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Understanding Calculus Through Maple-Based Dynamic Visualization Tools

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This is to certify that the doctoral dissertation by

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Walden University
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Abstract

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by

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MS, Grand Canyon University, 2019

MS-IDT, Walden University, 2012

MBA, European University Toulouse, 1999

BS-Math, Northern Caribbean University, 1996

Dissertation Submitted in Partial Fulfillment

of the Requirements for the Degree of

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Abstract

First-year college students experience difficulties in understanding the concepts of derivatives and integrals. At the postsecondary level, the use of static visualization and other traditional instruction delivery methods often are unable to meet students' needs in calculus. This problem is current and essential in the field of education and needs consideration to enhance the method of teaching calculus. The rationale for this study was to scrutinize the effects of Maple dynamic visualization instructional activities, within the framework of the animation-visualization theory, on students' conceptual and procedural understanding of differential and integral calculus. The usage of a quantitative 2x2 factorial pretest-posttest control group quasi-experimental mixed design, with multivariate analysis of variance for data (de-identified list of 81 students' test scores on derivatives and integrals) analyses, helped examine the relationships between the research variables. Results showed that the Maple dynamic visualization group, significantly ($p < 0.001$), outperformed the non-Maple static visualization group with a significant interaction between the groups with a substantial effect size of at least 0.27. This study augments the body of evidence that supported the efficacy of animated visuals over static visuals in producing more exceptional academic performance. A future researcher should use the random assignment to groups to minimize the possibilities of nonequivalent groups and the same measure for pretest and posttest. This study provides a groundwork for positive social change to reach a shared vision in education, enable learners to gain skills in calculus, and prepare students in and for science, technology, engineering, and mathematics majors and careers.

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Dedication

I dedicate this dissertation to my loving and caring wife, Sherry Ulis L. Kossivi, who encouraged and supported me, in every way, to pursue my dreams and earn my Ph.D. Without supports from my dearest darling, friend, and partner Sherry, earning a Ph.D. would have been a shattered dream. Thank you, my supportive and beloved partner.

To my Princess Binah Kossivi, this is a legacy for you to emulate.

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Chapter 1: Introduction to the Study

Introduction

The students' conceptual and procedural understanding of derivatives and integrals in calculus, through Maple-based dynamic visualization (animated visualization) tools, within the framework of animation-visualization theory (Erlich & Russ-Eft, 2011; Lakhvich, 2012; Nossun, 2012) became the focus of this study. At the postsecondary level, available research on the use of static visualization (still pictures, graphs, PowerPoint slides) and other traditional methods of instruction show they are unable to meet the students' needs in calculus (see Sevimli, 2016a). Mathematics educators also recognized that college students often experienced difficulties in understanding the concepts of derivatives and integrals, due to the abstract nature of calculus (see Covington et al., 2017) and yet lacked in perception (see De Freitas, 2016). While some instructors used graphing calculators (GCs) with incorporated computer algebra system (CAS) to help students' learning, these teachers used GCs' features much more than CAS features due to the lack of motivation of learning innovative technologies and demands from external assessments (see Karadeniz & Thompson, 2018).

Prior research on the uses of GCs in calculus was mostly set at the postsecondary level and used descriptive methods. In this study, a potential approach to enhance and update teaching calculus at the postsecondary level is necessary to enable students to gain skills in calculus, which is a gateway subject to science, technology, engineering, and mathematics (STEM) education. According to Ellis et al. (2016), an innovative approach to teaching calculus could increase the performance of students in calculus, reduce the gender gap in STEM education, and enable the United States to compete globally. However, reducing the gender the gap was not the focus of this study.

Dynamic visualization constituted an essential pillar in an effective instructional system (see Pretorius et al., 2017; Soemer & Schwan, 2016; Verhoeff, 2020), especially in mathematics, enhancing students' spatial skills (see Verdine et al., 2017). Nevertheless, educational research had yet to ascertain the benefits of the animation-visualization theory in teaching calculus at the tertiary level. Also, available research, on GCs were mostly at the secondary level and were unsophisticated case reports (see Karadeniz & Thompson, 2018). The lack of applying this theory in teaching calculus at the postsecondary level was a current research gap.

The setting for the study was Lehman College (LC). The college's mathematics and computer science department took a leadership role to transform mathematics learning, with the ambition to digitize its mathematics degree programs, using one of the innovative technology tools, such as Maple software. Maple is a mathematical software package with graphics, computation, and programming tools, encompassing CAS, dynamic interactive graphing applets, and math palettes (see Meikle & Fleuriot, 2012). It possesses sophisticated functionality to assist with mathematical problem solving (see Bunt et al., 2013).

While instructors widely used GCs such as Texas Instruments (TI) in mathematics teaching, these GCs did not possess Maple technology tools (see Yu, 2014). The use of TI-Nspire calculators had helped provide some limited interactivity, but they still have limited processor speed, which is critical for dynamic and interactive math applets and palettes (see Meikle & Fleuriot, 2012). On the other hand, the Maple platform's use could help teaching analytic geometry and calculus, differential equations, and statistics (see Yu, 2014). Learners could benefit from the Maple three-dimensional (3D) tool to create, retain, retrieve, and transform structured visual images in learning calculus.

Instructors could use the Maple platform with its animation and visualization tools and resources to be more productive and effective by enriching students' learning (see Rusli & Negara, 2017; Salleh & Zakaria, 2016; Végh & Stoffová, 2017). Buneci (2014), Bunt et al. (2013), and Roanes-Lozano et al. (2014), in their research, provided evidence for or against an application of Maple as a computational software. This study and other studies sought to promote the benefits of visualization and animation that might foster increased learning software development that could be targeted and sold to households versus educational institutions. The study results indicated that the use of dynamic visualization could enhance the method of teaching calculus at the postsecondary level, to enable students to gain mathematical confidence and insight in calculus, a crucial subject to science, technology, engineering, and mathematics (STEM) education, reduce the gender gap in this field and to enable the United States to compete in the global market.

Chapter 1 contains the discussion on the background of the study, problem statement, and purpose of the study. It comprises of the research questions and hypotheses, theoretical foundations, and nature of the study. It also encompasses the construct definitions, assumptions, scope and delimitations, limitations, significance, and a chapter summary.

Background

Information and communication technologies have advanced to provide emerging software with animation and visualization techniques in computer science, meteorology, military, graphics, and medical field (see Agbatogun, 2013; Blazhenkova & Kozhevnikov, 2016; Karakus & Duressi, 2017; Kinkeldey et al., 2014; Lv et al., 2013; Opach et al., 2014; Persson, 2014; Sarlis & Christopoulos, 2014). However, people rarely found reports of using such software to teach calculus at the tertiary level. In a study on the use of Maple software for teaching calculus, Samson (2014) focused on the computational aspect of the software, while Roanes-Lozano et al. (2014) concentrated on using Maple codes, evidencing the benefits of Maple in visualizing and generalizing square arrays (n x n) of numbers to generate a formula for the arithmetic sum of the first n numbers. It is essential to teach calculus to go beyond this level of practice, by taking advantage of emerging software, such as Maple, with its animation and visualization tools, to challenge the traditional method of an instructor's delivery and enhance students' conceptual and procedural understanding of calculus.

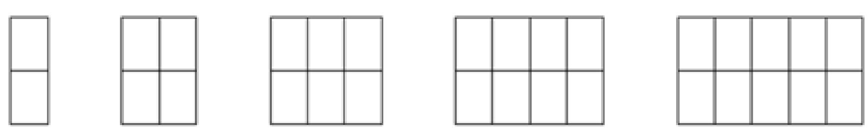
In mathematics education, effective interaction with visual representations using CAS-based GC could enhance students' intellectual skills (Ghani et al., 2012; Prahani et al., 2016). At SRI International, study results on CAS graphing calculators, TI, and networked graphing calculators (TI-Navigator system) showed that TI technology incited innovative ways to engage the classroom learning (Leng, 2011). Ghani et al. (2012) postulated that advanced GCs with registered marks TI-84 plus, TI-Voyage (in Europe) or TI-Nspire and Casio-ClassPad 330 possessed powerful software with the programming options, supported the undergraduate students' difficult problem solving and facilitated student-centered teaching. Yildiz Ulus (2013) asserted that the pedagogical experimentation (numerical computation), educational (teaching) tool, and algorithmic (programming) aspects of advanced calculators and their functionality in linear algebra could extend to other domains of mathematics. Nevertheless, in the high school mathematics education, the students' deficiency of operating skills and teachers' approaches to the use of the equipped advanced CAS-based GC contributed to the ineffective use of the technology (Bardini & Pierce, 2015; Brown, 2015a; Brown, 2015b; Karadeniz & Thompson, 2018; Moy et al., 2015). Thus, students and teachers limited the use of those GCs to quick algebraic and numeric computations and consequently reduced the active interaction with visual representations with CAS-based GC (Solares & Kieran, 2013). Individuals could recognize that CAS-based GCs might support calculus students' critical thinking and increase their learning of the abstract nature of calculus (Ghani et al., 2012), with their visualization representation capability. Hence, despite extensive research on visualization in math education using GCs, a less comprehensive study on CAS was prominent (Hitt, 2011).

Nevertheless, Maple encompasses an advanced symbolic computation engine with powerful numeric algorithms, advanced visualization tools, and intuitive interfaces designed to enrich calculus teaching and learning experiences (see Salleh & Zakaria, 2016). Moreover, CAS-based Maple might provide a dynamic learning environment with more student-centered pedagogy than traditional instruction (see Milovanović et al., 2016). The use of Maple technology might support users in experiencing an active learning environment, in explaining some difficult concepts of calculus, in facilitating mathematical notation (Bali et al., 2016; Kumar et al., 2019; Salleh & Zakaria, 2016; Samková, 2012; Vieira, 2015), and in promoting the visualization of scientific and mathematical concepts, without the limitations of the Microsoft equation editor.

Graphical representations (GR) in math (number lines, strips, graphs) help individuals encode and respond to general information through the visual sensory channel (see Solares & Kieran, 2013). GR assists individuals in establishing the means of solving a mathematical problem (see Anderson et al., 2014). One of the advantages of GR in math was to assist the learner in understanding the concept of magnitudes as locations, lengths, areas, and volumes (see Pyke et al., 2015). For instance, students could generate a function rule (formula) of a sequence by observing given patterns and counting the number of boxes in Figure 1.

Figure 1

Graphical Representation of a Sequence



Looking at Figure 1, people could count 2, 4, 6, 8, and 10 boxes in the first, second, third, fourth, and fifth positions, respectively, and note that the next number or boxes consisted of adding two boxes to the number of previous boxes or multiplying the number of positions by 2. That was, if n represented the position, then the number of boxes would be $2n$. Therefore, the set of numbers: 2, 4, 6, 8, 10, …, $2n$, is a sequence, which was a series of numbers that followed a definite pattern. The visual representation of this sequence might assist individuals' conceptual and procedural understanding of these numbers.

According to Dobler and Klein (2002), Descartes might be the founder of visual representation in math. Descartes observed the movement of a fly jumping and landing from place to place on the ceiling of his room. He decided to put a grid on the ceiling. As the fly moved from one point to another, Descartes would mark the spot on the grid, noting the distance between points across, counting the number of units horizontally, and vertically. Thus, from a dynamic movement of the fly and a visual representation of such movement, Descartes developed the Cartesian coordinate system (Dobler & Klein, 2002), which is foundational to the visualization of mathematical relationships. Furthermore, graphical representations in math might help engage the learner's mental processes, which were necessary for conceptual understanding of math and problem-solving.

Scholarly articles have provided evidence of Maple's potential as an instructional medium. However, they offered less information on animation-visualization theory in the teaching of calculus at the postsecondary level (Buneci, 2014). Jahanshahi et al. (2015), in their study, detailed the use of the trapezoidal rule and CAS Maple to solve, numerically, Abel integral equations of the first kind. Moreover, Yurttas et al. (2012) asserted that Maple was an efficient tool to calculate the Minimal Polynomial of $2\cos(\pi/n)$ over Q (the set of rational numbers).

Unfortunately, research was unavailable on the application of animation-visualization theory to the teaching of calculus using Maple's capabilities. However, Haciomeroglu (2016) found that a relationship between visualization correlated with spatial, verbal-logical reasoning, and mathematical problem solving. Carden and Cline (2015) and Kidron and Tall (2015) used a sequence of visual graphs to demonstrate the convergence of a series of functions to a fixed limit of functions using Mathematica. These authors concluded that the software helped in blending dynamic perception and symbolic operation as tenets of mathematical reasoning. Like in the case of Mathematica, it was essential to apply visualization theory to the teaching of calculus using Maple's capabilities to ensure students' readiness to embrace STEM-related careers. The proper techniques could enable instructors to use dynamic visualization to tie together the verbal, symbolic, and graphical representations of math concepts at every level from numbers through calculus.

Problem Statement

Given the difficulties that college students experience in understanding the concepts of derivatives and integrals due to the abstract nature of calculus (see Covington et al., 2017; Katsioloudis et al., 2016; Salleh & Zakaria, 2016), it is essential to understand the role of dynamic visualization in teaching the concept of derivatives and integrals. Moreover, the literature on the use of animation-visualization theory in teaching calculus at the postsecondary level is scarce, despite the theory's use in other fields (see Kinkeldey et al., 2014; Opach et al., 2014; Persson, 2014; Sarlis & Christopoulos, 2014). The problem is current and relevant in math education. It needs attention to enabling learners to gain a mathematical understanding of calculus to prepare students in and for STEM majors and careers.

While educators can use hand-held graphing calculators to support highly interactive and student-centered pedagogy capabilities of a new generation of the classroom-based interactions, the superior power of Maple provides the opportunity to overcome the limitations of the prior tools. Consequently, the problem is to understand further the role of animation and visualization tools within the animation-visualization theory framework in teaching math at the postsecondary level. Specifically, this study seeks to understand the potential role of Maple dynamic visualization (animated visualization) tools to assist students in their conceptual and procedural understanding of derivatives and integrals in first-year college calculus.

Purpose of the Study

The purpose of this quantitative quasi-experimental was to ascertain the impact of Maple-based dynamic visualization lessons, designed within the framework of the animation-visualization theory (see Erlich & Russ-Eft, 2011; Paik, 2012; Zurita & Nussbaum, 2007), on college students' conceptual and procedural understanding of derivatives and integrals in calculus. The independent variable is the type of visualization (non-Maple static visualization vs. Maple dynamic visualization). The dependent variable is the type of understanding (conceptual and procedural understanding of derivatives and integral in calculus).

Research Questions and Hypotheses

Four research questions guided this investigation. The research questions examined the effects of the Maple dynamic visualization activities on students' conceptual and procedural understanding of derivatives (RQ1 and RQ2) and integrals

(RQ3 and RQ4) in calculus. The instrumentation comprised students' pretest (prerequisite skills for derivatives and integrals), quiz (posttest1), and end of term exam (posttest) scores on the derivatives and integrals' concepts and procedures in calculus, with the use of multivariate analysis of variance (MANOVA) for statistical analysis.

RQ1: Was there a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?

H_01_1: There was no significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

H_11_1: There was a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

H_01_2: There was no significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test.

H_11_2: There was a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test.

RQ2: Was there a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?

$H_0 2_1$: There was no significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

$H_1 2_1$: There was a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

$H_0 2_2$: There was no significant difference in pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test.

$H_1 2_2$: There was a significant difference in pretest (prerequisite skills for derivatives' procedure), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test.

RQ3: Was there a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?

$H_0 3_1$: There was no significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

$H_1 3_1$: There was a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

$H_0 3_2$: There was no significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test.

$H_1 3_2$: There was a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test.

RQ4: Was there a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?

$H_0 4_1$: There was no significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1) and end of term exam (posttest) scores on the integrals' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

$H_1 4_1$: There was a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1) and end of term exam (posttest) scores on the integrals' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

$H_0 4_2$: There was no significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' procedure calculus test.

$H_1 4_2$: There was a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1) and end of term exam (posttest) scores on the integrals' procedure calculus test.

Theoretical framework

The animation-visualization theory was the basis for the theoretical framework of this study. Visualization was essential in learning, as Mayer (2014) observed that mental processes, which formed the cognitive procedure, stemmed from visual models. Two studies added detail and evidence: Nossun (2012) and Pyke et al. (2015) showed that learners constructed knowledge from visuals models, as learning encompassed the somatic and psychosomatic pillars of the theoretical cognitive process of visualization, and interactions between the two. According to Nossun (2012) and Pyke et al. (2015), the advantages of dynamic visualization (animated visualization), which transmitted instructional contents, realistically, in video form and procedural-motor-type of knowledge, exceeded those of static visualization (still picture). These two studies lead to infer that dynamic visualizations could enhance the learning process. Also, learners' spatial ability, and 3-D animations offered an environment that supported a learner's inadequate mental model (see Castro-Alonso et al., 2016; Katsioloudis et al., 2016; Sarlis & Christopoulos, 2014). Consequently, it was essential to optimize the combination of realistic animation and visualization to explore college students' conceptual and procedural understanding of derivatives and integrals in calculus. The fully described animation-visualization theory, in Chapter 2, provided a sound theoretical framework for the research question on teaching and learning calculus through Maple technology tools.

Nature of the Study

This study's was a quantitative 2x2 factorial pretest and posttest control group quasi-experimental design (QED). The design was appropriate for examining the relationship between constructs. The study consisted of using the logic model (deductive approach): theory-hypothesis-observation-confirmation to guide the path of students' conceptual and procedural understanding of derivatives and integrals in calculus. The use of multivariate analysis of variance (MANOVA) would contrast the intervention and comparison groups and establish students' performance gains on test questions related to derivatives and integrals.

The instruments were the instructors' generated test scores on derivatives and integrals, which would serve as distinct elements in interactive time that might help learners understand the abstract nature of derivatives and integrals in calculus (see Aurigemma et al., 2013; Lv et al., 2013). In Figure 2 and Table 1, I displayed the study's variables. While Figure 3 illustrated a logic model diagram of the constructs that would interact in a predictive relationship, using design principles from animation-visualization theory, Table 2 featured learning gain and triangulation plan.

Figure 2

Definition of Criterion and Response Variables

Let X1 = Non-Maple-based visualization (static visualization) tests (pretest/diagnostic, posttest1/quiz, and end of term exam/posttest) scores on derivative and integrals,
X2 = Maple-based animated visualization (dynamic visualization) tests (pretest/diagnostic, posttest1/quiz, and end of term exam/posttest) scores on derivative and integrals,
Let IV = Independent variables (X1 and X2),
DV = Dependent variable (Students' conceptual and procedural understanding of derivatives and integrals),
OV = My observation notes on class activities and interview response from intervention group professors on derivative and integral,
Y11 = Pretest scores on derivative and integral-related questions,
Y12 = Posttest (end of term exam) scores on derivative and integral- related questions,
Y12 – Y11 = Posttest and Pretest gain,
OV and Y12 = My observation descriptive notes on any variation between OV and Y12 for triangulation (on dynamic visualization).

Table 1

Theoretical Constructs

	Independent Variables (IV)	Fidelity of Implementation Variables (OV)	Dependent Variable (DV)
Non-Maple Static Visualization	X1	OV	Y11
Maple Dynamic Visualization	X2	OV	Y12

Figure 3

Logic Model Diagram

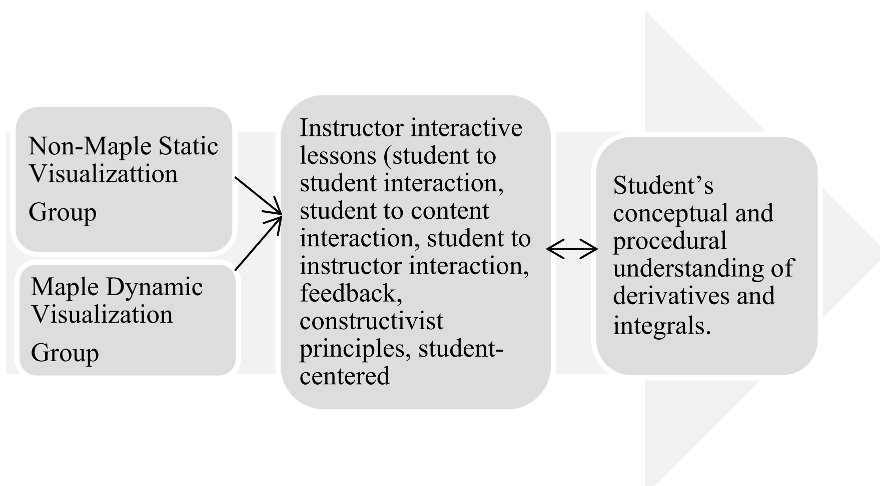


Table 2

Gain and Triangulation

	Measuring Gain	Triangulation	Fidelity Measure
Attainment difference	Y12 - Y11	OV and Y12	Compare and contrast OV and Y12 for triangulation (example)
Ascertain and analyze any difference between OV and Y12 via my observation note and interview response			Compare and contrast OV and Y12 for triangulation (example)

This quantitative 2 x 2 factorial QED with pretest and posttest control group, as the initial research design, was an excellent fit to control factors that could affect the internal validity (see Campbell & Stanley, 1963). This design helped reduce bias, establish the construct and content validity, and avoid internal validity threats. The use of MANOVA, as statistical analysis for all four research questions, helped assess any statistically significant interaction between the comparison and intervention groups and the pretest, quiz, and posttest periods. The use of a sampling strategy, sample size, and power provided a way to deter any biases and empower generalizability (see Guilleux et al., 2014). The choice of QED stemmed from the impossibility of randomization due to the conditions surrounding students' registration to the calculus classes. I detailed those conditions in Chapter 3.

Construct Definitions

Animation: Animation is the creation of a slow or rapid series of representations, which individuals use to represent dynamic relationships (Haciomeroglu, 2016).

Conceptual Understanding: Conceptual understanding (CU) encompasses the students' ability to describe and explain the factors and variables related to the unknown to the known in the given calculus problem for an adequate solution (Ocal, 2017; Rittle-Johnson & Schneider, 2015)

Derivatives: Derivatives of a function are the average and instantaneous rate of change of a function concerning a specified index (Park, 2015; Patwardhan & Murthy, 2015).

Integrals: Integrals of a function are the primitives of that function.

Maple: Maple is an essential tool for researchers, teachers, and students in any mathematical discipline. It allows users to explore, visualize, and solve even the most complicated mathematical problems, reducing errors, and providing greater insight into the learning of math (Salleh & Zakaria, 2016).

Procedural Understanding: Procedural understanding (PU) depicts the students' model solution, description, or explanation of the model (Ocal, 2017; Rittle-Johnson & Schneider, 2015).

Visualization: Visualization is a visual representation, via static, interactive, or animated (2-D or 3-D) of an object's structure, execution, behavior, and evolution, using software systems (Serenko, 2007).

Assumptions

I considered the following assumptions for this study: (a) the participants' honest behavior, following the standard administration protocols in administered tests, and generalizing the results to the study group, (b) the theoretical assumptions, which encompassed animation-visualization theory (see Erlich & Russ-Eft, 2011; Lakhvich, 2012; Nossun, 2012), (c) student's self-efficacy about calculus, teacher content knowledge, which is necessary for effective teaching and learning of mathematical concepts, (d) an increase in students' intrinsic and extrinsic motivational tools for subject affinity, as they relie on content, communication, and collaboration, (e) a provision of the necessary resources to enable learners to meet established specific, measurable, accepted, realistic, time-bound (SMART) goals and objectives for desired learning outcomes, and (f) the alignment of assessments with the learning objectives.

Scope and Delimitations

In this study, I focused on college students' conceptual and procedural understanding of derivatives and integrals of functions, through instructors' use of Maple-based animation and visualization lessons. It was unfeasible to study the levels of technology implementation in the college, administrators', teachers' confidence, proficiency in technology use, and the technology integration process in the college. The focus was not on learners' mathematical abilities and demography.

Regarding potential generalizability, one logical fallacy I avoided was to assume that the animation and visualization theory found in cartography or engineering courses functioned equally in calculus. Another fallacy was the assumption that all math instructors could use Maple to teach calculus. I avoided those overgeneralizations and dealt with the experts in using Maple software for the intervention group.

Limitations

Some limitations, discussed in Chapter 3, Chapter 4, and Chapter 5 of the study, related to design involved the sample size, no randomization, and inequality of the identified instruments in the theoretical framework session for internal, external, and construct validity. Threats to validity comprised internal and external threats. Relevant factors jeopardizing internal validity (see Campbell & Stanley, 1963) might include the following:

- History: any events that might occur between the pretest and posttest scores could be the learning through the intervention implementation, which was the focus of this study. Students' performance scores were analyzed according to the study design for interpretation to mitigate the threat to validity.
- Maturation: in this study, the time was a portion (when students learned the concepts of derivatives and integrals) of a semester, where students might increase their scores on the measurement regardless of the intervention. During the 15 week-duration, five students dropped the course, thus reducing the sample size, which became a limitation of the study.
- Testing dealt with the effects of taking a test on the outcomes of taking a second test. In this study, the posttest (end of term exam on derivatives and integrals) paralleled the pretest (prerequisite skills before the instructors taught derivatives

and integrals) and quizzes on derivatives and integrals, without repetition of questions, for a fair and valid interpretation of students' performance gain.

- Instrumentation: class observation notes and instructors' e-mailed interview responses served as a source for triangulation. The instructors were responsible for the students' graded scores for accuracy, reliability, and validity to avoid instrumentation as a threat to validity.
- Differential selection biases: the biases that might result in the selection of the comparison group, might occur. However, resorting to a convenience sample, the use of an adequate sample size with G* Power 3 (Faul et al., 2009) computation might mitigate these biases.
- Experimental mortality might reduce the sample size. The use of the provided attrition and the sample size for adjustment assisted in mitigating this threat. However, in this study, the sample size was smaller than the required G* Power 3 computation, after data cleaning and checking for error, and this threat became a limitation in discussing generalizability in Chapter 4 and Chapter 5.
- Selection-maturation interaction might lead to confounding outcomes, and erroneous interpretation might be a threat to the study. Nevertheless, there were no confounding outcomes.

The use of convenience sampling in a quasi-experiment inherently sacrifices some internal validity in favor of external validity (see Handley et al., 2018).

Significance of the Study

The study's uniqueness resided in its contribution to an underresearched area of the blending of activity, animation-visualization theory in students' conceptual and procedural understanding of derivatives and integrals in calculus as a gateway subject to other disciplines such as sciences and engineering. The study results might provide college math instructors and learners insights and methods of considering math as an organization tool for problem solving in a real-life situation and transfer of knowledge, through students' activity. The study was significant, as learners gained skills to equip them to enroll in programs that might lead them to embrace mathematics-related careers.

In terms of social change, at the micro- and macrolevels, the study provided evidence to enhance the method of teaching calculus at the postsecondary level and enable learners to gain mathematical skills in calculus. At the megalevel, learners could gain mobile learning skills, which was necessary for a transdisciplinary approach to reach a shared vision in education for a better and informed society, as Covid-19 has forced individuals into this time, for example. Moreover, the intended positive outcomes of this study might help increase the number of graduates in STEM and enable the United States to compete in the global market with a better STEM workforce (see Hutton, 2019). Furthermore, the research study might contribute to educational technology and add to the existing literature in academia.

Summary

Calculus instructors' simultaneous use of Maple-based dynamic visualization lessons and animation-visualization theory might enhance college students' learning and understanding of calculus. Maple-based animation and visualization activities might help LC students grasp the concept of graphs, envelope, and rotational-generated solids in calculus better than the still images of these objects they found in textbooks. Maple, with its applets and mathematics palettes, might become a wild card and disruptive technology that supports asynchronous and synchronous education and revolutionizes the American education system. The components of Chapter 2 included the literature review, the gap in the literature, animation-visualization theory, computer-based learning, and Maple technology.

Chapter 2: Literature Review

Introduction

College calculus is often a prerequisite requirement for advanced coursework in many STEM majors (see Cohen & Kelly, 2019). Students who experience difficulties understanding calculus (see Wismath & Worrall, 2015), a gateway subject to the STEM field see (Smolinsky et al., 2019), might shy away from the math-related field (see Mau & Li, 2018; Persaud & Burns, 2018).

The literature is scarce on the use of animation and visualization tools within the framework of animation-visualization theory in teaching calculus at the postsecondary level. Studies on the use of Maple's dynamic visualization are scarce. It is essential to examine the effect of Maple technology, within the framework of animation-visualization theory, on college students' learning of calculus, especially in their conceptual and procedural understanding of derivatives and integrals. This study's rationale is to establish the impact of Maple technology on college students' development of the conceptual and procedural understanding of derivatives and integrals in calculus. Chapter 2 comprises the literature search strategy, theoretical foundation, and ascertaining a gap in the literature, a related literature review on the main concepts, use of animation and visualization theory at the postsecondary education in calculus, Maple technology, and a summary of the chapter.

Literature Search Strategy

Library research strategies on *Maple technology, mathematics education, calculus, animation and visualization theory, interactive learning, geometric modeling,* *visualization, computer algebra system, graphing calculators, dynamic geometry environment,* and *Maple*, generated many peer-reviewed articles on the research topic. Through a search of multiple databases with Thoreau and multidisciplinary databases (Academic Search Complete, ProQuest Central, EBSCO, and Science Direct), I obtained numerous articles on synthesizing literature and ascertaining the detected gap in calculus understanding, including those from the International Congress on Mathematical Education , Psychology of Mathematics Education, Association for the Advancement of Computing in Education, and International Society for Technology in Education.

Theoretical Framework

The information communication technology has advanced to provide emerging software with animation and visualization techniques in the medical field, computer science, and others, especially in, cartography, imaging, and graphics (see Kinkeldey et al., 2014; Opach et al., 2014; Persson, 2014; Sarlis & Christopoulos, 2014). However, there was little evidence of the use of such software in teaching calculus at the postsecondary level, especially in differential and integral calculus. Recent theoretical research on visualization in math focused only on visualizing and generalizing with square arrays (n x n) of numbers to generate a formula for calculating the arithmetic sum of the first n numbers (Samson, 2014) in teaching calculus. It has become essential to take advantage of the emerging software, such as Maple, with its animation and visualization tools to challenge the traditional method of an instructor's delivery and enhance students' conceptual and procedural understanding of calculus.

Extensive research on the use of graphing calculators with computer algebra systems (CAS) has contributed to effective teaching and learning of calculus (Jarvis et al., 2014; McCulloch et al., 2013; Persson, 2014; Solares & Kieran, 2013). More than a decade ago, a limited version of CAS gained popularity on some hand-held calculators to handle complex numbers (Vincent et al., 2017).

Math instructors faced challenges in teaching with technology (Bunt et al., 2013). According to Vincent et al.'s (2017) study, teachers and students reduced the use of CAS and its rare usage to examining and graphing functions and missed strategies that might stimulate mathematical thinking and understanding. Furthermore, studies on CAS tended to be vague about the treatment and were often small in scale with weak methodologies; sometimes, the researchers did not articulate the theoretical framework. Maple technology offered more functionality than standalone CAS. Consequently, there was a need to further the understanding of the role of Maple animation and visualization tools, in teaching calculus at the postsecondary level, in learning differential and integral calculus (Salleh & Zakaria, 2016) especially, to bring across the abstract nature of derivatives and integrals in calculus to college students. Because static pictures could not directly depict these changes, Salleh and Zakaria (2016) investigated whether the corresponding informational disadvantage of static pictures could be compensated by describing the missing information in a text. Results revealed that animations still led to a deeper understanding of the content. Thus, according to Salleh and Zakaria, carefully designed animations for educational purposes could possess an informational advantage over static pictures, for instance, by directly depicting dynamic changes such as changes in the velocity of an object.

Literature Review Related to Key Variables and Concepts

The fundamental concepts and variables included non-Maple static visualization, Maple dynamic visualization, students' conceptual and procedural understanding of derivatives and integrals, and animation-visualization theory.

Animation and Visualization

Many have questioned the effectiveness of animation and visualization in learning, and several previous empirical studies have given reasons to detractors to support their negative views on understanding many concepts through animation and visualization, as those research findings suggested that animation and visualization were not necessarily superior to static visualizations (Ghani et al., 2012). However, other prior studies have shown that in various disciplines such as atmospheric science, biology, cartography, engineering, and physics, animation and visualization have played a crucial role in the delivery of instructional materials about nonconcrete concepts that were difficult to understand, or that encompassed abstract content such as calculus (Lin, 2011; Nossun, 2012). From the other accounts, people could infer that individuals' precise understanding of the effect of animation on learning was still unclear (Ghani et al.). These blurred perceptions accentuated when people experienced a core problem of using animation and visualization to tie together the verbal, symbolic, and graphic representations of math concepts in calculus. The unsynchronized presentation of these representations of a function, for instance, made it all but impossible for a student to solve a problem that required shifting from one representation system to another. The solution to this fundamental problem was to avoid teaching these three representations separately. Animation and visualization provided opportunities for calculus learners, through geometrical representations, to understand the mathematical concept of rate of change of objects on their trajectory (application of derivatives), and the computation of the area under the curve of a function, which required an application of integrals (see Salleh & Zakaria, 2016), which will constitute the objects of analysis in this study, according to the sample lesson plan (see Appendix B).

In this study, the instructors taught the three representations concurrently. Much of calculus has to do with the rate of change and optimization. These concepts were inherently dynamic, and thus, calculus facilitators could use dynamic visualization (animated visualization) to help students in their learning. Hence, a well-constructed graphic that visualized relevant concept attributes might improve instruction.

In a computer-based instructional (CBI) environment, accessible animations as pictures in motion were dynamic visual graphics that facilitated instructional and learning processes. In a posttest, only factorial experimental design, Lin (2011) examined the effect of static and animated visuals on students' learning of different educational objectives in a CBI environment and found that there was superior effectiveness of animated visuals on students' learning over static visuals. This statement was consistent with some previous studies that found significantly superior effects with animation than with static visuals, for a sampling of 80 analyzed items. Current research (Kühl et al., 2018) echoed Lin's study results that animations promoted a deeper understanding of the concept of velocity than still pictures, especially in students with low spatial abilities, which were essential in visualizations. Lin and Kühl et al. (2018) have contributed to research and practice, in providing insight to teachers to view visualizations as a suitable support for teachers' design inquiry of location-based learning activities, and enabled students to make an adequate diagnosis of their performance (Melero et al., 2015). These studies were significant and applicable to Maple-based animation and visualization interactive instructional materials.

Seeking to propose a solution to the concern of visualizing temporal and spatial information in cartography, Nossun (2012) has deviated from the discussion on static versus animated maps that previous researchers have undertaken and proposed combining qualities from both and introduced the concept of semistatic. Nossum found that dynamic visualizations were useful for learning human and non-human movements, helping students remember and understand the materials they studied (De Koning & Tabbers, 2013). The animation-visualization theory presented numerous advantages for a learner to gain insight into the abstractness of some mathematical concepts, using a content-rich and activity-based course. It might supplement the traditional non-interactive technologies use in teaching math at the postsecondary level. From this section, it was clear that the animation and visualization theory supported students' active engagement for conceptual and procedural understanding, which were some requirements of efficient learning.

Animation and Visualization: Teaching and Learning in STEM Education

At the International Carpathian Control Conference, presenters ascertained that the use of animation was suitable for solving local extrema (minimum or maximum) for functions of two-real variables x and y (Mojžišová & Pócsová, 2018). Impelluso (2018) found that students were able to experience 3-D dynamics through the visualization of interactive animations that favored students in solving physics problems. Correlational analysis revealed that spatial ability, verbal-logical reasoning ability, and mathematical performance correlated significantly. High spatial visualizers had significantly higher spatial ability and mathematical performance scores than high object visualizers. However, there were no significant differences between verbalizers and high spatial visualizers in their verbal-logical reasoning ability and analytical performance scores. Results provided support for the existence of two different groups of visualizers concerning their spatial ability.

Solving calculus-related problems such as finding limits, maximum and minimum (see Mojžišová & Pócsová, 2018), and related-rate problems, tangent lines of a function at a given point, or rates of changes requires the application of derivatives. Finding the area under a curve or calculating the volume of a solid of revolution entails the application of integrals. These types of problems stand for the components of calculus and helping learners understand derivatives, and integrals can help students in their success in other advanced analysis courses (see Ocal, 2017). Solving optimization problems, such as finding the dimensions of a rectangular fence, requires the use of variables, function, which serve as an equation relating the defined variables, constructing a table of values, graphs, visualization, and even animation, and derivatives to calculate the maximum or minimum values (see LaRue & Infante, 2015). Therefore, in this study, in helping students understand calculus, the intervention group's instructors, concurrently, used verbal, symbolic, and graphic representations, animating all the visuals for students to emulate. That meant the students modeled the used Maple animation and visualization tools in class, their homework, the quizzes, and the end-of-term exam. The intervention instructors and students used the maple applets, programing codes, and palettes in their learning and teaching (see Appendix B).

The Role of Multiple Representations in Learning Calculus

The use of multimedia presentations remained of prime importance to lessen the irrelevant cognitive load and increase the relevant cognitive load, coherent with the multimedia, modality, and spatial-contiguity principle (Jung et al., 2016; Mayer, 2014). The modality principle, which required presenting words as speech rather than on-screen text (Jung et al., 2016), could deepen students' understanding of the presented material when an instructor explained current information by audio narration rather than on-screen text. The spatial-contiguity principle consisted of placing related graphics of learned concepts in proximity with text to minimize cognitive processing by positioning-related graphics and narration (text) in proximity to ensure students' undivided attention (see Jung et al., 2016; Mayer, 2014). The use of words and meaningful graphics (multimedia principle) has contributed positively to students' learning. Moreover, the narration and animation of learned concepts in a synchronized manner with visual or analytic processing might contribute to the students' conceptual, procedural, and strategic understanding (see Foshay & Silber, 2009) of derivatives and integrals.

Learners need to develop the aptitude of transmuting between intangible conceptual representations (see Özkan et al., 2011) of mathematical concepts and real-world representations via a body's kinesis. Advanced technologies have increased individualized learning opportunities to integrate animation and visualization theories in teaching and to learn (Ghani et al., 2012; Lin, 2011) to enhance learners' aptitude. Erlich and Russ-Eft (2011), Lakhvich (2012), and Nossun (2012), in their studies, showed that animation and visualization were essential elements in an effective instructional system that promoted student-centered education, especially in mathematics.

Learners could benefit from the 3-D representation of a generated solid of revolution in calculus, in terms of understanding, creating, retaining, retrieving, and transforming structured visual images (see Allendoerfer et al., 2014; Choi et al., 2013; Nathan et al., 2013) in learning of STEM. For example, in this study, one calculus problem that students solved was to find the volume of the obtained figure from rotating the area under a curve of the given function $y = f(x) = 1+\cos(x)$, with $0 \leq x \leq 3$ and $0 \leq y \leq 2$. The solution to this problem involved the concept of a definite integral. Students used their prior knowledge to graph the given function and use their learned integral concepts to arrive at the solution. The calculus instructor modeled the concept using Maple animation and visualization tools and engaged students to arrive at the solution. Solids of revolution resulted from rotating portions of curves between functions, about an axis (see Swift, 2017), with the displays below. Figure 4 was the tabular representation, and Figure 5, the graphical representation, of the symbolic representation of f(x). Figure 6 was a result of rotating a part of Figure 5 about the x-axis. While Figure 7, Figure 8, and

Figure 9 were the static visualization, Figure 10 was the dynamic representation of the solid of revolution of f(x).

Figure 4

Tabular Representation of the Function f(x) = 1 + cos(x)

x	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
y	2	1.87	1.54	1.07	0.58	0.20	0.01	0.06	0.34	0.80	1.28	1.71	1.96

Figure 5

Graphical Representation of the Function f(x) = 1 + cos(x)

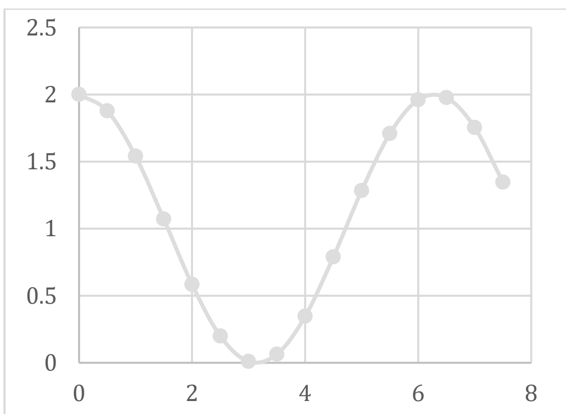


Figure 6

Graphical Representation of Rotation of Figure 5

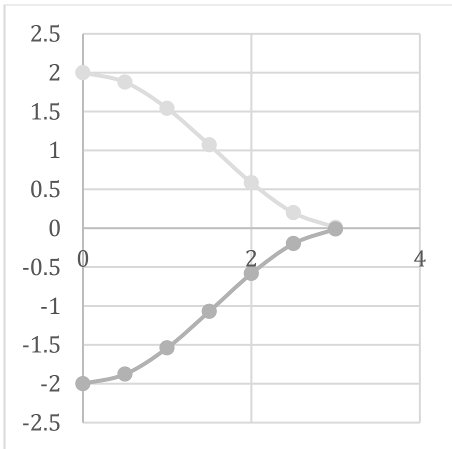


Figure 7

3D Static Visualization of Figure 6

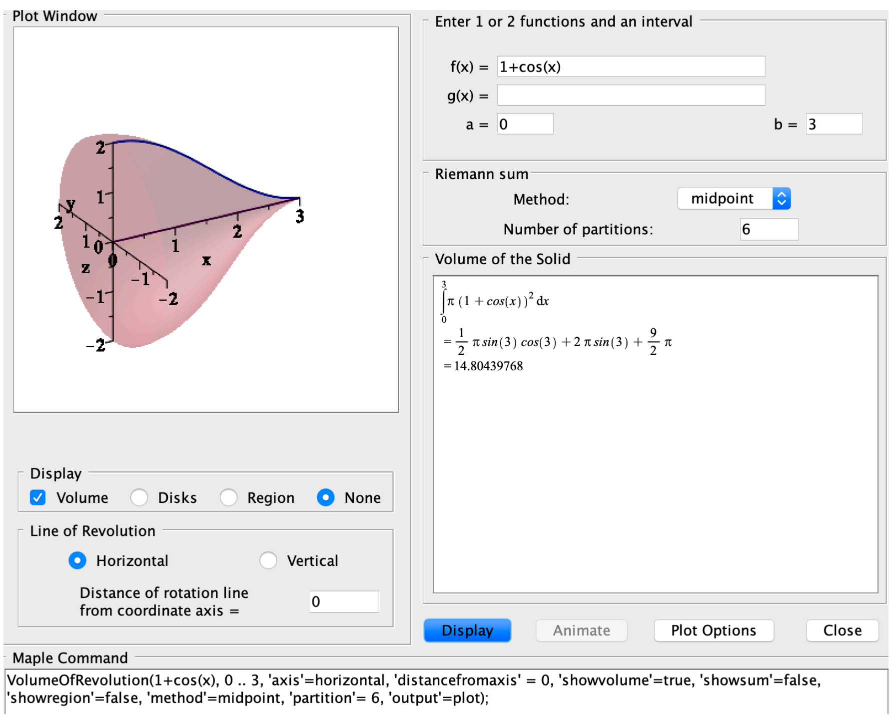


Figure 8

Static Visualization of Regional Representation of Solid of Revolution

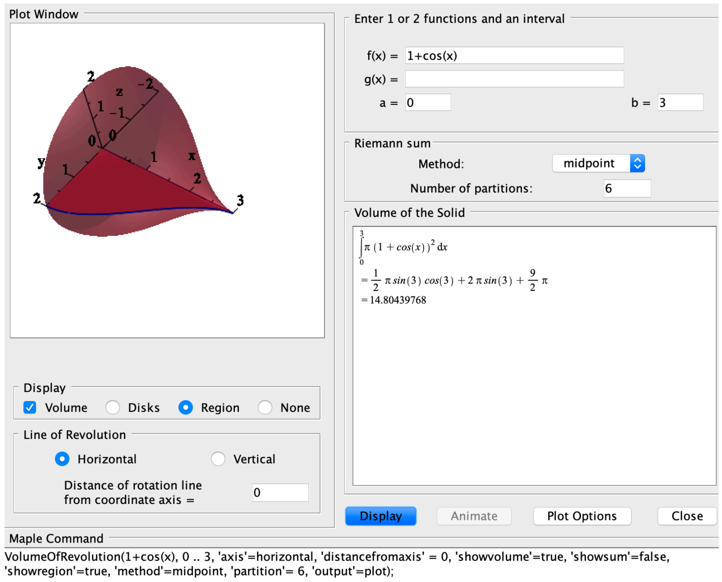


Figure 9

Disc Method- Static Visualization of the Solid of Revolution

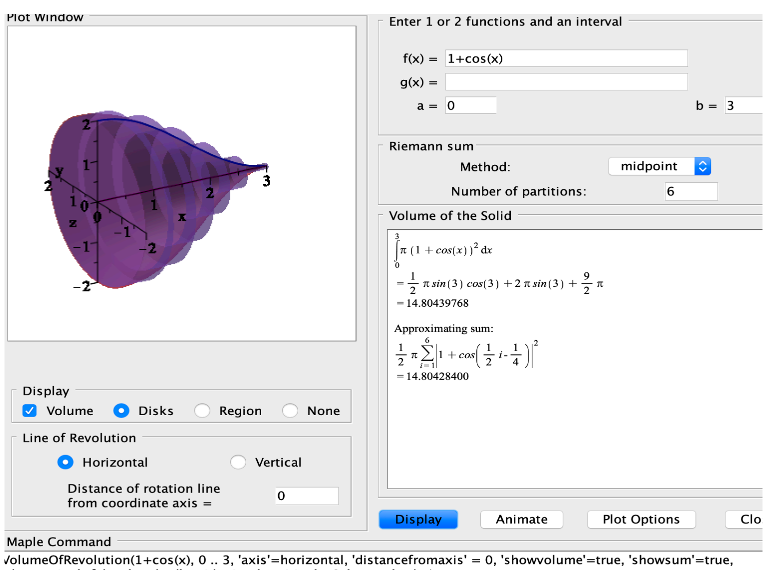
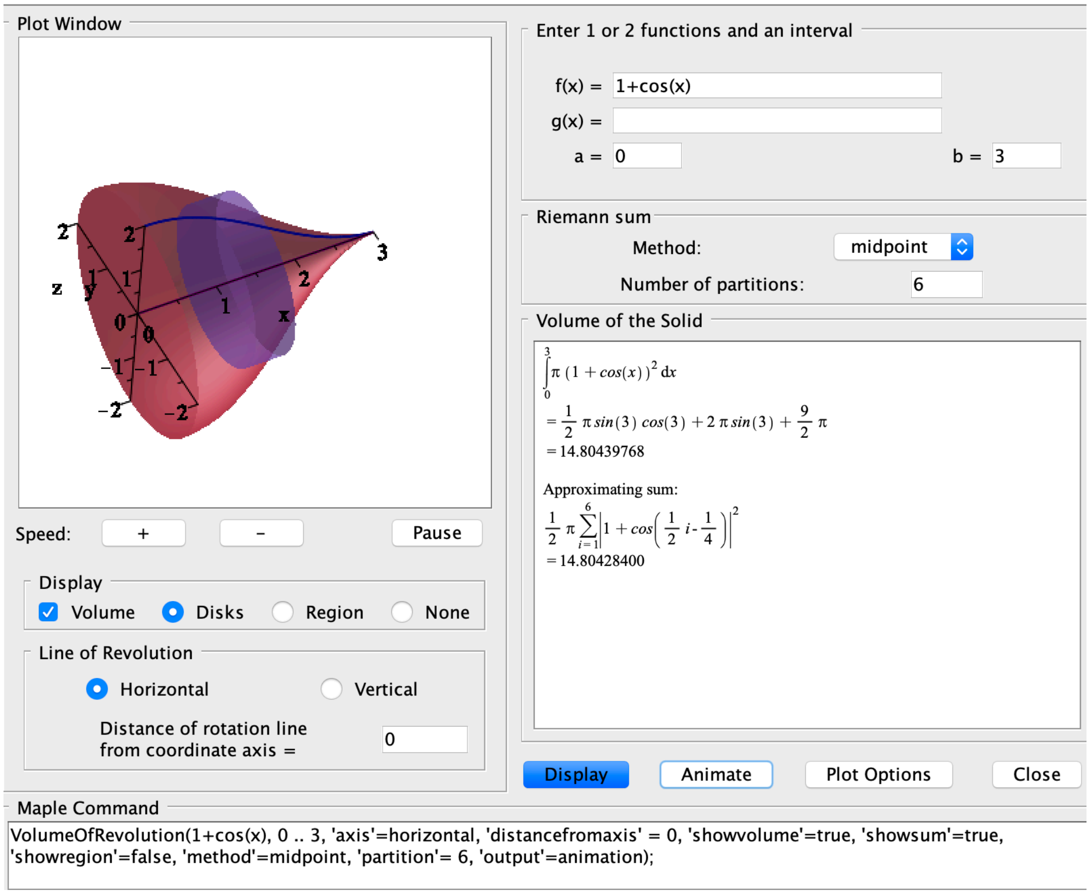


Figure 10

Dynamic Visualization of the Solid of Revolution



Thus, the geometric diagrams in Figures 4 through 10 were the multiple representations, which represented the teaching of mathematical concepts with the use of different procedures (see Özkan et al., 2011), illustrating the nature of a given real-world problem, its verbal representation (as the instructor narration), tabular representation (numerical table of values of x and y), algebraic representation, and graphical representation with Cartesian connections. These representations might activate students' mental processing as they constructed knowledge.

Dick's and Edwards' work (2008) on multiple representations and local linearity, Foshay's and Silber's study (2009) on improving performance, and many calculus reform efforts stressed the use of multiple representations instructional strategy. Those efforts emphasized the fundamental idea of functions in calculus by examining key concepts such as limits, derivatives, and integrals (Dick & Edwards, 2008) in verbal, analytic (symbolic formula), graphic, numeric, and tabular representations. This strategy could provide learners with robust support for learning, understanding derivatives, and integration in calculus and assist students in communicating mathematical ideas through the practice of proper notations in mathematics (see Shahbazi & Irani, 2016), as the Figure 4, Figure 5, Figure 6, Figure 7, Figure 8, Figure 9, and Figure 10 showed. These are the core principles of visualization for understanding calculus. Maple supported the concept of multiple representations, which was part of hybrid approaches of teaching mathematics (see Wilkie, 2016) and could enhance students' conceptual and procedural understanding of mathematical problems.

Maple Technology

Maple is a compact technology that is conducive for communication, information search, and teaching aid (see Awang & Zakaria, 2012) and runs on any operating system. Students could attain meaningful mathematics learning with active participation in hands-on activities. With a plot command, Maple defaults to a 2-D animation for animated visualization graphs to reinforce abstract concepts in mathematics. Maple has editing capabilities with desired colors. Previous research on module for learning integral calculus with Maple revealed that reflective activities were able to trigger metacognitive awareness among the engineering technology mathematics students (Awang & Zakaria, 2012; Salleh & Zakaria, 2016).

Unlike the TI networked CAS-based graphing calculators and TI-Navigator system, Maple technology offered various instructional tools such as powerful mathematical software package, which embodied graphic, computation, programming tools, and a spreadsheet (see Siddique, 2010), or PowerPoint presentation (see Wiwatanapataphee et al., 2010), which were missing on graphing calculators. Moreover, CAS graphing calculators run virtually on Maple. Furthermore, Maple carried many math apps that were missing on CAS graphing calculators. Maple carried increased computational power with active animation and visualization and functioned like video cameras for the development of vision-based intelligent monitoring systems, which could automatically extract useful information from visual data to analyze actions (see Padilla-López et al., 2015). The software provided links and nodes, which represented interactivity and its effects on learning, between contents and students' activities, using hypertext and hypermedia techniques in computer-based learning (CBL) to enhance geometric modeling (see Padilla-López et al., 2015).

Maple encompassed powerful symbolic manipulations, which were programing languages that permitted users to implement their algorithms and constituted a powerful tool for teaching and research in geometric modeling problems (see Sozcu et al., 2013). The programing languages utilized CAS add-ons for use in applied mathematics such as physics, bioinformatics, computational chemistry, and packages for physical computation, graphic production, and editing such as computer-generated imagery and sound synthesis (see Sozcu et al., 2013). The use of Maple in a math classroom could assist in modeling constructivist and connectivist instructional principles to assist students' learning.

Maple as Computational Tool

The available literature on Maple's use as an instructional tool featured Maple's computational nature (see Ozturk et al., 2013; Samková, 2012; Zamuda, & Brest, 2013), sparing its animation and visualization capabilities. Previous research focused on the effectiveness of Maple for providing solutions to the characterization of parametric equations (see Thompson, 2013), Differential Geometry (see Anderson & Torre, 2012), Abel equations (see Jahanshahi et al., 2015), for instance. Anderson and Torre used Differential Geometry, a Maple software tool to solve equations symbolically, analyze a family of hypersurfaces, isolate values of functions and parameters, and solve advanced calculus problems.

Ozturk et al. (2013) used Maple to solve the system of a nonlinear algebraic equation and compute the coefficients of the truncated Taylor sum in matrix form, by collocation method for solving fractional Riccati differential equation with delay term. Meikle and Fleuriot (2012) integrated Maple into the Prover's Palette and found that Maple was a powerful and popular CAS with its plotting capabilities to provide significant insight into proving theorems. They discovered that Maple was useful as a presentation tool that could replace the chalkboard lectures and static PowerPoint slides, permitting users to accelerate the process of proving and verifying interactively sophisticated theorems and complex algorithms (Meikle & Fleuriot, 2012). Despite all the contributions and benefits of Maple technology, the researchers have identified that none of them has dealt with animated visualization in calculus in this section.

Awang and Zakaria (2012), in their quasi-experimental nonequivalent control group design, with randomly selected 101 participants on the process of integrating Maple software in the teaching of the first-year integral calculus topic, found that from a pretest-posttest gain in integral calculus, the experimental group outperformed the control group significantly. Moreover, Vieira (2015) used Maple to solve Euler' type of nonhomogeneous fractional differential equations and ascertained that Maple's visual representation enabled students to view the roots of polynomial functions in a complex variable.

Additionally, Salleh and Zakaria (2016), in their research, using a quasi-experimental nonequivalent control group design, investigated the effectiveness of a learning strategy using Maple in integral calculus. The research data analyses revealed that first-year university students who underwent the integral calculus lesson using Maple software outperformed the control group in terms of procedural and conceptual understanding. Referring to this study, one could infer that there are significant differences between those using Maple software with those using the conventional method in learning integral calculus and that the study is significant to practice.

The rare study on the use of Maple animation and visualization was limited to its executable computer code in generating graphs of an elliptic paraboloid, hyperbolic paraboloid, and hyperboloid of one-sheet, in 3-D (Siddique & Mitchell, 2010). According to Siddique and Mitchell (2016), Maple codes were immensely helpful in visualizing and understanding the quadric surfaces (graphs of quadratic equations in three-dimensional Cartesian coordinates). Hence, Maple produced high-quality visualizations and animations. The strength of these Siddique's and Mitchell's (2016) study sprung from providing a written code for the graphs. These authors missed stating what the students did with the animation and visualization tools.

Most of the available peer-reviewed articles were more descriptive than analytical, and no statistic availed to support the postulated claims that Maple software technology-enabled the visualization of the motion of a material point on its trajectory and assisted students to improve their understanding of calculus (see Aan & Heinloo, 2012). This study focused on Maple-based animation and visualization lessons and their impact on college students' conceptual and procedural understanding of derivatives and integrals in calculus.

Many research studies discussed CAS Maple and the appropriate software package for effective classroom delivery. However, the literature on the use of animation and visualization to teach calculus in college was scarce. The new Maple encompasses a software package (math applets, wolfram alpha demonstrations, GeoGebra, MathCad, LaTex, MathLab) for active learning (see Ozturk et al., 2013; Salleh & Zakaria ,2016; Samková, 2012; Zamuda, & Brest, 2013). Maple technology embraces a multimedia presentation platform that could help students understand abstract concepts through learner-centered pedagogy, constructivist, and connective teaching and learning principles.

Conceptual and Procedural Understanding of Calculus

Serhan (2015) asserted that a salient principle of understanding was the ability to connect conceptual and procedural knowledge. The conceptual understanding consisted of a knowledge that was rich in relationships, and procedural skills were algorithms or sequences of steps related to problem types. Dick and Edwards (2008) viewed conceptual knowledge as an associated web of knowledge, networked with linking relationships. The conceptual understanding of mathematics was the comprehension of mathematical concepts, operations, and relations (see Ocal, 2017; Rittle-Johnson & Schneider, 2015). While procedural understanding was the fluency (skill) in carrying out procedures flexibly, accurately, efficiently, and appropriately, strategic competence was the ability to formulate, represent, and solve mathematical problems (see Ocal, 2017).

Instruction focused on conceptual understanding tended to improve students' procedural skills. However, the converse was not necessarily true (see Hodara, & Xu, 2016; Rittle-Johnson & Schneider, 2015). Cox's (2015) research into students' conceptual understanding of fundamental concepts of calculus has provided comprehensively designed calculus tasks to measure the students' preference for a visual method of solution, which included graphic representations and analytic processing, which required algebraic representations, and analyses of students' difficulties (Quarles & Davis, 2017).

Quarles and Davis (2017) expounded on criteria for a mathematical proficiency to include conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. They noted that conceptual understanding, as knowledge of mathematical concepts, included operations, and relations, while procedural understanding involved aptitudes in performing procedures compliantly, competently, and applicably. These authors shed light on a deeper mathematical comprehension, which derived from students' conceptual, procedural, and strategic understanding that facilitated retrieval and improved retention. In their study on learning in developmental math, using pretest and posttest research design, with descriptive statistics, linear regression, and logistic regression analyses, Quarles and Davis postulated that, while conceptual mathematics proficiency correlated with higher grades, procedural algebra skill did not. Their study bore some implications, in terms of practice: (a) learning math that stressed on procedural skills did not prepare learners for college-level math and (b) students with procedural skills could fail to recall within a few months. In terms of research, the study added to the literature to include the need for further research on students' assessments.

Despite some limitations (variances in students' scores by assessment tools), Quarles' and Davis' (2017) study was significant for this study on students' conceptual and procedural understanding. The Maple platform offered both the static visualization and dynamic visualization (animated visualization), which were the requirements for a conceptual and procedural understanding of calculus.

Summary and Conclusions

The animation-visualization theory constituted the theoretical framework for this study. Researchers highlighted the advantages of dynamic visualization calculus instructors could use to enhance students' conceptual and procedural understanding. Calculus encompasses abstract concepts that are difficult to grasp and requires dynamic visualization activities to mitigate learning difficulties. From literature, instructors could improve students' spatial visualization skills with appropriate content, using Maple-CAS (see Höffler & Leutner, 2007; Kadunz, & Yerushalmy, 2015; Karakus & Duressi, 2017). Maple was useful for improving spatial visualization skills (see Kühl et al., 2018). Despite the advantages of the animation-visualization theory in other fields of study, postsecondary calculus students suffered from its disuse in calculus. It might be imperative, then, that calculus instructors, as subject matter experts, use Maple animation and visualization tools in the framework of animation and visualization theory and principles to increase students' intrinsic motivation in their teaching model to help students' learning of calculus. The study's uniqueness resided in its social significance and the way it might address a gap in the literature. This research study examined the impact of Maple-based animation and visualization activities on college students' understanding of calculus via the described quantitative 2x2 factorial pretest and posttest control group quasi-experimental design.

The use of multiple representations could activate students' mental processing as they constructed knowledge to enhance their performance (see Geiger et al., 2016; Ghani et al., 2012). Dick's and Edwards' work (2008) on multiple representations and local linearity, Foshay's and Silber's (2009) study on improving performance, and many calculus reform efforts stressed the use of multiple representations instructional strategy. Those efforts emphasized the fundamental idea of function in calculus by examining key concepts such as limits, derivatives, and integrals (see Dick & Edwards, 2008) in verbal, analytic (symbolic formula), graphic, numeric, and tabular representations. This strategy could provide learners with robust support for learning, understanding derivatives and integration in calculus, and assist students in communicating mathematical ideas through the practice of proper notations in mathematics (see Shahbazi & Irani, 2016). These are the core principles of visualization for understanding calculus. Maple supported the concept of multiple representations, which was part of hybrid approaches of teaching mathematics (see Wilkie, 2016) and might enhance student's conceptual and procedural understanding of mathematical problems. Chapter 3 presents the instructors' use of written assessments to evaluate students' conceptual and process knowledge and performance-based assessments in the application of knowledge as a means of evaluation.

Chapter 3: Research Method

Introduction

The purpose of this study was to investigate the impact of Maple-based dynamic visualization (animated visualization) activities on college students' conceptual and procedural understanding of derivatives and integrals in calculus. This chapter comprises discussions on the research design and rationale, methodology, and threats to validity. It also includes a review of the procedures for data collection, analysis, and ethical considerations to protect participant rights.

Research Design and Rationale

This study was about comparing the performance of two groups, ascertaining between and within effect. Therefore, the use of the mixed between-group design, with a 2x2 factorial pretest and posttest control group quasi-experimental mixed-design, which analyzed the independent and joint effects of the constructs in the study, was a good fit. The choice of the design emerged from its ability to facilitate the control of internal factors such as the history, maturation, testing, instrumentation, regression, mortality, and interaction of selection, maturation, internal validity. The factorial design offered the flexibility for exploring the intervention in the study, using the causal relationship to reduce bias and aiding in the establishment of construct and content validity, avoiding threats to internal validity (see Frankfort-Nachmias & Nachmias, 2008). The strength of this factorial design resided in the causal attributions, which resulted in consequences in varying an intervention. The concept of internal validity was the nucleus of cause-effect inferences (see Trochim, n.d.) that were legitimate deductive and logical assertions. Thus, this research design was a reliable and efficient candidate as a choice for examining intervention variations (see Trochim, n.d.), and was consistent with research designs to advance knowledge in educational technology, concerning calculus.

Students' test scores on derivatives and integrals and their conceptual and procedural understanding were the study variables. Furthermore, the observation notes on class activities on derivatives and integrals and e-mailed interview responses from the intervention professors served as pillars for triangulation. The department curriculum did not require the use of Maple animation and visualization. The elements in Figure 11 and Table 1 exemplified the used variables to answer the research questions on the impact of Maple animation and visualization on students' understanding of calculus. The IV were non-Maple static visualization (X1), and Maple dynamic visualization (X2) and dependent variable (DV) was the CU and PU of derivatives and integrals (see Table 1). One test was the foundation for measuring the conceptual and procedural understanding of derivatives and integrals.

Figure 11

Section on Model to Respond to Research Questions

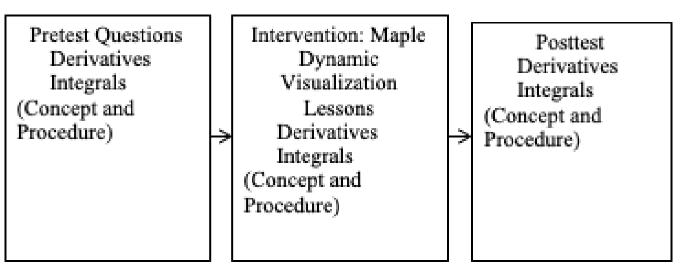


Table 3

Summary of Research Design

Group	Pretest/Prerequisite	Intervention	Posttest	Remarks (LO Met -1 or Not- (0)
Comparison	Application of Derivative (D) and Integrals (I)	Teaching without Maple	Application of Derivative (D) and Integral (I)	1/0
Intervention	Application of Derivatives (D) and Integrals (I)	Teaching with Maple	Application of Derivatives (D) and Integrals (I)	1/0

The study addressed the following research questions:

RQ1: Was there a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?

RQ2: Was there a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?

RQ3: Was there a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?

RQ4: Was there a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' procedures calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?

The choice of the intervention sprung from the failure to use static visualization (still pictures, graphs, PowerPoint slides) and other traditional methods of instruction to meet students' needs in learning calculus (see Sevimli, 2016b) at the postsecondary level. Besides, mathematics educators recognized that college students often have difficulties understanding the concepts of derivatives and integrals, due to the abstract nature of calculus (see Covington et al., 2017; Katsioloudis et al., 2016) and required dynamic visualization activities to mitigate learning difficulties (see LaRue & Infante, 2015). Nonetheless, the use of animation and visualization theory in teaching calculus at the postsecondary level was lacking. Still, there was a scarcity of studies on the use of Maple's dynamic visualization to enhance students' learning. However, Kühl et al. (2018) advised instructors to view visualizations as a suitable support for teachers' design inquiry of location-based learning activities and enabled students to make a valid diagnosis of their performance (see Melero et al., 2015).

Furthermore, other prior studies have shown that in various disciplines such as atmospheric science, biology, cartography, engineering, and physics, animation and visualization have played a crucial role in the delivery of instructional materials about abstract concepts that were difficult to understand, or that encompassed abstract content such as calculus (see Lin, 2011; Nossun, 2012). Animation and visualization provided opportunities for calculus learners, through geometrical representations, to understand the mathematical concepts, exceptionally, in computing the rate of change of a particle on its trajectory and the volume of a solid. The rate of change required the application of derivative. In contrast, the computation of the volume of a figure resulting from a rotation a portion of a curve (solid of revolution) required an application of integrals (see Swift, 2017), which constituted the objects of analysis in this study, per the sample lesson plan (see Appendix A).

Therefore, it was essential to examine the effect of Maple technology, within the framework of animation-visualization theory, on college students' learning of calculus, especially, in their conceptual and procedural understanding of abstract concepts of derivatives and integrals, in calculus.

Methodology

The inquiry was about the impact of Maple-based animation and visualization calculus lesson delivery on learners, using students' pretest, quiz, and posttest performance scores on questions on derivatives and integrals. Consequently, the quantitative 2x2 factorial pretest-posttest control group QED, with MANOVA for the statistical analysis, was a good fit for this study. Due to the students' choice of calculus classes, randomization was impossible, presenting the use of QED, which engendered some limitations to the study. However, despite these limitations, it was acceptable to use a QED, which was a feasible alternative to the original experimental design when actual experiments were impossible (see Campbell & Stanley, 1963; Tasker, 2014). Another limitation consisted of any differences in outcomes that could incur because of nonequivalent nature of groups, rather than the intervention (see Hodara & Xu, 2016). However, the use of MANOVA could detect any between and within effects.

Population

The target population included four calculus professors and 120 male and female students, with age ranging from 20 to 30 years from a multicultural population of Black American, White, Hispanic, and Asian, who registered for analytical geometry calculus; some for Maple dynamic visualization section and others for non-Maple animation visualization section (static visualization). The students received an explanation of the difference between the Maple and non-Maple sections when they registered for their calculus class. All available classrooms participated in the study. Thus, students in the Maple classroom received the intervention by default. However, students could opt out of the study's data collection (per research ethics).

Sampling and Sampling Procedures

Sampling and sample size can strengthen or weaken a study (see Campbell & Stanley, 1963; Tasker, 2014). Although the use of probabilistic sampling could have strengthened the study, I resorted to a nonprobability sampling (convenience sampling) due to the conditions surrounding students' selection. Students' preselection in their registered classes complicated random assignment, administratively. They knew the type of class (Maple or non-Maple technology) for which they registered, as the administration informed them during registration. While random assignment of students to comparison and intervention groups was impossible, the fixed effects for each instructor in this model might statistically adjust for mean differences between professors. Four classes, with 120 students, formed the study's sample.

Sample Size

The number of students who signed and returned the consent form constituted the sample size in the convenience. The sample size covered the intervention group (*n* = 86), which used Maple-based animated visualization lessons in calculus class and the comparison group (*n* = 34), which used the static visualization (non-Maple based) lessons in calculus class. Considering mixed-design MANOVA for statistical analysis, the use of G*Power 3 computation software (see Faul et al., 2009) presented an effective way of determining an adequate sample size. Factoring attrition problem, and using a statistical power (80%), level of significance alpha (0.05), and effect size (0.12), the G*Power 3 computation required a total sample size of 84 for both groups.

As per G*Power 3 computation, a convenience sampling of at least 84 students for both the comparison and intervention groups (all three subsamples) was necessary to achieve the accepted minimum power threshold (see Field, 2017; Pallant, 2016; Tabachnick & Fidell, 2018). However, after data cleaning and checking for errors, the number of participants reduced to 81 students with the comparison group (*n* = 29) and intervention group (*n* = 52). The reduction in participants became a source of limitation (McNeish, 2017) in this study as discussed in Chapter 5.

Procedures for Recruitment, Participation, and Data Collection

Before the data collection, I sought permission from the LC administrators and professors and followed the college policy for authorization to conduct research, with a confirmatory letter from the college, according to research requirements with appropriate signatures, in Appendix H. The application permission letter from LC depended on submitting the proposal approval from Walden University. Students knew in advance which sections would and would not use Maple animation and visualization. They had the ability to opt out, without any adverse consequences, just like the student consent form stipulated.

Before data collection, after receiving the approval from Walden University and LC institutional review boards (IRBs), I sought permission from the LC math department personnel, who introduced me to the calculus professors. After the introduction, I met the professors one on one and discussed my intentions about the study with them and gave them the consent form if they choose to participate. At the beginning of the academic term, I visited the professors and collected their signed consent forms. They allowed me into their classroom, introduced me to the students, and let me speak (1-2 minutes) to the learners about my intentions and handed to each one of them the consent form. I waited in the lobby until the end of class and collected some signed forms and left with the professors to get their schedule for my classroom observations. During my first observation, I collected some more students' signed consent forms.

Data collection was comprised of the following items:

- Students' de-identified list of scores on pretest (prerequisite skills on derivatives and integrals), quiz (posttest1), and end of term exam (posttest), on derivatives and integrals.

- Researcher's observations (two on derivatives and two on integrals, for each class, with photographic pictures of class activities, with no images of students and professors) notes on derivatives and integrals for implementation fidelity.
- Emailed interview response from intervention professors for triangulation.

There were no reports on participants' demographic information. Students exited the study when they took the end of term exam (full discussion in Chapter 4). There were no follow up for interviews apart from returning to share the study outcome with LC personnel, per research ethics.

Intervention

The professors, with a Ph.D. in Mathematics, conducted the intervention, using Maple-based animation and visualization tools in calculus class. I did not teach any part of the course and had no prior relationship with the instructors, apart from the professional connection for data collection. The instructors were experts in using Maple software and did not need any additional workshops. All students received standardized instructions and took all tests in their classrooms. Three professors taught the intervention group using Maple software, and one instructor taught the comparison group without Maple software.

Students learned to use the software during their teaching periods as the instructor modeled a topic in the intervention group. Table 1exemplified the independent variables. Chapter 2 exemplified the details on the definition of animation and its effect on the conceptual and procedural understanding of derivative and integral in calculus. The calculus professors used their instructional materials to teach the curriculum (see

Appendix J). Three professors used Maple animation and visualization tools to teach calculus for the intervention group. In Chapter 2, I illustrated a sample lesson activity, as per learning objectives Students' achievement tests on derivatives and integrals, served as a means of measuring the construct variables according to the displayed learning objectives (LO; see Figure 12).

Figure 12

Maple-Based Dynamic Visualization Learning Objectives

LO1	At the end of the topic on derivatives, students would be able to demonstrate the application of conceptual and procedural understanding of the derivatives of functions, to solve derivatives related problems, with at least 80% accuracy.
LO2	At the end of the topic on integrals, students would be able to demonstrate conceptual and procedural understanding of integrals of functions, to solve derivatives related problems, with at least 80% accuracy.

All instructors met the learning objectives and implemented the intervention as expected. Thus, the professors executed the intervention the expected outcomes. The observation and implementation fidelity rubric is located in Appendix C with full detail in Chapter 4.

Instrumentation and Operationalization of Constructs

The instruments were the students' tests (pretest, quiz, and posttest) scores on derivatives and integrals to measure the manipulated variables. These tests were relevant and valid and mirrored the learned concepts and aligned with the learning objectives for assessment. Instructors designed their lesson plans and assessment items. Tests were valid and reliable, aligning with learning objectives. While the tests were not the same across groups, quizzes and posttest (end of term exam) of each instructor mirrored the pretest (prerequisite skills for derivatives and integrals), thus establishing internal consistency (see Appendix J, Appendix K, and Appendix L). The constructs related to the established instruments (concurrent validity); the variables aligned with the constructs (convergent validity). The independent variables caused change in the dependent variable (internal validity). While measures predicted students' superior performance on posttest (predictive validity), the scale measured the theoretical constructs (construct validity), establishing sufficiency of instrumentation to answer the research questions. While Figure 12 displayed the objectives for assessment, Table 4 presented the summary of the defined variables the constructs, Table 5, the gain and implementation information and Table 6, the observation information.

Table 4

Variables Table

	Independent Variable	Fidelity of Implementation	Dependent Variable
Non-Maple static visualization	X1	OV	Y11
Maple dynamic visualization	X2	OV	Y12

Table 5

Monitoring Implementation Fidelity and Gain

Attainment difference	Measuring Gain	Triangulation	Fidelity Measure
Y12 – Y11	Posttest and Pretest Gain established	Expectations met	Valid/Reliable (Appendices B, J, K, L)
OV and Y12			
Ascertained and analyzed the existence of difference between OV and Y12 via my observation note learning objectives		Compared and contrasted OV and Y12 for triangulation	Validation of fidelity measure (Appendices B, J, K, L)
Maple animation lessons on derivatives and integrals	Students demonstrated and replicated similar taught concepts, with at least 80% performance regarding the conceptual and procedural understanding of derivatives and integrals		Descriptive and MANOVA statistical analysis
Depiction	Depiction of treatment skills monitors that students demonstrate treatment-related behavioral skills and cognitive strategies in relevant real-life settings as intended		Descriptive and MANOVA statistical analysis

Table 6

Observation Materials

Objectives	Instructor's Teaching Materials	Example	Instructor's Assessment	Researcher's Description of Observed Materials
Conceptual understanding of derivatives of a given function	Derivatives of functions, using Maple animation tool	See Appendix B	See Appendices J, K, L	See Appendix B
Procedural understanding of the derivative of the given function	Derivatives of a function, using Maple animation tool	See Appendix B	See Appendices J, K, L	See Appendix B
Conceptual understanding of a definite integral of a given function	Definite and Indefinite Integrals of a function, using Maple animation tools	See Appendix B	See Appendices J, K, L	See Appendix B
Procedural understanding of a definite integral of a given function	Definite and Indefinite Integrals of a function, using Maple animation tools	See Appendix B	See Appendices J, K, L	See Appendix D
Dynamic visualized materials created for student's engagement.				Learners interacted with the instructor's modeled activity

Data Analysis Plan

Using numbering as a coding system, for data analysis, I assigned randomized computer-generated 3-digit numbers such as 351, 945 (using RAND function in Excel) to each participant in the study (see Appendix E). For protection, I secured the collected and encrypted data on a laptop, assuring that the coded information match perfectly with the student's de-identified list of test scores (see Appendix F and Appendix I). The observation materials had neither instructors', students' image, name, nor identifiable symbol to preserve anonymity and confidentiality. The use of encrypted data helped preserve any used file and documents to avoid unwelcome intrusion. A collection of pretest and posttest scores on derivatives and integrals questions served as instruments for data analyses. I entered the collected data into SPSS version 24 for the variables of interest, examined and discarded those scores for values that were more than 3.29 standard deviations above or below the mean, and removed any detected outliers through descriptive statistics (see Tabachnick & Fidell, 2018). During data cleaning, there were no detected outliers. However, I disregarded invalid data (data with no pretest and quiz scores) and excluded such from any statistical analysis. I included a plan for checking the reliability of tests (see Appendix F).

The mixed-design MANOVA was the appropriate test for this research, to examine both between and within groups differences on a linear combination of dependent variables (see Pallant, 2016; Tabachnick & Fidell, 2018). The two groups were the non-Maple static visualization group and Maple dynamic visualization group. The within factor (time) comprised three measurements: (a) pretest, (b) quiz, and (c) posttest measures on the linear combination of derivatives and integrals calculus scores. Descriptive and inferential statistics with MANOVA analysis confirmed any relationship between the dependent variables (students' conceptual and procedural understanding of derivatives and integrals) and independent variables (non-Maple static and Maple dynamic visualizations). The parametric assumption for the mixed design MANOVA included normality and sphericity. Sphericity was the homogeneity of the error variances of the differences scores among the repeated measures (see Pallant; Tabachnick & Fidell). However, the multivariate version of the mixed-design MANOVA did not require the assumption of sphericity. Therefore, only normality warrant checking during the data analysis process (see Hair et al., 2018; Pallant; Tabachnick & Fidell).

The specific tested null hypotheses, using mixed-design MANOVA, encompassed:

RQ1: Was there a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?

H_01_1: There was no significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

$H_1 1_1$: There was a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

$H_0 1_2$: There was no significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test.

$H_1 1_2$: There was a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test.

RQ2: Was there a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?

$H_0 2_1$: There was no significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

$H_1 2_1$: There was a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

H_02_2: There was no significant difference in pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test.

H_12_2: There was no significant difference in pretest (prerequisite skills for derivatives' procedure), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test.

RQ3: Was there a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?

H_03_1: There was no significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

H_13_1: There was a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

H_03_2: There was no significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test.

H_13_2: There was a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test.

RQ4: Was there a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?

H_04_1: There was no significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1) and end of term exam (posttest) scores on the integrals' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

H_14_1: There was a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1) and end of term exam (posttest) scores on the integrals' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

H_04_2: There was no significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' procedure calculus test.

H_14_2: There was a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1) and end of term exam (posttest) scores on the integrals' procedure calculus test.

There were no potential covariates. There were no confounding variables. Interpretation of results would be *p*-values-based and effect size with MANOVA analyses for all the hypotheses.

Threats to Validity

Inconsistencies could originate from test scorers and lead to differences in the values of reliability and validity, thereby negatively impacting the accuracy of inferences based on test scores. Therefore, the use of students' class test scores, for autocorrelation, helped mitigate bias from between-teacher effects.

Generalizability depended on the sample size for valid generalization (see Campbell & Stanley, 1963; Trochim, n.d.). However, there was a possibility to generalize to the LC study group. The embedded pretest-posttest control group design was effective in controlling history, maturation, mortality, and instrumentation that could affect internal validity. A focus was on the assumptions, such as fidelity of implementation (see Creswell & Creswell, 2018; Frankfort-Nachmias & Nachmias, 2008).

The use of convenience sampling could introduce selection bias, as the sample could not represent the entire population, unlike the probability sample. However, the convenience sample fit the occasion of the availability and readiness of the participants because it allowed obtaining necessary data and trends regarding this study without randomization. The only students who were out of the selection process were those who did not sign and return the consent form or chose not to be part of the study.

For risks in the study protocol, this study required the use of numerical interpretation with quantitative responses. Consequently, following the procedures and specific research protocols, the facts were accurate, measurable, and precise to enhance the trustworthiness and validity of the current quantitative study (see Creswell & Creswell, 2018).

Ethical Procedures

In a study, risks might be inevitable. However, in this study, risks to students were minimal. Adhering to the guidelines of Walden University's and Lehman College's IRBs and stipulations throughout the study, before collecting any data, I sought permission from professors to observe their class activities on derivatives and integrals. I followed the ethical principles of beneficence, respect for persons, and justice as the Belmont Report outlined (see Horner & Minifie, 2011), while conducting the research study, and included a copy of the required research ethics training certificate (see Appendix H).

Before data collection, every study participant received an informed consent form with enough detailed information on the study. The form contained the purpose, expected duration, procedures, and information on the participants' right to opt-out as they please, without any adverse effect from the researcher, and the benefits and risks of the study. The inclusion and exclusion criteria covered all students who registered for calculus class. Before the study, any volunteer could participate in the equitable selection. However, with no perceived coercion, according to Walden University research compliance, the university's ethical standards, and United States' federal regulations, only those who signed and returned the informed consent form took part in the study. Appendix G included documents for agreements to gain access to participants or data, with ethical considerations on the part of the researcher, according to the Walden University IRB stipulations.

Instructors were responsible for the ethical concerns of their instructional materials. The intervention of human participants was of minimal if not inexistent associated risk to the study. LC instructors provided the necessary data according to the LC's ethical research processes. The data comprised a de-identified list of students' test scores with no name and were confidential with no publishing of participants' names or test scores. The classroom observation notes on derivatives and integrals beard no image of instructors or students. Participants' test scores were number-coded, with encrypted analyzed data stored on the researchers' password-protected laptop. There would be no data dissemination. However, the LC administrators and professors in the study would have access to the final research conclusions. Data would be destroyed after five years, according to IRB stipulations. There was no identifiable conflict of interest.

Summary

The quantitative 2x2 factorial pretest and posttest control group QED was a perfect fit for this study. The use of mixed-design MANOVA for data analysis helped ascertain the main and interaction effects that could exist between factors, and mediate threats to validity. It was necessary to adhere to accurate and reliable instrumentation with ethical considerations in investigating Maple-based animation visualization activities' impact on college students' conceptual and procedural understanding of derivatives and integrals. The use of IBM SPSS Statistics for statistical analyses of reliably collected data (Chapter 4) was crucial for valid interpretation and recommendation (Chapter 5).

Chapter 4: Results

Introduction

The reason for this quantitative study was to determine if Maple-based animation and visualization lessons, designed within the framework of the animation-visualization theory (see Erlich & Russ-Eft, 2011; Paik, 2012; Zurita & Nussbaum, 2007), make an essential difference in college students' conceptual and procedural understanding of derivatives and integrals in calculus. This quantitative study was causative and depended on the use of a 2x2 factorial pretest-posttest control group quasi-experimental mixed design. Four research questions, using the predictive assessment software IBM SPSS Statistics MANOVA to test the hypotheses and interpret research results, characterized this study.

This chapter contains the procedures and associated results with the data collection and analysis for the study. First was a reporting of participants' descriptive statistics. Next, there were three phases of the data analysis process. The first phase was the data preparation phase, which consisted of entering data into the SPSS Statistics software for checking for errors and missing values, conducting descriptive statistical analysis. In this phase, the computation of new variables was necessary. In phase two of the data preparation, the normality and sphericity test of parametric assumptions were effectual. The third phase was the primary analysis phase, which consisted of the statistical analyses used to test the null hypothesis. This section ended with a chapter summary.

The following research questions and hypotheses were used to guide and focus the study:

RQ1: Was there a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?

H_01_1: There was no significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

H_11_1: There was a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization animation).

H_01_2: There was no significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test.

H_11_2: There was a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test.

RQ2: Was there a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization animation)?

$H_0 2_1$: There was no significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

$H_1 2_1$: There was a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

$H_0 2_2$: There was no significant difference in pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test.

$H_1 2_2$: There was no significant difference in pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test.

RQ3: Was there a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?

$H_0 3_1$: There was no significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

H$_1$3$_1$: There was a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

H$_0$3$_2$: There was no significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test.

H$_1$3$_2$: There was a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test.

RQ4: Was there a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization animation?

H$_0$4$_1$: There was no significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1) and end of term exam (posttest) scores on the integrals' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

H$_1$4$_1$: There was a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1) and end of term exam (posttest) scores on the integrals' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization).

H_04_2: There was no significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' procedure calculus test.

H_14_2: There was a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' procedure calculus test.

Data Collection

After the LC IRB and Walden University's IRB approval (# 05-02-19-0196043), the LC mathematics department personnel introduced me to and briefed the MAT 155 professors about my intentions. Then, I met each of the four professors (three for the intervention group and one for the comparison group) in their office to remind them of my intentions, hand them the consent form, and assure them that the data would be kept private and anonymous. Each of the professors introduced me to their classes and allowed me (about 1-2 minutes) to hand the consent form to the students. I sat inconspicuously in the back of the classroom to avoid seeing any students' faces during the observation. After the first observation in each class, I stood at the door and collected some signed documents the students willingly returned to me. Like the first one, the subsequent observations proceeded for the entire lesson period.

Data collection spanned 15 weeks of four sections of the Fall 2019 MATH-155 course, which met once a week for 100 minutes a week, at LC. The data gathered included the following:

- Class observations (twice during the teaching of derivatives and twice during the teaching of integrals) for implementation fidelity.
- Students' pretest (homework questions on prerequisite skills for derivative and integral), quizzes, and end of term exam (posttest) scores, on derivatives, and integrals.
- Emailed-interview response from the professors who used Maple software (for triangulation).

The scores were deidentified lists from the participant instructors. The actual recruitment and response rates were 95.83% (115 out of 120 participant students) and an attrition of five students who dropped the course in the comparison group and whose data I excluded from the data analysis.

There were some discrepancies in data collection from the plan I presented in Chapter 3. There was no administered pretest as per the original pretest-posttest control design. Therefore, I could not measure direct gain. I filed for an amendment to my existing IRB to modify the original pretest-posttest design to use assignment prerequisite skills for derivatives and integrals (as pretest) and include interview response from the intervention instructors (see Appendix G). The interview response was to ascertain a triangulation. That adverse event precluded direct measurement of learning gains but still allowed a weaker test of group equivalence, hoping that the interview response from the instructors in the intervention group would triangulate with group equivalence analysis since direct measurement of learning gains was impossible. Moreover, data from one intervention professor had neither a pretest nor quiz scores and were entirely removed from the analysis. This loss of data reduced the sample size from 115 to 81respondents. Thus, the sample size of 81 participants was less than 84: the required sample size from G*Power 3 computations. This small sample size became a source of a discussed limitation in Chapter 5. Twenty-nine students formed the comparison (static visualization), and 52 learners constituted the intervention (dynamic visualization animation) group. Instructors took measures for each respondent across three time periods (pretest, quiz, and posttest). These professors, additionally, conducted tests for derivatives' concepts and procedures and integrals' concepts and procedures.

Preparation of Data

There were 81 respondents after data cleaning, in this study, with 29 participants in the comparison (static visualization) group and 52 students in the intervention (dynamic visualization) group. Using the SPSS detailed procedures (frequencies with no graphs) for all entered variables, I checked for errors and missing values (see Figure M1 in Appendix M). The SPSS descriptive procedures, using frequencies, revealed that there were no missing values or data errors. Hence, there was no need to recode or compute new variables to. So, the next step in the data analysis process was the test for assumptions phase.

Descriptive Statistics: Group Comparability Analysis

It is essential to gauge the performance outcome to ascertain gain. I used descriptive statistics to simplify data. Descriptive statistics consists of measures of central tendency and measures of variability (see Trochim, n. d.), such as the mean (*M*) and standard deviation (*SD*), to provide a summary of the sample and the measures (see Table 7).

Table 7

Comparability Analysis Table

	Comparison Group (Static Visualization)			Intervention Group (Dynamic Visualization)		
	N	*M*	*SD*	*N*	*M*	*SD*
Pretest_HWK_Derivative_Concept	29	66.72	12.86	52	66.56	12.82
Pretest_HWK_Derivative_Procedure	29	66.72	12.86	52	66.56	12.82
Pretest_HWK_Integral_Concept	29	71.97	8.58	52	65.50	10.91
Pretest_HWK_Integral_Procedure	29	71.97	8.58	52	65.50	10.91
Quiz_Derivative_Concept	29	87.34	11.22	52	82.10	10.16
Quiz_Derivative_Procedure	29	87.34	11.22	52	82.10	10.16
Quiz_Integral_Concept	29	91.34	13.81	52	84.58	10.13
Quiz_Integral_Procedure	29	91.34	13.81	52	84.58	10.13
Post_Derivative_Concept	29	87.24	11.25	52	91.98	6.59
Post_Derivative_Procedure	29	87.24	11.25	52	91.98	6.59
Post_Integral_Concept	29	86.66	11.50	52	95.50	6.48
Post_Integral_Procedure	29	86.66	11.50	52	95.50	6.48

It was crucial to note that the tests were not identical, so scores were not directly comparable. However, the contents on derivatives and integrals in both comparison and intervention groups were the same (see Appendix B). The *M* and *SD* pretest scores in both groups (see Table 8) were so close with a difference of 0.16 for the mean and 0.04

for the standard deviation between both groups for the pretest on derivatives' concept and procedure. The *M* and *SD* difference on the pretest for integrals' concept and procedure were, respectively, 6.47and 2.33. A similar analysis from Table 9 showed that the difference in the *M* and *SD* between the comparison group instructor and the Intervention Group Instructor 1 were 0.16 and 0.86, respectively for derivatives' concept and procedure; 7.24 and 3.43, respectively, for pretest on integrals' concept and procedure. The four means and standard deviations were close enough to justify combining the sections into one big sample for each group (see Table 7).

Table 8

Descriptive Statistics by Group and Instructor

	Comparison Group			Intervention Group					
				Instructor 1			Instructor 2		
	N	*M*	*SD*	*N*	*M*	*SD*	*N*	*M*	*SD*
Pret_HWK_DC	29	66.72	12.86	16	66.56	13.75	36	65.19	12.34
Pret_HWK_DP	29	66.72	12.86	16	66.56	13.75	36	65.19	12.34
PretHWK_IC	29	71.97	8.58	16	64.73	12.01	36	65.44	10.57
Pret_HWK_IP	29	71.97	8.58	16	64.73	12.01	36	65.44	10.57
Quiz_DC	29	87.34	11.22	16	80.94	14.25	36	81.58	7.91
Quiz_DP	29	87.34	11.22	16	80.94	14.25	36	81.58	7.91
Quiz_IC	29	91.34	13.81	16	85.60	13.32	36	85.31	8.47
Quiz_IP	29	91.34	13.81	16	85.60	13.32	36	85.31	8.47
Post_DC	29	87.24	11.25	16	92.17	9.06	36	92.17	5.29
Post_DP	29	87.24	11.25	16	92.17	9.06	36	92.17	5.29
Post_IC	29	86.66	11.50	16	94.13	9.25	36	96.11	4.83
Post_IP	29	86.66	11.50	16	94.13	9.25	36	96.11	4.83

Table 9

Difference in Mean and Standard Deviation Comparison by Instructor

	Comparison Instructor Versus Intervention Instructor1		Comparison Instructor Versus Intervention Instructor2		Intervention Instructor1 Versus Intervention Instructor2	
	M	*SD*	*M*	*SD*	*M*	*SD*
Pretest Derivatives Concepts	0.16	0.89	1.53	0.52	1.37	1.41
Pretest Derivatives Procedure	0.16	0.89	1.53	0.52	1.37	1.41
Pretest Integrals Concepts	7.24	3.43	6.53	1.99	0.71	1.44
Pretest Integrals Procedure	7.24	3.43	6.53	1.99	0.71	1.44

Implementation Fidelity and Triangulation: Observation Notes

The observation notes and the intervention group professors' interview responses served as pillars for the implementation of fidelity and triangulation. In this section, the qualitative part (implementation fidelity and triangulation) preceded the quantitative analysis. I reported on the activities of both groups' instructors firstly, then the intervention group professors secondly.

Comparison and Intervention Groups

Before the four observations (two on derivatives and two on integrals), I introduced myself to the instructors before the class started and reminded them what I was there for. I assured the facilitators that the data would be kept private and anonymous. In each class, the instructors routinely introduced me to the classroom. I sat inconspicuously in the back of the classroom, so I could not see the students' faces and observed each entire lesson period.

The instructors complied with the set learning objectives on derivatives and integrals in each class of both the comparison and intervention groups. The facilitators aligned all their assessments with the taught topics, adhering to the constructivism principles of learning, asking questions to enable students' engagement. Class activities, which fostered conceptual and procedural understanding started with definitions and examples. The professors continued modeling topical activities on derivatives and integrals, followed by students replicating what the professors have modeled in class.

In both the comparison and intervention groups, professors modeled and asked learners to use the concept of derivatives to find the extrema (minimum/maximum) of functions. Other activities related to finding the equations of the secant, tangent, and normal lines to a curve, then ascertaining the difference between the concept of average and instantaneous rate. Activities related to integrals consisted of finding the primitive, antiderivative, indefinite and definite integrals, of a given function, and using the concept of integral to find the area under a curve. Professors extended students' practice of taught and learned concepts, which were the bases for quizzes (postest1) and end of term exam (posttest) in a homework assignment.

The instructors used multiple representations, which represented the teaching of mathematical concepts using different procedures, formulating questions that targeted students' higher thinking skills (see Maharaj & Wagh, 2016). The activities illustrated the nature of a given real-world problem, its verbal representation (the professors' narration), tabular representation (numerical table of values of x and y), algebraic representation, and graphical representation with the Cartesian connection. These representations could activate students' mental processing as they construct knowledge (see Bakirtzoglou & Ioannou). The instructors delivered their lessons according to the set SMART goals and objectives. They demonstrated interactive lessons (student to student interaction, student to content interaction, student to instructor interaction), provided constructive feedback; thus, exemplifying constructivist principles, and student-centered learning pedagogy.

Intervention Group

In the intervention group, in addition to the above-presented information, the professors used Maple animation-visualization tools to enable students to visualize a particle's movement along a curve of a function, its secant, and tangent line. See Appendix B. In one of the intervention group classes, other class activities and examples fostering the conceptual and procedural understanding of derivatives and integrals 3-D representation to generate and compute the volume of a solid of revolution (see Figure I4 in Appendix I).

During class activities and discussions, the professors used words as speech rather than on-screen text (modality principle), placed related graphics of learned concepts near the text to minimize cognitive processing. They positioned related graphics and narration (text) close to ensure students' undivided attention (spatial-contiguity principle). The instructors' use of multimedia presentation could help lessen the irrelevant cognitive load and increase the relevant cognitive load, coherent with the multimedia, modality, and spatial-contiguity principle (see Jung et al., 2016). The use of words and meaningful graphics (multimedia principle) has contributed positively to students' learning. Moreover, the narration and animation of learned concepts in a synchronized manner with visual or analytic processing could contribute to the students' conceptual, procedural, and strategic understanding (see Foshay & Silber, 2009) of limits, derivatives, and integrals. However, the loss of invalid data from the intervention group caused a reduction in the sample size, which constituted a limitation of the study. The instructors administered the intervention as I expected, and there were no challenges that prevented the planned implementation, as I described in Chapter 3. However, during the data cleaning process, I found that 34 participants had no pretest and quiz scores, and I excluded their information from the data analysis. The reduction in the sample size had severe complications, such as the inability to generalize (external validity) I discussed in Chapter 5.

Interview Response

The interview response from the intervention group professors concurred that the use of Maple technology helped students grasp the taught concepts on derivatives, especially finding the equation of the secant, tangent, and normal lines, as well as establishing the difference between the average and instantaneous rate of change. However, some students experienced some difficulties in writing Maple codes. The professors testified to this occurrence of these experiences in their response to the interview question on what challenges did students experience using the Maple animation and visualization tools in learning the concepts of derivatives and integrals in calculus.

The facilitators also provided some examples (see Appendix G) of things students did well and get confused about using Maple software. The research findings across the data source accurately showed that the dynamic visualization group significantly outperformed the static visualization group for each RQ.

Results

The results section includes the report on descriptive statistics that appropriately characterize the sample. It comprises evaluation of statistical assumptions appropriate to the study. In addition, it contains report of statistical analysis findings, which I organized by research questions in the primary analysis.

Test for Assumptions

A mixed-design MANOVA, using the multivariate Wilks' Lambda test, did not require the assumption of sphericity, which was the homogeneity of variance between the pretest and the two posttest scores (see Field, 2017; Pallant, 2016). The assumption of normality was not a concern because the central limit theorem indicated that sample sizes above 30 produced a normal distribution of sample means (Field; Pallant; Tabachnick & Fidell, 2018). Therefore, no tests of assumptions were necessary. Another assumption of the mixed-design MANOVA was that the with-in subject repeated measures are the same at each time-period (Creswell & Creswell, 2018; Tabachnick & Fidell, 2018). Meaning, if there were two time periods where measurements were the same, the measure was given at both time-periods. In this study, there was a violation of this assumption, as the pretest, quiz, and posttest measures were all slightly different for the treatment and control groups. There was also an assumption of the comparison group and intervention group equivalence on the pretest. The hypothesis upheld on the derivative concept (RQ1) and the derivatives' procedure (RQ2) tests, but not on the integrals' concept (RQ3) and integrals' procedure (RQ4), thus presenting a limitation (with full details in chapter 5) to the study.

Primary Analysis Using Quantitative Data

Comparing the non-Maple static and Maple dynamic groups' outcome variables requires the use of inferential statistics, consisting of the statistical analyses used to test the null hypothesis (see Trochim, n. d.). The use of the probabilistic analysis assisted observing any difference between groups and ascertaining the main and interaction effects between factors. It was essential to recall that the scores were from slightly different tests; therefore, before executing the mixed-designed MANOVA in the rest of this chapter, the tests were equated. I classified the primary analysis by the research questions in the study:

RQ1: The first question asked, *Was there a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?* There were two categorical independent variables. The first was a group, which contained comparison and intervention groups. The second independent variable was time, which contained three factors: a pretest, quiz (posttest1), and an end-of-term exam (posttest). The continuous dependent variable was the scores on the derivatives' concept calculus test, where high scores represented superior performance.

Mixed-design MANOVA to determine any effect the intervention had on scores on the derivatives' concept calculus test indicated a statistically significant group by time interaction, $F(2,78) = 14.07$, $p < 0.001$. The eta square, effect size value ($\eta^2 = 0.265$), indicated that a 26.5% of the variability in derivative concept calculus test scores was accounted for by the group by time interaction. According to Cohen's eta squared effect size standards of 0.01 for small, 0.06 for medium, and 0.14 for large, the size of the effect was considerable.

For both the comparison and intervention groups, pretest scores were significantly lower than both the quiz and posttest scores (see Table 10). Additionally, for the intervention group only, posttest scores were significantly higher than quiz scores. However, there was no significant difference between the quiz and posttest scores for the comparison group.

Table 10

MANOVA -Pre_ Quiz_Post- Comparison Chart_Derivatives' Concept

	Pretest ^{a}		Quiz ^{b}		Posttest ^{c}				
	M	*SD*	*M*	*SD*	*M*	*SD*	Group	Time	Int
Der-Pr							0.13	125.97*	14.07*
Comp	66.72^{bc}	12.86	87.34^{a}	11.22	87.24^{a}	11.25			
Inter	66.56^{bc}	12.82	82.10^{ac}	10.16	91.98^{ab}	6.59			

Note. * - denote $p < 0.001$;

Letters indicated significant difference between mean score in the referencing column.

Moreover, results of the mixed-design MANOVA also indicated no significant difference in pretest derivative concept scores between the comparison and intervention groups. However, the comparison group had significantly higher scores on the quiz, but the intervention group had significantly higher scores on the posttest (see Table 11 and Figure 13).

Table 11

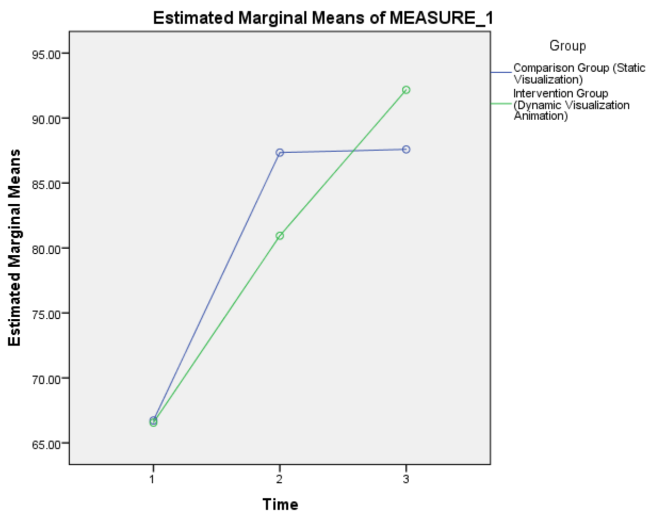
MANOVA – Comparison and Intervention Derivatives' Concept

	Comparison [a]		Intervention [b]				
	M	*SD*	*M*	*SD*	Group	Time	Int
Derivative Concept					0.13	1125.97*	14.07*
Pretest	66.72	12.86	66.56	12.82			
Quiz	87.34[b]	11.22	82.10[a]	10.16			
Posttest	87.24[b]	11.25	91.98[a]	6.59			

Note. * - denoted $p < 0.001$; letters indicated significant difference between mean score in the referencing column.

Figure 13

Profile Plots of MANOVA – Derivatives' Concept



Therefore, the results indicated that the intervention group had significantly ($p < 0.001$) higher scores on the posttest, with a large effect size of 0.27, but not on the quiz. Consequently, the results supported the expected outcomes from the animation-visualization theorists.

RQ2: The second research probed, *Was there a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?* There were two categorical independent variables: group (comparison and intervention) and time (pretest, quiz, and posttest). The continuous dependent variable was the scores on the derivatives' procedure calculus test, where high scores represented enhanced performance.

Results of the multivariate mixed-design MANOVA revealed a statistically significant group by time interaction effect, Wilk's Lambda = 0.74, $F(2. 78) = 14.07$, $p < 0.001$, $\eta^2 = 0.265$, where the interaction accounted for 26.5% of the variability in derivative procedure calculus scores. This effect size was a large effect, as it was larger than Cohen's 0.14 standard for a large effect. The pretest scores for the comparison group were significantly lower than the quiz and posttest scores (see Table 13). However, there were no significant differences between quiz and posttest scores among the comparison group. For the intervention group, the pretest was significantly lower than both the quiz and posttest. The quiz was also statistically lower than the posttest scores on the derivatives' procedure calculus test (see Table 12).

Table 12

MANOVA – Pre_Quiz_Post_Comparison Chart–Derivatives' Procedure

	Pretest [a]		Quiz [b]		Posttest [c]				
	M	*SD*	*M*	*SD*	*M*	*SD*	Group	Time	Int
Der-Pr							0.01	125.97*	14.07*
Comp	66.72[bc]	12.86	87.34[a]	11.22	87.24[a]	11.25			
Inter	66.56[bc]	12.82	82.10[ac]	10.16	91.98	6.59			

Note. * - denoted $p < 0.001$;

Letters indicated significant difference between mean score in the referencing column.

When comparing Groups, the mixed-design MANOVA revealed no significant difference between groups on the pretest. However, on the quiz, the comparison group had significantly higher derivative procedure calculus scores than the intervention group but had lower scores on the posttest (see Table 13 and Figure 14). Let us remember the scores were from different tests.

Table 13

MANOVA - Comp and Inter Comparison Chart – Derivatives' Procedure

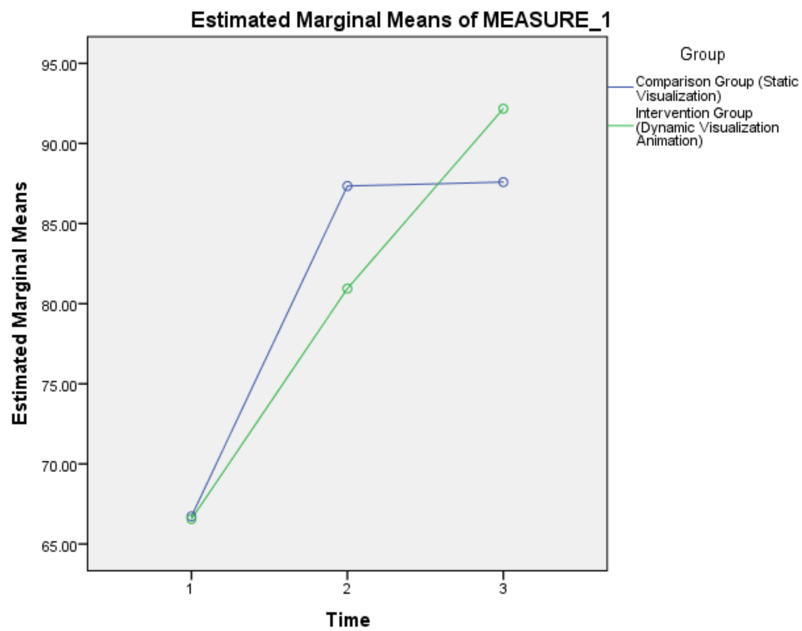
	Comparison [a]		Intervention [b]				
	M	*SD*	*M*	*SD*	Group	Time	Int
Derivative Procedure					0.01	125.97*	14.07*
Pretest	66.72	12.86	66.56	12.82			
Quiz	87.34[b]	11.22	82.10[a]	10.16			
Posttest	87.24	11.25	91.98	6.59			

Note. * - denoted $p < 0.001$;

Letters a and b indicated significant difference between mean score in the referencing column.

Figure 14

Profile Plots of Mixed Design MANOVA – Derivatives' Procedure



Based on data analysis, there was a significant ($p < 0.001$) positive effect of the dynamic visualization on the students' procedural understanding of derivatives in calculus, with a large effect size of 0.27. Again, **the results supported the expected outcomes from the animation-visualization theorists.**

RQ3: The third research question asked, *Was there a significant difference in the pretest (prerequisite skills for integrals), quiz, and posttest, scores on the integrals' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?* The continuous dependent variable was the scores on the integrals'concept calculus test, where high scores represented better performance on the test. The two categorical independent variables were group (comparison and intervention) and time (pretest, quiz, and posttest).

The results of the mixed-design MANOVA indicated that there was a statistically significant group by time interaction, Wilk's Lambda = 0.59, F (2, 78) = 26.87, $p < 0.001$, $\eta^2 = 0.408$. Based on the eta squared value, the interaction explained 40.8% of the variability in integral concept calculus scores, which was a substantial effect, as it exceeded Cohen's standard of 0.14 or 14%. For the intervention group, the pretest integrals' concept calculus scores were significantly lower than the quiz and posttest scores. The quiz calculus scores on integrals' concept were significantly lower than the posttest scores (see Table 14).

Table 14

MANOVA – Pre_ Quiz_Post Comparison Chart – Integral's Concept

	Pretest [a]		Quiz [b]		Posttest [c]				
	M	SD	M	SD	M	SD	Grp	Time	Int
Int Con							.72	130.13*	26.87*
Comp	71.97bc	8.58	91.34a	13.81	86.66a	11.50			
Inter	65.50bc	12.65	84.58ac	10.13	95.50ac	6.48			

Note. * - denoted $p < 0.001$;

Letters indicated significant difference between mean score in the referencing column.

The mixed design MANOVA indicated a statistically significant difference between the comparison and intervention groups on the pretest, quizzes, and posttests. Specifically, the comparison group had significantly higher scores than the intervention group on the pretest and quiz. However, the intervention group had significantly higher scores than the comparison group on the posttest (see Table 15 and Figure 15).

Table 15

MANOVA - Comp and Inter Comparison Chart – Integral's Concept

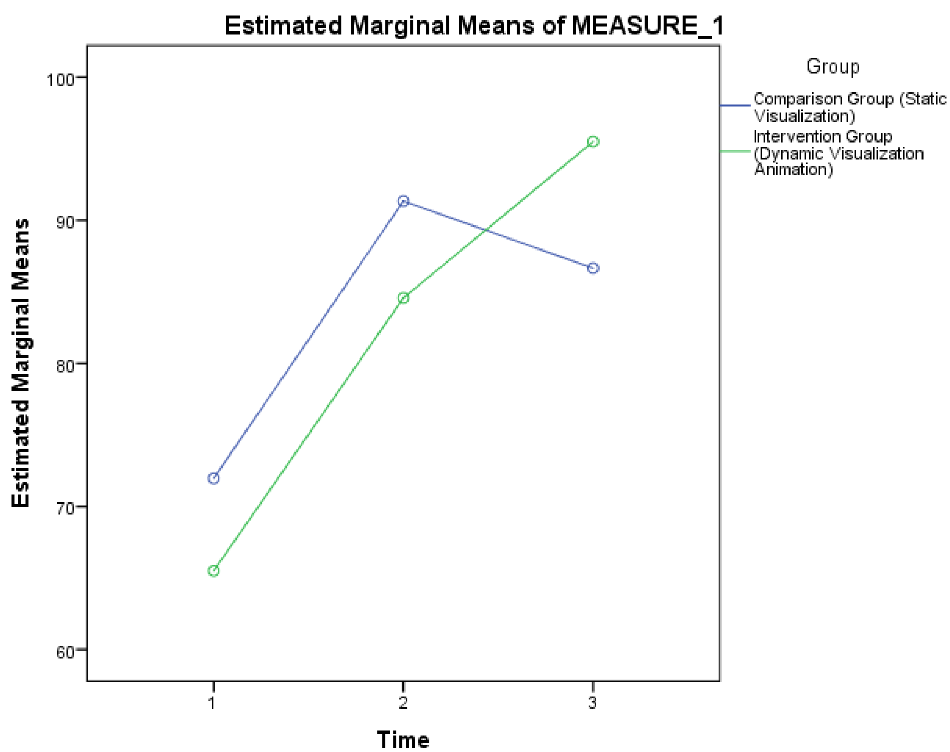
	Comparison [a]		Intervention [b]				
	M	*SD*	*M*	*SD*	Group	Time	Int
Integral Concept					.72	130.13*	26.87*
Pretest	71.97b	8.58	65.50a	10.91			
Quiz	91.34b	13.81	84.58a	10.13			
Posttest	86.66b	11.50	95.50a	6.48			

Note. * - denoted $p < 0.001$;

Letters indicated significant difference between mean score in the referencing column.

Figure 15

Profile plots of Mixed Design MANOVA-Integrals' Concept



Here again, there was a significant positive effect of the dynamic visualization on students' conceptual understanding of integral in calculus. The intervention group had significantly ($p < 0.001$) higher scores than the comparison group on the posttest on the integral's concept, with a substantial effect of 0.41. Thus, the analysis results concurred with the animation-visualization theorists' expectations.

RQ4: The fourth research question inquired, *Was there a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization animation?* The dependent variable was the scores on the integrals' procedure calculus test and was categorical. The two categorical independent variables were group (comparison and intervention) and time (pretest, quiz, and posttest).

The mixed design MANOVA test on the pretest, quiz, and posttest phases, and between the comparison and intervention groups, indicated that there was a statistically significant interaction between the comparison and intervention groups and the pretest, quiz, and posttest time periods, Wilk's Lambda = 0.72, $F(2, 78) = 15.00$, $p < 0.001$. The eta squared effect size value ($\eta^2 = 0.408$) indicated that 41% of the variability in integral's procedure calculus scores was explained by the group (comparison and intervention) and time (pretest, quiz, and posttest) interaction, which exceeds Cohen's standard of 0.14 for a large effect. For the comparison group, the integral procedure pretest scores were significantly lower than both the quiz and posttest scores. However, there were significant differences between the quiz and posttest scores. For the intervention group, pretest scores on integrals' procedure were also significantly lower than both the quiz and posttest scores. Unlike in the comparison group, the quiz scores in the intervention were also significantly lower than the posttest scores (see Table 16).

Table 16

MANOVA – Pret_Quiz_Post Comparison Chart – Integral's Procedure

	Pretest [a]		Quiz [b]		Posttest [c]				
	M	SD	M	SD	M	SD	Group	Time	Int
Integral Procedure							1.53	144.35*	15.00*
Comparison	71.97bc	8.58	91.34a	13.81	86.66a	11.50			
Intervention	64.67bc	12.65	82.45ac	10.74	94.50ac	7.66			

Note. * - denoted $p < 0.001$;

Letters (a, b, and c) indicated significant difference between mean score in the referencing column.

Additionally, there was a significant effect for the comparison and intervention groups across the three time periods. Specifically, the intervention group had significantly lower scores on the integral procedure test than the comparison group on the pretest and quiz. However, the comparison group had significantly lower scores on the posttest than the intervention group (see Table 17 and Figure 16).

Table 17

MANOVA - Comp and Int Comparison Chart-Integral's Procedure

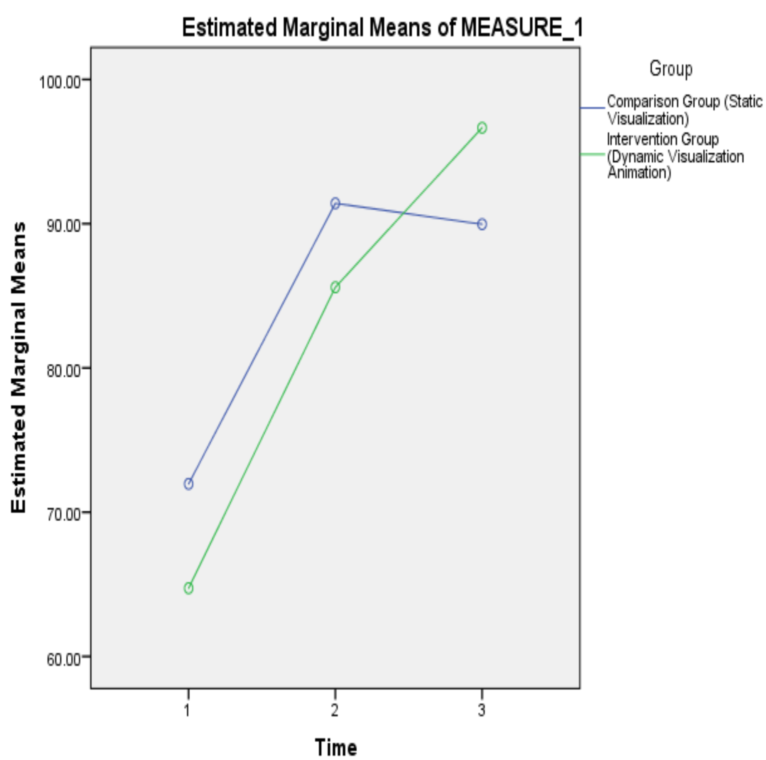
	Comparison [a]		Intervention [b]				
	M	SD	M	SD	Group	Time	Int
Integral Procedure					1.53	144.35*	15.00*
Pretest	71.97b	8.58	64.67a	12.65			
Quiz	91.34b	13.81	82.45a	10.74			
Posttest	86.66b	11.50	94.50a	7.66			

Note. * - denoted p < 0.001;

Letters (a and b) indicated significant difference between mean score in the referencing column.

Figure 16

Profile Plots of Mixed Design MANOVA – Integral' Procedure



Once more, there was a significant positive effect of the dynamic visualization of students' procedural understanding of integrals in calculus. The intervention group significantly ($p < 0.001$) outperformed the comparison group, with a substantial effect size of 0.41. Thus, the results supported the animation-visualization theorists' expectations.

While the tests were slightly different, the mixed-design MANOVA analysis indicated that the intervention group significantly ($p < 0.001$) outperformed the comparison group with a large effect size of 0.27 for the first and second questions, and substantial effect size of 0.41for the third and fourth question. Consequently, Maple's dynamic visualization, within the animation-visualization theory, had a positive impact on the students' conceptual and procedural understanding of derivatives and integrals in calculus. The implementation of fidelity and triangulation reflected performance outcomes, which showed that the tests aligned with the intervention's learning objectives.

Summary

There were 81 respondents in this study. The current study investigated four research questions. The first research question inquired, *Was there a significant difference in pretest, quiz, and posttest scores on the derivatives' concept calculus test between the static visualization and dynamic visualization groups?* The results indicated that there was a statistically significant group by time interaction, with a large effect size $\eta^2 = 0.27$. Meaning, there were no significant differences between the comparison and intervention groups on the pretest; however, there were statistically ($p < 0.001$) significant differences between the two groups on the quiz and posttest.

Research question two queried, *Was there a significant difference in pretest, quiz, and posttest scores on the derivatives' procedure calculus test between the static visualization and the dynamic visualization groups?* Inferential statistics indicated a statistically significant group by time interaction, with a large effect size $\eta^2 = 0.27$, where there were no significant differences between the comparison and intervention groups on the pretest and posttest. However, there were statistically significant differences between the two groups on the quiz and posttest, with a 27% variance.

The third research question inquired, *Was there a significant difference in pretest, quiz, and posttest scores on the integrals' concept calculus test between the comparison static visualization and dynamic visualization groups?* Again, inferential statistic results indicated a statistically significant group-by-time interaction, with a large effect size $\eta^2 = 0.41$, and the intervention group had significantly lower scores than the comparison group on both the pretest and quiz. However, the intervention group had significantly ($p < 0.001$) higher scores on the posttest with a substantial effect size of 0.41.

Finally, research question four enquired, *Was there a significant difference in pretest, quiz, and posttest scores on the integrals' procedure calculus test between the static visualization and dynamic visualization groups?* The results showed a statistically significant group-by-time interaction, with a large effect size $\eta^2 = 0.41$ and that the intervention group had significantly lower scores than the comparison group on both the pretest and the quizzes. However, the intervention group had significantly ($p < 0.001$) higher scores on the post-test, with a substantial effect of 0.41.

While the tests were slightly different, the MANOVA run on the assumption that the tests could be equated, a source of limitation of the study. Using noncomparable tests affected the assumption of group equivalence on the pretest, and the inability to compute a learning gain directly. Moreover, the comparison group was relatively small compared to the intervention group. There were significant differences between the comparison and intervention groups on the integral's procedure and integral's concept pretests. It is essential to point out that there were three instructors in the intervention group and only one facilitator in the comparison group; so, I could only check for a teacher effect in the intervention group. However, I found none.

The next chapter contained a discussion of the results.

Chapter 5: Discussion, Conclusions, and Recommendations

Introduction

The purpose of the study was to determine if an instructor's interactive Maple-based dynamic visualization lessons, designed within the framework of the animation-visualization theory (see Erlich & Russ-Eft, 2011; Paik, 2012; Zurita & Nussbaum, 2007), made an essential difference in college students' conceptual and procedural understanding of derivatives and integrals in calculus. Using the appropriate quantitative 2x2 factorial pretest and posttest control group QED mixed-design with MANOVA for data analysis, research results indicated that the intervention group has significantly ($p < 0.001$) outperformed the comparison group with a substantive effect size of at least 27%.

This chapter covers the discussion of the study results. The chapter also presents the limitations of the study, implications, and presented recommendations for future research. These implications included the impact of this study on positive social change, methodology, and practice. The concluding section consisted of the chapter conclusions.

Interpretation of the Findings

The interpretation of the findings occurs at two levels. The first domain includes the discussion of the study results in the context of the theoretical framework. The second level is the discussion of the findings related to the literature review. Thus, the interpretation covers the theoretical framework and previous research outcomes as I described in the peer-reviewed literature, Chapter 2.

Theoretical Framework

The theoretical framework in this study was the animation-visualization approach. This theory stipulated that carefully designed dynamic visualizations activities for educational purposes could possess an informational advantage over static pictures, for instance, by directly depicting dynamic changes such as changes in the velocity of an object (see Kinkeldey et al., 2014; Opach et al., 2014; Persson, 2014; Sarlis & Christopoulos, 2014). Given that static pictures could not directly provide these changes (see Salleh & Zakaria, 2016), the investigative inquiry on describing the missing information in a text could compensate for the corresponding informational disadvantage of static pictures was negative. Even when individuals described the dynamic changes in a text, animated visuals still led to a deeper understanding of the content. The expectation was that calculus students who used dynamic visualization learning techniques would learn concepts more effectively than those who did not.

The first research question asked, *Was there a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?* The mixed-design MANOVA analysis indicated a statistically significant group-by-time interaction, $p < 0.001$ with a substantial effect size value ($\eta^2 = 0.265$). Thus, 26.5% of the variability in derivative concept calculus test scores was accounted for by the group-by- time interaction, where there was no significant difference between the comparison and intervention groups on the pretest. According to Cohen's eta squared effect size standards of 0.01 for small, 0.06 for medium, and 0.14 for large. The size of the effect was considerable. The intervention group had significantly ($p < 0.001$) higher scores on the posttest, with a large effect size of 0.27, but not on the quiz. Therefore, the results supported the animation-visualization theorists' expectations, as the intervention group outperformed the comparison on the posttests.

The second research question asked, *Was there a significant difference in the pretest (prerequisite skills for derivatives), quiz (posttest1), and end of term exam (posttest) scores on the derivatives' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?* The results indicated that there was a statistically significant group by time interaction, with a substantial effect of, $\eta^2 = 0.265$; meaning the interaction accounted for 26.5% of the variability in derivatives' procedure calculus scores, where there were no significant differences between the comparison and intervention groups on the pretest. Still, there were significant differences between the two groups on the quiz and posttest. The intervention group had higher scores on the posttest, but lower scores on the quiz. Therefore, the results supported the animation-visualization theorists' expectations, as the intervention group had significantly ($p < 0.001$) higher scores on the posttest, with a considerable effect size of 0.27.

The third research question asked, *Was there a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' concept calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?* The mixed-design

MANOVA indicated a statistically significant group by time interaction, with a substantial effect of $\eta^2 = 0.408$ (40.8%). The test results showed that the intervention group had significantly lower scores on the pretest and quiz than the comparison group, but higher scores were on the posttest. The study's results indicated that the intervention group had significantly higher scores on the posttest but lower scores on the pretest and quizzes. Again, like with RQ1 and RQ2, the results supported what I expected, because the intervention group scores were significantly ($p < 0.000$) higher than the comparison group scores on the posttest, with a substantial effect size of 0.41.

Finally, the fourth research question asked, *Was there a significant difference in the pretest (prerequisite skills for integrals), quiz (posttest1), and end of term exam (posttest) scores on the integrals' procedure calculus test between the comparison group (static visualization) and the intervention group (dynamic visualization)?* The mixed-design MANOVA indicated a statistically significant interaction between the comparison and intervention groups and the pretest, quiz, and posttest time-periods, with a tremendous effect size value $\eta^2 = 0.408$ (40.8%). The results of the statistical analyses indicated that the intervention group had significantly ($p < 0.001$) higher scores on the posttest than the comparison group, with a considerable effect size of 0.41. These results supported what I expected, based on the animation-visualization theory.

Research from the Literature

In the literature review of Chapter 2, several studies examined the effects of visualizations and animations on educational processes and outcomes. In a posttest, only factorial experimental design, Lin (2011) examined the effect of static and animated visuals on students' learning of different educational objectives in a CBI environment. He found that there was superior effectiveness of animated visuals on students' learning over static visuals. Kühl et al.'s research (2018) echoed Lin's study results that animations promoted a deeper understanding of the concept of velocity than still pictures, especially in students with low spatial abilities, which are essential in visualizations. Additionally, Nossun (2012) found that dynamic visualizations were compelling for learning human and non-human movements, helping students remember and understand the materials they studied (De Koning & Tabbers, 2013). Impelluso (2018) found that students were able to experience 3-D dynamics through the visualization of interactive animations favored student in solving problems in physics. Correlational analysis revealed that spatial ability, verbal-logical reasoning ability, and mathematical performance were significantly correlated. High spatial visualizers had significantly higher spatial ability and mathematical performance scores than high object visualizers. Based on these findings, the expectations were that calculus students who used animation and visualization learning techniques would learn concepts more effectively than those who did not.

For RQ1, the results indicated a statistically significant ($p < 0.001$) group by time interaction, with a large effect of 26.5% variance, where there was no significant difference between the comparison and intervention groups on the pretest. However, the intervention group had significantly higher scores on the posttest, but not on the quiz. The results supported what I expected, based on the animation-visualization theorists' views, as the intervention group outperformed the comparison on the posttests.

For RQ2, the results indicated a statistically significant group by time interaction, where there were no significant differences between the comparison and intervention groups on the pretest. However, there were significant ($p < 0.001$) differences between the two groups on the quiz and posttest, with a large effect size of 26.5%. The intervention group had higher scores on the posttest, but lower scores on the quiz. Therefore, the results supported what I expected, as the intervention group had significantly higher scores on the posttest.

For RQ3, the results of the study revealed a group by time interaction, where the intervention group had significantly ($p < 0.001$) higher scores on the posttest, with a substantial effect size of 41%, but lower scores on the pretest and quiz. Again, with RQ1 and RQ2, the results supported what I expected, because the intervention group scores were significantly higher than the comparison group scores on the posttest.

Finally, for RQ4, the results of the statistical analyses indicated that the intervention group had significantly ($p < 0.001$) higher scores on the posttest than the comparison group, with a substantial effect size of 41%. These results supported what I expected, based on the animation-visualization theory.

Limitations of the Study

This study registered few limitations. First, the comparison group was relatively small compared to the intervention group. The statistical power was 1.0 for the within-subject analysis (pretest, quiz, posttest) and 0.996 for the interaction effects (comparison/intervention and pretest, quiz, posttest). These statistical power values indicated that there was a 100% (1.0) and 99.6% (0.996), respectively, probability of detecting a significant effect if one exists in the real world. Given that the sample size for the comparison group was small (n = 29) compared to the intervention group (n = 52), the between-subjects analysis (comparison/intervention group) was low (0.062), indicating there was only 6.2% chance of detecting a significant between-subjects effect. So, even though the study revealed significant differences between the intervention and comparison groups across the three time-periods, the mixed-design MANOVA was not statistically strong enough to detect the comparison group effects alone, only in combination with the time periods. Second, the tests used for the pretest, quiz, and posttest were slightly different for the comparison and intervention groups and within each class of the intervention group. The pretest, quiz, and posttests were also slightly different for the groups with the intervention group; that meant that there might have been variations in the difficulty of the pretest, quiz, and posttest, as they were all different. Given this possibility, the intervention might not induce the observed significant differences, but the variations in the measurements might.

Moreover, there was a lack of group equivalency on the pretest. There were significant differences between the comparison and intervention groups on the integrals' procedure and integrals' concept pretests. Theoretically, if one group was more skilled in calculus than the other, differences in math scores from pretest to posttest, or the lack there off, might be due to the level of competency of the groups and not the intervention.

As a result, the lack of pretest group equivalency provided an alternative explanation for the seen significant effects in this study.

Recommendations for Future Research

The first recommendation for future research is related to sample size. A future researcher should adopt an equal size of 84 minimum for both the intervention and comparison groups (see Faul et al., 2009). The second recommendation for future research is that the pretest, quizzes, and posttest measures should be the same measure for both the intervention and comparison groups; this way, the assumption of the repeated measures design would not be violated. Third, to minimize the possibility of violating the assumption of group equivalency on the pretest, I recommend that future research uses randomization for group assignment to minimize the possibilities of nonequivalent groups (see Creswell & Creswell, 2018).

Implications

There are several implications that are associated with this study. Firstly, the discussion relates to the social implications. Secondly, the discussion relates to the theoretical implications. Thirdly, the discussion continues with the implications for practice. And fourthly, the discussion ends with policy implications

Social Change Implications

Findings from this study and other studies promote the benefits of visualization and animation and may foster increased development of learning software that can be targeted and sold to households versus educational institutions. The study results indicate that the use of dynamic visualization can enhance the method of teaching calculus at the postsecondary level, to enable learners to gain mathematical skills in calculus, prepare students in and for STEM majors and careers, and enable the United States to compete globally. The educational software programs that incorporate visualization and animation programs can supplement the education the students are receiving at school and for students who homeschool.

Theoretical Implications

From a theoretical perspective, this study helps address the gap in the literature and provides additional evidence that carefully designed animations for educational purposes possess an informational advantage over static pictures and result in improved educational performance. Results for RQ1 and RQ2 indicate that the intervention group had significantly ($p < 0.001$) higher scores on the posttest than the comparison group, with a substantial effect of 26.5% variance. For RQ3 and RQ4, the intervention group has significantly lower scores on the pretest but significantly higher scores on the posttest. The results provide evidence for the animation and visualization theory, as the intervention group has significantly ($p < 0.001$) outperformed the comparison group with a 40.8% variance.

Implications for Practice

This study may have a positive impact on educational practice by influencing educators to employ animation and visualization learning approaches to various subjects at various educational levels. The current study applies dynamic visualization (animated visualization) to teach calculus concepts. Educators and facilitators may use the calculus animations to enhance teaching methods at both the secondary and postsecondary levels.

Additionally, other disciplines may use calculus animation training as a template for other topics and disciplines.

Policy Implications

From an educational policy perspective, the results of this study, along with other studies, can influence educational policy. As most of the students in the United States currently homeschool due to the Covid-19 pandemic, remote learning via computers has become the status quo. The use of educational technology will become more central, and curriculums may incorporate more animation and visualization in the lessons, based on findings from this study and other studies, to enhance the educational performance of studies.

Conclusions

The use of technology in the classroom will become more prevalent at all levels of education. Technology in the classroom allows for individualized learning and assessment, which provides students with a more adaptive and customized learning experience. As educational faculty and administrators look for effective ways to increase student performance, embedding visualization and animations in instruction can be an asset. Preliminarily, studies have shown that animated visuals were more effective than static visuals at fostering retention and positive learning outcomes. Only one previous study showed that visualization and animation could be useful in the STEM fields, and that was a study by Impelluso (2018). He showed that there were academic performance benefits when instructors incorporated animation into a physics curriculum. The current study adds to the body of evidence that supports the efficacy of animated visuals over static visuals in producing more excellent academic performance. Specifically, this study provides additional evidence that visualization and animation can be applied effectively to more challenging subjects like calculus, a STEM field. As the need for professionals in the science and technology field grows, so will the need for curriculums that incorporate visualization and animation, as this makes the challenging subject matter more accessible to the non-technically inclined.

References

Aan, A., & Heinloo, M. (2012). Visualization of the kinematics of a material point. *Engineering for Rural Development - International Scientific Conference*, *11*204-209.

Agbatogun, A. (2013). Interactive digital technologies' use in Southwest Nigerian universities. *Educational Technology Research & Development*, *61*(2), 333-357. https://doi.org/10.1007/s11423-012-9282-1

Allendoerfer, C., Wilson, D., Kim, M., & Burpee, E. (2014). Mapping beliefs about teaching to patterns of instruction within science, technology, engineering, and mathematics. *Teaching in Higher Education*, *19*(7), 758-771. https://doi.org/10.1080/13562517.2014.901962

Anderson, I. M., & Torre, C. G. (2012). New symbolic tools for differential geometry, gravitation, and field theory. *Journal of Mathematical Physics*, *53*(1), 01351. https://doi.org/10.1063/1.3676296

Anderson, J. R., Lee, H. S., & Fincham, J. M. (2014). Discovering the structure of mathematical problem solving. *NeuroImag*e, 97, 163–177. https://doi.org/10.1016/j.neuroimage.2014.04.031

Aurigemma, J., Chandrasekharan, S., Nersessian, N. J., & Newstetter, W. (2013). Turning experiments into objects: The cognitive processes involved in the design of a lab-on-a-chip device. *Journal of Engineering Education, 102*(1), 117-140. https://doi.org/10.1002/jee.20003

Awang, T. S., & Zakaria, E. (2012). Module for learning integral calculus with Maple: Lecturers' views. *Turkish Online Journal 11*(3), 234-245.

Bali, J. S., Kumar, A., & Nandi, V. (2016). An experience, using software based tools for teaching and learning mathematically intensive signal processing theory concepts,*2016 IEEE 4th International Conference on MOOCs, Innovation and Technology in Education (MITE)*, 100-104. https://doi.org/10.1109/MITE.2016.029.

Bardini, C., & Pierce, R. (2015). Assumed mathematics knowledge: the challenge of symbols. *International Journal of Innovation in Science and Mathematics Education, 23*(1), 1-9.

Blazhenkova, O., & Kozhevnikov, M. (2016). Types of creativity and visualization int of different educational specialization. *Creativity Research Journal*, *28*(2), 123-135. https://doi.org/10.1080/10400419.2016.1162638

Brown, J. P. (2015a). Complexities of digital technology use and the teaching and learning of function. *Computers & Education, 87*, 112–122. https://doi.org/10.1016/j.compedu.2015.03.022

Brown, J. P. (2015b). Visualisation tactics for solving real world tasks. In G. Stillman, W. Blum, & M. S. Biembengut (Eds.), *Mathematical modelling in education research and practice: Cultural, social and cognitive influences* (pp. 431–442). https://doi.org/10.1007/978-3-319-18272-8_36

Buneci, M. R. (2014). Using Maple to represent the subgroupoids of trivial groupoid x×z×x. *Fiability & Durability / Fiabilitate Si Durabilitate*, (1), 446-454.

Bunt, A., Terry, M., & Lank, E. (2013). Challenges and opportunities for mathematics software in expert problem solving. *Human–Computer Interaction, 28*, 222–264. https://doi.org/10.1080/07370024.2012.697020

Campbell, D. T., & Stanley, J. C. (1963). *Experimental and quasi-experimental designs for research*. Cengage Learning.

Carden, J., & Cline, T. (2015). Problem solving in mathematics: the significance of visualization and related working memory. *Educational Psychology in Practice*, *31*(3), 235-246. https://doi.org/10.1080/02667363.2015.1051660

Castro-Alonso, J., Ayres, P., & Paas, F. (2016). Comparing apples and oranges? A critical look at research on learning from statics versus animations. *Computers & Education*, *102*234-243. https://doi.org/10.1016/j.compedu.2016.09.004

Choi, M. G., Noh, S., Komura, T., & Igarashi, T. (2013). Dynamic comics for hierarchical abstraction of 3D animation data. *Computer Graphics Forum*, *32*(7), 1-9. https://doi.org/10.1111/cgf.12206.

Cohen, R., & Kelly, A. M. (2019). Mathematics as a factor in community college STEM performance, persistence, and degree attainment. *Journal of Research in Science Teaching, 57*(2), 145-307. https://doi.org/10.1002/tea.21594

Covington, M., Chavis, T., & Perry, A. (2017). A scholar-practitioner perspective to promoting minority success in STEM. *Journal for Multicultural Education,11*(2), 149-159. https://doi.org/10.1108/JME-01-2016-0001

Cox, E. (2015). Coaching and Adult Learning: Theory and Practice. *New Directions for Adult and Continuing Education, 2015*(148), 27-38. https://doi.org/10.1002/ace.20149

Creswell, J. W., & Creswell, J. D. (2018). *Research design: qualitative, quantitative, and mixed methods approaches* (5th ed.). SAGE.

De Freitas, E. (2016). Material encounters and media events: what kind of mathematics can a body do? *Educational Studies in Mathematics*, *91*(2), 185-202. https://doi.org/10.1007/s10649-015-9657-4

De Koning, B. B., & Tabbers, H. K. (2013). Gestures in instructional animations: A helping hand to understanding non-human movements? *Applied Cognitive Psychology, 27*, 683–689. https://doi.org/10.1002/acp.2937

Dick, T. & Edwards, B. (2008). MR and local linearity: research influences on the use of technology in calculus curriculum reform. In G. Blume & M. K. Heid (Eds.), *Research on technology and the teaching and learning of mathematics (Cases and perspectives, 2*, 255–278). Information Age.

Dobler, C. P., & Klein, J. M. (2002). First graders, flies, and a Frenchman's fascination: Introducing the Cartesian coordinate system. (Links to Literature).*Teaching Children Mathematics*, *8*(9).

Ellis, J., Fosdick, B. K., & Rasmussen, C. (2016). Women 1.5 times more likely to leave stem pipeline after calculus compared to men: Lack of mathematical confidence a potential culprit. *PLoS ONE 11(7)*: e0157447, 1-14. https://doi.org/10.1371/journal.pone.0157447

Erlich, R. J., & Russ-Eft, D. (2011). Applying social cognitive theory to academic advising to assess student learning. *NACADA Journal, 31*(2), 5-15. https://doi.org/10.12930/0271-9517-31.2.5

Faul, F., Erdfelder, E., Buchner, A., & Lang, A. G. (2009). Statistical power analyses using G*Power 3.1: Tests for correlation and regression analyses. *Behavior Research Methods*, *41*, 1149-1160.

Field, A. (2017). *Discovering statistics using IBM SPSS statistics: North American edition* (5th ed.). Springer. https://books.google.com/books?hl=en&lr=&id=CPFJDwAAQBAJ&oi=fnd&pg=PT16&dq=field+2017+spss&ots=FKBqjGqyP7&sig=4--E7oGtZc7f2uhoJZmyg6BSCgw#v=onepage&q=field%202017%20spss&f=false

Foshay, W. R., & Silber, K. H. (2009). *Handbook of improving performance in the workplace, instructional design and training delivery*. (1st ed.). John Wiley & Sons.

Frankfort-Nachmias, C., & Nachmias, D. (2008). *Research methods in the social sciences.* (7th ed.). Worth.

Geiger, V., Calder, N., Tan, H., Loong, E., Miller, J., & Larkin, K. (2016). Transformations of teaching and learning through digital technologies. In K. Makar, J. Visnovska, M. Goos, A. Bennison, & K. Fry (Eds.), *Research in Mathematics Education in Australasia 2012-2015*, (pp. 255-280). Springer. https://doi.org/10.1007/978-981-10-1419-2_13

Ghani, S., Elmqvist, N., & Yi, J. S. (2012). Perception of animated node-link diagrams for dynamic graphs. *Computer Graphics Forum*, *31*(3pt3), 1205-1214. https://doi.org/10.1111/j.1467-8659.2012.03113.x

Guilleux, A., Blanchin, M., Hardouin, J., & Sébille, V. (2014). Power and sample size determination in the Rasch model: Evaluation of the robustness of a numerical method to non-normality of the latent trait. *PLoS ONE*, *9*(1), e83652. https://doi.org/10.1371/journal.pone.0083652

Haciomeroglu, E. E. (2016). Object-spatial visualization and verbal cognitive styles, and their relation to cognitive abilities and mathematical performance. *Educational Sciences: Theory & Practice*, *16*(3), 987-1003. https://doi.org/10.12738/estp.2016.3.0429

Hair, J. F., Ringle, C. M., Gudergan, S. P., Fischer, A., Nitz, C., & Con Menictas, C. (2018). Partial least squares structural equation modeling-based discrete choice modeling: an illustration in modeling retailer choice. *Business Research 12*. https://doi.org/10.1007/s40685-018-0072-4

Handley, M. A., Lyles, C. R., McCulloch, C., & Cattamanchi, A. (2018). Selecting and improving quasi-experimental designs in effectiveness and implementation research. *Annual Review of Public Health 39*(1), 5-25. https://doi.org/10.1146/annurev-publhealth-040617-014128

Hitt, F. (2011). Construction of mathematical knowledge using graphic calculators (CAS) in the mathematics classroom. *International Journal of Mathematical Education in Science and Technology*, *42*(6), 723-735.

Hodara, M., & Xu, D. (2016). Does developmental education improve labor outcomes? Evidence from two states. *American Educational Research Journal, 53*, 781-813. https://doi.org/10.3102/0002831216647790

Höffler, T., & Leutner, D. (2007). Instructional animation versus static pictures: A meta-analysis. *Learning & Instruction, 17*, 722–738. https://doi.org/10.1016/j.learninstruc.2007.09.013

Horner, J., & Minifie, F. D. (2011). Research ethics III: Publication practices and authorship, conflicts of interest, and research misconduct. *Journal of Speech, Language, and Hearing Research: JSLHR*, *54*(1), S346–S362. https://doi.org/10.1044/1092-4388(2010/09-0263)

Hutton, C. (2019). Using role models to increase diversity in STEM: The American workforce needs every capable STEM worker to keep America in a global leadership position. *Technology & Engineering Teacher*, *79*(3), 16–19.

Impelluso, J. T. (2018). The moving frame method in dynamics: Reforming a curriculum and assessment. *International Journal of Mechanical Engineering Education*, *46*(2). https://doi.org/10.1177/0306419017730633

Jahanshahi, S., Babolian, E., Torres, D. F., & Vahidi, A. (2015). Original article: Solving Abel integral equations of first kind via fractional calculus. *Journal of King Saud University - Science*, *27,*161-167. https://doi.org/10.1016/j.jksus.2014.09.004

Jarvis, D. H., Buteau, C., & Lavicza, Z. (2014). Computer algebra system (CAS) usage and sustainability in university mathematics instruction: Findings from an international study. *Electronic Journal of Mathematics & Technology*, *8*(4), 286-299. f

Jung, J., Kim, D., & Na, C. (2016). Effects of WOE presentation types used in pre-training on the cognitive load and comprehension of content in animation-based learning environments. *Journal of Educational Technology & Society, 19*(4), 75-86.

Kadunz, G., & Yerushalmy, M. (2015). Visualization in the teaching and learning of mathematics in The Author(s) 2015 463 S.J. Cho (ed.). *The Proceedings of the 12th International Congress on Mathematical Education*. Springer. https://doi.org/10.1007/978-3-319-12688-3_41

Karadeniz, I., & Thompson, D. R. (2018). Precalculus teachers' perspectives on using graphing calculators: an example from one curriculum*. International Journal of Mathematical Education in Science and Technology,49*(1), 1-14. https://doi.org/10.1080/0020739X.2017.1334968

Karakus, M., & Duressi, A. (2017). Quality of service (QOS) software defined networking (SDN): Survey. *Journal of Network and Computer Applications, 80*(15), 200-218. https://doi.org/10.1016/j.jnca.2016.12.019

Katsioloudis, P., Dickerson, D., Jovanovic, V., & Jones, M. (2016). Use of dynamic visualizations for engineering technology, industrial technology, and science education students: implications on ability to correctly create a sectional view sketch. *Journal of Technology Education*, *28*(1), 19-36. https://doi.org/10.21061/jte.v28i1.a.2

Kidron, I., & Tall, D. (2015). The roles of visualization and symbolism in the potential and actual infinity of the limit process. *Educational Studies in Mathematics*, *88*(2), 183-199. https://doi.org/10.1007/s10649-014-9567-x

Kinkeldey, C., MacEachren, A. M., & Schiewe, J. (2014). How to assess visual communication of uncertainty? A systematic review of geospatial uncertainty visualization user studies. *Cartographic Journal*, *51*(4), 372-386. https://doi.org/10.1179/1743277414Y.0000000099

Kühl, T., Stebner, F., Navratil, S. C., Fehringer, B. C. O. F., & Münzer, S. (2018). Text information and spatial abilities in learning with different visualizations formats. *Journal of Educational Psychology, 110*(4), 561–577. https://doi.org/10.1037/edu0000226

Kumar, K., Pandey, R. K., & Sharma, S. (2019). Approximations of fractional integrals and Caputo derivatives with application in solving Abel's integral equations. *Journal of King Saud University - Science 31*(4), 692-700. https://doi.org/10.1016/j.jksus.2017.12.017

Lakhvich, T. (2012). Visualization-assisted teaching: Can virtual give rise to real knowledge? *Problems of Education in the 21st Century, 42*, 5-7.

LaRue, R., & Infante, N. E. (2015). Optimization in first semester calculus: a look at a classic problem. *International Journal of Mathematical Education in Science and Technology*, *46*(7), 1021-1031. https://doi.org/10.1080/0020739X.2015.1067844

Leng, N. W. (2011). Using an advanced graphing calculator in the teaching and learning of calculus. *International Journal of Mathematical Education in Science and Technology, 42*(7), 925-938. https://doi.org/10.1080/0020739X.2011.616914

Lin, H. (2011). Facilitating learning from animated instruction: Effectiveness of questions and feedback as attention-directing strategies. *Educational Technology & Society, 14* (2), 31–42. https://psycnet.apa.org/record/2012-24260-003

Lv, Z., Tek, A., Da Silva, F., Empereur-mot, C., Chavent, M., & Baaden, M. (2013). Game on, science - how video game technology may help biologists tackle visualization challenges. *PLoS ONE 8*(3), 1-13. https://doi.org/10.1371/journal.pone.0057990.

Maharaj, A., & Wagh, V. (2016). Formulating tasks to develop HOTS for first-year calculus based on Brookhart abilities. *South African Journal Of Science*, *112*(11/12), 77-82. https://doi.org/10.17159/sajs.2016/20160139

Mau, W. J., & Li, J. (2018). Factors Influencing STEM Career Aspirations of Underrepresented High School Students. *The Career Development Quarterly*, *66*(3), 246-258. https://doi.org/10.1002/cdq.12146

Mayer, R. E. (2014). *The Cambridge handbook of multimedia learning (2nd ed.)*. Cambridge University Press. http://dx.doi.org/10.1017/CBO9781139164603

McCulloch, A. W., Kenney, R. H., & Keene, K. A. (2013). What to Trust: Reconciling Mathematical Work Done by Hand with Conflicting Graphing Calculator Solutions. *School Science and Mathematics*, *113*(4), 201-210. https://doi.org/10.1111/ssm.12016

McNeish, D. (2017). Challenging conventional wisdom for multivariate statistical models with small samples. *Review of Educational Research, 87*(6), 1117-1151. https://doi.org/10.3102/0034654317727727

Meikle, L. I., & Fleuriot, J. D. (2012). Integrating systems around the user: Combining Isabelle, Maple, and QEPCAD in the Prover's Palette. *Electronic Notes in Theoretical Computer Science 285,*115–119. https://doi.org/10.1016/j.entcs.2012.06.010

Melero, J. J., Hernández-Leo, D., Sun, J., Santos, P., & Blat, J. (2015). How was the activity? A visualization support for a case of location-based learning design. *British Journal Of Educational Technology*, *46*(2), 317-329. https://doi.org/10.1111/bjet.12238

Milovanović, M., Perišić, J., Vukotić, S., Bugarčić, M., Radovanović, L., & Ristić, M. (2016). Learning mathematics using multimedia in engineering education. *Acta Technica Corvininesis - Bulletin Of Engineering*, *9*(1), 45-49. http://acta.fih.upt.ro/pdf/2016-1/ACTA-2016-1-06.pdf

Mojžišová, A., & Pócsová, J. (2018). Visualisation of mathematical content using LATEX animations. International Carpathian Control Conference (ICCC), Szilvasvarad, 536-541, doi: 10.1109/CarpathianCC.2018.8399689. https://ieeexplore-ieee-org.ezp.waldenulibrary.org/document/8399689?arnumber=8399689

Moy, K., Li, W., Tran, H. P., Simonis, V., Story, E., Brandon, C., & ... Kim, H. (2015).
Computational methods for tracking, quantitative assessment, and visualization of C.

elegans locomotory behavior. *Plos ONE*, *10*(12), 1-22. https://doi.org/10.1371/journal.pone.0145870

Nathan, M. J., Srisurichan, R., Walkington, C., Wolfgram, M., Williams, C., & Alibali, M. W. (2013). Building cohesion across representations: A mechanism for STEM integration. *Journal Of Engineering Education, 102*(1), 77-116. https://doi.org/10.1002/jee.20000

Nossun, A. S. (2012). Semistatic animation – Integrating past, present, and future in map animations. *The Cartographic Journal, 49*(1), 43–54. https://doi.org/10.1179/1743277411Y.0000000014

Ocal, M. F. (2017). The effect of GeoGebra on students' conceptual and procedural knowledge: The case of applications of derivative. *Higher Education Studies, 7*(2), 67-78. http://doi.org/10.5539/hes.v7n2p67

Opach, T., Gołębiowska, I., & Fabrikant, S. I. (2014). How do people view multi-component animated maps? *Cartographic Journal*, *51*(4), 330-342. https://doi.org/10.1179/1743277413Y.0000000049

Özkan, E. M., Tepedeldiren, Y. T., & Ünal, H. (2011). Geometrical and algebraic approach to multiple representations. *Procedia - Social and Behavioral Sciences*, *15*(3rd World Conference on Educational Sciences - 2011), 72-75. https://doi.org/10.1016/j.sbspro.2011.03.053

Ozturk, Y., Anapali, A., Gulsu, M., & Sezer, M. (2013). A collocation method for solving fractional Riccati differential equation. *Journal of Applied Mathematics*, 1-9. https://doi.org/10.1155/2013/598083

Padilla-López, J. R., Chaaraoui, A. A., & Flórez-Revuelta, F. (2015). Visual privacy protection methods: A survey. *Expert Systems with Applications 42*, 4177–4195. https://doi.org/10.1016/j.eswa.2015.01.041

Paik, E. (2012). Should multimedia instruction be easier to understand? Implications of animation induced illusion of understanding. In P. Resta (Ed.). *Proceedings of SITE 2012-Society for Information Technology & Teacher Education International Conference* (pp. 748-751). Austin, Texas, USA: Association for the Advancement of Computing in Education (AACE). https://www.learntechlib.org/primary/p/39661/.

Pallant, J. (2016). *SPSS Survival Manual: A Step By Step Guide to Data Analysis Using SPSS Program (6th ed.).* McGraw-Hill Education.

Park, J. (2015). Is the derivative a function? If so, how do we teach it? *Educational Studies In Mathematics*, *89*(2), 233-250. http://dx.doi.org/10.1007/s10649-015-9630-2.

Patwardhan, M., & Murthy, S. (2015). When does higher degree of interaction lead to higher learning in visualizations? Exploring the role of 'interactivity enriching features'. *Computers & Education*,*82*292-305. http://dx.doi.org/10.1016/j.compedu.2014.11.018

Persaud, D., & Burns, J. (2018). First Nations People: Addressing the Relationships between Under-Enrollment in Medical Education, STEM Education, and Health in the United States. *Societies 8*(1), 1 – 11. https://doi.org/10.3390/soc8010009

Persson, P. E. (2014). How teachers can use TI-Nspire CAS with laptops in an upper secondary course. *The Electronic Journal of Mathematics & Technology*, *8*(4), 257–273. http://urn.kb.se/resolve?urn=urn:nbn:se:mau:diva-3625

Prahani, B. K., Limatahu, I., Soegimin, W. W., Yuanita, L., & Nur, M. (2016). Effectiveness of physics learning material through guided inquiry model to improve student's problem solving skills based on multiple representation. *International Journal of Education and Research Vol. 4*(12), 231 – 242. https://pdfs.semanticscholar.org/702e/16b50b835ee7348ab9d0297254d134f96682.pdf

Pretorius, A. J., Khan, I. A., & Errington, R. J. (2017). A Survey of Visualization for live cell imaging. *Computer Graphics Forum*, *36*(1), 46-63. https://doi.org/10.1111/cgf.12784

Pyke, A., Betts, S., Fincham, J. M., & Anderson, J. R. (2015). Visuospatial referents facilitate the learning and transfer of mathematical operations: Extending the role of the angular gyrus. *Cognitive, Affective and Behavioral Neuroscience, 15*(1), 229-250. https://doi.org/10.3758/s13415-014-0317-4

Quarles, C., & Davis, M. (2017). Is learning in developmental math associated with Community College outcomes? *Community College Review*, *45*(1), 33-51. https://doi.org/10.1177/0091552116673711

Rittle-Johnson, B., & Schneider, M. (2015). *Developing conceptual and procedural knowledge of mathematics. In Kadosh R., Dowker A. (Eds.), Oxford handbook of numerical cognition (2nd ed., pp. 1118-1134).* Oxford University Press.

Roanes-Lozano, E., Alonso, J. A., & Hernando, A. (2014). Revisiting four-valued logics from Maple using the Logics Explorer package. *Mathematics and Computers in Simulation 104*, 31–42. https://doi.org/10.1016/j.matcom.2012.06.012

Rusli, M., & Negara, I. Y. (2017). The effect of animation in multimedia computer-based learning and learning style to the learning results. *Turkish Online Journal Of Distance Education (TOJDE)*, *18*(4), 177-190. https://doi.org/10.17718/tojde.340409

Salleh, T. S., & Zakaria, E. (2016). The effects of Maple integrated strategy on engineering technology students' understanding of integral calculus. *Turkish Online Journal Of Educational Technology - TOJET*, *15*(3), 183-194. https://files.eric.ed.gov/fulltext/EJ1106377.pdf

Samková, L. (2012). Calculus of one and more variables with Maple. *International Journal Of Mathematical Education In Science & Technology*, *43*(2), 230-244. https://doi.org/10.1080/0020739X.2011.582248

Samson, D. (2014). Visualizing and generalizing with square arrays. *Australian Mathematics Teacher*, *70*(2), 4-12. https://files.eric.ed.gov/fulltext/EJ1093259.pdf

Sarlis, N. V., & Christopoulos, S. G. (2014). Visualization of the significance of receiver operating characteristics based on confidence ellipses. *Computer Physics Communications, 185*(3), 1172-1176. volume 185 (3): 1172–1176. http://dx.doi.org/10.1016/j.cpc.2013.12.009

Serenko, A. (2007). The development of an instrument to measure the degree of animation predisposition of agent users. *Computers in Human Behavior 23* (2007) 478–495. https://doi.org/10.1016/j.chb.2004.10.042

Serhan, D. (2015). Students' understanding of the definite integral concept. *International Journal Of Research In Education And Science*, *1*(1), 84-88. https://files.eric.ed.gov/fulltext/EJ1105099.pdf

Sevimli, E. (2016a). Do calculus students demand technology integration into learning environment? Case of instructional differences. *International Journal Of Educational Technology In Higher Education*, *13*(1), 1-18. https://doi.org/10.1186/s41239-016-0038-6

Sevimli, E. (2016b). Evaluating views of lecturers on the consistency of teaching content with teaching approach: traditional versus reform calculus. *International Journal of Mathematical Education in Science and Technology, 47*(6), 877-896. https://doi.org/10.1080/0020739X.2016.1142619

Shahbazi, Z., & Irani, A. (2016). Online calculus and pre-calculus modules. In *Proceedings of EdMedia 2016-World Conference on Educational Media and Technology* (pp. 42-51). Vancouver, BC, Canada: Association for the Advancement of Computing in Education (AACE). https://www-learntechlib-org.ezp.waldenulibrary.org/p/172930/

Siddique, M. (2010). Symposium: Using Maple to visualize mathematical concepts. *AIP Conference Proceedings*, *1281*(1), 705. https://doi.org/ 10.1063/1.3498575

Smolinsky, L., Olafsson, G., Marx, B. D., & Wang, G. (2019). Online and handwritten homework in calculus for STEM majors. *Journal of Educational Computing Research, 57*(6), 1513 – 1533. https://doi-org.ezp.waldenulibrary.org/10.1177/0735633118800808

Soemer, A., & Schwan, S. (2016). Task-appropriate visualizations: Can the very same visualization format either promote or hinder learning depending on the task requirements? *Journal of Educational Psychology, 108*(7), 960–968. https://doi.org/10.1037/edu0000093

Solares, A., & Kieran, C. (2013). Articulating syntactic and numeric perspectives on equivalence: The case of rational expressions. *Educational Studies In Mathematics*, *84*(1), 115-148. https://doi.org/10.1007/s10649-013-9473-7

Sozcu, O. F., Ziatdinov, R., & Ipek, I. (2013). The effects of computer-assisted and distance learning of geometric modeling. *European Researcher, 39*(1 -2), 175 – 181. https://www.researchgate.net/publication/236683023_The_Effects_of_Computer-assisted_and_Distance_Learning_of_Geometric_Modeling

Swift, R. (2017). Solids of revolution and the Herschel-Maxwell theorem. *Mathematical Scientist*, *42*(3), 127-130. https://www.researchgate.net/publication/322340245_Solids_of_revolution_and_the_Herschel-Maxwell_theorem

Tabachnick, B. G., & Fidell, L. S. (2018). Using multivariate statistics, (7th ed). Pearson.

Tasker, R. (2014). Research into Practice: Visualizing the molecular world for a deep understanding of chemistry. *Teaching Science*, *60*(2), 16-27. https://www.rsc.org/images/Tasker-Dalton%20paper%20final_tcm18-52113.pdf

Thompson, S. (2013). On the classification of parametric cubic curves. *Electronic Journal Of Mathematics & Technology*, *7*(3), 195-204. https://www.radford.edu/~thompson/RP/classification.pdf

Trochim, W. M. K. (n.d.). Research method: Knowledge based: Sampling http://www.socialresearchmethods.net/kb/sampling.php

Végh, L., & Stoffová, V. (2017). Algorithm Animations for Teaching and Learning the Main Ideas of Basic Sortings. *Informatics in Education, 16*(1), 121-140. http://dx.doi.org/10.15388/infedu.2017.07

Verdine, B. N., Golinkoff, R. M., Hirsh-Pasek, K., & Newcombe, N. S. (2017). VI. Discussion and implications: How early spatial skills predict later spatial and mathematical skills. *Monographs Of The Society For Research In Child Development*, *82*(1), 89-109. https://doi.org/10.1111/mono.12285

Verhoeff, N. (2020). Surface Explorations: 3D Moving images as cartographies of time. In Sæther S. & Bull S. (Eds.), *Screen Space Reconfigured* (pp. 179-200). Amsterdam University Press. https://doi.org/10.2307/j.ctv12pnt9c.11

Vieira, A. F. (2015). Viewing the roots of polynomial functions in complex variable: The use of GeoGebra and the CAS Maple. *Acta Didactica Napocensia* *6*(4), 45-58. https://files.eric.ed.gov/fulltext/EJ1053713.pdf

Vincent, J., Pierce, R., & Bardini, C. (2017). Structure sense: A precursor to competency in undergraduate mathematics. *Australian Senior Mathematics* Journal, 31(1), 38-47 2017. https://files.eric.ed.gov/fulltext/EJ1153295.pdf

Wilkie, K. J. (2016). Students' use of variables and multiple representations in generalizing functional relationships prior to secondary school. *Educ Stud Math 93*, 333–361. https://doi.org/10.1007/s10649-016-9703-x

Wismath, S., & Worrall, A. (2015). Improving University Students' Perception of Mathematics and Mathematics Ability. *Numeracy: Advancing Education In Quantitative Literacy*, *8*(1), 1-17. http://dx.doi.org/10.5038/1936-4660.8.1.9

Wiwatanapataphee, B., Sada, N., & Yong Hong, W. (2010). An integrated PowerPoint-Maple based teaching-learning model for multivariate integral calculus. *International Electronic Journal Of Mathematics Education*, *5*(1), 5-31. https://www.iejme.com/download/an-integrated-powerpoint-maple-based-teaching-learning-model-for-multivariate-integral-calculus.pdf

Yildiz Ulus, A. (2013). Teaching the diagonalization concept in linear algebra with technology: A case study at Galatasaray University. *TOJET: The Turkish Online Journal of Educational Technology,12*(1), 119 – 130. https://files.eric.ed.gov/fulltext/EJ1008874.pdf

Yu, C. (2014). Solving the partial differential problems using Maple. *Journal of Applied Research and Technology, 12*. 1144-1153. https://doi.org/10.1016/S1665-6423(14)71673-6

Yurttas, A., Ozgur, B., & Cangul, I. N. (2012). Calculation of the minimal polynomial of 2cos(π/*n*) over Q with Maple. *Numerical Analysis and Applied Mathematics ICNAAM,* 1*479*, 371-374. https://doi.org/10.1063/1.4756141

Zamuda, A., & Brest, J. (2013). Environmental framework to visualize emergent artificial forest ecosystems. *Information Sciences 220*, 522–540. https://doi.org/10.1016/j.ins.2012.07.031

Zurita, G., & Nussbaum, M. (2007). A conceptual framework based on activity theory for mobile CSCL. *British Journal of Educational Technology, 38*(2), 211–235. https://doi.org/10.1111/j.1467-8535.2006.00580.x

Appendix A: Sample Lesson Plan

I . Application of Derivatives

Learning Objective (L O 1)

At the end of this unit, students will be able to use the concept of derivative, with at least 80% accuracy, to compute the:

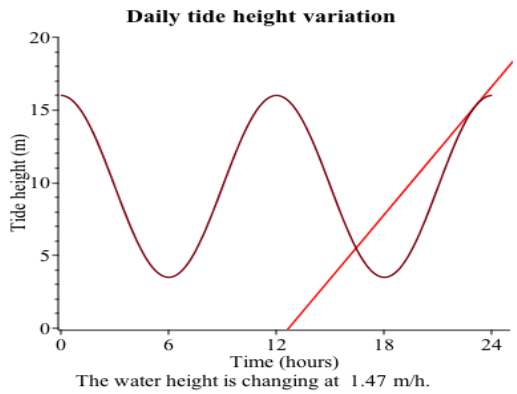
- Difference quotient.
- Instantaneous rate of change
- Average rate of change
- Equation of the tangent and normal of a line to a curve.

Vocabulary and Definitions

Function, curve, difference quotient, derivatives, instantaneous rate of change, average rate of change, derivatives, extrema, minimum, maximum, point of inflection, equation of the tangent and normal of a line. Figure B1 exemplified a sample problem with a secant and tangent line to a curve.

Figure B1

Graph-Secant and Tangent Line to a Curve



II Application of Integrals

LO 2

At the end of this unit, students will be able to use integral, to:

- Find the area under a curve of a function
- Solve problem involving
- Calculate the volume of a solid of revolution.

Vocabulary and Definitions

Integral, definite integral, indefinite integrals, primitive, anti-derivative, area under a curve of a function, volume, solid of revolution.

A s ample problem instructors and students might work on was presented in chapter 2.

Appendix B. Observation and Implementation Fidelity Rubric

Table B1

Comparison Group

Items	Procedural Understanding Yes (1)	Procedural Understanding No (0)	Conceptual Understanding Yes (1)	Conceptual Understanding No (0)	Illustration
Non-Maple Static visualization: lesson and activities on derivative	1		1		See Appendix A and Appendix I
Non-Maple Static visualization: lesson and activities on integral	1		1		See Appendix A and Appendix I
Instructor's questions engaging students' understanding of derivative	1		1		See Appendix A and Appendix I
Instructor's questions engaging students' understanding of integral	1		1		See Appendix A and Appendix I
Instructor encourages students to ask questions on concepts (integral) taught	1		1		See Appendix A and Appendix I
Instructor asks students to explain results of questions on derivatives	1		1		See Appendix A and Appendix I
Instructor asks students to explain results of questions on integrals	1		1		See Appendix A and Appendix I
Reinforcement activities on derivatives	1		1		See Appendix A and Appendix I
Reinforcement activities on integrals	1		1		See Appendix A and Appendix I

Table B2

Intervention Instructor 1

Observation Table Rubric

Items	Procedural Understanding		Conceptual Understanding		Illustration/ Definitions
	Yes (1)	No (0)	Yes (1)	No (0)	
Dynamic visualization: lesson and activities on derivative	1		1		Definitions and class work on: Function and Curve, Difference quotient, Instantaneous and average rate of change, Equations of the tangent and normal lines to a curve. See Appendix B and Appendix I
Dynamic visualization: lesson and activities on integral	1		1		Definition and class work on Primitive, Antiderivative, Definite and Indefinite Integrals, Area under a curve of functions, Volume of a Solid of Revolution. See Appendix A, Appendix, B and Appendix I
Reinforcement activities on derivative	1		1		The assigned homework questions reflected the activities done in class
Instructor's questions engaging students' understanding of integrals	1		1		See Appendix B and Appendix I
Instructor asks students to explain results of questions on Integral	1		1		See Appendix B and Appendix I
Reinforcement activities on derivative	1		1		See Appendix B and Appendix I
Reinforcement activities on derivative	1		1		See Appendix B and Appendix I

Table B3

Intervention Instructor 2

Observation Table Rubric					
Items	Procedural Understanding		Conceptual Understanding		Illustration/ Definitions
	Yes (1)	No (0)	Yes (1)	No (0)	
Dynamic visualization: lesson and activities on derivative	1		1		See Appendix A, Appendix, B and Appendix I
Dynamic visualization: lesson and activities on integral	1		1		See Appendix A, Appendix, B and Appendix I
Reinforcement activities on derivative	1		1		The assigned homework questions reflected the activities done in class
Instructor's questions engaging students' understanding of integrals	1		1		See Appendix A, Appendix, B and Appendix I
Instructor asks students to explain results of questions on Integral	1		1		See Appendix A, Appendix, B and Appendix I
Reinforcement activities on derivative	1		1		See Appendix A, Appendix, B and Appendix I
Reinforcement activities on derivative	1		1		See Appendix A, Appendix, B and Appendix I

Appendix C. Sample Pretest Questions

Question 1

Consider the function $f(x) = \dfrac{1}{x^2 - kx}$, where k is a nonzero constant. The derivative of f is given by

$f'(x) = \dfrac{k - 2x}{\left(x^2 - kx\right)^2}$.

(a) Let $k = 3$, so that $f(x) = \dfrac{1}{x^2 - 3x}$. Write an equation for the line tangent to the graph of f at the point whose x-coordinate is 4.

(b) Let $k = 4$, so that $f(x) = \dfrac{1}{x^2 - 4x}$. Determine whether f has a relative minimum, a relative maximum, or neither at $x = 2$. Justify your answer.

(c) Find the value of k for which f has a critical point at $x = -5$.

(d) Let $k = 6$, so that $f(x) = \dfrac{1}{x^2 - 6x}$. Find the partial fraction decomposition for the function f.

Find $\int f(x)\,dx$.

Question 2

At time $t \geq 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = \left(\cos\left(t^2\right), e^{0.5t}\right)$. At $t = 1$, the particle is at the point $(3, 5)$.

(a) Find the x-coordinate of the position of the particle at time $t = 2$.

(b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

(c) Find the time at which the speed of the particle is 3.

(d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.

Question 3

Question 3

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem. Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

(c) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.

(d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

Appendix D. Proof of NIH Web-based Training Course Completion

Certificate of Completion

The National Institutes of Health (NIH) Office of Extramural Research certifies that **Segla Kossivi** successfully completed the NIH Web-based training course "Protecting Human Research Participants."

Date of Completion: 09/24/2018

Certification Number: 2931668

NIH National Institutes of Health
Office of Extramural Research

Appendix E: Randomized Number for Coding

fx =RANDBETWEEN(10,1000)

B	C	D	E	F	G	H	I	J
594	51	297	675	247	203	265	100	601
602	950	300	239	732	226	704	426	178
802	526	871	730	74	137	78	193	369
152	641	555	510	729	437	397	401	936
828	58	128	764	571	742	905	995	576
224	418	594	757	331	579	616	620	34
754	117	833	575	346	230	81	53	42
200	897	803	459	305	504	168	427	92
399	988	816	793	999	743	204	859	312
219	307	595	86	588	131	401	405	447
685	305	165	787	654	83	534	798	573
168	457	919	320	475	807	68	836	170
444	506	637	424	252	671	782	291	296
194	251	73	107	950	684	807	582	842
83	481	856	442	113	667	386	98	534
411	183	718	773	669	33	37	704	140
20	276	160	276	260	429	16	827	517
360	719	322	90	65	150	26	811	85
350	673	463	514	886	448	485	695	671
923	121	46	636	538	494	61	538	794
87	635	345	215	902	367	891	36	247
338	718	325	544	824	347	251	829	854
348	763	229	449	42	223	803	149	536
337	553	675	302	950	146	477	584	789
884	53	453	112	951	832	449	149	843
787	591	828	539	100	688	1000	347	795
160	650	66	87	228	284	172	23	30
241	115	268	108	599	365	371	615	982
70	354	428	927	802	247	254	297	545
503	60	56	627	927	825	179	900	260
757	250	383	335	423	86	882	189	388
35	934	38	914	936	885	416	601	226
422	263	362	234	321	652	226	343	744
110	172	745	502	211	347	48	449	105
544	734	134	716	514	228	916	45	[illegible]
922	349	564	829	472	328	120	742	[illegible]

Appendix F: Sample of Coded Data for Participants

Figure F1

Coded Data

#	CU		PU		CU		PU		Instructor #
	Pretest		Pretest		Quiz (Potest1)		End of Term Exam (Posttest)		
	D	I	D	I	D	I	D	I	
###	x%%	x%	x%	x%	x%	X%	X%	X%	#

Note. CU = Conceptual Understanding; PU = Procedural Understanding
D = Derivatives; I = Integrals

Appendix G: Interview Response from Intervention Group Instructors

Instructor 1

1. How do you think the Maple animation and visualization tools help students understand conceptually and procedurally the concepts of derivative and integral in calculus?

I use the Maple to teach the difference quotient, derivatives as tangents, Newton's approximation, continuity of functions and Riemann sums to understand integrals. The regular classes are all hand calculations and traditional methodology so the Maple program enables them to easily see how the concepts look.

2. What challenges did students experience using the Maple animation and visualization tools in learning the concepts of derivative and integral in calculus?
3. Can you give an example of things students did well, get confused about in using Maple software?

One of the main problems is simply getting students to carefully key in formulae and programming words. They often forgot to put in a multiplication sign or didn't have parentheses correctly placed. Students seemed satisfied with the tool. If there is one issue, it is that the tools of Maple aren't used in any of their other classes so the task of learning to get around in Maple seems like it may not be worth the effort to them. My main goal is to support the students in their regular calculus class so some times, Maple has to be pushed aside to help with he paper and pencil work necessary to advance in the class.

4. Do you think you are going to use Maple software next year?

I am using Maple again but I am making a few changes to incorporate some other software.

Instructor 2

Questions

1. How do you think the Maple animation and visualization tools help students understand conceptually and procedurally the concepts of derivative and integral in calculus?
2. What challenges did students experience using the Maple animation and visualization tools in learning the concepts of derivative and integral in calculus?
3. Can you give an example of things students did well, get confused about in using Maple software?
4. Do you think you are going to use Maple software next year?
5. Do you have any other suggestions?

Responses:

> Responses to questions:
> 1/ Did little animation/ I think the visualization helps very much
> 2/Challenges: getting syntax correct
> 3/ Plotting finding intersections and tangent lines and computing integrals all done well. Determining intervals where functions concave/convex most difficult
> 4/ I will use Maple again. Other instructors like other software
> 5/No...sorry.

Appendix H: LC Approval Letter to Conduct Research

Dear Segla,

I have been in touch with Dr. Prohaska, who approved your initial request to conduct research at Lehman College under your Walden University IRB application (protocol #05-02-19-0196043, "Understanding Calculus Through Maple-Based Animation and Visualization Tools"). The changes that you propose below have also been approved. Please accept this email as confirmation of that.

Best,

Appendix I: Observed Class Activities on Derivatives

Intervention Instructor 1-Derivatives and Integrals: Concept and Procedure

Figure I1

Derivatives: Concept and Procedure: Tangent and Normal Lines

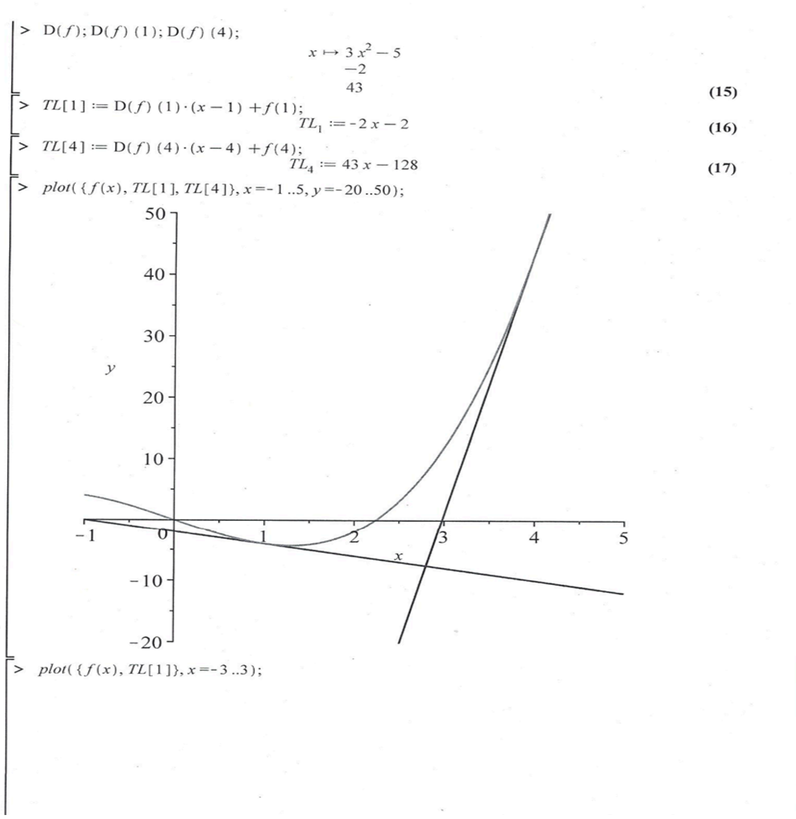


Figure I 2

Integrals: Concept and Procedure: Area under a curve

SurfaceOfRevolution, SurfaceOfRevolutionTutor, Tangent, TangentSecantTutor, TangentTutor, TaylorApproximation, TaylorApproximationTutor, Understand, Undo, VolumeOfRevolution, VolumeOfRevolutionTutor, WhatProblem]

We can use Riemann sums to approximate the area given by the integral

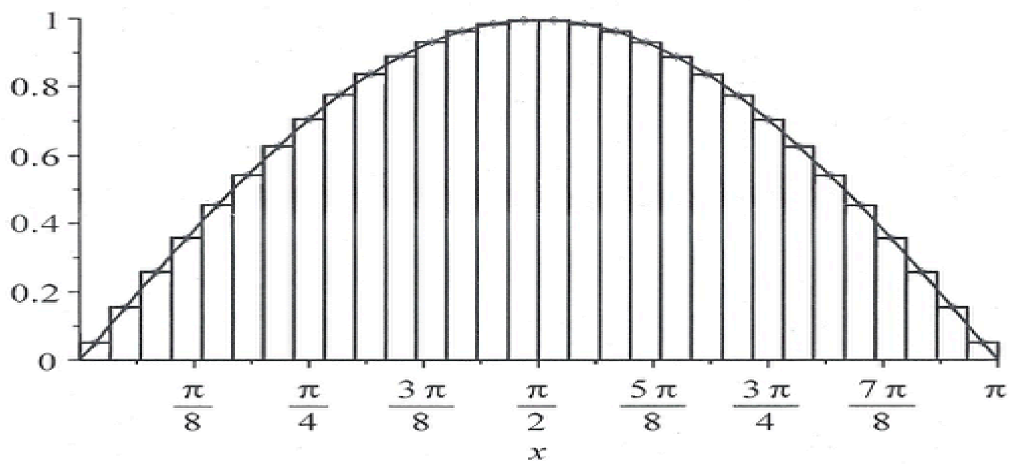
Example:

Approximate the area under y = sin(x) from x = 0 to x = Pi

$f := x \rightarrow \sin(x)$

$$f := x \mapsto \sin(x)$$

RiemannSum($f(x)$, $x = 0$ ..Pi, *partition* = 30, *method* = *midpoint*, *output* = *plot*)



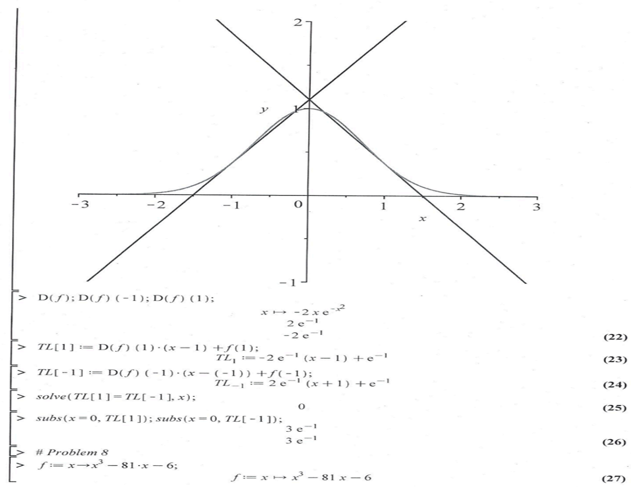
A midpoint Riemann sum approximation of $\int_0^{\pi} f(x)\, dx$, where $f(x) = \sin(x)$ and the partition is uniform. The approximate value of the integral is 2.000914145. Number of subintervals used: 30.

RiemannSum($f(x)$, $x = 0$ ..Pi, *partition* = 60, *method* = *midpoint*, *output* = *plot*)

Intervention Instructor 2-Derivatives and Integrals: Concept and Procedure

Figure I 3

Derivatives: Concept and Procedure-Tangent Lines



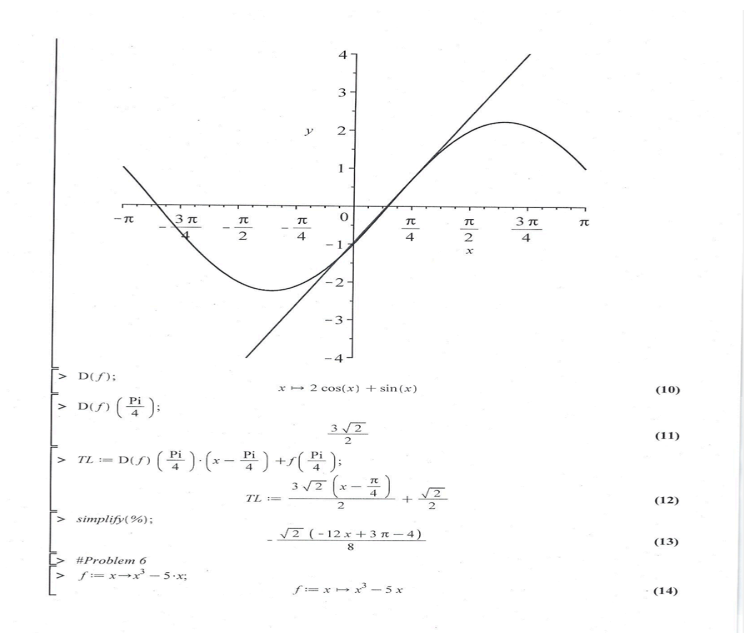
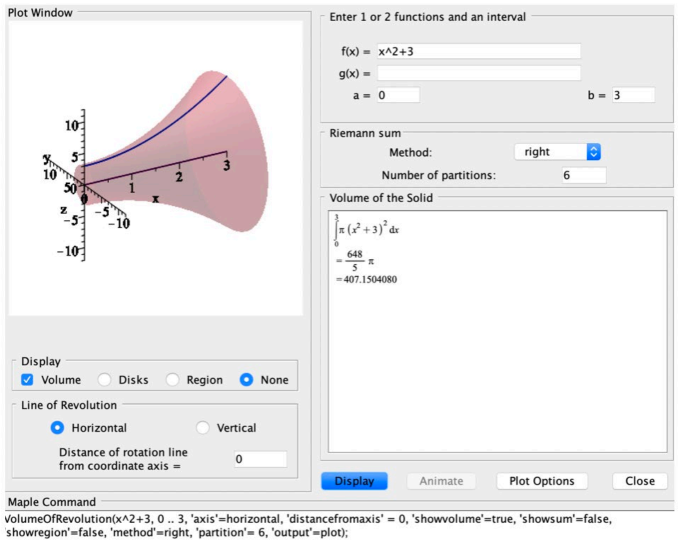


Figure I 4

Integrals: Concept and Procedure- The Volume of a Solid of Revolution



Comparison Group -Derivatives and Integrals: Concept and Procedure

Class activities: Finding the equation of the tangent line; explore the concavity o a curve and finding the point of inflection.

1. Graph the equation given by $x^2 + 4y^2 = 4$ plot the coordinate $(\sqrt{2}, -\frac{1}{\sqrt{2}})$.

find the slope of the tangent line to the curve at this point. Verify that you get the same result using implicit differentiation.
Hint: Use the tangent line in $y = mx + b$ form to see its slope.

2. Determine the points of inflection (if any) of and the open intervals on which the graph of *f(x)* = $x^4 - 4x^3$ is concave upward or downward by hand. Then, sketch the function's graph and indicate this information.

a) Answer the following questions about the definite integral $\int_0^4 5e^x dx$
(b) Estimate the definite integral using lower sums and 25 rectangles. Then, use 250 rectangles.
(c) Compute the definite integral using the Fundamental Theorem of Calculus.

Appendix J: Pretest-Prerequisite Skills on Derivatives and Integrals

Comparison Group

Derivatives

Homework

Instructions. Answer all questions

Problem	1	2	3	4	5	Total
Points	5	5	5	5	5	25

1. Given $f(x) = x^2 + 4$, find:
   a) $f(x+h) - f(x)$
   b) $[f(x+h) - f(x)]/h$ (This is the formula for difference quotient
   c) The limit of $[f(x+h) - f(x)]/h$ as h tends to 0.
   (This is the instantaneous rate of change of f)
2. For the following function, find the equation of the tangent line at $x = 1$.
   $f(x) = e^x$
3. Find the equation of the line passing through the points (2,3) and (3,6).
4. For the function below, find the average rate of change on the interval [-2,3].
   $f(x) = x^3$
5. For the function below, find the instantaneous rate of change at $x = e$
   $f(x) = \ln(x)$

Integrals

Homework

Instructions. Answer all questions

Problem	1	2	3	4	5	Total
Points	5	5	5	5	5	25

For the following functions, $f(x)$, *find* $F(x)$ *such that* $F'(x) = f(x)$

1) $f(x) = 2x$

2) $f(x) = \dfrac{1}{x}$

3) $f(x) = e^x$

4) $f(x) = \cos(x)$

5) $f(x) = 1$

Intervention Group- Instructor 1

Prerequisite Skills- Derivatives

Homework
Total point 25

1) $f(x) = x^3$
   a) Find the difference quotient of f(x). (5 points).
   b) Find the limit of the difference quotient, as h tends to 0. (5 points).
2) Use the technique in (1) to find the equation of the tangent to $f(x) = x^2$. (5 points)
3) Find the equation of the line passing through A(0,4) and B(1,5). (5 points)
4) Use the technique in 1to find the slope of the tangent line of $f(x) = e^{x2} + 3$. (5 points).
5) A particle moves along a curve of y = ln(x). Find its average rate on [1 2]. (5 points).

Prerequisite Skills- Integrals

Homework
Each question worth 5 points.

Find the primitive of the following functions:

1) f(x) = sin(x)
2) $f(x) = 1/(x + 2)^2$
3) $f(x) = e^x$
4) $f(x) =6x^5$
5) f(x) = 1/x

Intervention Group Instructor 2

Prerequisite Skills- Derivatives

Homework
Each question worth 5 points.

Given $F(x) = x^n$, use the binomial theorem

1.

$$(p+q)^n = p^n + np^{n-1}q + \frac{n(n-1)}{2}p^{n-2}q^2 + \ldots + npq^{n-1} + q^n$$

and difference quotient to show that

$$\lim_{h\to 0} of\ the\ difference\ quotient\ of\ x^5 = 5x^4$$

2. *Find the equation of the tangent line to the curve of $g(x) = 2e^x$ at x = 1*

3. *Find the equation of the line passing through A(1,3) and B(2,4).*
4. *Alan travelled 125mi for 3hours, then 160mi for 4hours. What was Alan's average speed?*
5. *Find the average rate of $f(x) = x^2 + 1$ on the interval [0 2]*

Prerequisite Skills- Integrals

Homework

Each question worth 5 points.

Refer to the rules of derivatives and find the antiderivatives (primitive) of the following functions:

1) $f(x) = x$.
2) $f(x) = 3/x$.
3) $f(x) = e^x$.
4) $f(x) = \cos(x)$.
5) $f(x) = 0$.

Appendix K: Quiz (Postest1)-Derivatives and Integrals

Comparison Group

Derivatives

MAT 155: Quiz
Fall, 2019

Instructions Answer all questions

Problem	1	2	3	4	5	Total
Points	5	5	5	5	5	25
Score						

Problems.

1. Let $f(x) = x^3 - \frac{3}{2}x^2$. Find the open interval(s) on which the function is increasing or decreasing. Then, find and classify all relative extrema on this interval.
   Plot the zeros of $f'(x)$ and show how/if they correspond to the relative extrema of f.
   Your sketch should indicate intervals of increase/decrease, as well.

2. Determine the points of inflection of and the open intervals on which the graph of
$f(x) = \sin(x) + \cos(x)$ is concave upwards or downwards on the interval $[0,2\pi]$.

3. Compute the slope of the tangent line to the curve $y^3 + y^2 - 5y - x^2 = -4$ at the point $(1, -3)$. Make sure that you are looking at the correct tangent line and that you view the line's equation in $y = mx + b$ form.

4. Determine the points of inflection (if any) of and the open intervals on which the graph of $f(x) = x^4 - 4x^3$ is concave upward or downward.

5. Consider the function $l(x) = e^x sin(x)$ over the interval $[0, \pi]$. First, compute the $l'(x)$. Then, graph both $l(x)$ and $l'(x)$ over the given interval. Estimate the critical points of $l(x)$ and then its extrema.

Integrals

MAT 155
QUIZ
Topic: Riemann Sums and Definite Integrals

Instructions. **Answer** the questions below will be assigned or reviewed in class. Pay attention to the instructions given; you may be asked to submit solutions for grading.

Name:

Problem	1	2	3	4	5	Total
Points	5	5	5	5	5	25

1. Answer the following questions about the definite integral $\int_2^4 x^3 + 4\,dx$.
   (a) Estimate the definite integral using lower sums and 10 rectangles. Then, use 100 rectangles.
   (b) Estimate the definite integral using upper sums and 10 rectangles. Then, use 100 rectangles.
   (c) Compute the definite integral by hand using the Fundamental Theorem of Calculus.

2. Answer the following questions about the definite integral $\int_0^4 5e^x\,dx$.
   (a) Estimate the definite integral using lower sums and 25 rectangles. Then, use 250 rectangles.
   (b) Estimate the definite integral using upper sums and 25 rectangles. Then, use 250 rectangles.
   (c) Compute the definite integral using the Fundamental Theorem of Calculus.

More Problems On Back

4. Answer the following questions about the definite integral $\int_0^{2\pi} \cos(x)\, dx$.

(a) Estimate the definite integral using lower sums and 10 rectangles. Then, use 250 rectangles.

(b) Estimate the definite integral using upper sums and 10 rectangles. Then, use 250 rectangles.

(c) Compute the definite integral using the Fundamental Theorem of Calculus.

(d) Explain why the definite integral is not the same as the *area under the curve* in this case.

5. Answer the following questions about the definite integral $\int_0^{5} t\sqrt{t^2+2}\, dt$.

(a) Estimate the definite integral using lower sums and 10 rectangles. Then, use 100 rectangles.

(b) Estimate the definite integral using upper sums and 10 rectangles. Then, use 100 rectangles.

(c) Compute the definite i ntegral by using the Fundamental Theorem of Calculus.

Intervention Group

Instructor 1

Derivatives

1 Find the equation of the line with slope $-37/43$ that passes through the point $(3/22, 59/18)$. Plot the line. Find the x and y intercepts of the line. Write the answers here: (5 pts each)

2. Compute the first and second derivatives of the function

$$f(x) = \cos\left(9 - e^{-x}\right)$$

Write the answer here:

3 Compute the tangent line and the normal line to the curve

$$y = \sqrt{x^4 - 7x^2 + 17}$$

at $x = 2$. Write the equation of the tangent line here:

Write the equation of the normal line here:

4. On the same graph, use "scaling constrained" to plot the curve

$$y = \sqrt{x^4 - 7x^2 + 17}$$

and the tangent and normal lines at $x = 2$.

Let

$$f(x) = \sqrt{x^4 \quad 7x^2 + 17}$$

Find the local minima and local maxima of the function $f(x)$:

Integrals

5 points for each question.

1. Compute the indefinite of $\int e^x \cos x$

2.

Compute the area under the curve $y = 5 + 2\sin(x/3)$ for $0 \le x \le 6\pi$.

3.

Compute the area under the curve $y = 5 + 2\sin(x/3)$ for $0 \le x \le 6\pi$.

4. *Compute the area under the curves of* $f(x) = e^x$ *et* $g(x) = -x + 1$ *between* -1 *and* 1.

5. $\int_1^{10} e^x dx$

Instructor 2

Derivatives

MAT 155 Quiz Derivatives

Read direction carefully and use comments to explain your solutions. Each question worth 5 points.

1. For the function $f(x) = e^x$,
   a) Write the equation of the tangent line at $x = 2$.
   b) Write the equation of the normal line at $x = 2$.

2. For $f(x) = -x^4 + x^3$:
   a) Find the critical numbers and relative extrema of the function.
   b) Find the points of inflection and the concavity of the function.

3. For $y = \frac{x+4}{x-1}$, Find the 4th derivative of y.

4. Find the instantaneous rate of change of $f(x) = 3x^5 + 6$ at $x = 3$.

5. What is the average rate of $f(x) = \ln(x)$, between $x = 2$ and $x = 4$?

Integrals

MAT 155 Quiz Integrals

Read direction carefully and use comments to explain your solutions. Each question worth 5 points.

1. Compute the area under the curve f(x) = 2 +5sin(x/2) for $0 \leq x \leq 6\pi$.
2. $\int_1^6 e^x$dx.
3. Find the indefinite integral of y = $\frac{2}{x}$
4. Find the primitive of y = $e^x \cos x$.
5. Find the indefinite integral of f (x) = $\sqrt{1+x^2}$.

Appendix L: Posttest

Comparison Group

MAT 155: Final Exam
Fall, 2019
Form A

Instructions. Answer all questions

Problem	**1**	**2**	**3**	**4**	**5**	**Total**
Points	**5**	**5**	**5**	**5**	**5**	**25**
Score						

Problems.

1. Compute the following limit:

$$\lim_{x \to -4} \frac{x^2 + 7x + 12}{x^2 + 2x - 8}$$

   Verify that your calculation is correct by graphing the function and plotting the limit's coordinate.

2. Determine the value of A that makes the following piecewise defined function continuous on all of $\mathbb{R}$:

$$g(x) = \begin{cases} Acos(x) + 2, & x < 2\pi \\ 4\sin(x), & x \geq 2\pi \end{cases}$$

   Verify that your solution is correct by graphing $g(x)$ using the value of A calculated in your computation.

3. Compute the slope of the tangent line to the curve $5x^4 + 2y^3 = 7xy$ at the point (1,1). Make sure that your image shows the curve, the point, and the *correct* tangent line in the correct form.

4. Let $h(x) = x^4 + 4x^3 - 20x^2$. Find the open interval(s) on which the function is increasing or decreasing. Then, find and classify all relative extrema on this interval.

5. Answer the following questions about the definite integral $\int_{-1}^{1} 12x^3 + 24x^2 dx$.
   a) Estimate the definite integral using lower sums and 30 rectangles. Then, use 300 rectangles.
   b) Estimate the definite integral using upper sums and 30 rectangles. Then, use 300 rectangles.
   c) Compute the definite integral using the Fundamental Theorem of Calculus.

Intervention Group-Instructor 1

Math 155 - Final Exam NAME:

December 16, 2019

FOLLOW THESE INSTRUCTIONS:

(1) Write your name on this paper.

(2) The exam is graded on a 100 point scale. Each problem counts 10 points. Do as many problems as you can.

(3) It is sufficient to compute numerical answers with 3 decimal digits (for example, 7.123).

(4) Write numerical answers to problems in the boxes provided on this exam.

(5) You will turn in this exam and the printout of your Maple worksheet.
Make sure your name is on the computer printout.
Put this exam on top of your computer printout and staple them together.

(6) You may not use notes, books, calculators, or the internet.
You may use the Maple help feature in the Maple program.

(7) The set of boxes below is for grading. Do not write in it.

1		6	
2		7	
3		8	
4		9	
5		10	

(1) Plot the following lines on the same graph:

$$8x - 13y = 5$$
$$7x + 21y = -3$$

Find all solutions of this system of equations. Write the answer here:

(2) Compute

$$\lim_{x\to\infty} \frac{1000x^5 - 63e^x}{x^{1000} + 7e^x}.$$

Write the answer here:

Compute

$$\lim_{x\to\infty} \frac{900x^5 - 19x^3 - 8}{225x^5 + 18x^4 + 11x^2}.$$

Write the answer here:

(3) Compute the first and second derivatives of the function

$$f(x) = \sin(5 + e^x)$$

Write the answer here:

(4) Compute the tangent line and the normal line to the curve

$$y = \sqrt{x^4 + 2x^2 + 6}$$

at $x = 2$. Write the equation of the tangent line here:

Write the equation of the normal line here:

(5) On the same graph, use "scaling constrained" to plot the curve

$$y = \sqrt{x^4 + 2x^2 + 6}$$

and the tangent and normal lines at $x = 2$.

(7) Let

$$f(x) = \sqrt{x^4 - 7x^2 + 17}$$

Find the local minima and local maxima of the function $f(x)$:

Find the inflection points of the function $f(x)$:

(8) Compute the indefinite integral

$$\int e^x \sin 2x dx$$

and the definite integral

$$\int_0^{2\pi} e^x \sin 2x dx.$$

(9) Compute the area under the curve $y = \tan x$ for $0 \leq x \leq \pi/4$.

(10) Use implicit differentiation to compute dy/dx on the curve

$$x^3 - xy + 2y^2 = 16.$$

Compute the tangent line to the curve at $(1, 3)$.

Intervention Group-Instructor 2

MAT155 Final Exam Fall 2019

Read direction carefully and use comments to explain your solutions.

1. Find the equation of the line tangent to the function y = sin(θ), where θ =110 degrees. (10Pt)
2. a) Define a function with a slope of 2 and goes through point (3,4).
   b) Find the inverse of this function.
   c) Plot both the function and its inverse on the same axes and confirm that they reflect each other through the line y = x. (10 Pt)
3. For the function f(x) = $\frac{x^2-6x}{x^3-36x}$, find the values of x for which this function is undefined, then find the limit as x approaches these values. (10 Pt).
4. For the function f(x) = ln(x) + 2x,
   a) Write while loops to estimate the root of f to two decimal places.
   b) Using Newton's method of approximation, write a loop to estimate the root. (10 Pt)
5. When a rock is thrown upward on the planet Killjoy, the height of the rock is given by h(t) = $\frac{48}{19}t^2 + 48t - 11$, where *h* is measured in feet and *t* is measured in seconds.
   a) What is the rock's maximum height?
   b) What is the rock's final velocity right before hitting the ground?
   c) Find an expression for the acceleration of the rock. (10 Pt).
6. For the function f(x) = -7x^4 + 5x^3:
   a) Determine the proper window to display the interval in which local extrema occur.
   b) Find the critical numbers and the relative extrema of the function.
   c) Find the points of inflection and the concavity of the function. (10 Pt)
7. Consider the function z(x) = 2 + sin(x -1) from x = 1 to 2.
   a) Write a loop to display the left-hand displays and sums and right-hand displays and sums 2, 4, 8, 16, 32, 64, 128, 256, 516, 1024 subintervals.
   b) Find the value of the integral of the function from x = 1 to 2. (10 Pt).
8. Use implicit differentiation to compute dy/dx on the curve x^3 - xy + 2y^2 = 16 to compute the tangent line to the curve at (1, 3). (10 Pt).
9. Compute the indefinite integral and definite integral of $\int_0^{\pi/4} e^x sinx\, dx$. (10 Pt).
10. Compute $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$. (10 Pt).

Appendix M: Descriptive Statistics and MANOVA

Figure M1

Descriptive Statistics: Checking Errors and Missing Values

	N	
	Valid	Missing
ID	81	0
Instructor	81	0
Pre_HWK_Der-Concept	81	0
Pre_HWK_Der-Procedure	81	0
Pre_HWK_Int-Concept	81	0
Pre_HWK_Int-Procedure	81	0
Quiz _Der-Cncept	81	0
Quiz _Der-Procedure	81	0
Quiz _Int-Concept	81	0
Quiz _Int-Procedure	81	0
Post _Der-Concept	81	0
Post _Der-Procedure	81	0
Post _Int-Concept	81	0
Post _Int-Concept	81	0

Figure M2

Descriptive Statistics-RQ1

Descriptive Statistics

	Group
Pretest_HWK_Derivative_Concept	Comparison Group (Static Visualization)
	Intervention Group (Dynamic Visualization Animation)
	Total
Quiz_Derivative_Concept	Comparison Group (Static Visualization)
	Intervention Group (Dynamic Visualization Animation)
	Total
Posttest_Final_Derivative_Concept	Comparison Group (Static Visualization)
	Intervention Group (Dynamic Visualization Animation)
	Total

Figure M3

Tests of Within-Subjects Effect-RQ1

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Time	Sphericity Assumed	21826.502	2	10913.251	155.419	.000	.663	310.838	1.000
	Greenhouse -Geisser	21826.502	1.759	12411.584	155.419	.000	.663	273.314	1.000
	Huynh-Feldt	21826.502	1.818	12004.209	155.419	.000	.663	282.589	1.000
	Lower-bound	21826.502	1.000	21826.502	155.419	.000	.663	155.419	1.000
Time * Group	Sphericity Assumed	1130.996	2	565.498	8.053	.000	.093	16.107	.954
	Greenhouse -Geisser	1130.996	1.759	643.138	8.053	.001	.093	14.162	.934
	Huynh-Feldt	1130.996	1.818	622.029	8.053	.001	.093	14.643	.940
	Lower-bound	1130.996	1.000	1130.996	8.053	.006	.093	8.053	.800
Error(Time)	Sphericity Assumed	11094.477	158	70.218					
	Greenhouse -Geisser	11094.477	138.926	79.859					
	Huynh-Feldt	11094.477	143.641	77.238					
	Lower-bound	11094.477	79.000	140.436					

a. Computed using alpha = 0.05

Figure M4

Tests of Between-Subjects Effects-RQ1

Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Intercept	1437731.442	1	1437731.442	6358.083	.000	.988	6358.083	1.000
Group	24.381	1	24.381	.108	.744	.001	.108	.062
Error	17863.998	79	226.127					

a. Computed using alpha = 0.05

Figure M5

Descriptive Statistics-RQ2

	Group	Mean	Std. Deviation	N
Pretest_HWK_Derivative_ Procedure	Comparison Group (Static Visualization)	66.7241	12.86162	29
	Intervention Group (Dynamic Visualization Animation)	66.5577	12.81912	52
	Total	66.6173	12.75399	81
Quiz_Derivative_ Procedure	Comparison Group (Static Visualization)	87.3448	11.21630	29
	Intervention Group (Dynamic Visualization Animation)	80.9423	11.05674	52
	Total	83.2346	11.46764	81
Posttest_Final_Derivative Procedure	Comparison Group (Static Visualization)	87.5862	11.42215	29
	Intervention Group (Dynamic Visualization Animation)	92.1731	7.05085	52
	Total	90.5309	9.06930	81

Figure M6

Multivariate Tests-RQ2

Multivariate Tests[a]

Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[c]
.750	116.725[b]	2.000	78.000	.000	.750	233.449	1.000
.250	116.725[b]	2.000	78.000	.000	.750	233.449	1.000
2.993	116.725[b]	2.000	78.000	.000	.750	233.449	1.000
2.993	116.725[b]	2.000	78.000	.000	.750	233.449	1.000
.245	12.626[b]	2.000	78.000	.000	.245	25.252	.996
.755	12.626[b]	2.000	78.000	.000	.245	25.252	.996
.324	12.626[b]	2.000	78.000	.000	.245	25.252	.996
.324	12.626[b]	2.000	78.000	.000	.245	25.252	.996

a. Design: Intercept + Group
Within Subjects Design: Time

b. Exact statistic

c. Computed using alpha = .05

Figure M7

Tests of Between-Subjects Effects-RQ2

Tests of Between-Subjects Effects

Measure: MEASURE_1

Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Intercept	1437731.442	1	1437731.442	6358.083	.000	.988	6358.083	1.000
Group	24.381	1	24.381	.108	.744	.001	.108	.062
Error	17863.998	79	226.127					

a. Computed using alpha = .05

Figure M7

Descriptive Statistics-RQ3

Descriptive Statistics

	Group	Mean	Std. Deviation	N
Pretest_HWK_Integral_Concept	Comparison Group (Static Visualization)	71.9655	8.57522	29
	Intervention Group (Dynamic Visualization Animation)	64.7308	12.51214	52
	Total	67.3210	11.73545	81
Quiz_Integral_Concept	Comparison Group (Static Visualization)	91.4138	13.69963	29
	Intervention Group (Dynamic Visualization Animation)	85.5962	10.35120	52
	Total	87.6790	11.91095	81
Postest_Final_Integral_Concept	Comparison Group (Static Visualization)	89.9655	11.08565	29
	Intervention Group (Dynamic Visualization Animation)	96.6538	6.26814	52
	Total	94.2593	8.85830	81

Figure M8

Multivariate Tests-RQ3

Multivariate Tests[a]

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[c]
Time	Pillai's Trace	.787	144.350[b]	2.000	78.000	.000	.787	288.701	1.000
	Wilks' Lambda	.213	144.350[b]	2.000	78.000	.000	.787	288.701	1.000
	Hotelling's Trace	3.701	144.350[b]	2.000	78.000	.000	.787	288.701	1.000
	Roy's Largest Root	3.701	144.350[b]	2.000	78.000	.000	.787	288.701	1.000
Time * Group	Pillai's Trace	.278	15.001[b]	2.000	78.000	.000	.278	30.003	.999
	Wilks' Lambda	.722	15.001[b]	2.000	78.000	.000	.278	30.003	.999
	Hotelling's Trace	.385	15.001[b]	2.000	78.000	.000	.278	30.003	.999
	Roy's Largest Root	.385	15.001[b]	2.000	78.000	.000	.278	30.003	.999

a. Design: Intercept + Group
Within Subjects Design: Time

b. Exact statistic

c. Computed using alpha = .05

Figure M9

Tests of Between-Subjects Effects-RQ3

Tests of Between-Subjects Effects

Measure: MEASURE_1

Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Intercept	1553461.563	1	1553461.563	9422.501	.000	.992	9422.501	1.000
Group	251.341	1	251.341	1.525	.221	.019	1.525	.230
Error	13024.511	79	164.867					

a. Computed using alpha = .05

Figure M10

Descriptive Statistics-RQ4

Descriptive Statistics

	Group	Mean	Std. Deviation	N
Pretest_HWK_Integral_Concept	Comparison Group (Static Visualization)	71.9655	8.57522	29
	Intervention Group (Dynamic Visualization Animation)	64.7308	12.51214	52
	Total	67.3210	11.73545	81
Quiz_Integral_Concept	Comparison Group (Static Visualization)	91.4138	13.69963	29
	Intervention Group (Dynamic Visualization Animation)	85.5962	10.35120	52
	Total	87.6790	11.91095	81
Postest_Final_Integral_Concept	Comparison Group (Static Visualization)	89.9655	11.08565	29
	Intervention Group (Dynamic Visualization Animation)	96.6538	6.26814	52
	Total	94.2593	8.85830	81

Figure M11

Multivariate Tests-RQ4

Multivariate Tests[a]

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[c]
Time	Pillai's Trace	.787	144.350[b]	2.000	78.000	.000	.787	288.701	1.000
	Wilks' Lambda	.213	144.350[b]	2.000	78.000	.000	.787	288.701	1.000
	Hotelling's Trace	3.701	144.350[b]	2.000	78.000	.000	.787	288.701	1.000
	Roy's Largest Root	3.701	144.350[b]	2.000	78.000	.000	.787	288.701	1.000
Time * Group	Pillai's Trace	.278	15.001[b]	2.000	78.000	.000	.278	30.003	.999
	Wilks' Lambda	.722	15.001[b]	2.000	78.000	.000	.278	30.003	.999
	Hotelling's Trace	.385	15.001[b]	2.000	78.000	.000	.278	30.003	.999
	Roy's Largest Root	.385	15.001[b]	2.000	78.000	.000	.278	30.003	.999

a. Design: Intercept + Group
Within Subjects Design: Time

b. Exact statistic

c. Computed using alpha = .05

Figure M12

Tests of Between-Subjects Effects-RQ4

Tests of Between-Subjects Effects

Measure: MEASURE_1

Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Intercept	1553461.563	1	1553461.563	9422.501	.000	.992	9422.501	1.000
Group	251.341	1	251.341	1.525	.221	.019	1.525	.230
Error	13024.511	79	164.867					

a. Computed using alpha = .05