which New York State participants were able to devote to covering the challenging Common Core math curriculum emerged as a major challenge. New York State participants' daily math lessons averaged 60 minutes while the Asian contingent spent approximately 150 minutes each day on math instruction.

Lopez-Agudo and Marcenaro-Gutierrez (2020) defined instructional time as the period when students are attending to a learning task and attempting to learn. Patall et al. (2010) as cited in Lopez-Agudo and Marcenaro-Gutierrez (2020) indicated that recent international test results showed that students in the United States did not fare as well as those nations with longer school days or school years. The investigators reported that American students receive hundreds of hours less schooling each year than many of their European or Asian peers. The study concluded that the resulting harm may be cumulative and lasting. However, the goal of increasing the length of the school year seems chimeric. Today, the strongest opposition to extending school in the United States is derived from middle-class and affluent parents who value the summer vacation for their children and question the value of additional school time (Lopez-Agudo & Marcenaro-Gutierrez, 2020). Generally, teachers, teacher unions, and school administrators also failed to emphatically support initiatives to increase school time citing the increased workload as well as costs (Patall et al., 2010) as cited in (Lopez-Agudo & Marcenaro-Gutierrez, 2020). However, the most persuasive argument for increasing the number of days in the school year or lengthening the school day is its potential to increase the amount learned by students (Patall et al., 2010) as cited in (Lopez-Agudo & Marcenaro-Gutierrez, 2020). Proponents emphasize that additional time can promote learning and achievement by increasing time on task, enabling teachers to extend broader and deeper coverage of curriculum, as well as offer more opportunities for experiential learning and deepening the adult–child relationships (Patall et al., 2010) as cited in (Lopez-Agudo & Marcenaro-

Gutierrez, 2020). Despite current opposition, the authors concluded that extending school time can be an effective way to support student learning, particularly for at-risk students. Considerations must also be made for how the time is utilized (Patall et al., 2010) as cited in (Lopez-Agudo & Marcenaro-Gutierrez, 2020).

RQ:3 How do other determinant influences, such as teachers' preservice education, and PD affect math instruction in the United States, Singapore, and Shanghai?

Analysis of the New York State participants' responses to the third research question divulged eight codes/categories and three themes. Analysis of the Asian cohort's responses revealed six codes/categories and three themes.

Teacher quality is an essential component of successful education (Darling-Hammond et al., 2019). Currently, colleges face the challenge of preparing teachers with the content and curricular knowledge, instructional strategies, and affective dispositions which will allow them to effectively teach 21st century skills to their students (Darling-Hammond et al., 2019). However, research has shown that teacher education in many colleges in the United States offer curricula which feature theory with tenuous links to practice (Darling-Hammond et al., 2019). For that reason, my New York State practitioners cited the need for more contact with students during their college training. They also lamented the paucity of math courses.

Further investigation of the attending themes revealed the following. The New York State compeers' college training was rated poor by two participants and good by the remaining two. All believed that they had not received adequate content knowledge in math-related topics. The Asian contingent graded their own college training as good to excellent. Darling-Hammond (2021) revealed that teacher qualifications may influence the quality of math instruction. Singapore limits the acceptance rate at universities to obtain higher-quality applicants (Darling-Hammond, 2021; Stewart, 2011). For that reason, standards for admission to teacher preparation are quite elevated in Singapore and they include demonstrations, on the part of the candidate, of strong academic ability as well as a passion to teach (Darling-Hammond, 2021). Darling-Hammond (2021) and Jaciw et al. (2016) reported that Singaporean teachers must show evidence of high levels of mathematics skills before embarking upon their teacher education programs. Singapore's MOE, which oversees education in the nation, mandates that educational institutions be held accountable for establishing initial teacher competencies that directly relate to the national standards (Darling-Hammond, 2021). Finally, Singapore also actively recruits mid-career candidates, believing their work experiences are valuable affordances (Darling-Hammond, 2021; Stewart, 2011).

East China Normal University and Shanghai Normal University, in Shanghai, recruit academically advanced students for their teacher preparation programs (Cer & Solak, 2018). After graduation, these applicants must then pass national examinations in psychology, pedagogy, and teaching methods as well as a district level test of content mastery (Cer & Solak, 2018; Darling-Hammond et al., 2017). Preparation programs in Shanghai emphasize foundational instruction in academic content, pedagogical knowledge, and professional standards (Cer & Solak, 2018; Darling-Hammond et al., 2017). The university curriculums also highlight research and prepare teachers to conduct ongoing scholarly investigations. Finally, Shanghai, like Singapore, has developed hiring systems which recruit teachers from the top of their college classes (Retna & Pak, 2016) as cited by (Koh-Chua et al., 2021).

The United States, in contrast, does not limit the numbers of people who train to become teachers (Darling-Hammond, 2021; Stewart, 2011). In addition, many scholars have noted the increasing proliferation of ineffective teacher preparation programs in the United States (Cochran et al., 2015; McDiarmid, 2019). The United States does subsidize older, work-experienced candidates in United States' schools (Darling-Hammond, 2017; Thomas & Mockler, 2018). These individuals enter teaching through alternative programs, such as Teach for America (Thomas & Mockler, 2018). These non standard avenues often entail only a few weeks of preservice training. As a consequence, these short-circuited pathways into teaching have precipitated the lowering of teacher requirements in many inner-city schools in the United States (Darling-Hammond, 2017; Thomas & Mockler, 2018).

Concerning PD there was a clear demarcation of viewpoints between the two international groups of participants. The New York State teachers perceived PD as neither systematic nor ongoing. Indeed, one of the participants had not taken part in a PD session in years. The New Yorkers also reported that PD was not linked to classroom activities. The New York State participants' overall verdict was that PD had not increased their teaching capacity. However, all reported that there was systematic weekly collaboration among grade-level teachers which often adopted the guise of informal PD sessions.

The views of my New York State participants concerning PD may well serve as a microcosm for the United States as well. American schools, despite significant and continued efforts, have not witnessed a sustained improvement in student-achievement linked to PD (Nelson & Bohanon, 2019). There are many reasons for this. Matherson and Windle (2017), as well as Bentley and Cason (2019), found that many United States' teachers who participated in PD sessions believed that this training was not connected to classroom problems and therefore had little positive impact on their pedagogical practice or on student achievement.

The lack of productive PD programs facing my New York State participants as well as many other teachers in the United States has profound implications for student learning. Investigations have confirmed that teachers and the quality of their teaching practices are the single-most influential variable on student learning (Birgit et al., 2017; Sharp et al., 2019; Sun-keung et al., 2017; Woessmann, 2016). Moreover, in their study, Rodriguez-Lopez et al. (2019) specified that the quality of teacher education programs and ongoing PD are determinant factors that influence instructional practices as well as student achievement. Additionally, PD, which systematically focuses on student learning and helps teachers develop skills to teach specific kinds of content, such as math, has been found to have strong positive effects on pedagogical expertise (Bentley & Cason, 2019; Zhang et al., 2017).

Research continually adds new knowledge to the field of education (Tuncel & Cobanoglu, 2018). As a consequence, it is imperative that teachers efficaciously update their knowledge and skills on curricula, psychology, and pedagogy as well as investigate new theories on teaching and learning (Tuncel & Cobanoglu, 2018). Requisitely, PD should not be limited to a few in-service training sessions. Rather PD must be offered as a continuous process (Ren & Smith, 2018; Tuncel & Cobanoglu, 2018). Furthermore, PD, to be most advantageous, should avoid abstract discussions. Instead, active participation and interactive engagement, linked to andragogic principles and classroom practices, should be the norm (Tuncel & Cobanoglu, 2018). Professional development sessions should incorporate the tenets of andragogy which describes how adults learn best (Tuncel & Cobanoglu, 2018). These principles include opportunities for individuals to work with and learn from others in small groups on an ongoing basis, chances to interact with colleagues of similar position, as well as offering a degree of autonomy which includes a choice in work roles and tasks. Additionally, PD, to be a cause of change in teacher behavior, should be continuous, connected to practice, and at least 50 hours in duration (Tuncel & Cobanoglu, 2018). Professional development should focus on a single subject, center on participants' needs, actively engage participants in the pursuit of answers to authentic problems, help participants develop collegial relationships, integrate Information and Communication Technology (ICT) tools with their current knowledge, and encourage instructors to reflect on their teaching (Tuncel & Cobanoglu, 2018).

Finally, the training process should be led by experts in the field (Tuncel & Cobanoglu, 2018).

Another approach to PD has been styled “the math workshop.” A math workshop facilitates a student-centered method to teach mathematics to instructors. The math workshop therefore develops knowledge of the concept as it fosters a constructivist, inquiry-based approach for small groups of learners (Sharp et al., 2019). The goals of the math workshop are to promote the following principles for teaching math students: communicate mathematics objectives to focus learning, create tasks that promote reasoning and problem solving, ask purposeful open-ended questions, facilitate meaningful mathematical discourse, cultivate procedural fluency from conceptual understanding, and elicit evidence of student-practitioners’ thinking (Sharp et al., 2019). Finally, math curriculum specialists should provide ongoing support for teachers who were implementing the math workshop approach in their classrooms. This bolstering includes encouraging and implementing attendance at state- and national-level math workshop training, inviting speakers to conduct math training at schools, and facilitating classroom observations of master teachers who efficaciously implement math workshops (Sharp et al., 2019).

Studies have also evidenced that successful PD programs must not only be systematic, ongoing, and related to classroom activities but must also focus on the “whole teacher” (Ren & Smith, 2018). This departs from the traditional renditions which focus primarily on teachers’ acquisition of knowledge and skills. Rather, “whole teacher” PD promotes holistic aspects of a teacher's development, including their attitudes and beliefs (Ren & Smith, 2018).

Ren and Smith (2018) posited that an instructor's past educational experiences, both positive and negative, strongly influence their mathematical attitudes. To exemplify, teachers' mathematical anxiety may be caused by the difficulty of certain content, as well as past experiences in a rigid classroom or with officious teachers. In addition, timed tests and the fear of making mistakes may also have affected their attitudes. Ren and Smith (2018) found that many primary teachers reported having anxiety toward mathematics. Teachers' anxiety may have negative effects on students' attitudes toward learning mathematics as well. In addition, teachers' attitudes toward learning mathematics also have important implications for their instructional practices. Teachers' negative attitudes may impact the amount of time allotted to math instruction, result in a lack of confidence when responding to student queries, and influence instructional methods used in the classroom. Current research has indicated that a high-quality teaching staff, benefitting from extensive rigorous coursework in college, and an ongoing teacher training system (which enhances the educator's attitude and confidence) resultantly plays a significant role in Singapore and Shanghai's educational achievement (Jiang et al., 2018; Tonga et al., 2019; Zhang et al., 2017; Zhang & Zheng, 2020).

The Asian contingent in my study rated their teacher training in college as good to excellent. Moreover, they received extensive training related to their goal of becoming math specialists. Professional learning communities in both Singapore and Shanghai focus on student learning and outcomes through improvements in teaching (Zhang et al., 2017). Professional learning communities in Singapore and Shanghai are generally embedded in teachers' work sites and complement their schedules (Zhang et al., 2017; Zhang & Zheng, 2020).

The Asian participants revealed that they took part in PD sessions on a monthly or bi-monthly basis. Suitably, they felt that their PD, which was often linked to classroom activities, resulted in improved pedagogical practices. Finally, the Asian teachers met every four weeks with their grade-level peers.

Both the New York State and Asian cohorts endorsed and practiced grade-level collaboration among teachers. All participants of my study have affirmed that their principals have taken an active role in providing teachers with adequate time and other necessary supports for collaborative engagement. Scholarly investigation has established that collective efficacy is often related to effective teaching (Miller & Anthony, 2021). Bryk et al. (2010) as cited in Miller and Anthony (2021) showed evidence that teaching with purpose and coordination supports not only teachers' professional growth but also student learning and academic outcomes. Often, top-down approaches, such as traditional PD programs, are disconnected from daily practices (Tuncel & Cobanoglu, 2018). Grade-level collaboration, therefore, offers a necessary corrective. In this milieu, teachers become constructive participants in their professional growth as they identify problems of practice, create unified goals, cooperate on lesson planning, and evaluate teaching and learning outcomes (Collet, 2019).

New York State participants in my study affirmed that they met on a weekly basis with grade-level peers. The participants utilized these venues to coordinate lesson plans, share collected data, and discuss teaching strategies. All confirmed that these collaborative sessions were integral to increasing their expertise as well as promoting beneficial learning outcomes for their students.

Sharing personal information about daily practices is the norm among teachers in Singapore and Shanghai (Hairon & Tan, 2017; Jiang et al., 2018; Zhang & Zheng, 2020). School leaders in both sectors also promote the intermingling of teachers' experiences among different schools. Teachers in these venues discuss their classroom experiences, exchange ideas about new theories, create exam questions, and conduct research (Zhang et al., 2017; Zhang & Zheng, 2020).

RQ:4 "How does SLT influence fourth-grade math instruction in the United States, Singapore, and Shanghai?"

The data from the New York State participants produced 10 codes/categories and two themes concerning the fourth research question. Analysis of the Asian teachers' responses also elicited seven codes/categories and one theme. I will expand upon these themes in the following paragraphs.

Vygotsky's SLT advanced the idea that students construct knowledge interactively in a social setting. The ZPD is a central maxim of Vygotsky's theory of social learning (Lasmawan & Budiarta, 2020; Vygotsky, 1978). This zone describes a student on the cusp of advancing from prior, mastered, and independently applied knowledge to new understanding. To achieve this higher level, students must elicit support from more capable others (Hardman, 2020; Nguyen, 2017). Therefore, SLT substantiates an atmosphere that cultivates collaboration, communication, critical thinking, scaffolding, and shared problem solving (Hardman, 2020; Pie-Ling Tan et al., 2017; Vygotsky, 1978). Student-centered activities are informed by SLT. Student-centered learning provides students with opportunities to be actively involved in the process of generating mathematical knowledge through inquiry and problem solving (Ediger, 2018; Lattimer, 2015).

Operationalizing the practices inherent in social learning-infused classrooms find students discussing the problems in small groups, sharing a number of strategies to solve the problem, and reflecting on their efforts (Adeleye, 2021; Leon & Castro, 2017). During this process, students also compare and contrast strategies, resolve conflicts, negotiate agreements, synthesize new ideas, and reach a consensus (Leon & Castro, 2017; Mathews, 2020).

Student-centered practices, supported by SLT, also subordinate, to a lesser level, the importance of procedural fluency (Chen et al., 2018; Leon & Castro, 2017). Instead, these SLC approaches place a greater emphasis on understanding the underlying mathematical content and concepts (Chen et al., 2018). However, teachers may encounter obstacles when implementing a student-centered modality. These challenges include concerns about timeframes, anxieties over students’ performance on standardized exams, resistance to change from traditional methods, peer pressure from other teachers, the tendency to teach as they were taught, and problems in related to classroom management (Keiler, 2018)

Alternatively, teacher-centered instruction has been the traditional conduit for imparting knowledge in Singapore and Shanghai (DeSouza, 2018; Zhao et al., 2014; Serin, 2018). Teacher-centered instruction is distinguished from student-centered instruction by the degree of teacher control and the level of student participation in classroom activities (Yoonjeon, 2018). Serin (2018) surmised that explicit instruction, as practiced in teacher-centered classrooms, places a high priority on teacher autonomy. The goal of teacher-directed instruction, as a consequence, is for the instructor to don the mantle of authority and transmit information, skills, and concepts to students in a didactic manner (Serin, 2018). In consonance with Engelmann (2014), teacher autonomy, an important facet of DI, also reduces the chances of students' misinterpreting ideas (Stockard et al., 2018). This is largely due to the unambiguous nature of the instruction, as well as frequent questioning and feedback by the teacher (Stockard et al., 2018). To bolster this explicitness, the direct instructive approach promotes the use of clear, specific examples which are sequenced to encourage correct inferences from the students (Stockard et al., 2018). Further, as part of their advocacy for teaching for mastery, practitioners of DI revisit previously learned knowledge and skills in recursive cycles (Rosenshine, 1979; Stockard et al., 2018).

Studies have identified explicit, systematic math instruction as an effective instructional approach which supports the efforts of struggling students (Cohen, 2018;

Kaymakamoğlu et al., 2017; National Mathematics Advisory Panel, 2008). According to Cohen (2018) these practices often place less demand on attention, working memory, language, and general cognitive resources. However, since 2001, Shanghai and Singapore have begun to make incipient changes in the content of its mathematics examinations to focus more attention on assessing students' understanding rather than rote memorization (Hardy et al., 2020; Lee, 2017). There has also been a corresponding emphasis on implementing collaborative group work which places more responsibility for learning on the students (Hardy et al., 2020; Lee, 2017). These "hybrid" approaches may constitute the "best of both worlds" philosophy where teachers activated constructivist approaches to help students discover effective strategies to learning, then implemented direct teaching to pass on knowledge and skills to struggling pupils. Axiomatically, in this modality, instructors used both teacher-centered and learner-centered applications to adapt pedagogical approaches to match their students' needs (Kaymakamoğlu et al., 2017; Serin, 2018).

The New York State participants, with the exception of the one discrepant case, expressed the thought that SLT, with its emphasis on collaboration and communication, played an essential role in how math was taught in their classrooms. The teachers often began lessons with DI, which was followed by collaborative activities. Independent group work was then assigned once the foundational knowledge was internalized.

Returning to the theme of collaboration, these participants concurred that students learn best when they learn with someone else. As a consequence, the New York State group felt that collaboration was an effective method for teaching math. The participants believed that small-group discussions, which ensued among students, reinforced concepts. These participants also agreed that the students were usually active participants as they collaborated, gained knowledge from each other, and discovered effective problem-solving strategies through communal discussions. In this milieu, pupils witnessed a variety of methods and absorbed an array of explanations. According to my New York State participants, the unique perspectives and different views often expressed in groups, created better strategies and solutions. Group discussions also encouraged each student to be more flexible, expansive, and creative. Moreover, the communications skills that were honed, such as brainstorming, will be valuable assets in the workforce.

Group work is an important asset embedded in SLT. Three of the four New York participants required individual accountability in group work, often assigning specific roles for each student. Many of these collaborative lessons also incorporated life-skills such as communication, cooperation, conflict resolution, consensus building, higher order thinking, and problem solving. During the course of the lesson, three teachers from the New York State cohort offered students the opportunity to coach each other as well as afforded learners occasions for self- and peer-assessments. The teachers believed that seeing samples of other students' work would have the beneficial effect of raising the standard for their own performance. However, the teachers felt it necessary to model peer- assessment's purpose as well as behaviors and language prior to implementing the practice. The remaining New York State participant (discrepant case) demurred from both practices.

Assigning group leaders was endorsed by three of the four New York State participants. Students who understood the concepts might be assigned as group leaders or co-teachers. Assessment of collaborative activities was often informally rendered through observation or during a whole class reflection which commenced at the conclusion of the lesson. However, all four New Yorkers disagreed with the notion that collaboration led to classroom management issues. They felt the key to a controlled environment was thoughtful reflection on the best mix of students when creating the groups.

The Asian participants were unanimous in their appraisal that SLT, with its emphasis on collaboration and communication, played a valuable, yet subordinate role in their math instruction. Rather, they all felt that DI, via a teacher-centered motif, was the more efficient and effective method to teach fourth-grade math. Although most knowledge was transmitted by the teacher and students adopted a more passive than their American counterparts, there was a nuanced approach to active learning in Asian schools. Asian participants implied that they were adept at holding control of the class; therefore, students' were often encouraged to orally respond to higher-order questioning while, at the same time, learners, in a whole class forum, were invited to respond to each other's ideas. However, most knowledge was transmitted by the instructor. Therefore, students only intermittently engaged in small groups or in pairs to explore math problems. As a direct result, math lessons often failed to incorporate life-skills such as communication, cooperation, conflict resolution, and consensus building. Students were encouraged to coach each other however, and peer- and self- assessments were routinely utilized. Then too, collaborative activities were used by all Asian participants when time allowed. The groups were comprised of a mixture of fast and slow learners. The Asian participants believed that small group activity facilitated further insights into the topic as well as allowing shy students to feel more comfortable when learning. Each student would add to the other students' knowledge and that would facilitate their understanding of the concept.

Group captains often served the role of co-teachers when collaboration was required in Asian classrooms. Moreover, individual responsibilities were assigned to each students during group work. Assessments of collaborative work were rendered through informal observations as well as general reflections at the end of the lesson. The teachers concurred as one that collaborative work was not disruptive and did not lead to enhanced disciplinary issues.

Discrepant Case

There was one discrepant case among my 12 participants. P2, from New York State, was an anomaly whose perspectives differed widely, in many respects, from not only the Asian teachers and their New York State colleagues, but also from many research-based practices. Discrepant cases occur when a code does not fit into any of the understood categories (Nowell et al., 2017; Rose et al., 2020). This indicates that a participant’s perception or experiences differ from the mainstream evidence. P2 chose direct teaching as their mode for transmitting math knowledge as opposed the student-centered approaches advocated by New York State participants in the best practices category. P2 also refrained from implementing other best practices such as group skills and life skills due to the time constraints. Further dichotomies abounded. P2 abnegated peer- and self-assessments as well as student coaching as viable strategies. P2 also refrained from using hands-on manipulatives to develop concepts. Instead, P2 immediately modeled procedural rules. In addition, P2 offered little feedback on the students' work, disavowed differentiation and cross-curricular themes, and again cited time circumscriptions for the lack of spiraling in her lesson plans. All of these choices by P2 created a compelling dichotomy between her practices and those supported not only by her colleagues but by scholarly research as well.

Limitations of the Study

The design of this study, as well as the consequences of the unanticipated pandemic, subjected it to possible limitations. The data collection process took place during the COVID-19 pandemic which limited my access to certain participants. Although I had foreseen using Zoom to facilitate my interviews with teachers in Singapore and Shanghai, I had originally planned to conduct face-to-face interviews with participants located in schools in New York State. However, as a result of the unforeseen circumstances, all interviews with New York State teachers were conducted by telephone. To address the dynamics related to the telecommunications, I asked participants to designate a timeframe that was convenient and recommended that they choose a private setting that would be appropriate for the audio transcription of their interview. As a result, this potential limitation did not interfere with my ability to solicit meaningful responses to my interview and research questions.

There were other limitations related to my doctoral study. The 12 fourth-grade teachers who participated in this exploratory case study were purposely chosen and represented three schools in the United States, four schools in Singapore, and four schools in Shanghai. As a result of the small number of subjects, I cannot confirm that the results of my study are applicable to broader populations. As a result, the transferability of the findings may not be fully realized. However, there were advantages to having a purposeful sampling. Research has documented that a deep, careful, and complete exploration of the cases can illuminate substantive findings relevant to the discipline (Creswell, 2018). Moreover, advocates of qualitative studies have contended that multiple, experienced-based interpretations of reality exist (Creswell, 2018). As a result, each person's perception may exhibit a unique understanding of reality, as opposed to subscribing to universal characteristics and therefore add to the knowledge pool (Creswell, 2018). Subsequently, the perspectives of my cohort of teachers may serve to inform larger populations of instructors, administrators, and policy makers..

Other limitations involved the veracity of participants' responses, as well as the fidelity of the recorded transcriptions. To address these issues related to credibility, I repeated the same questions, albeit with rephrasing, to certify similar responses. At all junctures throughout the study, I utilized unique identifiers, such as Participant 1, to mask the real names of the participants and school sites. This layer of privacy was implemented to encourage participants to be more forthright in their responses. I also effectuated member checking as a means to ensure the fidelity of the transcriptions.

A third limitation, characteristic to qualitative studies, was researcher bias (Creswell, 2014; Ravitch & Carl, 2020). Studies have indicated that in qualitative explorations, the researcher adopts the role of the principal collector, as well as analyst of information (Ravitch & Carl, 2020). In this role, I therefore became part of the research. Accordingly, I utilized a researcher reflective log to review my questions and tone thereby sustaining an unbiased position. As a direct result, I was able to authenticate the exactitude of the participant's responses as well as my subsequent interpretations. Lastly, to help ensure the robustness of the participants' views, I enlisted the support of colleagues who reviewed the proposed questions to confirm they were clearly posed, non judgmental, and couched in neutral language.

A final limitation involved the design of my study which was based primarily on discovering teacher perceptions through interviews. After my research in the literature review segment produced evidence of the lower cognitive abilities of many teachers in the United States, as compared to those of certain nations, I determined the inappropriateness of asking my New York State participants to respond to this line of inquiry. Therefore, I will utilize the Recommendation section to elucidate on the possibility of future research into this topic.

Recommendations

The problem of this study, identified in current research literature and supported by standardized tests results was that fourth-grade math students in the United States were not performing as well on standardized tests as students of other nations and international cities (Pew Research Center, 2018; TIMSS, 2019; Woessmann, 2016). To compound the problem a large percentage of fourth-grade math students have consistently failed to reach proficient levels in national tests over a 10-year period (The Nation's Report Card, 2018)**.** Essentially, the intent of my study was to offer rich and comprehensive descriptions of 12 fourth-grade teachers' perceptions concerning math instructional strategies, inherent challenges to effective instruction, as well as the quality of their prior and ongoing training. These participants represented school located in Singapore, Shanghai, and New York State. Based on the strengths, findings, limitations, and literature review of my current study I recommend the following two actions, both of which will entail further research.

My first recommendation arose from unexpected findings during the ongoing literature review which has accompanied the development of each section of the study. As background, research has confirmed that teacher quality is an essential component of successful education and that teachers' cognitive abilities may have a direct impact on student achievement (Darling-Hammond et al., 2019; Hanushek et al., 2019). However, studies have shown that the United States teachers have lower cognitive abilities in math than those who practice in other high scoring nations (Hanushek et al., 2019). Teachers from the United States scored in the 47th percentile in math compared to the 74th percentile accrued by teachers from high scoring nations (Hanushek et al., 2019). Moreover, education majors in the United States have consistently recorded lower SAT scores compared to students entering other domains (Alisov et al., 2020). In contrast, teacher-candidates from Singapore are recruited from the top third of the college class. Shanghai, with an overabundance of educator-candidates applying for every position, selects only the best candidates (Çer and Solak, 2018; Darling-Hammond et al., 2017; Tonga et al., 2019). However, in the United States, just 23 percent of teacher-candidates emanate from the top third (Hanushek et al., 2019). More specifically, studies have posited two reasons for the cognitive differences among teachers in the United States and those in Singapore and Shanghai. First, in America, women now have greater job opportunities outside the field of teaching (Hanushek et al., 2019). Secondly, teacher salaries are low in the United States in relation to those earned by other four-year college-degree holders (Hanushek et al., 2019). United States teachers earn 22% less than those with comparable degrees (tied for last with Sweden in one study) (Hanushek et al., 2019). Consequently, to correct this discrepancy, local, state, and the federal government in the United States must find the financial resources to provide math teachers with the same level of compensation that is offered to other math professions (SREB Teacher Preparation Commission, 2019).

Summarily, teaching is a complex and intellectually demanding job (Bardach & Klassen, 2020). Intelligence or cognitive ability can be defined as the ability to reason logically, plan, solve problems, think quickly and abstractly, understand complex ideas, and learn from experience (Bardach & Klassen, 2020). However, research on teacher effectiveness has largely ignored intelligence as a potential predictor of how well teachers enable student learning (Bardach & Klassen, 2020). One study, however, found that a teacher's high cognitive ability, in isolation, does not predict higher achievement among students (Baier et al., 2019). However, the teacher's extraversion (characterized by positive and enthusiastic qualities for teaching), combined with pedagogical/psychological knowledge and emotional intelligence (empathetic and creating a safe learning environment) can be significant predictors of learning (Baier et al., 2019; Esan-Aygun, 2018). The Singapore and Shanghai recruitment processes places emphasis on the candidates' passion for teaching, high cognitive ability, as well as pedagogical and psychological knowledge (Hanushek et al., 2019; Tonga et al., 2019). Based on my literature review, I recognize that the limits of my study precluded a more thorough exploration on the effect that a teacher's high cognitive ability, when combined with other salient traits, has on student achievement. I would, consequently, recommend additional scholarly research into this issue.

My second recommendation encourages further explorations of the efficacy of ability grouping. My study, based on participant responses as well as the review of literature, recognized the dichotomy between the assets and drawbacks of differentiation (Barohny, 2019; Crutchfield & Inman, 2020; Deunk et al., 2018; McGillacuddy & Devine, 2018). Differentiation can be described as an adaptive practice in which teachers modify the curriculum, instructional methods, resources, learning activities, and student products to address the various needs of students and maximize student achievement (Deunk et al., 2018). Deunk et al. (2018) also reported that homogeneous groupings, related to differentiation, may result in reduced learning opportunities for low-ability students since they are not privy to the input of higher-ability classmates. Moreover, instructors may convey lower expectations for the lower performing groups. As a consequence, establishing a hierarchy of groupings and learning goals may result in a widening gap between higher-end and lower-end students (Deunk, 2018). However, proponents of ability grouping have emphasized its capacity to cater to a wide range of student abilities when they are placed in homogenous settings. Advocates also assert that differentiation provides an effective means to align the ability, interests, and the pacing needs of students with the pedagogical requirements of the learning standards (McGillacuddy & Devine, 2018). Therefore, the connection between the positive aspects of personalized learning and the relevant drawbacks of homogenous grouping, and lower expectations often associated with diverse groupings, should offer a rich area for subsequent research.

Implications

A core belief of Walden University is to cultivate social change for the benefit of every individual. As a change agent, my study has important implications for improving math instruction which, in turn, may help close the achievement gap, and help marginalized students better prepare for the workforce (Brinkmann, 2019; Hanushek et al., 2019). Numerous studies have determined that teacher quality is directly linked to student achievement (Darling-Hammond et al., 2019). Therefore, the dissemination of results from this study, which extensively focused on teacher quality and were distilled through interviews with a dozen fourth-grade math teachers, may improve many American teachers' pedagogical knowledge and instructional abilities. The enhanced pedagogical reforms may have salutary implications. These practices, relative to improved instruction, include math specialists in fourth-grade teaching positions, closer scrutiny of assessment methods related to student-centered approaches, and increased time spent by the students on math tasks. To prepare teachers to efficaciously actualize these approaches my study also described the importance of re-conceptualizing preservice education and PD. Concomitantly, the subsequent increase in teachers' knowledge and skills, with an accompanying intensification of their cultural sensitivity, which, incidentally, acknowledges and addresses the social, emotional, and intellectual needs of all learners, can reanimate efforts to increase teacher capacity (Abacioglu et al., 2019). Manifestly, Wright (2016), as well as Vakil and Ayers (2018) contended that successful mathematics education and the accompanying increase in job opportunities for the marginalized play a role in addressing difficulties faced by our society, including equity and the upward mobility of the economically disadvantaged.

In the interest of extending and amplifying my discussion of the study's implication, I offer the following insights. Firstly, each school should optimally have math specialists teaching at the fourth-grade level. Data analysis revealed that teachers in my study strongly favored specializing in fourth-grade math rather than teaching a panoply of subjects. This philosophy accords with the recommendations of the NCTM, the Association of Mathematics Teacher Educators (AMTE), and the Association of State Supervisors of Mathematics (ASSM) (NCTM, 2022). The NCTM (2022) cited research which suggested that elementary math specialists exert a positive effect on student achievement. The NCTM recommended that the training of elementary math specialists place especial emphasis on attaining a deep understanding of how students learn accompanied by the belief that all students are capable of mastering math. Preparation of math specialists also requires a particular focus on math content, pedagogy, and leadership skills. The intent, therefore, is to produce instructors who can not only teach at an advanced level but can elevate the instructional skills of the entire staff (NCTM, 2022). In a commensurate manner, math specialists would utilize their bailiwick to demonstrate, to their colleagues, effective and equitable instruction.

All Asian participants in my study were strong proponents as well as practitioners of specialized math instruction. The four New York State participants, although describing themselves as generalists, recognized the prospective advantages of specialization. Research has supported the idea of math specialization (Bastian & Fortner, 2020; NaYoung & Kisida, 2021). Studies have suggested that many elementary school teachers typically have more expertise in certain subjects (Bastian & Fortner, 2020; NaYoung & Kisida, 2021). This indicated that assigning teachers to subjects based on their comparative abilities may enhance student learning (Bastian & Fortner, 2020;

Campbell et al., 2014; Condie et al., 2014). The potential benefits for learners, taught by math specialists, include increased exposure to expert instruction and preparation for departmentalization that will take place in middle school (Markworth et al., 2016) as cited in (Mills et al., 2020).

Another implication concerns the reconceptualization of PD programs. Analysis of my study's collected data found that all the New York State participants were categorical in expressing profound misgivings concerning their PD in math. Moreover, as a group, they questioned the ability of current PD models to support improvements in math instruction. The American teachers summarily felt that PD in math was, in their experience, stagnant or non existent. Therefore, this implication provides an extension of the previous implication in that mathematics specialists should not only have conspicuous input in teaching fourth-grade math, but should be key players in the PD of the entire staff. Both the NCTM and the Council for the Accreditation of Educator Preparation (CAEP) asserted that math specialists, in their roles as teachers, lead instructors, and mentors, are able to provide a touchstone between theoretical knowledge and real-world applications. Math specialists can also function as vanguards in the PD of teachers as they analyze and apply research-based practices (NCTM, 2020). These individuals possess myriad credentials ranging from content specialization and pedagogical expertise to an abundance of professional experiences (Tuncel & Cobanoglu, 2018). Math specialists can therefore initiate positive changes in instructional goals while enhancing teacher capacity (NCTM, 2020). Math specialists can also promote a positive disposition toward mathematical processes and learning (NCTM, 2020). Moreover, the reconfigured PD, led by math specialists, should be onsite and ongoing, remain connected to classroom practices, and highlight sensitivity to cultural diversity as well as andragogical principles such as active learning, hands-on material, self-direction, and collaboration (Bentley & Cason, 2019; NCTM, 2020; Nelson & Bohanon, 2019; Tuncel & Cobanoglu, 2018).

Other iterations of PD may be considered. Shanghai, for example, utilizes collective learning and collaborative lesson planning in the form of open classrooms (Ye & Zhou, 2022; Zhang & Zheng, 2020). In this milieu, teachers opened their classrooms to peers, master teachers from in and outside their school, and the university cognoscenti (Ye & Zhou, 2022). The observed lessons were then followed by discussions (Zhang & Zheng, 2020). The resulting counsel helps to improve the teachers' instruction. Other recommendations for PD enhancement emanate from Shanghai. For Shanghainese teachers, research competence has become an outstanding feature of their PD (Ye & Zhou, 2022). As a result of their personal investigations, teachers develop independent, autonomous approaches to pedagogy. This produces a range of ancillary outcomes such as deep curriculum reform as well as more prestigious professional titles. Master Teachers, for instance, not only oversee the PD of colleagues but receive monetary remuneration as well (Ye & Zhou, 2022). Finally, PD in Shanghai was advanced by system-wide, resource-based support for teachers accompanied by the cultivation of a caring, trustful, and respectful school atmosphere (Zhang & Zheng, 2020).

A third implication concerns the reconfiguration or increase in the time devoted to daily math instruction. The New York State contingent pointed to limited timeframes as a referent for ongoing challenges and constant concern. Analysis of the data accrued from my participants revealed the following. New York State teachers unanimously endorsed the idea that they were not allotted adequate time to teach the entire math curriculum as mandated by Common Core. Typically, New York State teachers provided students with one hour of math instruction each day. As a result, the geometry strand was generally ignored and mastery of subject matter was rarely attempted. One of the specific problems contributing to these limited timeframes was reform-oriented teaching. Reform-oriented instruction is at the center of the Common Core Curriculum (Common Core State Standards Initiative, 2022). These approaches involve SCL, with the concomitant group work and discussions (Kaput, 2018). Student-centered learning prioritizes interactive teaching where pupils play an active role by exploring problem-centered activities in small groups, while contributing points to discussions, resolving conflicts, and explaining and demonstrating their methods and solutions to others in the class (Kaput, 2018). Compared to direct or traditional teaching methods, these activities, as well as mastering behaviors conducive to collaboration, require an investment of additional class time (Mathews, 2020; Koh-Chua et al., 2021). However, curriculum planners, school boards, administrators, and teachers' unions in the United States have failed to reconcile the dichotomy between the expanded curriculum demands and traditional daily time frames for math instruction (Leong & Kaur, 2019).

A further conundrum related to timeframes, as revealed by my participants, was the number of students who were not exhibiting readiness for fourth-grade math. This may have been partly the result of the aggregated time constraints encountered by teachers and students during previous years. More specifically, because of the cumulative nature of mathematics and the fact that new skills and concepts often require foundational knowledge, effective math instruction, in prior years, is critical to fourth-grade mathematics learning.

Lack of reading readiness was another ancillary challenge. According to my New York State participants, most authentic math problems, as well as those found on standardized tests, involved reading as well as math. Taking that into account, extra time was invested in scaffolding below-grade level performers. As a consequence of all these factors, New York State teachers may require new ways of thinking about using class time. Policy makers and administrators, in turn, will need to be supportive of changes in class scheduling to overcome these challenges. However, such options as expanding the length of the school year or school day would entail sizeable augmentations of school budgets as well as securing the compliance of teacher unions. Lastly, the use of flipped classrooms, where students prepare at home for the next day's lesson, may not be practical due to the lack of Internet connections or parental supervision for many students. Considering the totality of factors, I would encourage the New York State teachers to re-institute a half-hour of math homework each night. However, my New York State participants revealed a lack of parental involvement in the students' homework assignments due primarily to unfamiliar algorithms. To ameliorate this situation, teachers can use the students' folders to send instructions and models of current algorithms and strategies to the parents. Teachers can also create community nights where parents can be apprised of the math curriculum. Teachers can use this opportunity to assure the caregivers of an open communication policy via telephone or by email. In addition, time allotments in the school day may be recomposed. Perhaps more can be taught in a 120-minute session held every other morning than in 60-minutes taught every day. Finally, researchers have suggested that the peripheral problem of tight timeframes may be ameliorated by an increase in teacher quality (Teig et al., 2019). Studies have determined that teachers who are confident in their abilities consider time limitations as less of a challenge than those teachers with low self-efficacy (Depaepe & Konig, 2018). In congruence, teacher self-efficacy (the teacher's belief in their ability to successfully enact instruction) has been closely linked to quality instruction and student progress (Teig et al., 2019). Unfortunately, there is scant research on the time issues that arise when teachers use innovative curricula.

The study's next implication is related to teachers' preservice college or university training. Currently, colleges face the challenge of preparing teachers with the content and curricular knowledge, instructional strategies, and affective dispositions which will allow them to effectively teach 21st century skills to an increasingly diverse body of students (Darling-Hammond et al., 2019). However, many researchers have noted the increasing proliferation of ineffectual teacher preparation programs in the

United States (Cochran et al., 2015; McDiarmid, 2019). Moreover, scholarly explorations have shown that teacher education in many colleges in the United States offer curricula which feature theory albeit with tenuous links to practice (Darling-Hammond et al., 2019). These less productive approaches were accompanied by a precipitous decline (25% reduction between the years 2010 and 2020) in students attaining master degrees in education (NCES, 2020). Accordingly, my New York State practitioners cited the need for more contact with students during their college training. They also lamented the paucity of math courses as well as their lack of depth. Therefore, my recommendation concerns a reconsideration of the preservice training teachers receive at colleges and universities. More precisely, my suggestion encompasses a set of research-based goals, first mastered by teacher-candidates and thereupon practiced by their students (Darling Hammond, 2010; Darling-Hammond et al., 2019). These goals for preservice education are: Create learning that is personalized and can be transferred and applied to other contexts. This adult learning should occur in collaborative communities of practice with a strong nexus to social justice (Darling Hammond, 2010; Darling-Hammond et al., 2019). At the center of this reform is a concerted effort to link coursework directly to classroom teaching thereby connecting theory and empirical practice. This may be accomplished by inculcating more extended practicum settings where master instructors adopt a supervising role (Darling Hammond, 2010; Darling-Hammond et al., 2019). Furthermore, these practicums should *accompany* rather than *follow* the concomitant coursework. As a case in point, coursework in math methods courses might include systematic analysis of practice via videotaped lessons involving the student-teachers (Santagata et al., 2019). The subsequent observations and co-constructed interpretations of videotaped episodes may result in the cultivation of learning-from-teaching competencies (Santagata et al., 2019). Consequently, analysis of practice can be achieved by viewing classmate-created videos of instructional exercises with the expressed goal of placing learning in the context of practice (Santagata et al., 2019). In this manner, student-candidates may acquire further insights into student thinking by conducting interviews with pupils concerning their problem-solving strategies. These interviews will allow student-candidates to hone skills related to listening carefully to student responses as well as pose questions which make student thinking visible. As a result of these communal analyses, candidates can learn to surmount authentic challenges to teaching, while eliciting the support of mentors (Santagata et al., 2019). Next, universities should attempt to complement the context of student teaching with the candidates' subsequent teaching assignments, relevant to grade levels, subject matter, and student demographics (Darling-Hammond et al., 2019). Moreover, there should be an increase in the robustness of reading and mathematics content areas. Furthermore, teacher-candidates should become proficient with techniques such as performance assessments, organizing group work, planning student inquiry-based problem solving, and other long-range projects (Darling Hammond, 2010; Darling-Hammond et al., 2019). Finally, teacher-candidates should be evaluated using performance assessments that appraise day-to-day teaching skills rather than memorized information (Darling Hammond, 2010; Darling-Hammond et al., 2019).

These assessments would require teacher-candidates to create a unit of instruction, videotape lessons, and exhibit evidence of student learning. In addition, candidates should demonstrate that they have calculated the developmental and readiness levels of the students (Darling Hammond, 2010; Darling-Hammond et al., 2019). Adaptations and modifications for English language learners and for special education students should also be incorporated into the lessons. Rubrics may be used to assess their performance. Afterwards, teacher-candidates should be given explanatory feedback and, if necessary, afforded numerous opportunities to improve their work (Darling Hammond, 2010; Darling-Hammond et al., 2019). However, restructuring teacher education will necessitate a large increase in academic funding. Federal, state, and local governments should encourage the recruitment of high-caliber math students with student loan forgiveness for those who commit to teaching (SREB Teacher Preparation Commission, 2019). Government should also provide stipends for extended year-long practicums.

The final implication appertains to assessments of SCL. Often, teachers from both cohorts revealed that following group activities, they assessed student understanding with informal observations, whole class sharing, written reflections, or a plethora of end-of-unit presentations. However, these evaluations were generally not aligned with standardized assessments (Buchs et al., 2017; Luitel & Pant, 2019; Polly et al., 2014). Therefore, further research should be conducted on the topic of SCL assessments.

Conclusion

The problem of this study, identified in current research literature, was that fourth-grade math students in the United States are not performing as well on standardized tests as students of other nations and international cities (Pew Research Center, 2018; TIMSS, 2019; Woessmann, 2016). To compound the problem a large percentage of the nation's fourth-grade math students have consistently failed to reach proficient levels in national tests in the last ten years (The Nation's Report Card, 2019). These failures represented a gap in the practice. My purpose, therefore, was to collect and analyze the perceptions of 12 fourth-grade teachers representing districts in Singapore, Shanghai, and New York State, regarding the teaching of math as well as the attending factors which influenced math instruction.

Four research questions served as the fountainhead for the derivative interview queries. Participants' beliefs, elicited during the course of the semi structured interviews, divulged insights regarding best practices related to math instruction, as well as inherent obstacles to sound instruction. In addition, participants disclosed how preservice college training, PD, and Vygotsky's SLT, which served as the study's conceptual lens, affected the form and quality of their instruction. The literature review highlighted a strong connection between teacher capacity and student achievement. Therefore, my interviews focused on many aspects of instruction as well as the background and current training of the participants. Results of the study showed that both the New York State and Asian participants agreed on the efficacy of many best practices such as reflections, encouraging multiple approaches to problem solving, real-world applications, scaffolding, mini lessons, formative tests, concrete manipulatives, concept development, parental support, critical thinking, justifying thinking, cross-curricular subject matter, self-and peer-assessments, and ongoing feedback. In addition, both groups of participants described the positive assets of both SCL and DI. However, my study identified distinctions between the New York State and Asian participants on the following issues. Teacher preservice and current training, teaching for mastery, differentiation, math specialists as instructors, student-centered versus teacher-centered instruction, student input into exams, class size, issues concerning personal chrome books and interactive software programs, ability grouping, teacher expectations, grading as perquisites, competitive atmosphere, homework, timeframes for math, mainstreamed students, rote learning, and study aides.

The following issues arose which will require improvement through systematic reform. First, the New Yorkers were explicit in their strongly held view that, because of the investment of additional time required for SCL, as compared to DI, they were not able to teach the entire Common Core-based math curriculum. Hence, teachers may have to make changes in in their instructional approaches limiting collaborative activities to concept development or policy makers and administrators will need be supportive of changes in class scheduling or extending timeframes. Secondly, teachers in the study unanimously cited specialized math instructors, at the fourth-grade level, as palliative to ameliorate the gap in the practice. Thirdly, teachers from New York State reported that the prevalence of ability grouping in differentiated instruction often lead to lower expectations regarding the performance of some students. Fourthly, analysis of the data highlighted the deficiencies of many current PD practices in the United States. In congruence, a review of scholarly research has bolstered the study's contention that math specialists conduct on-site, ongoing PD in math while utilizing andragogic tenets. These PD sessions should be primarily concrete, collaborative and self-directed and should be directly related to classroom practices.

Next, further analysis of my participants' responses suggested that improvements were deemed necessary in the quality of their preservice education. This included the need for extended practicums which were aligned with characteristics relevant to their future classroom assignment (Darling-Hammond et al., 2019). Moreover, in the course of their college experience, teacher-candidates should be offered more robust math courses, as well as opportunities to observe the lessons of master teachers. Additionally, teacher-candidates should become proficient with techniques such as cultivating procedural fluency from conceptual development, performance assessments, organizing collaborative inquiry-based problem solving, and formulating open-ended questions which elicit evidence of student learning. Finally, my research identified the need to ensure that the most proficient college students are recruited as education majors. Although this particular challenge was beyond the parameters of my study, this goal was aligned with the premise of improving math instruction. Accordingly, due to the limitations of my study, further research will be needed to address this challenge and expand on my work.

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Appendix: Interview Questions

What challenges do you face in helping students attain proficiency in fourth-grade math?

What are the most challenging math concepts to teach? What strategies do you use to meet those challenges?

Is there adequate time in the school year to cover the math curriculum? Please explain.

What is the length of your school day?

How much time do you devote to math lessons in the classroom each day?

How many students are in your class?

Is there another adult in the classroom when you teach math?

Can you describe how you differentiate math instruction to account for students' different abilities?

How do you utilize concrete manipulatives?

How do concrete manipulatives affect students understanding?

In what ways are students asked to justify their answers? How frequently do you ask them to justify their answers?

What opportunities are provided for student reflection at the end of the lesson?

How much emphasis is placed on grades?

What role does technology play in math instruction?

Does each student have a personal computer in class?

How does this affect learning?

How do you feel about teachers specializing in math instruction rather than teaching other subjects as well?

Please explain the role that DI plays in math lessons.

How often do you convey information while students watch and listen?

Can you describe the process when you introduce a new math concept?

How do you encourage critical thinking in your math lessons?

What role does student collaboration play in math instruction?

How do you feel about using collaborative learning as the predominant method of teaching math?

How is DI used to support collaborative instruction?

Can you describe how collaborative activities affect classroom management?

What role does memorization of basic facts play in your instruction?

What are your thoughts about designing lessons using a discovery-oriented approach?

Do you assign group leaders in collaborative activities? What effect does that have?

What role do assessments play in your classroom?

How do you incorporate individual accountability in group work?

What challenges are associated with creating assessments for group work?

How is each child in the group assessed?

What are the benefits or drawbacks in allowing students to offer input into assessment criteria?

What is your opinion of self- and peer-assessments?

How do students demonstrate higher order thinking in the formative/ summative math exams?

How often do you offer formative exams?

How is the data from formative exams used to calibrate your lessons?

How do you incorporate mini lessons or other types of scaffolding?

How frequently do you teacher for mastery?

How much emphasis is given to revisiting previously-learned material in the formative and summative assessment?

What role do summative assessments play?

Describe how you communicate feedback following formative and summative assessments?

Can you describe the demographics of the student body? By that I mean are there special needs students in the class? Students with a different native language? Students on reduced or free lunch programs?

Tell me about the Special Education students in your class.

Are the Special Ed students assigned an aide?

Do Special Ed students have to take standardized tests?

Based on your experiences what percentage of Special Ed parents opt out of the test?

Does the inclusion of Special Ed students significantly affect the cumulative average of scores on standardized tests?

What learning opportunities are provided for gifted students?

How much time is required for students to complete daily homework assignments for math?

What role do parents play in students' math success?

How do you encourage parent participation?

How has PD affected your capacity as a fourth-grade math teacher?

Would you describe the type of PD programs that is most advantageous?

Do you find that PD sessions that focus on classroom strategies to be of particular interest?

Why is that?

How often do you participate in PD sessions?

How would you describe the mentoring program for new teachers in your school?

What changes could you visualize to improve workplace
conditions in your school?

How often are you provided with opportunities to share information about daily practices with other grade-level teachers? How has this affected instruction?

How would you describe the atmosphere in the school?

How is competitiveness encouraged or discouraged among students and staff?

What opportunities are there, during the school day, to eliminate unnecessary or less necessary activities in order to devote more time to math?

How would you rate your teacher training in math instruction in college?

What changes would lead to improvement in teacher training at the college level?