Decomposition Heuristic with partial assignment approach for MLCLSP-M problem

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Abstract

This research proposes a heuristic method, which decomposes the Multi-level Multi-item Capacitated Lot Sizing Problem with Multi-workstation (MLCLSP-M) into two phases which are an assignment with given lot size and a partial lot size with given assignment. Each iteration, the sub problem mathematical models are solved with AMPL/CPLEX 8.0.0 solver. An example is present for demonstration of the heuristic. The result indicate that the proposed heuristic (Partial Assignment – Lot size: PA-LS) give a satisfactory solution within faster solving time on comparison with the original mathematical model solving.

Keywords

Production planning; Lot sizing; Capacitated; Decomposition heuristics;

Introduction

The problem we focus on is a multi-item multi-period in finite planning horizon with availability and price of raw materials vary over time periods situation. The production has given Bill of materials (BOM) composed of both dependent demand (components and raw materials required by products production) and independent demand (customer's demands). The firm also needs to take into account of factors which change over time period, i.e. workstation capacity; raw material availability; production cost; raw material cost; etc. As a result, the firm has to find a purchasing or procurement plan and a production plan with minimum cost while keeping customer order due date or to find lot size of purchasing and production order in each period for all items. In nutshell, it is a major challenge for a manager to provide the optimal plan for any activity that effects manufacture of the firm. In addition, warehouse storage space is another constraint for managers to decide to hold inventories of what item and how much. In this multi-period limited resource environment, every parameter can change or vary by time period; decision for each period can be affected by how these parameters change. Moreover, with multi-level multi-item consideration, demands can be either dependent or independent demand. Thus, this situation becomes more complex and is hard to solve. The purpose of this research is to find a solution for the problem with the aforementioned situations. The holistic view model (considering purchasing planning situation together with production planning situation with warehouse capacity constraint) is appropriated and needed in order to find an optimal plan. Such problem can be defined as Multi-level Multi-item Capacitated Lot Sizing Problem with Multi-workstation (MLCLSP-M) (Drexl & Kimms, 1997; Karimi, Ghomi, & Wilson, 2003; Pochet, 2001; Suerie, 2005).

Generally, the lot-sizing MIP models are often very large in practice even advanced solvers such as CPLEX are unable to identify provably-optimal solutions in acceptable computational time (Clark, 2003; Silvio, Marcos, & Alistair, 2008). That the developed model might be classified as a capacitated lot sizing with setup time model (Brahimi, Dauzère-Pérès, Najid, & Nordli, 2006; Chinprateep & Boondiskulchok, 2007; Drexl & Kimms, 1997; Karimi et al., 2003; Kimms, 1996; Pochet, 2001; Suerie, 2005; Zangwill, 1996), which is NP-hard problem it can be solved to optimality only with a huge computational effort. Clearly it takes an impracticable amount of computer time and memory, motivating the development of the alternative approaches. Therefore, this paper proposes a heuristic algorithm that can solve large-scale problems to near-optimality with a reasonable computational time. Although there is more than one way to tackle the problem, the effective one is decomposition (Aardal & Larsson, 1990; See-Toh, Walsh, Shah, Marquardt, & Pantelides, 2006; Vercellis, 1999; Wu, Hartman, & Wilson, 2003; Zapfel, 1996), consequently, the problem can be solved more efficiently. In this paper, we proposed the heuristics in two phases which are the partial assignment with given lot size and the lot size with given the assignment. The target of the first phase is to find the assignment matrix to be used as input data for the second phase which used data from the first as given assignment and then solve the lot size problem. The extensive experiments conducted in this paper show empirical evidence that the resultant algorithms outperform their original versions while keeping low computational demands.

The structure of the paper is as follows. Section 2 formulates and explains the planning model and then in Section 3 heuristic is proposed and description. In section 4 an example of the heuristic is shown. Finally section 6 concludes the paper with a discussion of the test results and provides some points for further research. The conclusion is that, for the situation and data of the manufacturer, the proposed heuristic gives a satisfactory solution for the problem with low computational time efforts.

The Mathematical Model

With MLCLSP-M situation, we aim to find a minimum lot size of particular item in a period over multi-period planning horizon consideration. Once a product is produced, the setup time of the production will occur and be considered as sequence independent between different productions. The setup activities for each production workstation incur setup costs and consume setup time. Within a limited capacity expressed in hours during each time period of the workstation, the operation time and the setup time will reduce capacity in period. Assume that the demands for the parts processing vary with time in a deterministic manner and there is no order for a component item from the customers. Therefore, production quantity of a component item can be computed from ordering quantities of end items that require the component item. If an item will not be used in the next period, it must be kept in warehouse and charged holding costs. In this paper, we neglect the quality aspect of production. Moreover, all unit costs, prices, and setup times/costs are assumed to be known but dynamic by period. Further assume that the processing, setup, and resource consumption costs do not depend on the planning period. To address this multiple time period formation problem, a mixed integer programming (MIP) model is formulated. The objective of the model is to minimized total cost (setup cost, purchasing cost, production cost and inventory cost) for the entire planning time horizon with given time varying parameters (external demand, availability of raw materials, capacity of workstations, time parameters and cost parameters).

These are the notations which we used in our model.

Sets

T	represents the set of discrete period time in planning horizon, whereby $T=\{1,\ldots,NT\}$
R	represents the set of raw material items
$COMP$	represents the set of component items
E	represents the set of end-items
I	represents the set of items, whereby $I=\{1,\ldots,NI\}=R\cup COMP\cup E$
WR	represents the set of purchasing workstations
WP	represents the set of production workstations
WS	represents the set of workstations, whereby $WS=\{1,\ldots,NK\}=WR\cup WP$
$S(i)$	represents the set of successor items of item i (By given BOM)
$W(i,t)$	represents the set of successor workstations of item i (By given route sheet) in period t
$I(k,t)$	represents the set of item that workstation k can operate in period t

Indices

t,l	represents the time period index in planning horizon, whereby $t,l\in T$
i,j	represents the item index, whereby $i,j\in I$
k	represents the workstation index, whereby $k\in WS$

Decision Variables

$x_t^{i,k}$	represents the amount of production item i on workstation k in period t
s_t^i	represents the amount of stock item i at the ending of period t
$y_t^{i,k}$	represents the setup decision of the production item i on workstation k in

period t, whereby $y_t^{i,k} \in \{0,1\}$

Costs

$p_t^{i,k}$	represents the production unit cost of item i on workstation k in period t
$f_t^{i,k}$	represents the setup cost per unit of production item i on workstation k in period t
h_t^i	represents the holding cost per unit of stock item i at the ending of period t

Parameters

d_t^i	represents the independent demand item i in period t
V	represents the warehouse capacity
$M_t^{i,k}$	represents $\min\left\{\left(\dfrac{c_t^k - g_t^{i,k} \cdot y_t^{i,k}}{o_t^{i,k}}\right), \sum_{l=1}^{t} d_l^i, \left(V - \sum_{i \in I} s_t^i\right)\right\}$
$Avail_t^k$	represents the availability of workstation k during period t
cap_t^k	represents the capacity of workstation k during period t
$o_t^{i,k}$	represents production usage resource of item i on resource k during period t
$g_t^{i,k}$	represents setup usage resource of item i on resource k during period t
$u^{i,j}$	represents the usage item i for producing item j, whereby $j \in S(i)$

The model of multi-level multi-item multi-workstation lot-sizing with capacity (MLCLSP-M) constraints for purchasing and production planning under warehouse limited consideration can be formulated as follows:

$$Min \sum_{t \in T} \sum_{i \in I} \sum_{k \in WS} \left(p_t^{i,k} \cdot x_t^{i,k} + f_t^{i,k} \cdot y_t^{i,k}\right) + \sum_{t \in T} \sum_{i \in I} \left(h_t^i \cdot s_t^i\right) \tag{1}$$

Subject to

$$s_{t-1}^i + \sum_{k \in W(i,t)} x_t^{i,k} = d_t^i + s_t^i, \forall i \in E, \forall t \in T \tag{2}$$

$$s_{t-1}^i + \sum_{k \in W(i,t)} x_t^{i,k} = \sum_{j \in S(i)} \left(u^{i,j} \cdot \sum_{k \in W(j,t)} x_t^{j,k}\right) + s_t^i, \ \forall i \in I - E, \forall t \in T \tag{3}$$

$$\sum_{i \in I(k,t)} x_t^{i,k} \le Avail_t^k \,,\ \forall k \in WR, \forall t \in T \tag{4}$$

$$\sum_{i \in I(k,t)} \left(o_t^{i,k} \cdot x_t^{i,k} + g_t^{i,k} \cdot y_t^{i,k} \right) \le cap_t^k \,,\ \forall k \in WP, \forall t \in T \tag{5}$$

$$\sum_{i \in I} s_t^i \le V \,,\ \forall t \in T \tag{6}$$

$$s_0^i = s_{NT}^i = 0 \,,\ \forall i \in I \tag{7}$$

$$\sum_{k \in WS} y_t^{i,k} \le 1 \,,\ \forall i \in I, \forall t \in T \tag{8}$$

$$\sum_{i \in I} y_t^{i,k} \le 1 \,,\ \forall k \in WS, \forall t \in T \tag{9}$$

$$x_t^{i,k} \le M_t^{i,k} \cdot y_t^{i,k} \,,\ \forall i \in I, \forall t \in T, \forall k \in W(i,t) \tag{10}$$

$$x_t^{i,k} \ge 0 \,,\ \forall i \in I, \forall k \in W(i,t), \forall t \in T \tag{11}$$

$$s_t^i \ge 0 \,,\ \forall i \in I, \forall t \in T \tag{12}$$

$$y_t^{i,k} \in \{0,1\} \,,\ \forall i \in I, \forall k \in W(i,t), \forall t \in T \tag{13}$$

Here $M_t^{i,k}$ is a big value or an upper bound of $x_t^{i,k}$, which can be determined as the maximum amount of the lot size. Let the index k represents each workstation in the workstation set, WS ($k \in WS$), which is the set composed of all elements of the set of purchasing workstations (WR) and all elements of the set of production workstations (WP), understanding by this the set $WS = WR \cup WP$. The set I includes all items which composed of all elements of the set of raw material items (R), all elements of the set of components ($COMP$) and all elements of the set of end-items (E). The index i represents each item in the set I, and the index t represents each period in the planning period set T, understanding by this $i \in I$ and $t \in T$. With multi-level product structure, all end-items have only external demand and others have only internal demand or dependent demand. Besides, we introduce two new set which are $W(i,t)$ and $I(k,t)$. The first set represents the set of capable workstations of item i in period t. The second set represents the set of capable items of workstation k in period t. The $x_t^{i,k}$ and $y_t^{i,k}$ are lot size and ordering (setup) decision variables for item i on workstation $k \in W(i,t)$ in period t, respectively. The s_t^i are ending inventory decision variables for item i in period t. The cost parameters are the purchasing unit price or production unit cost, $p_t^{i,k}$, ordering cost or setup cost, $f_t^{i,k}$,and the holding cost, h_t^i. The time parameters are the operation time, $o_t^{i,k}$ and setup time, $g_t^{i,k}$. The capacity parameters for all workstation $k \in WP$ in period t are represented as cap_t^k and for all $k \in WR$ in period t are represented as $Avail_t^k$.

The objective function (1) aims at minimizing purchasing, production, setup, and holding cost. The constraints (2) and (3) are the balance constraints for end-items and immediate items, respectively. It is important to have a BOM relation representation for the balance material constraints. Let $S(i)$ represents the set of immediate successors of item i. The usage parameters, $u^{i,j}$, represent the number of units of item i required for producing one unit of the immediate successors item j whereby $j \in S(i)$. In constraints (4), purchasing lot size must not exceed the availability of raw materials on a workstation k in period t, $Avail_t^k$. Capacity production constraints (5) make sure that required capacity for setup and operation must not exceed available capacity. For each period, all holding-items are kept in one capacitated warehouse. The constraints (6) are the limited warehouse capacity (V). We assume that there is no item in warehouse at the first period and the end of planning horizon as represented in constraints (7). To ensure that only one item will be assigned to one workstation in each period and one workstation will operate one item in each period, the constraint (8) and (9) are respectively added. The constraints (10) ensure that the setup variables are set to be 1 if there is positive production or purchasing lot-size in period t. Constraints (11) and (12) restricted all variables $x_t^{i,k}$ and s_t^i to non-negative, respectively. Finally, constraints (13) enforce $y_t^{i,k}$ to binary.

The Heuristic Method

The proposed heuristic consists of two sub procedures: an assignment solution with given lot size (P1) and a lot size solution with given assignment (P2). This is because the characteristic of this problem as shown in Figure 1.

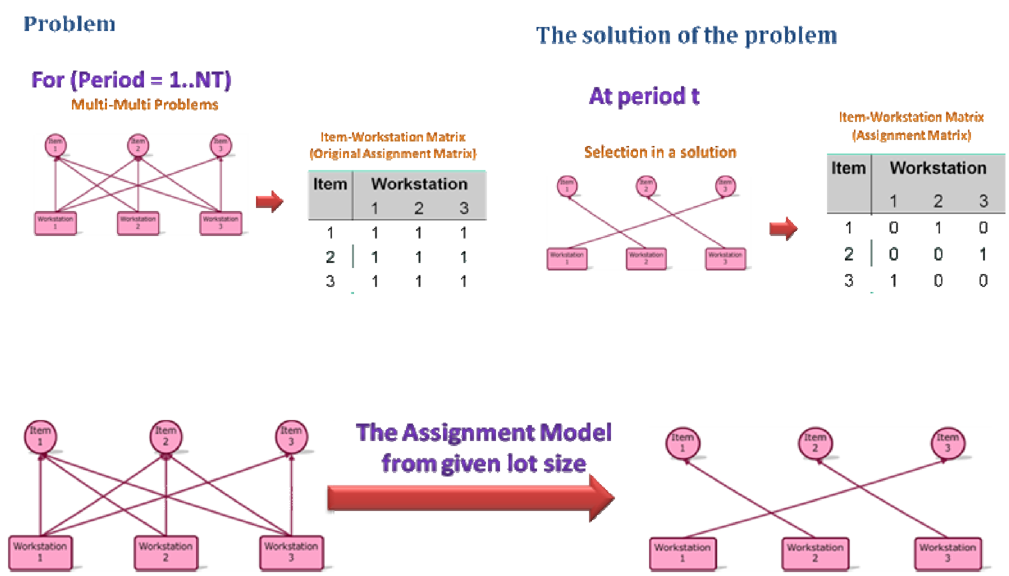


Figure 1. The demonstration of the problem item-workstation matrix and the solution item workstation matrix

In Figure 1, the situation is 3-item-workstation with NT planning horizon. In the problem, all items have a capability to be operated on all workstations and all workstation also have a capability to operate all items. However, in the solution environment assumption, in each period, only an item can be operated on one machine and a machine can operate only one item. This problem then has a characteristic of an assignment model. Other findings are that

- The amount of the lot size needed can be defined for all periods.

- In each sub-problem, on given lot size, the assignment can be done. Likewise, on given assignment, the lot sizing can be done.
- With echelon demand of item i in period t (e_t^i), the demand of each item in each period can be calculated form end-item demand with the equation (e)

$$e_t^i = d_t^i + \sum_{j \in S(i)} u^{ij} e_t^j, \forall i \in I; \forall t \in T \qquad \text{(e)}$$

- There is an assumption initiating the present heuristic approach, namely, all echelon demand lot size are feasible for P2. In the other word, Lot-for-Lot policy is feasible for this problem.

With given an assignment, the lot size will be find the smaller model as it was reduced to similar to one-one item-workstation model. As aforementioned, this research proposes a heuristic method which decomposes the MLCLSP-M into the following two phases:

Phase 1. Assignment problem with given lot-size.

Let $w_t^{i,k}$ represents the status of assignment ($w_t^{i,k} = 1$ if in period t item i is assigned to be operated on workstation k, and $w_t^{i,k} = 0$ for otherwise) and D_t^i represents the given lot size. The formulated model can be represented as follows

$$Min \;\; \left(p_t^{i,k} \cdot D_t^i + f_t^{i,k} \right) \times w_t^{i,k} \qquad \text{(P1)}$$

Subject to

$$D_t^i = d_t^i + \sum_{j \in S(i)} u^{ij} D_t^j \;, \; \forall i \in I; \forall t \in T \qquad (14)$$

$$\left(\sum_{i \in I(k,t)} D_t^i \right) w_t^{i,k} \le Avail_t^k \;, \; \forall k \in WR, \forall t \in T \qquad (15)$$

$$\left(\sum_{i\in I(k,t)}\left(o_t^{i,k}\cdot D_t^i+g_t^{i,k}\right)\right)w_t^{i,k}\le cap_t^k,\ \forall k\in WP,\forall t\in T \tag{16}$$

$$\sum_{k\in WS} w_t^{i,k}\le 1,\ \forall i\in I,\forall t\in T \tag{17}$$

$$\sum_{i\in I} w_t^{i,k}\le 1,\ \forall k\in WS,\forall t\in T \tag{18}$$

$$w_t^{i,k}\in\{0,1\},\ \forall i\in I,\forall k\in WS,\forall t\in T \tag{19}$$

In this section, we introduce a set $Q(t)$ to represent original assignment item-workstation in period t. Let $w_t^{i,k}$ represents the status of assignment ($w_t^{i,k}=1$ if in period t item i is assigned to be operated on workstation k, and $w_t^{i,k}=0$ for otherwise) and is calculated from the assignment model in the assignment with given lot size phase. Set $A(t)$, which is a set of the assignment item-workstation in period t, can easily defined by $A(t)=\{(i,k)\mid w_t^{i,k}=1\}$. Similarly, for the original problem, the assignment matrix has been already defined, as a result, $w_t^{i,k}$ can be defined by the equation (O) as follows:

$$w_t^{i,k}=\begin{cases}1 & \forall i\in I(k,t)\\ 0 & otherwise\end{cases} \tag{O}$$

Then the set $Q(t)=\{(i,k)\mid w_t^{i,k}=1\}$ is a set of the original assignment item-workstation in period t. The relaxation will begin in the second iteration. The assignment matrix will be used with relaxation to original problem assignment for all this iteration until max number of iteration. In another word, a given assignment matrix that will be sent to lot sizing part split into 2 types which are the “Full assigned” (as the result from assignment model with given lot size or set $A(t)$) and the “Partial assigned” (as the result from assignment model with given lot size only in iteration period or set $A(t)$, other periods will be replaced with the original assignment of the problem or set $Q(t)$).

The result of this part is the assignment matrix or the pair of item-workstation to be used as input data for the lot sizing part.

Phase 2. Lot sizing problem with given partial assignment matrix.

The result of the input data is the both set $W(i,t)$ and set $I(k,t)$ have only one member (Use the MLCLSP-M model for solving). Let set $A(t) = \{(i,k) \mid w_t^{i,k} = 1\}$ is a set of the assignment item-workstation in period t.

$$Min \sum_{t \in T} \sum_{(i,k) \in A(t)} \left(p_t^{(i,k)} \cdot x_t^{(i,k)} + f_t^{(i,k)} \cdot y_t^{(i,k)} \right) + \sum_{t \in T} \sum_{i \in I} \left(h_t^i \cdot s_t^i \right) \tag{P2}$$

Subject to

$$s_{t-1}^i + \sum_{(i,k) \in A(t)} x_t^{(i,k)} = d_t^i + s_t^i , \forall i \in E, \forall t \in T \tag{20}$$

$$s_{t-1}^i + \sum_{(i,k) \in A(t)} x_t^{(i,k)} = \sum_{j \in S(i)} \left(u^{i,j} \cdot \sum_{(j,k) \in A(t)} x_t^{(j,k)} \right) + s_t^i , \ \forall i, j \in I - E, \forall t \in T \tag{21}$$

$$\sum_{(i,k) \in A(t)} x_t^{(i,k)} \le Avail_t^k , \ \forall k \in WR, \forall t \in T \tag{22}$$

$$\sum_{(i,k) \in A(t)} \left(o_t^{(i,k)} \cdot x_t^{(i,k)} + g_t^{(i,k)} \cdot y_t^{(i,k)} \right) \le cap_t^k , \ \forall k \in WP, \forall t \in T \tag{23}$$

$$\sum_{i \in I} s_t^i \le V , \ \forall t \in T \tag{24}$$

$$s_0^i = s_{NT}^i = 0 , \ \forall i \in I \tag{25}$$

$$\sum_{(i,k) \in A(t)} y_t^{(i,k)} \le 1 , \ \forall i \in I, \forall t \in T \tag{26}$$

$$\sum_{(i,k)\in A(t)} y_t^{(i,k)} \leq 1\,,\ \forall k \in WS, \forall t \in T \tag{27}$$

$$x_t^{i,k} \leq M_t^{i,k} \cdot y_t^{i,k}\,,\ \forall t \in T, \forall (i,k) \in A(t) \tag{28}$$

$$x_t^{(i,k)} \geq 0\,,\ \forall (i,k) \in A(t), \forall t \in T \tag{29}$$

$$s_t^i \geq 0\,,\ \forall i \in I, \forall t \in T \tag{30}$$

$$y_t^{(i,k)} \in \{0,1\}\,,\ \forall (i,k) \in A(t), \forall t \in T \tag{31}$$

In the first iteration, the assignment problem will be solved with echelon demand ($D_t^i = e_t^i$). The solution of this phase (the assignment matrix) will be given to the lot sizing problem and then we will get the initial solution. This procedure can be demonstrated as Figure 2.

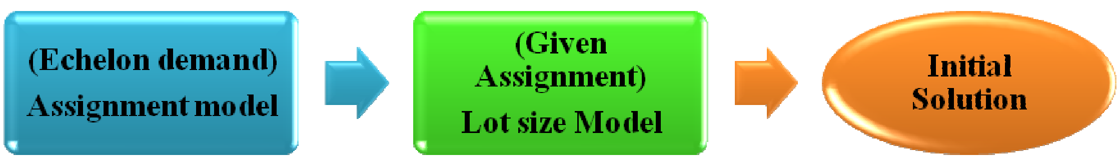
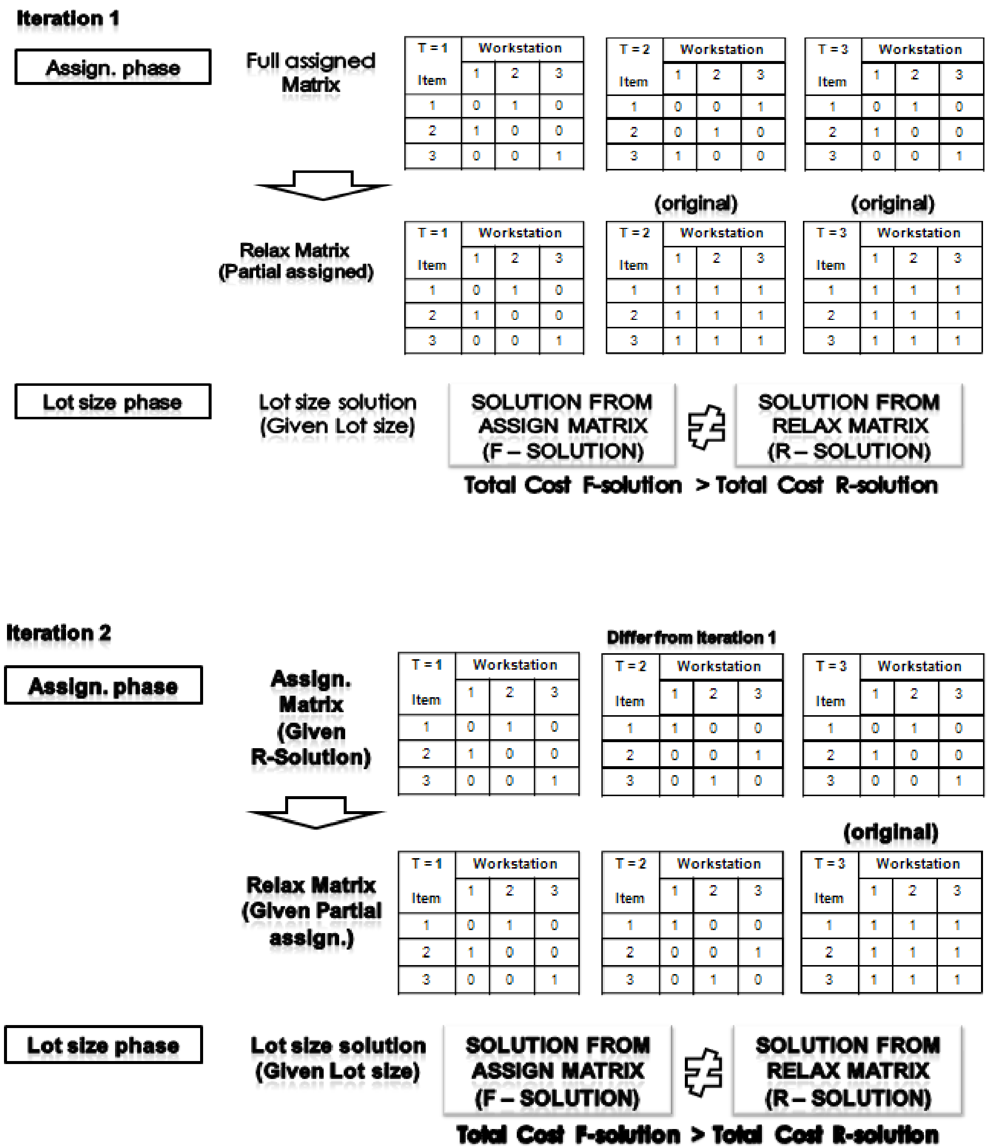


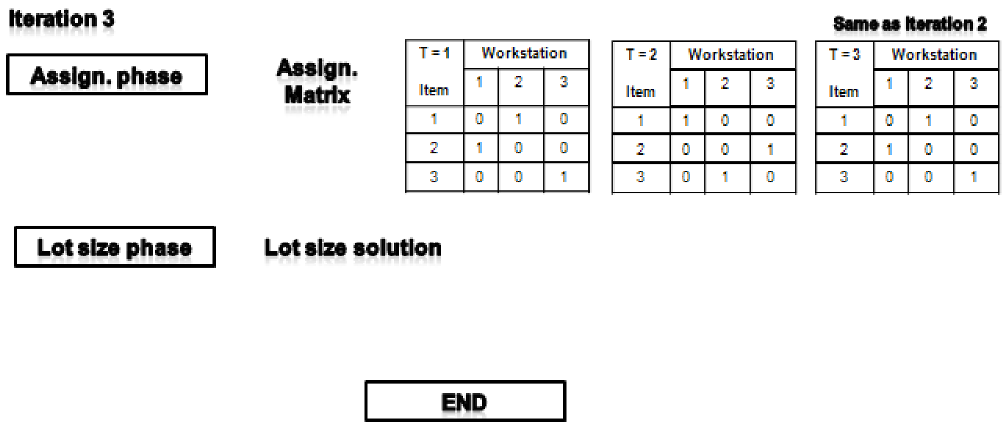
Figure 2. A demonstration of the first iteration procedure

Iteration.

The two sub-problems will be done iteratively. Firstly, the problem with the Full assignment result from the first model is solved and kept as the first solution (F-solution). Secondly, the problem is solved with the Partial assignment matrix that changes the data of the first period to the iteration period (using the number of iteration as period number) to the same as the result of the assignment part and the solution will be kept as the P-solution. Thirdly, the F-solution and the P-solution will be compared. If they are equal or the first one is lesser or the lot size solution is equal, we stop and this is the solution for this problem. Otherwise, we continue to next iteration.

For example, if the planning horizon covers 3 periods, the maximum number of the iteration will be 3. The illustration of this example is shown in Figure 3.





T = 1	Workstation		
Item	1	2	3
1	0	1	0
2	1	0	0
3	0	0	1

T = 2	Workstation		
Item	1	2	3
1	1	0	0
2	0	0	1
3	0	1	0

T = 3	Workstation		
Item	1	2	3
1	0	1	0
2	1	0	0
3	0	0	1

Figure 3. The illustration of three period problem

In the first iteration, a given assignment matrix that will be sent to lot sizing part will be split in 2 types which are the “Full assigned” (as the result from assignment model with given echelon demand lot size) and the “Partial assigned” (as the result from assignment model with given echelon demand lot size only in period 1, other periods (period 2 and 3) will be replaced with the original assignment of the problem). The Full assigned matrix will be sent and then solved in the lot size part and the solution will be kept as the first solution of the problem. On other hand, the Partial assigned will also sent and solved in the lot size part, the solution will be sent as given lot size in the next iteration. Likewise, the second iteration given lot size will be used the data from Partial assigned and the assignment matrix will be split in two types. The Partial in the second iteration will use the result from assignment model with given previous lot size only in period 1 and 2, other periods (period 3) will be replaced with the original assignment of the problem. This will go on and on until the number of iteration equals to max number of iteration. In this example will be end in the third iteration. In the third iteration, the assignment matrix will not be split and the solution will be the best solution.

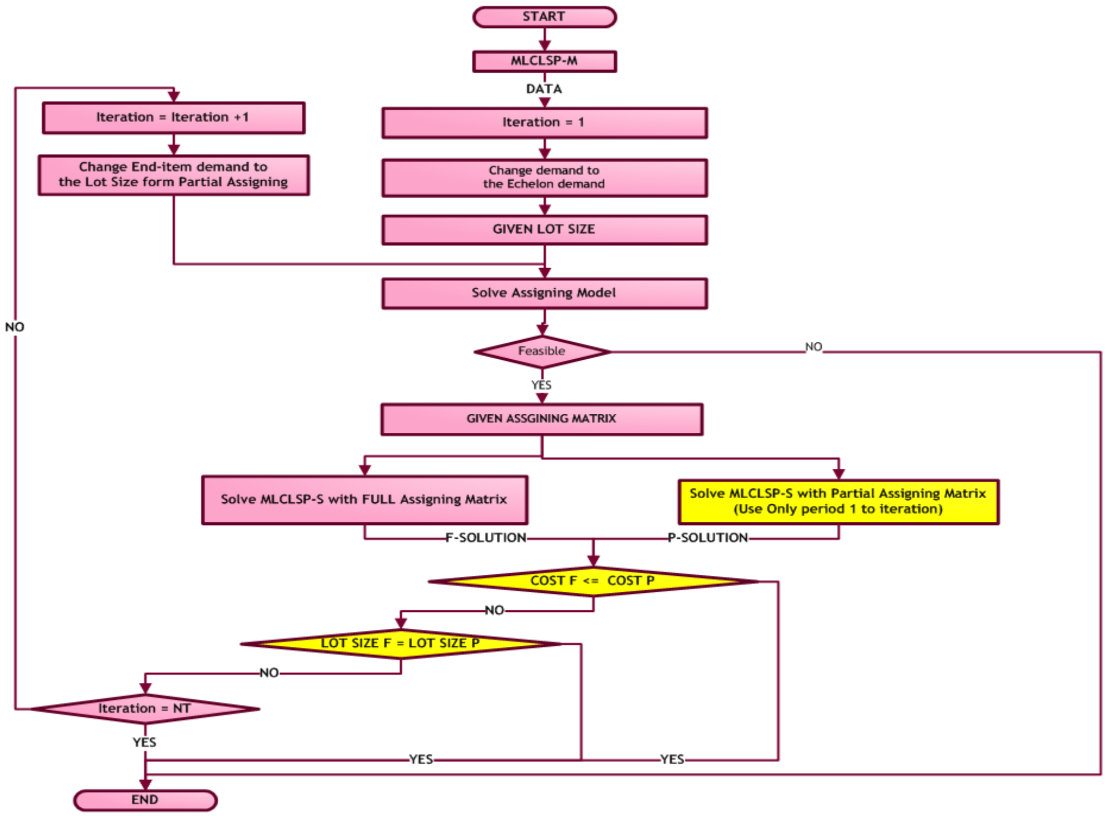


Figure 4. The flow of the proposed heuristic

With the concept of lot sizing problem, it is obvious that Lot-for-Lot policy gives only no holding solution and, especially in the case of setup consideration, the solution with more holding has a potential to be the better solution. On basis of this characteristic, in this paper propose the heuristic method that can change the Lot-for-Lot solution to the solution with consideration holding. The heuristic is called Partial Assignment- Lot size (PA-LS).

An Example

Our goal in this section is to demonstrate how the heuristic developed in the earlier section may be implemented in practice in order to obtain a solution. For the sake of clarity we assume a ten-product ten-workstation five-period situation. An example can be seen in Figure 5. There are ten items (seven production items and three raw materials), and ten workstations (seven production workstations and three purchasing workstations). The capacity of each workstation is limited by the availability of its resources. For example, the capacity of production workstations are limited by capacity of available operating time of machines or workers and the capacity of a purchasing workstation are limited by available raw materials in the supply workstation. For the production workstation the capacity will be charged both from lot-size operation time and setup time. On the other hand, the purchasing operation will be charged by only the purchasing lot-size. By way of illustration let us look at the case of item 1; the item 1 can be produced only on workstation 1 which is a production workstation with limited capacity. Whenever item 1 is produced, the capacity of workstation 1 will be charged setup time and operation time (both of which varying by period) for the lot size of item 1. In this case, it has to be a trade-off between fixed cost and variable cost if we want to keep the total cost low. As only one workstation per one item, we aim to find a minimum lot size of particular item in a period over multi-period planning horizon consideration. Here, once a product is ordered, the setup time of the production will be occurred and will be considered as sequence independent between orders of different productions. The setup activities for each production workstation incur setup costs and consume setup time. Within a limited capacity expressed in hours during each time period of the workstation, the operation time and the setup time will reduce capacity in period. Assume that the demands for the part processing vary with time in a deterministic manner and there is no order for a component item from the customers. Therefore, production quantity of a component item can be computed from order quantities of end items that require the component item. If an item will not be planed to be used in the next period, it must be kept in warehouse and charged holding costs. Here, the quality of production is neglected. Moreover, all unit costs, prices, and setup times or costs are assumed to be known with dynamic by period. To address this multiple time period formation problem, a mixed integer programming (MIP) model is formulated. The objective of the model is to minimized total cost (setup cost, purchasing cost, production cost and inventory cost) for the entire planning time horizon with given time varying parameters (external demand, availability of raw materials, capacity of workstations, time parameters and cost parameters).

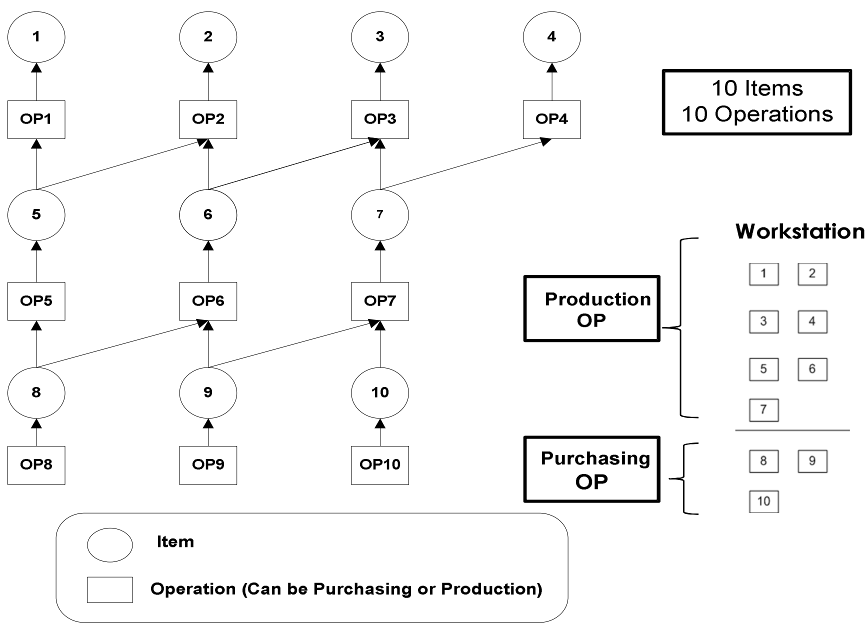


Figure 5 BOM for the example of heuristics implemented

The parameters are shown in Table 1 to Table 9. Item 1 to 7 can be produced on workstation 1 to 7 and item 8 to 10 can purchased on workstation 8 to 10.

Table 1 Primary demand, d_t^i.

i	*T*				
	1	2	3	4	5
1	38	32	41	50	47
2	18	15	17	19	23
3	25	27	30	33	43
4	48	55	53	74	80
5	0	0	0	0	0
6	0	0	0	0	0
7	0	0	0	0	0
8	0	0	0	0	0
9	0	0	0	0	0
10	0	0	0	0	0

Table 2 Usage parameters, $u^{i,j}$ (BOM in Figure 5).

i	J									
	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	1	1	0	0	0	0	0	0	0	0
6	0	1	1	0	0	0	0	0	0	0
7	0	0	1	1	0	0	0	0	0	0
8	0	0	0	0	1	1	0	0	0	0
9	0	0	0	0	0	1	1	0	0	0
10	0	0	0	0	0	0	1	0	0	0

Table 3 Ordering/Setup Cost parameters, $f_t^{i,k}$.

t = 1	K									
i	1	2	3	4	5	6	7	8	9	10
1	75	111	60	108	104	57	62	56	73	76
2	34	42	20	24	21	47	50	39	49	28
3	60	70	58	34	31	68	71	73	43	66
4	101	96	120	53	95	125	131	88	85	100
5	359	290	443	522	540	569	500	467	436	452
6	236	355	297	456	276	300	208	199	173	243
7	841	370	413	407	677	505	717	547	343	388
8	683	1244	928	1506	1083	1178	1034	1189	867	1340
9	1897	1665	2061	2466	2222	2248	1759	953	2392	1577
10	2849	1398	1843	1691	2794	3089	2262	1959	2460	1088

Table 4 Purchasing /Production Cost parameters, $p_t^{i,k}$.

t = 1	K									
i	1	2	3	4	5	6	7	8	9	10
1	9	19	7	16	18	7	7	8	12	12
2	12	17	6	6	14	12	13	10	11	6
3	19	11	16	9	5	16	14	7	16	6
4	5	9	12	16	18	17	11	13	5	8
5	18	16	11	13	5	10	17	13	18	18
6	13	17	10	6	8	14	16	12	7	9
7	6	16	11	5	11	14	11	6	17	8
8	34	59	48	34	30	54	53	50	46	54
9	43	38	46	49	57	57	37	59	57	47
10	49	44	57	36	58	36	36	35	36	47

Table 5 Setup Time parameters, $g_t^{i,k}$.

t = 1	K									
i	1	2	3	4	5	6	7	8	9	10
1	65	57	50	50	79	56	87	94	82	60
2	60	77	67	99	61	68	89	88	59	97
3	66	60	76	61	67	89	80	67	77	81
4	84	83	53	73	91	50	56	71	51	95
5	85	82	62	66	74	79	67	68	56	79
6	98	94	93	57	95	67	91	95	66	98
7	74	90	76	92	79	79	88	64	52	52
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0

Table 6 Operation Time parameter

t = 1	K									
i	1	2	3	4	5	6	7	8	9	10
1	16	11	16	18	17	11	17	10	14	14
2	17	15	10	10	10	12	15	15	12	10
3	12	12	14	13	17	11	15	13	16	14
4	19	17	11	18	14	14	18	17	19	12
5	14	19	19	10	11	14	13	14	11	16
6	19	16	12	15	19	17	19	12	10	18
7	13	15	18	12	19	12	12	13	18	18
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0

Table 7 Holding Cost parameters, h_t^i .

	T				
i	1	2	3	4	5
1	2	3	1	2	1
2	3	1	2	2	2
3	1	1	3	1	3
4	2	2	2	1	2
5	1	3	1	1	2
6	3	2	1	3	2
7	1	2	1	3	1
8	3	3	1	1	1
9	3	2	3	1	3
10	3	2	3	3	2

Table 8 Capacity parameters, cap_t^k .

k	*T*				
	1	2	3	4	5
1	6673	5890	7739	9660	9519
2	5577	6381	6553	9954	9009
3	6059	6006	6596	8107	8697
4	6871	6591	6222	9527	9597
5	5771	5878	6349	8743	10203
6	7542	6030	6524	7029	9688
7	6217	5764	7081	8766	8901
8	-	-	-	-	-
9	-	-	-	-	-
10	-	-	-	-	-

Table 9 Availability parameters, $Avail_t^k$.

k	*T*				
	1	2	3	4	5
1	-	-	-	-	-
2	-	-	-	-	-
3	-	-	-	-	-
4	-	-	-	-	-
5	-	-	-	-	-
6	-	-	-	-	-
7	-	-	-	-	-
8	542	520	586	698	790
9	542	520	586	698	790
10	542	520	586	698	790

The firm's manufacturing structure has the four products (item 1, item2, item 3 and item 4) and three raw material items (item 8, item 9, and item 10) according to the BOM as shown in Figure 5. There is zero initial stock for all items and the warehouse can keep only 542 units. The quantities of the items required to make a unit of another item are assumed to be 1 unit for all items. Table 2 shows the quantities of the required items or the usage parameters of all items. Table 3 and Table 4 show the ordering or setup cost and the purchasing/production cost parameters, respectively. In Table 3, the cost in column workstation 8, 9 and 10 are ordering costs, while the costs in column workstation 1 to workstation 7 are setup cost. Similarly, in Table 4, the cost in column workstation 8, 9 and 10 are purchasing costs, while the costs in column workstation 1 to workstation 7 are production costs. Table 5 and Table 6 show the setup time and operation time parameters, respectively. Only production workstations, which are workstation 1 to workstation 7, have the value of setup time and operation time. Table 7 shows the holding cost per period for each item. In addition, there are ten workstations in this problem and each workstation has capacity per period of time as shown in Table 8 and availability per period of time as shown in Table 9.

In summary, the problem considers a ten-component product structure of four end-item constrained by ten workstations over a 5-period planning horizon with the parameters and data presented in Table 1 through Table 9. The heuristics can be presented as follows:

Iteration 1

Assignment Phase

1. Using the data from Table 1 through Table 9
2. Calculating the echelon demand.
3. Solving with assignment model. The result will be set as assignment for Lot-size Part

Lot-size Phase

1. Solving the single model with assignment from the assignment part. The total cost is 87109.
2. Solving the single model with assignment from the first iteration only for period 1 and unchanging for others. The total cost is 82854.
3. Comparing the result from 1 and 2. We found that the solutions are unequal. Go to Iteration 2.

Iteration 2

Assignment Phase

1. Using the data from Table 1 through Table 9 for parameters.
2. Using the Lot-size result from the Lot Size Part of iteration 1 instead of echelon demand.
3. Solving with assignment model.

Lot-size Phase

1. Changing the demand to the result lot size from iteration 1 and solving the single model with assignment from the assignment part. The total cost is 81738.
2. Solving the single model with assignment from the first iteration only for period 1 through 2 and unchanging for others. The total cost is 81617.

3. Comparing the result from 1 and 2. We found that the solutions are unequal. Go to Iteration 3

Iteration 3

Assignment Phase

1. Using the data from Table 1 through Table 9 for parameters.
2. Using the result of the Lot-size Part of iteration 2.
3. Solving with assignment model, the result can be presented in Table 10.

Table 10 The assignment form iteration 3, $w_t^{i,k}$

t = 1	*k*									
i	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	1	0	0	0
2	0	0	0	0	0	1	0	0	0	0
3	0	1	0	0	0	0	0	0	0	0
4	1	0	0	0	0	0	0	0	0	0
5	0	0	0	0	1	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	1	0
9	0	0	0	0	0	0	0	0	0	1
10	0	0	0	0	0	0	0	1	0	0

t = 2	*k*									
i	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	1	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0	0
3	0	0	0	0	0	1	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	1	0	0	0	0	0	0	0	0
6	0	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	1	0
9	0	0	0	0	0	0	0	0	0	1
10	0	0	0	0	0	0	0	0	0	0

t = 4	*k*									
i	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	1	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	1	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0

t = 5	*k*									
i	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	1	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0

t = 3	k									
i	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	1	0	0	0
2	0	0	0	0	0	1	0	0	0	0
3	0	0	0	1	0	0	0	0	0	0
4	0	0	0	0	1	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	1	0	0	0	0	0	0	0	0
7	1	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	1

Lot-size Phase

1. Changing the demand to the result lot size from iteration 2 and solving the single model with assignment from the assignment part. The result can be presented in Table 11 and the total cost is 80187 (F-solution).
2. Solving the single model with assignment from the first iteration only for period 1 through 2 and unchanging for others. The result can be presented in Table 12 and the total cost is 80081 (P-solution).
3. Comparing the result from 1 and 2. We found that the solutions are unequal but the matrixes are same.
4. Then, we stop. The result can be presented in Table 13 through 14 and the total cost is 83273.

Table 11 F-solution of Iteration 3

	T				
i	1	2	3	4	5
1	38	32	138	0	0
2	18	15	17	42	0
3	25	27	63	0	43
4	103	0	53	154	0
5	56	244	0	0	0
6	43	42	165	0	0
7	155	0	313	0	0
8	56	244	0	0	0
9	99	451	0	0	0
10	240	0	478	0	0

Table 12 P-solution of Iteration 3

	T				
i	1	2	3	4	5
1	38	32	138	0	0
2	18	15	17	42	0
3	25	27	63	0	43
4	103	0	53	154	0
5	56	244	0	0	0
6	43	42	165	0	0
7	155	0	313	0	0
8	56	244	0	0	0
9	99	451	0	0	0
10	240	0	478	0	0

Table 13 The optimal solution of lot size, $x_t^{i,k}$

t = 1	*k*									
i	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	38	0	0	0
2	0	0	0	0	0	18	0	0	0	0
3	0	25	0	0	0	0	0	0	0	0
4	103	0	0	0	0	0	0	0	0	0
5	0	0	0	0	56	0	0	0	0	0
6	0	0	43	0	0	0	0	0	0	0
7	0	0	0	155	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	56	0
9	0	0	0	0	0	0	0	0	0	99
10	0	0	0	0	0	0	0	240	0	0

t = 2	*k*									
i	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	32	0	0	0	0	0
2	0	0	15	0	0	0	0	0	0	0
3	0	0	0	0	0	27	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	244	0	0	0	0	0	0	0	0
6	0	0	0	42	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	244	0
9	0	0	0	0	0	0	0	0	0	451
10	0	0	0	0	0	0	0	0	0	0

t = 3	*k*									
i	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	138	0	0	0
2	0	0	0	0	0	17	0	0	0	0
3	0	0	0	63	0	0	0	0	0	0
4	0	0	0	0	53	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	165	0	0	0	0	0	0	0	0
7	313	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	478

t = 4	*K*									
i	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	1	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	1	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0

t = 5	*K*									
i	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	42	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	154	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0

Table 14 The optimal solution of stock, s_t^i

	T				
i	1	2	3	4	5
1	0	0	97	47	0
2	0	0	0	23	0
3	0	0	33	0	0
4	55	0	0	80	0
5	0	197	42	0	0
6	0	0	85	43	0
7	27	0	197	43	0
8	0	0	0	0	0
9	0	165	0	0	0
10	42	0	0	0	0

The solution from the research algorithm is equal to the optimal solution from AMPL/CPLEX 8.0.0. For more clarification of the solution approach, the relation between the number of iterations and objective function as illustrated in Figure 6.

	Full Assignment	Partial Assignment
1	87,109	82,854
2	81,738	81,617
3	80,187	80,081

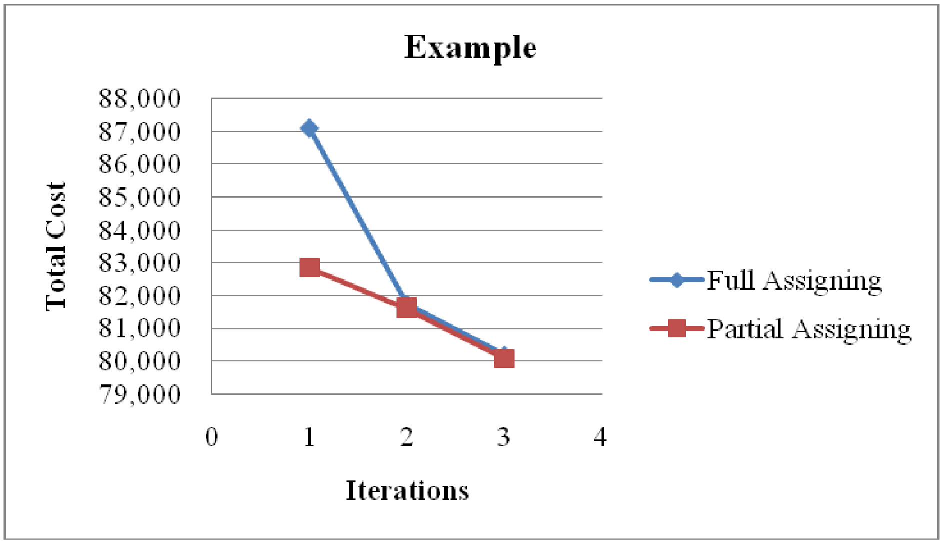


Figure 6 The relation between the number of iterations and objective function

It shows that the objective will go to the same point. It's obvious that the solution is very close to the optimal solution. However, the time for solving is very large.

Conclusion

This research proposes a new decomposition heuristic based on all assumptions of the MLCLSP-M model including that lot-for-lot policy is a feasible solution. The heuristics composes of two phases which are partial assignment with given lot size phase and partial lot size with given partial assignment phase. These phases are solved iteratively and sequentially with AMPL/CPLEX 8.0.0 solver. The assignment solution of the first phase is used as input data for the lot size phase and the lot size solution will be used as input for the assignment phase in the next iteration. The termination of this heuristic will happen whenever the iteration numbers is the maximum iteration limited or the lot solution of the previous iteration equal to this iteration.

As seen in the example, the drawbacks of this heuristic is the Partial steps make the problem reduce the problem size only in period dimensions (The Problem is large in the early iteration). Although the solution is great, it needs long solving time in the early iteration. It's clear that there is a need to find the way to reduce to solving time. Therefore, the heuristic that has lesser solving time and satisfactory solution should be developed in further research.

References

Aardal, K., & Larsson, T. r. (1990). A benders decomposition based heuristic for the hierarchical production planning problem. *European Journal of Operational Research, 45*(1), 4-14.

Brahimi, N., Dauzère-Pérès, S., Najid, N. M., & Nordli, A. (2006). Single item lot sizing problems. *European Journal of Operational Research, 168*(1), 1-16.

Chinprateep, S., & Boondiskulchok, R. (2007, October 25-26, 2007). *Developing the minimization cost lot-sizing model of purchasing & production planning system with on-hand inventory consideration* Paper presented at the Proceeding of IE Network Conference 2007 Royal phuket city Hotel, Phuket, Thailand.

Clark, A. R. (2003). Hybrid heuristics for planning lot setups and sizes. *Computers & Industrial Engineering, 45*(4), 545-562.

Drexl, A., & Kimms, A. (1997). Lot sizing and scheduling — Survey and extensions. *European Journal of Operational Research, 99*(2), 221-335.

Karimi, B., Ghomi, S. M. T., & Wilson, J. M. (2003). The capacitated lot sizing problem: a review of models and algorithms. *Omega, 31*(5), 365-378.

Kimms, A. (1996). Multi-level, single-machine lot sizing and scheduling (with initial inventory). *European Journal of Operational Research, 89*(1), 86-99.

Pochet, Y. (2001). Mathematical Programming Models and Formulations for Deterministic Production Planning Problems. In *Computational Combinatorial Optimization: Optimal or Provably Near-Optimal Solutions* (Vol. 2241/2001, pp. 51-111). Berlin: Springer Berlin / Heidelberg.

See-Toh, Y. C., Walsh, S. P. K., Shah, N., Marquardt, W., & Pantelides, C. (2006). Multi-period capacitated lot sizing with variable batch sizes. In *Computer Aided Chemical Engineering* (Vol. Volume 21, Part 2, pp. 1949-1954): Elsevier.

Silvio, A. d. A., Marcos, N. A., & Alistair, R. C. (2008). Lot sizing and furnace scheduling in small foundries. *Computers & Operations Research, 35*(3), 916-932.

Stadtler, H., & Surie, C. (2000). *Description of MLCLSP Test Instances*. Retrieved December 20, 2000, from http://www.bwl.tu-darmstadt.de/bwl1/forschung/ti_mlclsp/ ti_mlclsp.php?FG=bwl1

Suerie, C. (2005). *Time Continuity in Discrete Time Models New Approaches for Production Planning in Process Industries* (Vol. 552): Springer Berlin Heidelberg.

Vercellis, C. (1999). Multi-plant production planning in capacitated self-configuring two-stage serial systems. *European Journal of Operational Research, 119*(2), 451-460.

Wu, P., Hartman, J. C., & Wilson, G. R. (2003). A demand-shifting feasibility algorithm for Benders decomposition. *European Journal of Operational Research, 148*(3), 570-583.

Zangwill, W. I. (1996). A Deterministic Multiproduct, Multifacility Production and Inventory Model. *Operations Research, 14*(3), 486-507.

Zapfel, G. (1996). Production planning in the case of uncertain individual demand Extension for an MRP II concept. *International Journal of Production Economics, 46-47*, 153-164.