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Understanding the Effects of Mathematics Professional Development on Teachersâ Perceptions of Mathematics

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Abstract

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Perceptions of Mathematics

by

Michael Flynn

MA, Lesley University, 2000

BSE, Westfield State University, 1998

Project Study Submitted in Partial Fulfillment

of the Requirements for the Degree of

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Abstract

Negative mathematics perceptions may affect how teachers teach the subject, how much time they spend on the subject, and how well they teach it. The problem that grounded this study was that although anecdotal evidence showed that teachers' experience with professional development (PD) courses in mathematics teaching improved their perceptions of mathematics, there was a lack of data regarding how PD affects changes in teachers' perceptions. The study purpose was to gain a deeper understanding of what elements of a PD course have the greatest effect on improving teachers' perceptions of mathematics. Piaget's constructivist and Vygotsky's socio-constructivism theories made up the conceptual framework that guided this study. Teachers' attitudes, perceptions, and experiences with mathematics; content and pedagogical knowledge; and understanding of effective instruction before and after attending a PD course were examined in this case study. Five teachers who attended a weeklong PD course focused on building their content and pedagogical knowledge were interviewed before and at the conclusion of the course to gauge how their perceptions of mathematics changed as a result of their experience. Data were analyzed using open coding to generate themes leading to 7 major findings. The findings indicated that the course had a positive effect on teachers' perceptions of mathematics and gave insight into which elements were most effective, such as opportunities to engage in rich tasks in small groups while an experienced facilitator guides the learning. The findings from this study may affect positive social change by helping to design math PD that equips teachers with mathematical content and pedagogical knowledge in ways that relieve their anxiety about teaching mathematics and ensure better instruction for students.

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Dedication

To Megan, Daniel, Allison, Sean, Collin, and Cora for your love, support, and patience throughout this whole process. I could not have done it without you.

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I would like to thank my wife, Megan Allen, for her encouragement and support throughout this whole process. I would also like to thank my children, Dan, Alli, Sean, Collin, and Cora for their love, patience, and support.

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Section 1: The Problem

Elementary mathematics instruction in the United States has been criticized for not developing a strong mathematical foundation for students for more than 40 years (Taton, 2015). A large part of the challenge is that many elementary teachers are generalists responsible for teaching all subjects. Teacher training programs are designed to give preservice teachers a well-rounded experience with all aspects of elementary education. This broad approach may not allow for extensive courses in mathematics. Therefore, many teachers lack a broad conceptual understanding of mathematics, which affects their perceptions of the subject (Chapman, 2015). When teachers have negative perceptions of mathematics, it can affect how they approach teaching it and their desire to seek out professional learning experiences to deepen their own understanding of it (Andrews & Brown, 2015; Malinsky, Ross, Pannells, & McJunkin, 2006). But the Common Core State Standards for Mathematics push for a focus on building students' conceptual understandings through meaningful, rich experiences (Peterson & Ackerman, 2015), so teachers who lack a conceptual basis may need to build their mathematical content and pedagogical knowledge.

Professional development programs may not take teachers' perceptions into account so that they have long-lasting, sustainable effects on mathematics instruction (Polly, Neale, & Pugalee, 2014). Therefore, it is critical that professional development providers understand how different aspects of these experiences affect teachers' perceptions of mathematics because if they do not see themselves as math capable, they are less likely to engage in sustained professional growth in mathematics (Chapman, 2015). This information can help providers of professional development design more effective experiences by incorporating and emphasizing the elements that help teachers feel more positive about the teaching of mathematics (Wenig, 2016). For example, if teachers report feeling better about math after gaining critical insights about a challenging mathematical idea from an exploration via visual representations, then developers of professional learning may want to include more opportunities for teachers to explore with visual representations.

In Section 1, I discuss the relevant issues that make up the local problem and then explain the rationale for the study. Key terms will then be defined followed by discussion of the significance of the study. Next, I will discuss the research questions (RQs) and a review of the current literature related to the proposed study. Section 1 will end with a discussion of the implications of the study and a summary of Section 1.

The Local Problem

Negative perceptions about mathematics may affect how teachers teach the subject, how much time they spend on the subject, and how well they teach it. The professional and graduate programs in mathematics teaching at a local liberal arts college in Massachusetts, which I organize and facilitate for the Master of Arts in Mathematics Teaching programs, are designed to develop teachers' mathematical content and pedagogical knowledge. The target audience is kindergarten through eighth grade teachers from across the United States who attend the trainings in person or online. The programs serve hundreds of classroom teachers each year and gives a sense of teachers' needs and the challenges facing professionals working in K to 8 mathematics. For instance, the elementary teachers who attend the professional development courses at the college often express a dislike for mathematics, and some anxiety about teaching mathematics is evident among approximately 25% of participants. These teachers' math anxiety often manifests itself as apprehension around engaging in mathematical tasks in front of colleagues and a general nervousness around the course material, which is frequently present at the start of the seminars (see Andrews & Brown, 2015). However, teachers overall report a positive experience by the end of the training, though there are no specific data to suggest what influenced their perceptions of mathematics.

In addition to teachers having a general dislike and anxiety for the subject, state testing data reveal that local students' struggles with mathematics. For example, several of the graduate students and participants in the professional learning offered at the college teach in the local community. The data reported from the Department of Elementary and Secondary Education in Massachusetts for the 2016 to 2017 academic year indicate a gap in practice because 72% of fifth graders are not meeting the state mandated scores in mathematics (Massachusetts Department of Elementary and Secondary Education, 2017). The administrators in the district are sending more teachers to one or more of the courses offered in the local college program in the summer in hopes of addressing this local problem. One principal who wrote to me on January 24, 2017 said,

> It's frustrating because when it comes to professional development for language arts, teachers flock to it and I cannot get enough subs to cover the number of teachers that want to go. But nobody wants to do math. Our ELA scores are fine

> but everyone wants to go to reading workshops or writing workshops. How can we get teachers just as excited about math? That's what I'm looking for.

This sentiment is shared by many administrators and math coaches who contact me looking for professional learning experiences in mathematics for their teachers. However, although I have anecdotal evidence to say that teachers' experiences with the college professional development courses leave them feeling positive, there are no specific data that show how the courses affect teachers' perceptions of mathematics.

In addition to this evidence from my experience, researchers have identified a problem in teachers' negative perceptions of mathematics (Andrews & Brown, 2015; Finlayson, 2014; Jameson & Fusco, 2014). Many elementary teachers have high levels of math anxiety (Andrews & Brown, 2015; Malinsky et al., 2006), and teachers with high levels of math anxiety tend to have a poor attitude toward math and math instruction (Jameson & Fusco, 2014). Studies have shown that math-anxious teachers spend significantly less time teaching math concepts and problem-solving strategies and spend 50% less time teaching math than those comfortable with the subject (Finlayson, 2014; Sloan, 2010). This significantly affects student achievement because students with anxious teachers receive less instruction (Finlayson, 2014; Sloan, 2010). Teachers' math anxiety may also affect the quality of mathematics instruction because math anxious teachers tend to focus on teaching procedures for solving problems rather than developing students' understanding of the concepts underlying those procedures (Finlayson, 2014; Geist, 2015; Sloan, 2010).

Additionally, many elementary teachers lack the deep content and pedagogical knowledge needed to teach mathematics well (Campbell & Malkus, 2011; Ma, 2010; Reid & Reid, 2017). This knowledge gap is partly a result of the requirements of teacher training courses that prepare elementary teachers to be generalists regarding curriculum (Bell, Wilson, Higgins, & McCoach, 2010). Elementary teachers are responsible for teaching all subjects; therefore, teacher preparation programs give preservice teachers broad and brief experiences in the various content areas (Novak & Tassell, 2017; Sloan, 2010). But when teachers have a solid conceptual understanding of math and how their students learn, they are more effective in teaching the subject and their students perform better in the classroom (Bartell, Webel, Bowen, & Dyson, 2013; Hill, Rowan, & Ball, 2005; Santagata, Kersting, Givvin, & Stigler, 2011). Having one or two courses focused on mathematical concepts and math instruction is not enough time to help teachers develop the content and pedagogical knowledge needed to teach math well (Malinsky et al., 2006; Reid & Reid, 2017). For teachers to develop their knowledge and skills in mathematics, it is imperative that they participate in ongoing, high-quality professional development.

The professional development program that the local college offers is designed to develop teachers' mathematical content and pedagogical knowledge. During the program teachers work on mathematics tasks that have been designed specifically for adult learners to help them develop a deep conceptual understanding of the mathematical ideas in the course. When teachers develop a deeper understanding of mathematics, their negative attitudes are reduced (Looney, Perry, & Steck, 2017). Then the participants explore written and video-based case studies of kindergarten through eighth-grade students engaging in similar mathematical tasks. This experience is enhanced because the participants have both the lens of the learner and the teacher as they interact with the course materials. Finally, teachers then reflect on their own curriculum materials and consider how they will change elements of their teaching practices to help students develop a deeper understanding of the material. By focusing on developing mathematical content and pedagogical knowledge, participants gain a rich understanding of the mathematics they teach and a clearer sense of how students learn.

As teachers persist in the professional development experience and persevere through challenging experiences, they begin to make sense of a subject that for so long may have been a source of stress and anxiety in their lives (Thornton, Crim, & Hawkins, 2009; Wenig, 2016). At the end of each course many teachers report that they had a positive experience. But what is not understood is which aspects of the professional learning experience have the greatest effect on teachers’ perceptions of mathematics and how those perceptions have changed. The only feedback the local college receives is from a generic feedback form that asks participants to rate the instructor, the course material, and course methodologies. Thus, this study aimed to gather more specific data about how each element of the course affected participants’ perceptions of mathematics.

Rationale

Principals and superintendents who send teachers to the professional development program also share my concern that more than half of their elementary teachers have negative perceptions of mathematics and believe that these views are preventing them from making more gains with students in their classrooms. The principals and superintendents have expressed a desire for teachers to develop more positive dispositions related to mathematics in addition to increasing teachers' the mathematical content and pedagogical knowledge. In consultation with local and national administrators who send teachers to the program along with feedback from my teaching staff, it became evident that teacher perception is an area that I should address. Teachers' negative perceptions of mathematics was a common theme during discussions, but the professional development team have been unable to determine what elements of the courses contribute to changing teachers' perceptions. Thus, there was a need to address the problem of the study to more effectively support the teachers who join the program.

The problem of teachers having negative perceptions of mathematics is not limited to the schools and districts that send teachers to the program. This is a systemic challenge that exists throughout the country. Khaliqi (2016) and Center for Research in Mathematics and Science Education (2010) described a problematic cycle in elementary mathematics education. The author contended that teachers with an inadequate mathematics background using weak K to 12 materials produce high school graduates who have deficits in their mathematical understanding. Some of these graduates decide to become teachers and are not given adequate training in mathematics; therefore, the cycle continues (Viadero, 2010; Zager, 2017).

As schools continue to struggle with mathematics and as national and international test scores continue to indicate a strong need for improvement in mathematics, the research community works to understand the problem and investigate possible solutions to it (Blank & de las Alas, 2010; Reid & Reid, 2017). Over the last two decades, much has been uncovered about the importance of elementary teachers' mathematical content knowledge and comfort with the subject (Hill, Rowan, & Ball, 2008; Taton, 2015). Providers of professional learning in mathematics are also gaining a better understanding of how professional development can improve teachers' mathematical content and pedagogical knowledge (Bartell, Webel, Bowen, & Dyson, 2013; Doerr, Goldsmith, & Lewis, 2010; Hill, Rowan, & Ball, 2008). Researchers point to the importance of professional development to address negative perceptions of mathematics among elementary teachers. For example, Novak and Tassell (2017) as well as Cave and Brown (2010) discussed the positive effects of content-focused professional development for teachers. There is also an indication of the strength in partnerships between school and colleges for mathematics professional development (Reid, 2013; Trent, 2012). Professional development networks can also have a positive effect on the development of mathematics teacher leaders which in turn support school improvement goals (Barrett, 2013; Rieckhoff & Larsen, 2012).

The purpose of this study was to gain a deeper understanding of what elements of professional development aimed at deepening teachers' mathematical content and pedagogical knowledge have the greatest effect on moving teachers toward more positive perceptions of mathematics. It was important to know what professional learning experiences resonated with teachers and actually shifted how they view mathematics. A greater understanding of positive and negative learning experiences of participants in my courses can influence how these courses are revised and how new courses are created. By redesigning the course to maximize experiences that influence positive dispositions toward mathematics, teachers will be better prepared and motivated to make shifts in their teaching practices.

Definition of Terms

Conceptual understanding: Knowledge that is rich in relationships among key facts and ideas in mathematics. All key ideas and understandings are linked in a network (Bartell et al., 2013).

Math anxiety: A form of state anxiety that is prompted and exacerbated by mathematical experiences (Isiksal et al., 2009). Baloglu and Kocak (2006) described the three sources of math anxiety: situational factors that are external and immediate; dispositional factors that are internal and personalized; and environmental factors that include individuals' attitudes, prior perceptions and experiences.

*Professional developmen*t: Teaching and learning experiences that are facilitated by an instructor and are designed to support the development of professional knowledge, skills, and dispositions. These experiences could also include learning experiences that address the application of this knowledge in practice (National Professional Development Center on Inclusion, 2008).

Significance of the Study

Elementary principals in the local area reported that they were not seeing significant shifts in teachers' practices, which they attributed to a lack of professional learning in mathematics. However, they also reported that it was hard to convince teachers to enroll in courses in mathematics because many had negative perceptions of the subject. When faced with the choice of going to a mathematics conference or a language arts conference, their teachers overwhelmingly registered for language arts. If local teachers need professional development in mathematics, but they may be reluctant to take mathematics courses because of negative perceptions of the subject, then it was imperative that I find ways to change teachers' perceptions of mathematics (see Wenig, 2016). Understanding what elements of these experiences result in changes to teachers' perceptions of mathematics can help develop course structures and modify existing courses to address the problem of negative perceptions.

Improving mathematics professional development is also important because students in the United States consistently underperform in mathematics compared to their international counterparts in other developed countries (Hanushek, Peterson, & Woessmann, 2010; Stephens, Landeros, Perkins, & Tang, 2016; Vistro-Yu, 2013). In a globally competitive economy, success in the fields of science, technology, mathematics, and engineering are essential to the economic stability of the United States (Harris, Sithole, & Kibirige, 2017). The lag in performance on state mathematics tests presents a major challenge to educational leaders looking to bolster students' achievement and the United States' standing.

One way to address low performance in mathematics is through professional development for teachers to boost their mathematical content and pedagogical knowledge and to increase quality of their instruction (Garet et al., 2016; Reid & Reid, 2017). Teachers who engage in professional development aimed at increasing their understanding of their students' thinking showed a significant increase in the proportion of time spent devoted toward classroom discussions to boost students' understanding (Lachner & Nückles, 2016; Perry & Lewis, 2010). Professional development for teachers in mathematics can contribute greatly to students' mathematical achievement (Cave & Brown, 2010; Lachner & Nückles, 2016). Therefore, it is imperative to understand the components and qualities of these experiences that are most effective in shifting teachers' perceptions of mathematics.

Research Questions

If teachers have an aversion to mathematics and mathematics instruction, they may be less likely to enroll in a professional development program emphasizing that subject, which means they may be less likely to change their attitude toward mathematics and how they approach teaching it. This problem led to the following RQs:

RQ1. Prior to attending the professional development course, how do teachers describe their attitudes, perceptions, and experiences with mathematics?

RQ2. How do teachers who attended a professional development course describe their mathematical content knowledge?

RQ3. How do teachers who attended a professional development course describe their understanding of effective mathematics instruction?

RQ4. How do teachers who attended a professional development course describe their attitudes, perceptions, and experiences with mathematics?

RQ5. How do teachers who attended a professional development course describe the effect of the experience on their learning, teaching practices, their perceptions of mathematics, and their perceptions of mathematics instruction?

The answers to these questions helped me plan mathematics professional development courses and modify existing ones to address the gap in practice in mathematics in the state. To date, no specific researchers have identified these elements in their studies and a qualitative case study supported the work to achieve this end.

Review of the Literature

The literature review was compiled after reading articles related to the following search terms: *math anxiety*, *professional development in mathematics*, *negative perceptions of mathematics*, *elementary mathematics teachers*, *constructivism in mathematics teaching*, *mathematics teaching*, and *professional learning in mathematics*. This literature reviewed contains a blend of current literature ranging from 2016 to 2020 along with seminal pieces from which the conceptual framework derives.

The review is organized in five sections. The first section outlines the conceptual framework for this study. The next section looks at literature referencing the effects of effective professional development in mathematics. Then I explore the range of support for teachers and how it affects teachers' professional learning and implementation of new teaching methodologies. Next, I review literature related to teachers' perceptions of mathematics as it relates to the problem identified in this study. Finally, as much of the literature focused on teachers' math anxiety, I review the current literature on the effects of math anxiety on mathematics teaching.

Conceptual Framework

There are two theories that form the conceptual base for this study. The first is the constructivist theory that originated from the work of Jean Piaget (1957) and has been developed by other theorists over the years such as Jerome Bruner (1966). Bruner like Piaget maintained that learning is an active process in which students draw upon prior knowledge and experience to construct new concepts or ideas. The other theorist whose work influences the conceptual base is Lev Vygotsky (1978), who also subscribed to the fundamentals of constructivism as he developed his own theory of socio-constructivism involving the zone of proximal development (ZPD). The ZPD in mathematics refers to the distance between the actual and the potential level of development in students (Danish, Saleh, Andrade, & Bryan, 2017). In other words, it is the area between what a student already knows and what he or she can reasonably learn next. When teachers provide students tasks that fall within their ZPD, students have more opportunities to make mathematical discoveries and develop a conceptual understanding of the material (Malik, 2017)

The mathematics professional development in this study is rooted in constructivism and emphasizes the importance of working within students' ZPD. This approach helps teachers see the value of instruction that focuses on students making meaningful mathematical connections (Mntunjani, Stanley, & Sibawu, 2018). Teachers who have a poor attitude toward mathematics do not use teaching methodologies that allow students to make sense of the material (Meryem, 2017). A constructivist approach to mathematics professional development can influence the attitudes of teachers toward the subject.

Using a constructivist approach in professional development promotes collaboration, participation, active learning, and reflection among participants especially when they are provided authentic tasks that explicitly connect to prior knowledge and understanding (Toraman & Demir, 2016). This has implications for teacher engagement in the sessions and more broadly to shifts in their learning. Additionally, when professional development providers use the constructivist approach with adult learners, they are modeling the same teaching methodologies that teachers should be using in their classrooms. Thus, participating teachers can experience constructivist teaching from the learner's perspective. For instance, though most teachers have experienced teacher-centered learning when they were students, they prefer teaching and learning that was student-centered, involved group work and collaboration, and aligned with constructivist views that promote active learning (Sahin & Özpinar, 2020). These findings should be considered by educators that design professional development programs.

Ulrich, Tillema, Hackenberg, and Norton (2014) emphasized the active and interactive nature of humans and how they affect learning. They contend that participants continually construct their own mathematical schemes from active experiences building upon and modifying prior systems of schemes. Another assertion from this study is that changes in schemes take time and multiple experiences. Therefore, any meaningful professional development programs must take these findings into account.

Using the constructivist framework, the professional development in mathematics must build on teachers' prior knowledge and experiences. Teachers should be engaged in meaningful tasks that allow them to collaborate with their colleagues and continually construct their learning. The professional development must also provide a variety of learning experiences over time, so teachers have opportunities to shift their thinking. With these structures in place, the professional development should provide teachers with the support they need to develop a more favorable view of mathematics.

In addition to the theories that inform the conceptual framework, there are many elements related to professional development and teachers' perceptions of mathematics that are applicable to this project study. These include elements of effective professional development in mathematics, support for teachers, elementary teachers' perceptions of mathematics, and math anxiety in elementary teachers. The following review of the literature will touch on each of these critical elements and discuss what is currently known in the field regarding this work.

Elements of Effective Professional Development in Mathematics

Many forms of professional development are available to teachers, and researchers have been working to identify which forms and elements are most successful. There is a lot of emphasis on outcomes of student performance in the classroom and on standardized testing. For example, Polly et al. (2018) found a correlation between teachers engaging in professional development in mathematics and an increase in their students' achievement. In an earlier study, Lindvall (2017) found that when professional development models are designed specifically for a particular audience (e.g., elementary teachers), there are positive results on students' achievement, but when the same model is applied to teachers from another grade band (e.g., middle school teachers), the results can be negative. A similar positive effect on student achievement was noted when teachers engaged in professional learning and had support through coaching (Hill, Bicer, & Capraro, 2017).

Other researchers also identified a link between teachers' mathematical content knowledge and its effect on students' achievement in mathematics (Reid & Reid, 2017). Reid and Reid (2017) and Garet et al. (2016) found that increasing teachers' mathematical knowledge for teaching had a positive effect on students' scores in elementary grades. Tchoshanov et al. (2017) also noted that students who had teachers with a strong conceptual understanding of mathematics scored far better on mathematics assessments than students with teachers who were considered to have low content knowledge for teaching mathematics. Additionally, Cueto, Leon, Sorto, and Miranda (2017) found a link between teachers' mathematical content knowledge and student achievement and stressed the importance of teachers having strong pedagogical knowledge as well.

Another key element of professional development that has had a positive effect on student achievement is the direct application of the work to the actual classroom (Jayanthi, Gersten, Taylor, Smolkowski, & Dimino, 2017). Though making pedagogical shifts in teaching practices can create more work as teachers need to revise or recreate lesson plans, when teachers create lessons during the professional development sessions that have an immediate application to their classrooms, teachers have used higher-level problem-solving instructional strategies, and students have performed better on assessments (Jayanthi et al., 2017). Allowing teachers to plan for instruction while engaged in professional learning reduces some of the workload of teachers and encourages them to implement the new ideas in their classrooms.

Structures for Professional Learning

Other researchers have focused on the effect professional development had on teachers and identified critical elements professional development providers should consider. Prast, Van de Weijer-Bergsma, Kroesbergen, and Van Luit (2018) found that multidimensional professional development focused on math knowledge, diagnostic competence, teaching methods, and classroom management improved teachers' skills at differentiating instruction for their students. This approach modeled how to differentiate instruction for students by giving teachers an opportunity to experience differentiated and dynamic professional learning. When teachers are given time to collaborate with grade level teams in a workshop format, they are more likely to implement new teaching practices in their classrooms (Timmons-Brown & Warner, 2016).

Further, guided inquiry during professional development has improved teachers' mathematical knowledge and teaching practices (Chin, Lin, & Tuan, 2016). When teachers experience guided inquiry from the lens of a student, they develop a deeper appreciation of such methodologies and are more likely to implement them in their classrooms. Additionally, explicit work on guided inquiry can lead to teachers learning to develop and facilitate engagement with inquiry-based mathematical tasks (Cosby, Horton, & Berzina-Pitcher, 2017).

Intensive mathematics institutes or mathematics studios can also help teachers learn the necessary skills to teach mathematics effectively in the classroom (Lesseig, 2016). The confidence gained by learning the mathematics content through specific constructivist approaches has increased the integration of these methodologies in participants' classrooms. The institutes and studios give teachers access to many different instructional strategies with which they previously had little or no experience. There is a positive correlation between these professional learning models and teachers' confidence in teaching various mathematics concepts (Lesseig, 2016).

Effect on Student Learning

Current research on professional development in mathematics and science has shown how it affects student learning. A combination of collaboration, professional reading, clear documentation, critical colleagues, and time can help teachers learn about and implement teaching practices that support student-centered learning (Davidson, 2019). This work is critical because districts and schools are spending a lot on professional development, and it is helpful for them to know what kinds of professional development have the strongest influence on student learning gains. Additionally, improving and enhancing professional development is an important goal in education.

Many researchers and practitioners have tried to merge theory and practical knowledge to provide specific suggestions for improving professional development, but there may be a lack of empirical evidence to back up these arguments (Hill, Beisiegel, & Jacob, 2013). Most short-term efforts in professional development lack empirical evidence of success and are not supported by expert opinion, creating the need for more comprehensive, content-focused programs (Hill et al., 2013). However, evidence of more robust professional development on student achievement requires further research (Hill et al., 2013).

Though there is a lack of research on which elements of professional development are most successful, there has been evidence that programs focused on both content and pedagogical knowledge had a more significant influence on student gains than those that focused on only content knowledge or only pedagogical knowledge (Hill, 2007). Another aspect of student learning that has been explored was students' abilities to engage in small group collaboration as learners (Tabach & Schwarz, 2018). When teachers have been given opportunities to experience facilitated learning, they have developed better skills at facilitating learning for their students (Tabach & Schwarz, 2018). Additionally, collaborative models of professional development have supported teachers' use of effective teaching strategies and have had a positive effect on student performance (McNeill, Butt, & Armstrong, 2016).

Support for Teachers

A critical component of professional development is the implementation of new learning in the classroom along with support from instructional leaders. Drust (2013) highlighted the effectiveness of having strong leadership in schools along with flexibility and support for new teaching methodologies in assuring professional development improved student achievement. Gersten, Taylor, Keys, Rolfhus, and Newman-Gonchar (2014) found that involving school leaders with a team of mathematics experts and professional development providers to design and implement the professional development experiences allowed for a more thoughtful and cohesive approach that resulted in long-lasting change. On-site professional development supported by administrators and other instructional leaders has also be shown to benefit students and encourage teachers to avoid isolated teaching practices (Gersten et al., 2014). Samford (2013) shared that for teachers to make a new teaching strategy part of their regular repertoire, they need to know the theory behind it, see the strategy demonstrated, relate the new practice to current strategies, and receive support on-site during their initial attempts to implement the new approach.

Another form of ongoing support for teachers is professional development with a mathematics coaching component. Simpson and Linder (2014) discovered that implementation of new teaching strategies learned in professional development sessions was greatly increased when supported by a coaching model. Matching coaches and teachers together in the same professional learning experiences also proved to be effective in supporting long-term implementation of new teaching methodologies because the coaches were able to use the experience to support the teachers in their work (Parker, 2014). Gibbons and Cobb (2017) found that many activities math coaches and teachers can do together, including book studies, co-planning, rehearsals, and observations with feedback fit the criteria for high quality professional learning. However, the authors noted that some activities, like coaching cycles are often not fully implemented because teachers and coaches find scheduling all the components of it difficult.

Effective teaching hinges on a process of continuous learning on the part of the teacher through professional development opportunities, support with implementation from the school system, and comprehensive efforts of the entire community to create a strong learning community (Day, 2013). Additionally, adding components that involve teachers reading research on innovative teaching practices is an essential part of ongoing professional growth for educators (Jansen & Bartell, 2013). Additionally, teachers must be allowed and encouraged to pursue personal professional development to improve their practice and pedagogical work (Postholm & Wæge, 2016)). This autonomy provides ownership of the professional learning and is more likely to lead to actual change in practice. School system administrators should welcome teachers' input into their own personal learning goals in an effort to get more teacher buy-in and strengthen the likelihood that the professional learning will lead to sustainable change.

Elementary Teachers' Perceptions of Mathematics

Research regarding elementary teachers' perceptions of mathematics indicates that negative feelings and even some degree of math anxiety are present even before teachers enter the profession. Mcdermott and Tchoshanov (2014) asked preservice teachers to draw themselves teaching and learning mathematics and most of the images were of a negative nature. Mcdermott and Tchoshanov also noted that a lack of real-world connections to mathematics was a large contributing factor of this negativity. Shriki and Patkin (2016) also found a lack of intrinsic motivation among elementary school teachers to attend professional development in mathematics due to teachers' inability to see the connections between the professional development and their own growth as math teachers.

Other contributing factors toward teachers' negative perceptions of mathematics and resistance to change are the cultural routines and patterns associated with schools, teaching, and learning that have been embedded in the culture of the United States as soon as students enter formal schooling (Le Cornu, 2016). In other words, teachers have spent thousands of hours in schools as students and have preconceived notions of what mathematics teaching looks like and what their role is in teaching it. Professional development and teacher education can work to help teachers shift those beliefs as Knaus demonstrated in a 2017 study. Knaus (2017) found that when teachers' content knowledge improved through professional learning, their perceptions of mathematics became more positive.

Teachers' perceptions and attitudes toward mathematics could be positively influenced through collegial reflections on mathematical structures and relationships (Luebeck, Roscoe, Cobbs, & Scott, 2017). In professional development settings, this means providing ample time for teachers to interact with colleagues to articulate their understanding of mathematical structures and relationships, connect these ideas to concrete representations, and apply these new understandings to their grade level content.

The research by Brown (2016) revealed that there is a positive relationship between elements of cognitively guided instruction in professional development settings and a change in teachers' negative perceptions of mathematics. Brown's research included 56 teachers involved in four 5-week cycles of intensive mathematics professional learning. When teachers engaged in this work during professional learning experiences, they developed a more positive disposition toward mathematics (Brown, 2016).

Interesting discoveries about teachers' perceptions on their own mathematical abilities came from Pain (2015). Pain found that significantly higher student achievement in mathematics is influenced by teachers' perceptions of the meaningfulness in a professional development program. Levi-Keren and Patkin (2016) discovered that teachers' perceptions of mathematics and professional development improved when they could see a clear benefit for their students. Sawyer (2017) found that a combination of professional development, classroom teaching experiences, and past experiences as a student contribute to teachers' positive perceptions of mathematics.

In another study, Gresham (2018) discussed the factors that contributed to a positive attitude toward mathematics. One key factor was the amount and quality of professional development experiences for teachers. Additional factors that supported positive attitudes were years of teaching, smaller grade ranges, departmentalization, and repeated years teaching the same grade level. The implications of Gresham's study for districts highlight the need to provide consistent and ongoing professional development as well as supporting teachers by allowing them to focus on one grade level for several years.

Literature pertaining to elementary teachers' perceptions of mathematics is limited. What is available raises the issue that negative perceptions do exist and there is some correlation between professional development experiences and a change in teachers' perception over time. However, there is a dearth of information regarding what specific elements of the professional development experience have the most influence over this change. This is a key reason the project study was designed to investigate this question.

Math Anxiety in Elementary Teachers

In searching for articles pertaining to teachers' perceptions of mathematics, I found studies about math anxiety and elementary teachers. One key factor influencing teachers' negative perceptions of mathematics is math anxiety (Andrews & Brown, 2015). Many of the researchers studied preservice teachers, but the findings are consistent with research on inservice teachers regarding the effects of math anxiety and methods for reducing it.

Math anxiety is common at colleges and universities, especially among elementary preservice teachers (Karunakaran, 2020). Math anxiety manifests itself when the sufferer encounters quantification or other forms of mathematics (Andrews & Brown, 2015). Symptoms include nausea, hearth palpitations, and paralysis of thought. To understand the factors that contribute to math anxiety, Andrews and Brown (2015) examined math anxiety of preservice teachers. Andrews and Brown used the 9-Item Abbreviated Math Anxiety Scale with 180 college freshmen. The Abbreviated Math Anxiety Scale was developed by Vahedi and Farrokhi (2011). Andrews and Brown (2015) concluded that female students exhibited more anxiety than male students in college mathematics classes and non-traditional students exhibited more anxiety than students between the ages of 18 and 22. With regard to educational majors, the researchers found that students who majored in early childhood exhibited more math anxiety than secondary education majors. An implication of Andrews and Brown's study is that new early childhood and elementary teachers are likely to require significant professional development to overcome or reduce their math anxiety. Stoehr (2017) noted that a challenge for teachers with math anxiety is that while professional development may help to reduce anxiety, teachers with it may avoid professional learning experiences in mathematics.

Geist (2015) looked at the effects of math anxiety on teachers' perceptions of their mathematics abilities and how teachers approach the teaching of mathematics in the classrooms. The results of the study showed that the more anxiety a teacher possesses, the less he or she views himself or herself as capable in mathematics. Additionally, Geist found that the more confident a teacher is in his or her ability at mathematics, the more likely the teacher is to use developmentally appropriate methods of teaching in the classroom. Schubert (2019) found that a combination of poor teaching practices, math anxiety, and negative attitudes toward mathematics can be passed down to students from teachers.

The results of Geist's (2015) study showed a marked difference in anxiety levels among inservice and preservice teachers who participated in courses that developed mathematics content knowledge. Factors that specifically affected a decrease in anxiety included the instructor's methodology, field experience and peer group lessons, the instructor's disposition, and the classroom atmosphere. Ruef, Willingham, and Sweeny (2019) found that math anxious teachers can feel better about the subject when given opportunities to develop a conceptual understanding of it.

Math anxiety is a crippling condition for mathematical learners and teachers. The literature indicates that a cyclical structure exists whereby students with math anxiety who grow to become teachers with math anxiety, pass on their anxiety to students through poor instructional methodologies based on their limited knowledge and understanding of the mathematics they teach. The suggested treatments for math anxiety as presented in the research can be applied to professional development experiences to address this issue with inservice teachers.

Conclusion

The review of the literature confirmed the existence of negative perceptions of mathematics among elementary teachers and indicated that a correlation between professional development and changes in perception exists. However, research on how specific elements of professional development experiences affect teachers' perceptions is limited. Studies on the components of professional development focus on how the elements affect student achievement or teachers' knowledge. While this information is helpful, it does not help professional development providers who want to create courses that address the issue of negative perceptions.

The correlation between negative perceptions, math anxiety, and less effective instructional methodologies is strong. If professional development providers seek to change teachers' practices, then knowing how different elements from professional development experiences affect teachers' perceptions is critical. If providers of professional development can identify those elements that have the greatest influence, then they can highlight them when developing courses, workshops, seminars, and other forms of professional development for teachers.

Implications

This study has implications for me and my staff in the local context, but it also has implications in the larger context for mathematics coaches, professional development providers, and higher education faculty that work with preservice and in-service teachers. By learning the components and subtle nuances of mathematics professional development that contribute to either a positive or negative perception of mathematics, educators that design these experiences will have better knowledge of what works to create positive perceptions and what hinders teachers' perceptions of mathematics. This information can help professional development designers modify existing professional development experiences as well as create new versions in ways that develop more favorable perceptions of mathematics and mathematics teaching.

As a result of this project study, existing courses and professional development experiences in the program will be modified to better meet the needs of the teacher participants. The information from this study will inform revisions to individual courses as well as the overall structure of the graduate program. The findings from this study will also be shared with students in the courses because most of them are teacher leaders in mathematics within their schools and are responsible for creating and leading professional development in their districts.

It is possible that the findings could lead to creating a project that entails modifying the same course that the participants in the study took because the data will be most specific toward that experience. We run that course multiple times during the school year so changes could be implemented right away. Depending on the results of the study, modifications can be made to the mathematical explorations, the case study analyses, the teaching practices, and/or the teacher reflection elements of the course.

In addition to local implications for this study, there are a variety of options for how the information learned from this project study can be shared with the greater community of professional development providers in mathematics. One possibility is to present the findings at the National Council of Supervisors of Mathematics Annual Meeting. This information would be valuable in professional circles of teacher educators who are charged with creating engaging and effective professional development sessions in mathematics.

Summary

Educators have a crisis in mathematics education in the United States that is affecting students in the K to 12 systems (Jordan, 2014). This crisis is in part due to an inconsistent level of training and expertise on the part of K to 5 teachers (Tasdan & Koyunkaya, 2017). Elementary teachers tend to be generalists by trade and some lack the deep level on training in mathematics education needed to teach that subject well. Additionally, many elementary teachers report a dislike for the subject of mathematics (Andrews & Brown, 2015).

To address the inconsistent level of training mentioned above, it is imperative that teachers have significant professional development experiences to improve their mathematical content and pedagogical knowledge (Reid & Reid, 2017). Various forms of professional development in mathematics are available with varied levels of effectiveness and appeal to teachers (Barnes & Solomon, 2014). To design and implement the most effective forms of mathematics professional development, educational leaders must identify the components of these experiences that are most effective in altering teachers' negative perceptions of mathematics and mathematics instruction (Andrews & Brown, 2015). Therefore, this qualitative study is being proposed with the purpose of understanding the components of professional development in mathematics that contribute to positive shifts in teachers' perceptions of mathematics.

The following sections will address the remaining components of the project study. In Section 2, the methodology will be explained in detail. Section 3 will address the project itself along with its implications. Finally, the paper will conclude with Section 4 where I will include my reflections on what was learned and what still needs further study.

Section 2: The Methodology

The local problem prompting this study was teachers' negative perceptions of mathematics, which was a focal point for both me and my teaching staff at the local college as we considered how to revise the professional development courses. To gain a better sense of the key components of the courses that contribute to reducing negative perceptions and increasing positive ones, it was most helpful to have in-depth interviews with participants before and after their attendance in a course in last summer's session at the local college.

Research Design and Approach

This qualitative case study was conducted to understand the components, if any, that contributed to developing teachers' positive perceptions of mathematics. This method was selected over others because the purpose of the project study was to gain a deeper understanding about the courses by studying a group of teachers who had a common experience. Subjects were interviewed at the beginning and end of a mathematics course focused on developing teachers' mathematical content knowledge to gauge how the mathematics professional development affected their perceptions of mathematics. An analysis of the data was conducted to look for common themes.

I considered other qualitative research methods at the onset of this study; however, they did not seem as effective when considering the goal of understanding how the experience of a group of teachers affected their perceptions of mathematics instruction. An ethnographic study would not have been appropriate as I was not seeking to understand how a small group of people was influenced by society but to gain a better understanding of a group of people who were having the same experience at the same place and time (Creswell, 2009; Merriam & Tisdell, 2015). Similarly, a phenomenological study would not have been appropriate because I was not exploring a phenomenon and considering the experiences of those related to it (Merriam, 2009). Finally, grounded theory was also deemed inappropriate for this study because I was not seeking to develop a theory from the research but instead planned to gain a better understanding of teachers' perceptions (Creswell, 2009; Patton, 2015). None of these other methods would have supported my goal of developing a better sense of how a specific group of participants were affected by participating in a course and how that experience influenced their perceptions of mathematics.

The decision to use a qualitative study over a quantitative study was made primarily because my goal was to understand the subtle nuances of teachers' experiences in the program rather than a statistical analysis of those experiences. I had considered descriptive survey research because it does aim to describe people's behaviors, perceptions, attitudes, and beliefs about a current issue in education (see Lodico, Spaulding, & Voegtle, 2010). However, this approach would not have allowed me to look for themes that emerged from the way participants described their experiences, which was a key goal of my project study.

I also considered experimental research to see if one design of professional learning had a greater outcome over another design (see Lodico et al., 2010). Though these quantitative methods might have allowed me to measure the effectiveness of a particular course design or measure a correlation between certain variables, these approaches would not have supported my goal to develop a broad and rich understanding of a particular group and their experience in a specific setting (see Lodico et al., 2010). Thus, I made the decision to focus on a qualitative case study method for this work.

In this qualitative case study, I sought to understand specific aspects of professional learning in mathematics that have the greatest influence on teachers' perceptions toward mathematics and mathematics instruction. The qualitative case study method was ideal for this study because case studies provide individual insights around an event through multiple sources of evidence (Yin, 2014). Case study research is used to address a phenomenon like a program, activity, professional learning engagement, situation, or event (Hancock & Algozzine, 2006). In this study, I examined a professional learning course as a specific event and focused on how elements of this course affected the teachers participating in the course. Additionally, case studies are bound by place and time (Patton, 2015), which in this study was a specific professional learning course at a specific institution. Therefore, the characteristics of a case study were aligned to characteristics of the study.

Participants

Subjects for this study were selected from teachers who attended one of the mathematics professional development courses hosted at the local college during the 2018 summer session. Subjects were five teachers from Grades K to 5 from around the country. My goal was to have at least five teachers participate in this study to give a broad range of perspectives, and between four and 10 teachers is ideal for a qualitative case study if one is looking to conduct an in-depth analysis (see Stake, 1995).

Of the five who participated in the study, three were experienced teachers who have had a significant number of professional learning experiences in mathematics. They chose the program to deepen the knowledge and perfect their craft as skilled math teachers. One participant was a veteran teacher who did not have a lot of professional learning experiences in mathematics. The final participant was a new teacher who also did not have much experience with professional learning in mathematics.

Procedures for Gaining Access to Participants

Registration for the summer courses occurred between March and June of 2018. When teachers registered for the program, they provided their name, e-mail address, school mailing address, and phone number. Once I received Walden University institutional review board approval (Approval No. 06-28-18-0163288), I contacted the registered participants via e-mail and shared the participant letter to let them know about the study and asked if they would like to participate. The letter also included a statement of consent. They were instructed to submit the signed consent form to both me and my administrative assistant via e-mail.

This process continued until I had eight participants who agreed to the study. I was hoping for between six to eight participants for the study in case a few needed to back out. This was a good strategy because I had three teachers decide they could not participate. Each participant was then contacted to schedule an interview before the start of the course and another interview at the completion of the course.

Methods for Establishing Researcher-Participant Working Relationship

I am the director of the mathematics professional development program at the college. In this role, I create and modify courses; hire, train, and supervise the teaching faculty; recruit participants; and oversee all logistical planning for the courses and the summer program. I typically do not teach the courses, so I did not have any direct or perceived power over the participants. Grades and feedback came directly from the course instructors.

To establish personal connections with the summer program participants, I communicated with them prior to coming to campus via e-mail and through video messages. I also set up phone calls and video chats with those who preferred to meet to ask questions or work through some logistics. As a result of this early communication, most participants knew me and established a personal connection with me before the summer courses began. Additionally, once participants were on campus, I met with them informally as I visited classrooms and when we had breaks and ate lunch together.

Through these personal connections, I hoped to put participants in the study at ease, so they were comfortable talking with me during the interviews. Additionally, the interviews were conducted online or in the conference room outside my office. The online option and conference room location made it convenient and allowed participants to have some privacy from the colleagues during the interviews. If a participant was not comfortable for any reason prior to or during the interviews, they had the option to back out of the study at any time with no consequences. I had two participants back out, but both were due to scheduling issues.

Measures for Ethical Protection of Participants' Rights

Measures for ethical protection of the participants began when I received approval from the institutional review board at both Walden University and the local college where the study occurred. Other measures included informed consent and confidentiality. All participants were briefed on the details related to the study and had the option to withdraw from the study without penalty. I distributed and collected consent forms prior to the first session for active participants attending the course last summer. The course facilitators did not have knowledge of who chose to participate, and they did not have access to any data collected pertaining to the study. This ensured that participants' grades were not swayed as a result of participating or not participating in the study.

All participants' information was and will be kept confidential and under lock and key. Anytime I reference participants in the study, pseudonyms are used in place of real names. At the conclusion of the project, all identifiable information will be locked in a filing cabinet in my locked office. Only I have access to my office. After 5 years, the information will be destroyed. There is a low risk of harm for participants in this study because the coursework and professional development experience were the same that all participants received regardless of their participation in the study.

Data Collection

Interviews consisted of open-ended questions and were scheduled with the participants of a mathematics professional learning course at the college in 2018 who agreed to participate in the study. The interview questions were designed to help me gauge the perceptions of mathematics prior to taking the course and how their experiences in the course influenced their perceptions of mathematics and mathematics instruction (see Appendix B). The purpose of opened-ended questions was to capture details and nuances from participants (Patton, 2015). All participants who signed informed consents and who were interviewed are included in the study.

The one-on-one interviews were conducted by me prior to the start of the course and again at the conclusion of the course. The questions for the interviews were self-designed and contained questions aimed at understanding the elements of the professional learning that had the greatest influence on the participants. To establish sufficiency of the data collection instruments to answer the RQs, the interview questions were aligned to the RQs (see Appendix B).

Interviews were recorded and transcribed by me at the conclusion of each interview. Transcriptions were done in Microsoft Word, printed, and then put in a binder separated by dividers for each participant. The binder is kept in a locked file cabinet. Each recording was stored on an external hard drive that was password protected. The hard drive is currently stored in a locked filing cabinet. Once all the interviews were completed, I began coding them.

I read the transcripts from the printed pages in the binder and noted emerging understandings on sticky notes that marked passages of interest. These emerging understandings were then recorded in a Microsoft Word document and digital pictures of the sticky notes were included as a backup. Each subsequent review of the transcriptions was included in another section in the Word document and was cataloged accordingly.

Role of the Researcher

My role at the college began as a visiting lecturer in the Psychology and Education Department. In that capacity, I created and taught the mathematics methods course for the undergraduate students in the teachers' licensure program. I was then hired as the assistant director of Professional and Graduate Education Programs in mathematics and later promoted to director of the same program. In this capacity, I run the professional development program and the Master of Arts in Mathematics Teaching program.

My work involves creating and modifying courses; hiring, training, and supervising the teaching faculty; recruiting participants and graduate students; and overseeing all logistical planning for the courses and programs. My direct role with participants in the professional development courses is limited to scheduled appointments or calls prior to the start of the course and informal conversations during breaks and lunch once the course has begun. My role with graduate students is more formal as I also serve as their advisor and instructor at times.

I was concerned that my role as advisor and instructor in the graduate program might affect or influence how participants might respond. To address this concern, I ensured that the participants in the study were only teachers taking the course for professional learning or graduate students who were no longer taking courses from me. Additionally, I ensured that I would not teach the professional development courses and would not have any direct control over participants' learning experiences, grades, or any other type of feedback they received in the course.

Data Analysis

I used narrative data analysis techniques to analyze the data collected from the interview transcriptions (Hancock & Algozzine, 2006). This approach was chosen because it is an inductive process that allowed me to move from detailed data to generalized codes and themes (Connelly, 2013). A key component of this study was to gauge teachers' perceptions of mathematics. Therefore, it was important that data were coded and analyzed first in a way that considered the participants' perspective. The data also needed to be coded and analyzed using my expertise and perspective and then compared with the results from the first analysis (Merriam, 2009). Repeating these steps helped create comprehensive concepts that could be integrated into a coherent explanatory model (Bazeley, 2009).

To conduct this analysis, I transcribed each recorded interview using Microsoft Word and reviewed the transcriptions to ensure accuracy. This process took significantly longer than I anticipated, and I experimented with multiple formatting structures, but this process helped me analyze the data because of the number of times I needed to listen to each participant's responses. I read through each transcription to first get a general sense of the material and noted emerging understandings on sticky notes that marked passages of interest (Miles, Huberman, & Saldaña, 2013). Next, I went back through the transcriptions and divided the text into segments of information before labeling the segments with codes (Miles et al., 2013). These emerging understandings were then recorded in a Microsoft Word document and digital pictures of the sticky notes were taken as a backup. Each subsequent review of the transcriptions was included in another section in the Word document and each review was cataloged accordingly. I conducted another round of review then reduced overlap and redundancy of codes and then collapsed the final codes into themes (Patton, 2015).

Accuracy and Credibility

Several steps were taken throughout the process to ensure my findings were both accurate and credible. These assurances are important in a qualitative study because researchers must convey a sense of trustworthiness and promote confidence that they have accurately captured the event or phenomenon being studied (Shenton, 2003). The key step in this process is implementing tactics to help ensure honesty from participants when they are being interviewed (Shenton, 2003). These steps include establishing a rapport with participants to put them at ease, reminding participants that there are no right or wrong answers, conveying to participants that the researcher has an independent status and cannot affect their grades or credibility in the program based on responses, and reminding participants that they can withdraw from the study at any time. I made sure to do each of these steps with the participants (Shenton, 2003).

Additionally, I implemented peer debriefing to broaden my perspective and recognize any biases or preferences I may exhibit (Shenton, 2003). My colleague who is a well-known researcher in the field of mathematics education agreed to debrief with me after I conducted interviews and after rounds of data analysis. She also helped me identify flaws in this study (see Shenton, 2003). This colleague signed a confidentiality agreement before assisting with the study.

Establishing transferability is an important step in conveying the trustworthiness of the data. Transferability is challenging in a qualitative study because the small number of participants and environments (Merriam, 2009). However, it is important for the researcher to convey as much information about the setting, participants, researcher, methods employed, number and length of data collection sessions, and the time period over which the data were collected as a means of supporting attempts at transference (Cole & Gardner, 1979). Then when similar studies are done, the accumulation of the findings will contribute to a larger understanding that applies to a greater population (Shenton, 2003).

This issue of dependability is equally challenging in a qualitative study because the published descriptions apply to a specific event and population (Shenton, 2003). This challenge can be addressed by describing the elements of the study in great detail so future researchers can replicate it (Shenton, 2003). I included detailed descriptions of the design and implementation of the study as well as reflective appraisal of the project based on the recommendation of Shenton (2003).

To ensure confirmability, I followed the recommendation of Shenton (2003) who stressed the importance of detailed methodological descriptions so readers can determine if the data emerging from a study can be accepted. One method I employed was to include an audit trail so readers can trace my steps and perhaps replicate the methods in a future study (Shenton, 2003). The audit trail is in the form of a detailed diagram focusing on the data and how the data were gathered.

Discrepant Data

Accounting for discrepant data is a critical step that allows researchers to broaden and confirm patterns emerging from data analysis (Patton, 2015). I planned to address discrepant data through deviant case analysis where I would work to refine my analysis until I was able to specify the reasons for the discrepant data (Patton, 2015). This might have required multiple rounds of analysis but would have helped account for instances when data gathered from a participant differ greatly from the rest of the sample (Patton, 2015). Although I had a plan for discrepant data, there was not any in the study.

Data Analysis Results

The impetus of this study was the problem that many elementary teachers seem to have negative perceptions of mathematics and this, in turn, affects how they teach the subject. This problem led to the development of the RQs aimed at understanding why teachers feel the way they do about mathematics. From there, I designed interview questions for participants before they came to campus for the summer courses to gauge their perceptions of mathematics and to understand why they feel the way they do about mathematics.

To analyze the data, I began by reading each transcription and highlighting repeated words and phrases. Through continuously reading the text and reflecting on the similarities and differences between the responses, emerging patterns and themes developed. I examined and interpreted these themes and patterns to see how they contributed to answering my RQs (see Lodico et al., 2010).

Three themes emerged from the data for RQ1, two themes from RQ2, three themes from RQ3, two themes from RQ4, and three themes from RQ5. From these themes, I identified seven findings which address the five RQs and the local problem which initiated the study. In the section that follows, I discuss the findings and support them with examples from the data gathered from the interviews. I assigned pseudonyms (Participant 1-5) to protect the identity of the participants. Prior to the start of the course, the five participants fell into two groups. Three of them had a more positive disposition toward mathematics while two of them expressed negative feelings. The positive group was made up of more experienced teachers who had lots of experience with professional learning in mathematics while the other two (one veteran and one novice teacher) had no prior professional leaning in mathematics. I will describe these two groups as the experienced participants (Participants 1-3) and the inexperienced participants (Participants 4 & 5). In reviewing the transcripts from the interviews, I uncovered various themes and findings that relate to the local problem and the RQs. I organized the themes and findings in a table (see Table 1) to make the connections between the themes and finding clear. The narrative analysis that follows discusses the themes and findings in detail.

Table 1

Themes and Findings

Research questions	Themes	Findings
RQ1: Prior to attending the professional development course, how do teachers describe their attitudes, perceptions, and experiences with mathematics?	Less experienced teachers have negative perceptions of mathematics, Teachers' prior experiences with mathematics as students was didactic and emphasized rote memorization. Teachers did not have a love of mathematics as students.	1. Teachers' prior experience with mathematics contributed to a dislike of or ambivalence toward mathematics. 2. As teachers gain more experience with mathematics through professional learning and work with students, their perceptions of and attitudes toward mathematics improve.
RQ2: How do teachers who attended a professional development course describe their mathematical content knowledge?	Teachers feel they have an adequate understanding of basic mathematics. Teachers recognize their conceptual understanding of mathematics needs development, especially related to fractions and integers.	3. While teacher participants entered the profession with adequate knowledge to functionally do mathematics, they lacked the deep conceptual understanding needed to teach math at a much deeper level.
RQ3: How do teachers who attended a professional development course describe their understanding of effective mathematics instruction?	Teachers recognize the way they were taught (emphasis on rote memorization) is not effective in supporting students' conceptual understanding. Teacher see their role as being facilitators of learning rather than deliverers of knowledge. Teachers understand the importance of visual representations and concrete manipulatives as sense-making tools for students.	4. Teachers see that they need to teach for conceptual understanding but recognize that requires them to deepen their own conceptual understanding of mathematics as well. 5. Teachers need more experience with concrete tools and visual representations as learners, so they know how to better use them with students.
RQ4: How do teachers who attended a professional development course describe their attitudes, perceptions, and experiences with mathematics?	Teachers have a more positive perception of mathematics following their experience in the course. Teachers feel like they now see more possibilities for engaging their students in mathematics.	6. As teachers deepen their understanding of mathematics, their feelings and attitudes toward the subject become more positive.
RQ5: How do teachers who attended a professional development course describe the effect of the experience on their learning, teaching practices, their perceptions of mathematics, and their perceptions of mathematics instruction?	Small group work with colleagues was integral. Strong course facilitators that guide the learning rather than present knowledge. Rich mathematical tasks contributed to their engagement and learning as teacher participants.	7. Teachers want opportunities to engage in rich tasks in small groups, so they experience mathematics as learners while an experienced facilitator guides the learning.

It was clear from all the interviews, that everyone had math experiences as students that emphasized rote memorization, fact fluency, worksheets, lots of direct instruction from their teachers and mostly individual work with very little collaboration with classmates. While the inexperienced participants said this type of math created anxiety and a strong dislike for the subject, the experienced participants were more ambivalent about it. They did not hate math, but they also did not think it was very exciting. All participants agreed that this way of experiencing math was not what any of them want for their students.

Despite the traditional nature of their experiences as math learners, the three experienced participants did report having a positive perception of the subject as adults. They attributed it to developing a better understanding of mathematics as teachers which made them appreciate the subject more. The two inexperienced participants mentioned that math still gave them feelings of anxiety and it was not their favorite subject to teach. The inexperienced teachers attributed the high-pressure nature of how math was taught and the fact that they were never really taught to understand the content. The focus was on memorization, so math was not something ever really understood.

All participants reported that they believed their content knowledge of math was adequate but added that their knowledge was mostly procedural. None of them reported feeling they were experts in the field, but they felt they had enough content knowledge to teach elementary mathematics. A thread that seemed to emerge was a sense that participants were okay with memorizing procedures in school until the mathematics become more complex in high school and then things became difficult for them. This was a common story and led participants to develop the sense that math needs to be taught differently.

The three experienced participants expressed confidence when asked about how they feel about teaching math while the two inexperienced participants were more hesitant. It should be noted that the three participants have had significantly more professional learning experience than the other two and they referenced those experiences as having an influence on how they teach math. When asked what their teaching was like before those professional learning experiences, the experienced teachers reported that they taught in traditional ways that emphasized rote memorization rather than conceptual understanding.

None of the participants referenced any meaningful experiences in the teacher preparation that supported them to be more effective math teachers. Two of the experienced participants even said they did not have any math methods courses as part of their initial licensure program. It was difficult to glean much from this portion of the interview because the participants struggled to recall much from their undergraduate experiences in mathematics.

In general, prior to the start of the course, the experienced participants had a positive disposition toward mathematics but were not necessarily teaching the way they knew math should be taught. They registered for the course to learn how to teach math in a way that developed students conceptual understand and enjoyment of the subject. The inexperienced participants had more of a negative disposition toward math but were hopeful to change that through their experiences with the course.

Research Question 1: Teachers' Attitudes, Perceptions, and Perceptions Prior to Course

RQ1: Prior to attending the professional development course, how do teachers describe their attitudes, perceptions, and experiences with mathematics?

Finding 1. The first finding revealed that teachers' prior experiences with mathematics as students contributed to either a dislike of the subject or ambivalence toward it. As stated earlier this is largely due to the didactic nature of the math instruction the participants received when they were students along with an overemphasis on speed and accuracy. One participant, Participant 4, captured this sentiment by stating, "That aspect (emphasizing speed) always felt a little out of place to me because I've always been someone who liked to take time to consider things and our math program emphasized a lot of processing speed." This overemphasis on speed during early math experiences was consistent with each participant.

Participant 4 and other participants attributed this emphasis on speed to the feelings of math anxiety they have as adults. Another related aspect was the lack of relevance and contextual connections in mathematics class. All participants made references to mathematics being isolating and lacking real-world applications. During Participant 5's interview, the participant said, "Math was presented in a way where there weren't any relationships or connections to people or the world. So, everything seemed really out of context." Without direct connections to contextualized situations, participants were often taught mathematics as a series of abstract ideas without rich meaning.

Finding 1 is critical to the project because it reveals that many participants come to the courses with either neutral or negative perceptions of mathematics. Although participants come from various locations throughout the country and have all had unique experiences as mathematical learners, they had common negative experiences that contributed to their negative perceptions of mathematics. This finding also reinforced the local problem I identified which stated that many classroom teachers have negative perceptions of mathematics.

Finding 2. As teachers gain more experience with mathematics through professional learning and working with students, their perceptions of and attitudes toward mathematics improve. This finding emerged in both the pre and post-course interviews. Prior to the course starting, the inexperienced participants described their negative feelings toward the subject. These participants had not had many professional learning experiences in mathematics and lacked significant experience working with students. However, after the weeklong course, both inexperienced participants expressed a significant shift in their perceptions and attitudes toward mathematics.

In his post-course interview, Participant 4 stated, "Math now provokes more of a sense of possibility than in the past. I think probably because we dove very deeply into how these ideas developed in a very non-linear fashion." Participant 4's description highlighted a structure of the course that allowed for meaningful connections between lots of different mathematical ideas. The math work in the course is generative, meaning the participants generate many of the key mathematical ideas identified in the course objectives. As a result, the discussions weave together a mosaic of different yet related ideas about particular problems or areas of mathematics.

During the pre-course interviews, the experienced participants described how their feelings toward mathematics improved as they engaged in professional learning and gained more experience through their work with students. They all came to the course with prior experiences in mathematics professional learning, so their perceptions and attitudes were more positive at the onset compared to the inexperienced teachers. However, even the experienced participants reported a significant change in their perceptions based on their work in the course.

Participant 1 described her perceptions of math after taking the course in the following way: “I get excited. I feel challenged when I’m asked to do math problems or tasks and I really like that. I definitely like to work through things to figure them out so overall math is a pretty pleasant experience for me.” Participant 1’s description highlights the richness of the math tasks from the course. The math work teachers do in the course are either open-middle or open-ended tasks. Open middle refers to tasks where everyone starts with the same problem that has a specific solution, but they can approach the problem in any way that makes sense to them. Open-ended tasks have various solutions and/or conjectures and allow for lots of exploration. Participant 1 found this work to be extremely engaging and the participant became more excited about doing math this way.

In her post-course interview when asked about perceptions of mathematics, Participant 2 said, “I just love it. I’m excited to do it. I love getting a problem that I don’t know the answer to, and I really like doing the math with other people.” This was a significant change from the participant’s first interview where the participant said math was “often just something you did to get an answer.” By the end of the course, teachers saw math as more than answer-getting and appreciated the exploratory nature of the work.

Research Question 2: Teachers’ Descriptions of Their Mathematical Content Knowledge

RQ2: How do teachers who attended a professional development course describe their mathematical content knowledge?

Finding 3. While teacher participants entered the profession with adequate knowledge to functionally do mathematics, they lacked the deep conceptual understanding needed to teach math at a much deeper level. This sentiment was echoed across all interviews both before and after the course. Participant 2 said, “In middle school, I realized that while I always got really good grades and did really well enough, I didn’t really know what I was doing. If I made a mistake, I could not fix it.” The participant later reflected, “As a new teacher, my content knowledge was very surface level. I really didn’t have much of an understanding and I didn’t have a lot of tools. I’d like to go back and apologize to every single student that I had because it (my teaching) was very surface level.” This acknowledgement of the importance of teachers’ mathematical content knowledge is significant because it highlights how the more teachers know about the math they teach, the more confident they are in teaching it.

Participant 5 expressed similar feelings as Participant 2 in reflection. Participant 5 said, “When I came out of high school, the only thing that I felt very, very solid and confident in was statistics." The participant attributed this to the fact that statistics was contextualized and that is how the participant made sense of ideas. For Participant 5, most math was not presented in contexts or shown through representations that allowed the individual to see how ideas were related. As a result, Participant 5 did not really have a firm grasp on mathematics. When Participant 5 became a teacher, the participant had to relearn a lot of the mathematics needed to teach. This process helped give the participant deep conceptual understanding needed to teach math in a better way than the participant was taught.

All participants recognized how important it is for them to continue their learning of mathematics so they could get a deeper understanding of the concepts they teach. This was especially true for more difficult concepts like operating with fractions and integers. Participant 1 described thinking of those ideas as very formulaic before taking the course. The participant said the course helped to understand why the formulas work and now Participant 1 can help students develop the same understanding.

Research Question 3: Teachers Understanding of Effective Mathematics Instruction

RQ3: How do teachers who attended a professional development course describe their understanding of effective mathematics instruction?

Finding 4. Teachers saw that they needed to teach for conceptual understanding but recognized that requires them to deepen their own conceptual understanding of mathematics as well. This finding is related to Finding 3 because it addresses the gap in teachers' conceptual understanding of mathematics. However, this finding is different in that it describes how teachers believe they need to change their practice.

The biggest shift that each participant recognized was that math instruction really needs to be a facilitated learning experience versus an explained process if students are going to develop a conceptual understanding. Their conclusions were a result of experiences they had with the mathematics as learners, where the facilitators did not do direct teaching and instead led the participants through engaging tasks that required them to work to develop their understanding of the underlying mathematical ideas.

Participant 2 reflected on experience in the course by saying,

> The facilitators here are really good with wait time. That's when a lot of the learning happens, and the connections are exploding in our heads. That's where it happens and we [teachers] just don't take enough time to pause and do that with our students.

The wait time Participant 2 referred to happens when facilitators ask questions and allow the participants to have a significant amount of time to reflect and think about the questions before looking for a participant to share their ideas.

This idea of the facilitators asking questions, letting participants reflect, and then facilitating the discussion where the ideas are coming from the participants and not the facilitator resonated with many of the participants. Participant 1 commented, "I definitely want to try some of the things that we did in class and push my students' thinking the same way the facilitators pushed ours." This highlighted a useful structure from our courses where teachers get to experience this work as learners which helps them appreciate what it is like to engage in meaningful mathematical work.

Participant 4 reflected on experience collaborating with other colleagues during the course and how that affected his own learning. The participant said,

> We all learn differently and this course really raised that for me. When we were working together, one of my colleagues might have solved a problem in a way that I would not have considered. I learned from them that it could have been solved in a way that was more advanced than what I might have been wrestling with, but it made sense to me. I think children probably work in much the same way.

Participant 4's insight raises the importance of collaborative groupwork in mathematics. As with Participant 1's insight, Participant 4's insight shows that when teachers have positive learning experiences, they consider how to create similar experiences for their own students.

Finding 5. Teachers need more experience with concrete tools and visual representations as learners, so they know how to better use them with students. This finding was a result of all the positive comments participants gave about their work with concrete tools and manipulatives to help them deepen their own understanding of mathematics. Each participant referenced this work in their post-course interview. For participants like Participant 1 and Participant 5, who said they were more formulaic in the way they approached mathematics in the pre-course interview, this embrace of concrete tools and visual representations was a big shift for them. Participant 1 said, "I'm now very visual and hands-on as a learner. This experience has been really powerful for me." For some teachers, concrete tools and visual representations are seen as crutches students can use to help them solve problems. The work in the math course emphasizes the use manipulatives and visual representations as powerful tools for making sense of complex mathematics.

The other three participants (Participants 2, 3, & 4) indicated that they had been using tools like base-ten blocks, snap cubes, number lines, and other manipulatives and visual representations prior to attending the course because they learned about these tools through previous professional learning in mathematics. In their pre-course interviews, the participants did reveal that prior to having any professional learning, they did not make use of visual representations and concrete tools. Participant 4 credited experience with Cognitively Guided Instruction as the first time the participant really started to appreciate the use of manipulatives and representations in the math classroom. Participant 2 credited prior work with Math Recovery training for the shift in using these tools. Participant 3 did not cite specific trainings but said professional learning in general helped with the shift.

Research Question 4: Teachers' Attitudes, Perceptions, and Experiences with Mathematics?

RQ4: How do teachers who attended a professional development course describe their attitudes, perceptions, and experiences with mathematics?

Finding 6. As teachers deepen their understanding of mathematics during professional learning, their feelings and attitudes toward the subject become more positive after the experience. This shift in attitude was evident with both the experienced and inexperienced participants. There was palpable excitement from the participants as they talked about their work moving forward in mathematics.

A big factor that each participant said contributed to the change in his or her perceptions of mathematics was the type of mathematical work the participants were asked to do during the professional development course. The tasks and exercises required teachers to unpack the underlying structure of the mathematical ideas rather than focus on answer-getting exercises. For example, one task asked them to prove why the rule, "invert and multiple" worked for division of fractions. This type of work required participants to dig into mathematics they had procedural knowledge about but lacked the deep conceptual understanding. In other words, they knew that invert and multiply worked, but they did not understand why the procedure worked. Doing this kind of work allowed participants to gain a better understanding of mathematics and empowered them as mathematical learners because they uncovered the foundational ideas themselves. Participant 3 captured this greater degree of confidence in mathematics as a result of the work in the course by stating:

> I feel like I've got the confidence that I can attack any math problem…that I'll know something about any math problem. I may not know everything. I may not get the answer quickly. I may not get to the answer at all and that's okay. That's the piece I want to pass on to my students.

Participant 3's statements highlight a significant element of the work in the math course where teachers are engaging in mathematics as learners so they can develop a broader and deeper understanding of the content. In addition to contributing to their own content knowledge, the work appears to boost some teachers' confidence in the subject. Participant 2 shared a similar sentiment by stating:

> I had surface level understandings (prior to this course) and without this coursework, I'd probably still be status quo. I feel like I finally get it. I feel like my math ideas and my theoretical understandings of how math should be taught are matching what I now want to be doing in my classroom.

Participant 2's reflections indicate an increase in confidence with both mathematics and math teaching. The coursework is designed to engage teachers as mathematical learners and math teachers. Some of the work requires participants to work through complex mathematics designed to develop conceptual understandings. Other work requires participants to analyze video and print case studies to reflect on student thinking and instructional strategies implemented by teachers.

Participant 5 shared how the coursework contributed to her more positive disposition toward mathematics in the following reflection:

> I now have a deeper understanding of the relationship between the operations. prior to the class, I thought I already had a sufficient understanding of them. However, now I have a richer and more complex understanding. And now I feel like I can help my students have a better understanding.

Participant 5 conveyed the feeling other participants expressed regarding their experiences as mathematical learners. When teachers develop a greater degree of understanding of the mathematics, they are better positioned to support their students' mathematical learning. This sentiment was shared by all participants.

Research Question 5: Positive Elements of the Professional Learning Course

RQ5: How do teachers who attended a professional development course describe the effect of the experience on their learning, teaching practices, their perceptions of mathematics, and their perceptions of mathematics instruction?

Finding 7. Teachers want opportunities to engage in rich tasks in small groups, so they experience mathematics as learners while an experienced facilitator guides the learning. While professional development courses in mathematics can take on many forms and have several different elements ranging from lecture to hands-on, interactive activities, some elements have a greater effect on participants' learning and their perceptions of mathematics and math instruction. Participants in this study were asked to comment on the aspects of the course and how these aspects affected their learning and perceptions of mathematics and math instruction. Three elements stood out above the rest as being significant contributors to participants' learning: Small group work, strong facilitators, and rich mathematical tasks.

***Small group work*.** Small group work was the most surprising aspect of this study. I did not anticipate that the small group work and social aspect of collaborating with colleagues would be the highlight of the program for all participants. When asked what experiences had positively affected the participants' learning, they discussed how specific elements of the courses resonated with them. Patterns emerged with multiple references to the work they did with other people in the course. These experiences included doing math together, analyzing case studies, discussing instructional moves, discussing one another's students and classrooms, and informal conversations that developed in small groups. While different participants highlighted different elements, each element referenced occurred during small group work.

Every participant cited the power of the small group interactions that they experienced throughout the week. It was clearly the most exciting element for each participant, and it was referenced throughout the post-course interviews. It did not seem to matter what the specific task was for the small group interactions. Every reference to group work was positive. For example, Participant 2 said,

> When your learning becomes public, you learn so much more from it. You put yourself out there and say, "This is what I'm thinking." and after you start sharing your thinking, more people are like, "I never thought of it that way." You really start to respect yourself more and see yourself as a learner…and it's so interesting to consider how other people think about the same ideas.

Participant 2's comments capture the kind of learning that results from sharing one's ideas and gaining other people's perspectives on them.

The most common reason given for the positive reactions to the group work was in reference to how much the participants learned from each other in those moments. As Participant 1 said, "You can share more ideas because you have more time to talk…and with more voices, you get more perspectives." Again, the notion of different perspectives surfaced and contributed to greater learning.

There was a clear sense that participants appreciated the expertise of their colleagues and the nature of the group work allowed them to contribute to each other's learning. As a result, they could simultaneously support one another as they grappled with new ideas while collectively deepening their understandings. It was also stated by Participant 2 also stated that with fewer voices in the groups, it was easier to “wrangle” with ideas because everyone could have an active role.

Another interesting aspect of groupwork that arose was the perspective participants got as learners. Each participant referenced how important it was for them to collaborate with colleagues and it helped them realize how necessary it is for their own students. The participants knew group work was an effective instructional approach before taking the course but experiencing group work as learners where they had to struggle together through complicated tasks reinforced this understanding. Additionally, Participant 5 also highlighted how this group work helps teachers learn how to be better listeners and better learners.

***Strong facilitators*.** Numerous references were made to the strength of the facilitators in the course. Some statements were just broad compliments of the quality of the facilitators, but most references were about specific facilitator moves or attributes of the facilitators. A key aspect to this finding is that the quality of the small group instruction was very much tied to how well the course instructors facilitated the learning experiences. Participant 2 captured this idea by stating,

> The facilitators kind of swoop in but they don’t tell you how to work through a problem. The ones that are really good are the ones that give you a new question, one they probably thought up ahead of time, that will help you to get unstuck.

Participant 2 was addressing how strong facilitators do not rescue participants when they are stuck on a problem. Instead they ask more questions that reengage the participants and help them discover the solution on their own.

The most common compliment about the facilitators was regarding their ability to ask open-ended focusing questions that required participants to push the learning forward. The facilitators rarely lectured and never told the participants how to solve the problems with which they were working. Instead, the facilitators would allow the participants to engage in productive struggle and would provide support and encouragement to keep people moving forward in their explorations. The result of this format was captured with Participant 2’s following reflection on how the exploratory structure of the coursework contributed to more confidence with both doing mathematics as well as teaching it:

> I feel like you’re on your own journey as a learner and a teacher and (the) bottom line is, nobody can tell you how to do it. For me, you can tell me how to do it or what you think I should be doing, but I’m not going to do it until I understand it. And I think this course helped me understand what I should be doing for myself as both a learner and a teacher because we had to discover a lot of the learning for ourselves.

Participant 2’s comments solidify the idea that lecture-style professional learning courses are less advantageous than interactive courses that allow participants to gain a deep understanding of the content.

Facilitator strategies such as asking open-ended questions, engaging participants in exploratory tasks, and facilitating discussions that draw on participants' ideas rather than lecturing them made the experience harder for the all participants because it required more cognitive demand on their part. However, it is also why each participant said these courses pushed their thinking so far. According to Participant 2, because the heavy load of the work fell on participants, they were required to talk more and to refine their thinking so they could articulate their understandings to the larger group. The requirement of having participants handle most of the cognitive load during the course strengthened and deepened participants' learning.

Another key aspect of this work concerns the delicate nature of facilitating learning experiences without overly scaffolding the work or having the work so open-ended participants feel as if they are floundering. As Participant 3 said,

> The facilitators that I worked with were a huge part of this course. The encouragement, the support, and the gentle pushing out-of-the-box way they taught the course was most amazing and kept me going, even when it got really difficult and I was thinking 'I don't understand. I can't figure this out.' That was a huge part. That was a huge component.

Participant 3's comments highlight that balance that is needed to support participants through challenging mathematical work.

Rich mathematical tasks. Another central theme within this finding was the power of engaging in rich mathematical tasks to help participants develop the breadth and depth of the conceptual understanding of the mathematics in Grades K to 8. Numerous references were made to the delight participants experienced as they worked to make sense of complex ideas and to understand why certain mathematical procedures work. Rich tasks, as defined by the course syllabus, are tasks that involve complex thinking and reasoning and require more than a standard procedure to solve.

All five participants referenced how much deeper their knowledge and understanding of mathematics became from their participation in the professional development course. One participant went so far as to say, “We were forced to go deeper into the math than we were ever asked to before.” This work did push teachers out of their comfort zones, but it was done in supportive ways through dynamic group work with accomplished facilitators guiding the experience. As a result, participants were able to uncover lots of new ideas related to the mathematics and push their thinking forward.

One aspect of the work during the course that seemed to rise to the surface was the effectiveness of using visual representations and contexts to make sense of complex mathematical ideas. All participants expressed how helpful it was for them to have to solve problems using methods that included manipulatives and visual representations and then having to match the representations to the abstract mathematical procedures. Connecting concrete and representational thinking to abstract ideas helped solidify their conceptual understanding of the content.

One other aspect of the math work that surfaced was the fact that these experiences helped teachers understand what math is like for their students. The participants had to struggle and to work through confusion. They had to consider the ideas of others and then work collaboratively to construct some group consensus around the ideas being shared. They discovered the power of discovering mathematical ideas through purposeful exploration and rich mathematical tasks. These experiences helped deepen the participants' resolve that all students need to experience math this way.

After participants experienced the professional learning course, all of them expressed feelings of excitement at the thought of teaching mathematics. They also discussed the importance of teachers facilitating learning experiences for students. They cited their experiences as learners during the course and appreciated how the facilitators would guide their learning and not just lecture. The elements that stood out for participants from the course that they want to bring back to their classroom include: assigning fewer problems and going deeper with them, providing more time for students to work collaboratively in small groups, asking more intentional questions to get students talking about their mathematical thinking, using visual representations to help build conceptual understanding, and building off what students are doing and already know.

A key factor that each teacher referenced was the importance of them having positive experiences as learners from facilitators that were doing all the above instructional moves. For the participants who had been teaching for a while, the experience served as a reminder of why these instructional decisions are important. They knew these instructional strategies were important but did not always use them with their students. The course served as a reminder of why these strategies are important and why teachers should ensure they use them with their students.

Interpretation of the Findings

In this section, I will discuss the seven findings derived from the data and themes as they relate to the conceptual framework and the larger body of literature reported in Section 1. I will describe the ways the findings (see Table 1) confirm, disconfirm, or extend knowledge in the discipline.

Relationship of findings to conceptual framework. The findings from this study confirmed the conceptual framework that was based on two major theories in education. The first is constructivism developed by Piaget (1957) and expanded by Bruner (1966) and the second is socio-constructivism by Vygotsky (1978). These theories are closely tied to mathematics and the professional development course was designed with them in mind. Finding 2 indicates that "as teachers gain more experience with mathematics through professional learning and work with students, their perceptions of and attitudes toward mathematics improve." This finding connects experience with growth and affirms the underlying theory of constructivism. Bruner (1966) described learning as an active process in which students draw upon prior knowledge and experience to construct new concepts or ideas. Participants expressed how building off their own and others' prior knowledge and experience helped shape their learning.

Finding 4 indicates that "teachers see they need to teach for conceptual understanding but recognize that it requires them to deepen their own conceptual understanding of mathematics as well." This finding aligns with constructivism because this approach promotes collaboration where participants are active learners working on authentic math tasks that draw upon their prior knowledge and experiences (Chitanana, 2012; Rupam & Gujarati, 2013). Developing a deeper conceptual understanding of mathematics is supported with a constructivist approach and participants in this study attributed their deeper learning to this experience.

Finding 5 shows that "teachers need more experience with tools and visual representations as learners, so they know how to use them with students." This finding aligns with ZPD, which is a component of Vygotsky's socio-constructivist theory. In mathematics, ZPD refers to the distance between what students can currently do and their potential level of development (Danish, Saleh, Andrade, & Bryan, 2017). In other words, ZPD is the level between what students know and can do and what they can learn and do next. For participants in this study, their procedural math knowledge was already evident, but their conceptual understanding was not. Using visual representations to make sense of complex math ideas was an appropriate next step for their learning. According to the participants, they were able to connect this way of thinking to their own prior understandings and learn more as a result.

Finally, Finding 7 shows that "teachers want opportunities to engage in rich tasks in small groups so they experience the mathematics as learners while an experienced facilitator guides the learning." This finding aligns with Ulrich et al.'s (2014) research where the authors emphasized the active and interactive nature of humans and how they affect learning. The idea of humans wanting an active role in their learning along with opportunities to interact with others through the process was significant in this study. All participants expressed great appreciation for the social nature of the work and ascribed this experience as having the greatest influence on their learning.

Relationship of findings to prior research. Finding 1 connects teachers' prior experience with mathematics with negative feelings toward the subject and aligns with results from previous studies. Andrews and Brown (2015), and Karunakaran (2020) noted that math anxiety is common among preservice teachers and develops from teachers' prior experiences with mathematics as students. Participants in this project study all referenced some negative experiences with mathematics as learners and how it resulted in less positive perceptions of the subject. Participants also noted how this negative perception affected how they taught math to students. This admission connects with Geist's (2015) findings, that teachers with higher levels of math anxiety and negative perceptions of the subject were less likely to use developmentally appropriate teaching methodologies.

Mutodi (2014) identified connections between math anxiety and gender, noting a higher degree of math anxiety among female students. Participants in this project study were mostly female and the sample size was relatively small with only one male participant. This lack of representation and small sample size makes it hard to reflect on the gender differences. However, all five participants did report negative feelings toward math. Olson and Stoehr (2019) further identified high levels of math anxiety among preservice teachers and noted its negative effect on their confidence to be effective mathematics teachers.

Finding 2 notes that as teachers gain more experience with mathematics, their perceptions of the subject become more positive. This is consistent with findings that identified a positive relationship between cognitively guided instruction in mathematics and positive perceptions of the subject (Brown, 2016). The math work participants experienced in this study was consistent with cognitively guided instruction. All participants in the study also commented on the quality of the professional learning experience in the course. Gresham (2018) identified high-quality professional learning experiences as a key factor in developing more positive perceptions of mathematics among teachers.

Finding 3 indicates that while teacher participants entered the profession with adequate knowledge to functionally do mathematics, they lacked the deep conceptual understanding needed to teach math at a much deeper level. All five participants reported this assertion and it is consistent with the findings from other studies that noted many elementary teachers do not have the mathematical content knowledge necessary to teach for conceptual understanding (Novak & Tassell, 2017; Reid & Reid, 2017). Most participants also reported that they spent less time, if any, using concrete tools and visual representations prior to taking the professional development course. This relates to the findings in Finlayson (2014) noting that teachers with less positive perceptions of mathematics were more procedural in their approach to teaching math.

Finding 4 shows that teachers see that they need to teach for conceptual understanding but recognize that it requires them to deepen their own conceptual understanding of mathematics as well. This finding is closely aligned with Cueto et al.'s (2017) research noting that as teachers' mathematical content knowledge increased, their confidence increased which led to the application of effective teaching practices and increased student achievement. Participants in the project study recognized that as their knowledge grew, so did their confidence and abilities to facilitate mathematical learning experiences rather than directly teaching mathematical strategies to students. Cosby et al. (2017) also demonstrated that explicit work on guided inquiry can lead to teachers learning to develop and facilitate engagement with inquiry-based mathematical tasks.

Finding 5 indicates that teachers need more experience with concrete tools and visual representation as learners, so they know how to better use them with students. In some way this finding is related to Finding 4 because both have to do with teachers being learners. This one specifically references the visual aspect of conceptual math learning and is supported from prior research. Reid and Reid (2017) identified a link between teachers' mathematical content knowledge and its effect on students' achievement in mathematics. Lesseig (2016) found that professional learning that engages teachers in the use of visual representations gave teachers access to many different instructional strategies with which they previously had little or no experience. Participants in the project study all mentioned how important the use of concrete and visual tools were in supporting their own learning. They also noted how using them in the professional learning setting helped them see how they could be used in their own classrooms.

Finding 6 shows that as teachers deepen their understanding of mathematics, their feelings and attitudes toward the subject become more positive. Knaus (2017) found that professional learning and teacher education can positively shift teachers' beliefs. Similarly, Luebeck et al. (2017) found that teachers' perceptions and attitudes toward mathematics could be positively influenced through collegial reflections on mathematical structures and relationships. All participants in this project study commented on the power of the social dynamic of the work in the course and the focus on understanding the underlying mathematics behind the math they teach. Finally, Brown (2016) found that when teachers engaged in this work during professional learning experiences, they developed a more positive disposition toward mathematics. Brown's finding also aligns with Finding 6.

Finding 7 shows that teachers want opportunities to engage in rich tasks in small groups, so they experience mathematics as leaners while an experienced facilitator guides the learning. Prast et al. (2018) highlighted the importance of multidimensional professional development that includes opportunities for teachers to engage in work that develops their mathematical understanding while instructors modeled effective teaching practices. The nature of the work of the professional development course involved small groups engaging in mathematical tasks with low entry points and lots of room to expand. In other words, the math work participants did was differentiated based on the needs of the participants. The experienced facilitator was able to guide the learning based on the needs of the different small groups. Chin et al. (2016) found that when professional development instructors used methods such as guided inquiry during their sessions, teachers were more successful in implementing these new approaches. My study connects with those findings because the experienced facilitator was modeling the kinds of interaction necessary to facilitate this learning with students. Participants noted how helpful it was to have the opportunity to analyze the facilitator's instructional decisions and consider how they might use them in their classroom with their own students.

Project Deliverable as an Outcome

The findings provided a deeper understanding of the struggles elementary teachers face when teaching mathematics and what they need in terms of a supportive professional learning experience. I have a deeper understanding of which aspects of professional learning appeal to teachers and which aspects are less helpful. I also gained insight into how important the social component of learning is for teachers.

Based on the findings from this study, I created a 3-day professional development experience that is designed to meet the needs of elementary classroom teachers. For example, it was clear that some elementary teachers had a dislike of mathematics. Taking that finding into account, I will ensure that the professional learning experience has components that make the mathematics accessible and engaging. By having an engaging task with a low entry point, I can reduce the intimidation of the mathematics while getting teachers excited about the work. My findings suggest that this will likely increase teachers' positive perceptions of mathematics.

The 3-day learning experience will have 3 major goals. The first is to get elementary math teachers excited about learning and teaching mathematics. The second goal is to deepen teachers' conceptual understandings of the math they teach. The third is to deepen teachers' pedagogical knowledge and understanding of effective teaching practices in mathematics. Each of these goals relates the some of the findings from the study and will be discussed in depth in Section 3.

Conclusion

The goal of this study was to gain a more complete understanding of the specific components of professional development experiences that had the greatest effect on teachers' perceptions of mathematics and mathematics instruction. By gauging teachers' perceptions after the professional development experience, I was able to identify what significant changes occurred and what aspects of the course influenced their perceptions. The implications of this project study have informed me on the course elements that influenced teachers as well as the elements that had little effect in the professional development course attended by the research participants. As a result, I designed a 3-day course based on the results of the study.

Section 3: The Project

Introduction

The 3-day course called “Beyond Answer-Getting: Exploring Mathematical Practices and Developing Math Reasoning with All Students” is derived from the findings of this study and designed to help elementary teachers feel more comfortable with mathematics, deepen their mathematical content and pedagogical knowledge, and help them develop and implement effective mathematics teaching practices. The course draws on teachers’ interviews about their experiences from the professional development course, effective elements of professional learning in the course, and the elements that were identified as having the greatest influence on teachers’ learning. Taken as a whole, the key findings that influenced that project study are that teachers need opportunities to deepen their own conceptual understanding of mathematics through experiences with concrete tools and visual representations while working collaboratively in small groups and an experienced facilitator guides the learning.

In the following sections, I will explain in detail the scope and sequence of the course along with relevant information to clarify the purpose of selected assignments and experiences. I will begin by discussing the rationale for this project followed by a review of the relevant literature related to the project. Next I will provide a detailed description of the work that will happen on each day and then discuss how the effectiveness of the project will be measured. Finally, I will discuss the implications of the project for both the participants who take the course and for those individuals who teach the courses.

Rationale

Table 2 shows the seven findings for a quick reference. One element that was identified as significantly important to teachers in the findings of this study was the social aspect of their learning. Teachers want to interact and collaborate with colleagues to extend and deepen their understanding. Though this element existed in one form or another in the courses, none of them really leveraged this work as a focal point. I wanted to design a course that made this collective group work the driving force behind the professional learning.

Table 2

Findings

Finding #	Findings
1	Teachers' prior experience with mathematics contributed to a dislike of or ambivalence toward mathematics.
2	As teachers gain more experience with mathematics through professional learning and work with students, their perceptions of and attitudes toward mathematics improve.
3	While teacher participants entered the profession with adequate knowledge to functionally do mathematics, they lacked the deep conceptual understanding needed to teach math at a much deeper level.
4	Teachers see that they need to teach for conceptual understanding but recognize that requires them to deepen their own conceptual understanding of mathematics as well.
5	Teachers need more experience with concrete tools and visual representations as learners, so they know how to better use them with students.
6	As teachers deepen their understanding of mathematics, their feelings and attitudes toward the subject become more positive.
7	Teachers want opportunities to engage in rich tasks in small groups, so they experience mathematics as learners while an experienced facilitator guides the learning.

I also wanted to create a course that addressed Finding 1 that many teachers' prior experiences with mathematics contributed to a dislike of or ambivalence toward mathematics. To address this concern, I wanted a professional learning experience that had a high level of engagement with a low level of threat. In other words, I wanted to design a course that provided meaningful learning with work that is accessible and relatable.

Another finding that contributed to the development of this course is Finding 7: Teachers want opportunities to engage in rich tasks in small groups, so they experience mathematics as learners while an experienced facilitator guides the learning. Based on this finding, the tasks, activities, and overall structure of the course were designed to provide ample group time with participants engaged as learners. The tasks and activities were chosen based on the high level of interaction and engagement they provide. The flow of each day builds in time for lots of collaboration with the facilitator modeling how to support groups and pull their ideas together to further everyone's learning.

Another aspect of my study that contributed to the course development were Findings 3, 4, and 5 that addressed teachers' need to deepen their own mathematical content knowledge and they desire to do so when given the right experiences and tools. I wanted to make sure that teachers have meaningful mathematical tasks that would enrich their thinking, and I wanted them to gain more experience using concrete tools and visual representations to support their understanding. Giving teachers access to tools and meaningful work also supports Findings 2 and 6 indicating that when teachers have experiences like this, their perceptions of mathematics become more positive. My hope is that the course has this effect on the participants.

Review of the Literature

The findings of my study indicated that teachers are most likely to shift their perceptions of mathematics when they engage in professional learning that develops their mathematical and pedagogical content knowledge and allows them to collaborate with colleagues. Based on all my findings, I reviewed the current literature on professional learning in mathematics to ensure the components of my project were backed by research and data. I used the Walden University resources to search the following databases: Thoreau, Routledge, ERIC, Open Access, and ProQuest Central. The terms used in my search were *professional development, mathematics, elementary school, andragogy, facilitation,* and *math knowledge for teaching mathematics.* I searched and found a total of 108 sources within the publication dates of 2016 to 2020 and cited 25 for this review. The combination of these terms through a variety of databases allowed me to reach the saturation point with the literature review.

Deepening Mathematical Content Knowledge and Conceptual Understanding

A key component of effective professional learning in mathematics is the development of teachers' knowledge and understanding of the content they teach. Research has indicated that teachers have preferred professional learning that deepened content knowledge, connected theory and application, and pushed for greater understanding of mathematics (Martin, Polly, Mraz, & Algozzine, 2018). Additionally, teachers are more likely to change their practice as a result of gaining a deeper understanding of the mathematics (Martin et al., 2018). When teachers' content knowledge is developed alongside their experiences facilitating mathematical discourse, representing mathematical ideas visually, and organizing instruction, their professional competence is enhanced (Ahuja, 2019).

Although there are a variety of ways to deepen teachers' content knowledge, the use of visual representations and concrete models can support teachers in relearning the mathematics on a conceptual level (Barlow, Lischka, Willingham, Hartland, & Stephens, 2018). Additionally, teachers need experience looking at a variety of visual representations of mathematical ideas and making connections between them in order to develop their own knowledge (Brendefur, Thiede, Strother, Jesse, & Sutton, 2016). Teachers' beliefs about what is important in mathematics influences how they teach; therefore, when teachers experience the power of visual representations to deepen understanding, they are more likely to use these approaches with their students (Ahuja, 2019).

When teachers' math knowledge for teaching is developed, they are more likely to make changes in their instructional practices with students to emphasize conceptual understanding (Carney, Brendefur, Thiede, Hughes, & Sutton, 2016). Giving teachers time to explore mathematical content as learners along with time to critically explore curriculum and instructional decisions is not only preferable from the teachers' perspective, but it also increases the likelihood of changes in practice (Martin, Polly, Mraz, & Algozzine, 2019). One factor that contributes to changes in teaching practices is that teachers' confidence increases as their knowledge of mathematics increases (Kutaka et al., 2018).

Social and Collaborative Learning Experiences for Teachers

Successful professional development programs include teachers engaging collaboratively in solving rich tasks, exploring representations, and communicating mathematical thinking through argument (Biccard, 2019), as it is important to include a component of collaboration among colleagues as teachers construct new ideas (Holmqvist, 2017). Teachers draw on the ideas of others within their learning network, and they need time to work collaboratively with one another in professional learning settings (Anderson, 2019). This finding is particularly true for teachers' work around connecting visual representations, concrete models, and abstract ideas because the more variety of ideas teachers explore, the deeper their learning becomes (Barlow et al., 2018). Thus, providers of professional development need to include multiple opportunities for teachers to interact during their trainings (Walker, 2018). Active participation and collaboration not only supports teachers' learning, but it also can shape how their teaching practices evolve (Garcia et al., 2018). Collaboration through the sharing of ideas, lesson planning, and reflection on teacher and student learning has a significant influence on teachers' professional growth (Gee & Whaley, 2016). Additionally, having colleagues for support during professional learning can also ameliorate some anxiety teachers might feel as they engage in mathematics (Kutaka et al., 2018). For example, Perdomo-Diaz, Felmer, Randolph, and González (2017) found that teachers' peer interactions while solving problems involving fractions increased their comfort due to collaborative support.

Engaging as Learners with an Experienced Facilitator

A crucial component of effective professional learning is credibility and expertise of the leader facilitating the experience (Goos, Bennison, & Proffitt-White, 2018). When teachers have engaged in mathematical tasks with an experienced and knowledgeable facilitator, their math knowledge for teaching increased (Goos et al., 2018). Facilitators of professional learning need to have detailed knowledge of the subject matter and an understanding of how teachers' can apply their learning in their own classrooms (Jacobs, Seago, & Koellner, 2017).

Further, when teachers experience mathematics as learners and then practice facilitating these experiences with their peers, they shift their own instruction away from direct teaching methodologies to more student-centered approaches (Tabach & Schwarz, 2018). By giving teachers opportunities to engage in exploration-focused mathematics, there are gains in teachers' knowledge of how to facilitate this work with students and the amount they posed demanding tasks and asked high-level questions (Polly, 2017). When teachers have experienced teaching practices as learners, they were more likely to enact those same practices in their own classrooms (Perdomo-Diaz et al., 2017).

Connections to the Classroom

When professional learning includes components that help teachers connect their learning to their own classrooms and reflect on practical application of the new ideas, teachers shift their practices (Bonghanoy, Sagpang, Alejan, & Rellon, 2019). When professional learning and readings have been coupled with classroom application through collaborative lesson planning, observation, and debriefing, teachers' knowledge and sense of professional community grew (Lesseig, 2016). Effective professional learning draws from the collective and collaborative experiences of the teachers and connects the work to their classroom settings so teachers can apply the new ideas (Mhakure, 2019).

One way to build connections to the classroom and consider practical application is to draw upon teachers' stories of classroom experiences (Mohan & Chand, 2019). Researchers have found the teachers' instructional practices improve when they share stories of classroom experiences with colleagues in a professional learning setting (Mohan & Chand, 2019). Additionally, when teachers share their own experiences with colleagues, while also reflecting on the beliefs about inquiry-based learning, it can contribute to more positive perceptions of inquiry-based learning (Maass, Swan, & Aldorf, 2017). Therefore, teachers need ample opportunities to see connections between the work they do during professional learning and their own practice and teaching contexts (Kutaka et al., 2017).

Conclusion

This study aimed to address the local problem of elementary teachers having an unfavorable view of mathematics. The findings of this study indicated that teachers want experiences to collaborate with colleagues as they deepen their mathematical content knowledge. They want to learn from knowledgeable and experienced facilitators and consider how to apply their learning to their own teaching contexts. These findings are consistent with the current literature on professional learning in mathematics (e.g., Ahuja, 2019; Kutaka et al., 2018; Martin et al., 2018; Martin et al., 2019). In the project that I developed, I considered the study findings and the ideas from recent research in the field.

Project Description

The project is a 3-day course designed to deepen teachers' mathematical and pedagogical content knowledge (see Appendix A). The course is also designed to improve teachers' perceptions of mathematics and develop their confidence to teach it well. The course may be a positive experience for teachers and encourage them to invest more time and energy in mathematics professional learning. The course draws on my findings from this study as well as current research from the field.

Participants in the course will be active learners, and the instructor will serve mostly as a facilitator guiding participants through interactive and engaging activities and math tasks. The work is designed so that participants will uncover the key ideas and make discoveries as they collaborate with colleagues. The instructor will interact with groups and draw from the participants ideas to orchestrate meaningful discussions that bring key ideas together. This instructional design mirrors how teachers can facilitate their own mathematics classes so participants can experience this kind of learning from the perspective of the student.

The first day of the course will focus on the changes in math classrooms today compared to the way the participants were taught when they were students. Days 2 and 3 will focus on four of the eight Standards for Mathematical Practice (see National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and develop teachers' content and pedagogical knowledge. Part of the focus of the work from the course is on helping teachers apply these ideas in their own classrooms. Therefore, the instructor will make sure to pose questions that encourage participants to discuss how what they are learning can work within their classrooms. Details about the entire course can be found in Appendix A.

In the following sections, I will discuss the resources and supports needed for the course to run. I will also describe potential barriers that may impact the project. Next, I will describe a detailed timetable for what needs to happen before, during, and after the course. Then I will describe the roles and responsibilities of the participants and instructor. Finally, I will describe the project evaluation plan as well as the project implications.

Resources and Supports

This course will run for free at the local college the first time it is offered as a service to the local school districts. As it is not generating revenue, I will utilize existing resources available at the college rather than purchase new supplies and materials. For this course I will need interlocking cubes, base-ten blocks, base-five blocks, graph papers, rulers, colored pencils, chart paper, markers, index cards, and copies of all the math tasks and articles. I will also use a document camera and projector along with my laptop. I will facilitate the course so I do not have hire staff, but five of my staff from the professional development program that I also facilitate at the local college will participate in the course so they can learn how to facilitate it in the future.

A classroom space will be reserved on campus for 4 days, so that I have 1 day before the start of the course to set up the room. Access to the cafeteria will be arranged so participants can have lunch on campus each day. Access to the library will also be arranged so participants have a place to grab coffee and do their lunch reading if they want a change of scenery. I will coordinate with the college's Conference Services department to arrange this on-campus event and make sure the guidelines are followed. I will also work with campus security to arrange parking for participants.

I will market the free course by creating an advertising slide using Canva (an online graphic design platform) and will embed the created graphic in an email through the college's Slate system (a customer relations platform; see, https://technolutions.com/) so I can access the database of local school leaders. I will also post the advertisement on Twitter and Facebook, as many math teachers follow the college's math program via these social media and networking platforms. An online registration form will be created using the Slate system along with an automatic reply to confirm registration and provide participants information about all the logistics including where to purchase the course text.

Potential Barriers

I do not anticipate many barriers because the course will be free, and cost is usually the biggest barrier for teachers when it comes to professional learning. One challenge I do anticipate is that the course is likely to fill up fast but because it is free, it is easy for some participants to cancel or just not show up when the course runs. To address this challenge, I will create a waiting list for the overflow registrations after the course hits the maximum limit of 30 participants. As cancelations come in, I can reach out to the teachers on the waiting list to fill the spaces.

Another potential barrier is time. Summer can be a challenging time to recruit for professional learning that lasts more than one or two days. Additionally, this course does require a large time commitment because there is pre-course work, homework every evening, and a post-course reflection paper due at the end. Some teachers may not want to give that kind of commitment during the summer. To address this barrier, I will ensure that the registration form includes explicit descriptions of the work involved as well as the time commitment needed. That way, teachers know the expectations and that should reduce the amount of pushback from participants.

Finally, another potential barrier is the possibility of the Covid-19 crisis closing the campus for the summer. If this were to happen, I can run the entire course online via Zoom (a video-conferencing platform). Zoom is the preferred platform because I can still run breakout groups and there are a variety of interactive tools to enhance collaboration online. Small modifications will need to be made for some of the math tasks because teachers might not have access to the manipulatives. I will utilize virtual manipulatives for those participants that cannot get physical tools from their schools.

Proposal for Implementation and Timetable

I plan to run the course in August of 2020. Summer is the only time I can run a multi-day event that requires a significant time commitment from me. It is also much easier to secure classroom space on campus when the undergraduates are not in session. Below is the detailed timetable for the implementation of the project"

May

- I will arrange all the on-campus logistics with the various departments and address any questions or concerns raised during this process.
- I will create the advertising slide in Canva and share it with my assistant to proofread and offer feedback. Edits will be finalized by the end of the month.

- I will create the registration form and confirmation response in Slate.

June

- I will create and send an email to all local school leaders advertising the free course. The email will have a link to the registration form.
- I will create a social media post to use for Twitter and Facebook and link to the registration form. These will be posted in early June.

July

- My assistant will make all the copies needed for the August course.
- I will gather all the manipulatives and materials needed for the course and arrange to have the college facilities management staff deliver them to the classroom on the day before the course is scheduled to begin.
- I will review registrations and cancelations on an ongoing basis to ensure all the spaces filled.
- An email will be sent to participants to remind them to complete their pre-course assignment and reading. This email will also include the logistics again to make it easy for participants to access.
- I will also meet with my staff who will participate in the course to go over the findings of my study and discuss the implications of this work on this course and our future course development.

August

- I will review registrations and cancellations up until 2 days before the course begins. After that, I will close registration.

- I will review participants' pre-course assignment to gauge their knowledge and understanding and make any necessary modifications to the plan to meet their needs.
- I will set up the classroom space the day before the start of the course and make sure I have all the materials and copies I need.
- I will run the 3-day course and take notes at the end of each day on what worked and what needs revision. If any immediate revisions are needed to respond to the participants, these will be made.
- After the course, I will review the evaluations and post-course assignments to see if the course was successful and shifted teachers' knowledge and perceptions of mathematics.
- I will also debrief with my staff to get their feedback on the course.
- Revisions will be made to the plan right away while the ideas are still fresh in my head.

This is a four-month timetable and can be adjusted if the date of the project must be moved or modified. If the course needs to be done online, the plan will remain the same, but college officials will be notified that events are not happening on campus. If the course needs to run at another time other than the summer, it will need to be modified to occur over a few weekends or classes will be scheduled in the evenings and spread out over a longer period. The course content will not change.

Roles and Responsibilities of Participants and Others

My role as the researcher and course developer is to organize every aspect of this project, inform my staff of my research findings and project plan, lead the professional learning course, review and submit feedback to participants, and debrief with my staff on the course experience and explore further implications of this work on the courses the college offers in the future.

My assistant from the department at the college where I work will support this work by helping me prepare and distribute materials. She will also proofread and edit any materials that will be shared with participants. She is also the first point of contact when people try to reach me. She should be able to answer any logistical or technical questions that participants or school administrators might have. If she is unable to answer a question, she will connect me with the individual so I can support their needs.

Participants in the course will be expected to complete all assignments in a timely manner. I also expect them to fully participate in the course activities and discussions. Their primary role is as a learner in the course, but part of the course design is to draw upon the expertise of the participants so at times they will be sharing their ideas so others may learn from them. Participants will also provide feedback on the course through their course evaluations.

My staff that participates in the course have two roles. The first role is as learners. They will go through the course as participants and will be expected to fulfill all the responsibilities that participants have. This way they have an opportunity to experience the course from the perspective of the participant to gain a greater understanding of how the instructional decisions play out for learners. Their second role is as critical colleagues. I value my staff's insight and feedback and look forward to hearing their perspectives on the course design and implementation. In the future, they will all have to lead this course, so it is a good opportunity for them to participate in this process.

Project Evaluation Plan

To evaluate this project, participants will be given a pre-course assignment that consists of four problems (see resources in Appendix A). Participants will need to analyze the thinking of students' mathematical strategies and reflect on their own mathematical knowledge. This analysis requires that participants have a deep conceptual understanding of the mathematics. If teachers have more procedural knowledge, they are likely to reflect on the inefficiencies of the students' work and focus on the rightness and wrongness of the responses. This limited view of the mathematics is precisely the challenge I am trying to address in this course. I want teachers to shift from a focus on answer-getting in mathematics to focusing on the process of engaging in mathematical reasoning. The pre-course work will give me and the facilitators a clear sense of where teachers are in their thinking about these ideas.

At the end of the course, participants will be given back their pre-course assignment and will be asked to review and revise their analysis of the students' thinking. They must then write a short three to two-page reflection on how the ideas from the course contributed or did not contribute to their new analysis of the students' thinking. It is my hope that participants will demonstrate a much deeper understanding of the mathematics and the nuances in the students' thinking. I would also like to see specific references to experiences from the course that have helped shape their new understandings. However, the post-course assignment may reveal that the course did not have a significant effect on participants' thinking or understanding.

Regardless of the results of the post-course assignment, I will use the data from the assessments along with the course evaluations (see Appendix A) to make the necessary revisions to the course. I view courses as continual works in progress and I suspect that each year I run this course it will continue to grow and evolve. My hope is that the combined data sources that measure both the participants' growth as well as their own perceptions of the experience will help me make more target revisions for future iterations of the course.

Project Implications

The findings indicated that many elementary teachers have a negative perception of mathematics. As a result, they may be more hesitant to enroll in courses or attend professional development that focuses on mathematics. Hesitation to pursue professional learning in mathematics presents a challenge because teachers who do not have a favorable view of the subject tend to teach it less and their instruction tends to be less effective than a teacher who is more comfortable with the subject. If the course has the desired outcome, teachers will find it very accessible and transformative. Teachers' view of mathematics and the teaching of mathematics may shift to become more positive.

Based on my findings, a course that is easily accessible, provides a safe and supportive learning environment, and works on developing and deepening participants' content and pedagogical knowledge should result in creating more positive perceptions of mathematics. If this positive shift happens, my hope is that participants will be open to more professional development to deepen and broaden their learning. In essence, I see this course as a nice appetizer to much deeper learning through continual professional learning experiences.

Many other courses at the college can be intimidating because participants must explore how math ideas develop from kindergarten through eighth grade. The middle school content often scares elementary teachers and I believe it sometimes keeps people from enrolling in any of the courses. My project is designed to reduce the intimidation factor and to create a certain amount of buzz as participants return to their schools and talk about what a positive experience it was for them. This in turn may result in more teachers enrolling in the course the next time we run it. Ideally, this will create a cascade effect where more and more teachers gravitate toward more professional learning experiences in mathematics.

The other implication of the project is how it will affect my work and my staff's work as we revise and develop more courses for teachers. I plan to share the results of this study with my team so we can all use the results as plan for future learning experiences with teachers. If we design future courses and revise current courses with these findings in mind, we will be meeting the needs of our elementary teacher audience and will be more attractive to them when they are deciding on the professional learning needs.

Section 4: Reflections and Conclusions

The problem I addressed in this study was the fact that many elementary teachers have an unfavorable view of mathematics. As a result, some teachers have avoided professional learning in mathematics, which can have a negative effect on their students. As stated in the literature, when teachers lack a deep conceptual understanding of mathematics and when they have negative perceptions of the subject, they teach it for shorter periods of time and less effectively than teachers who view mathematics more positively (Grootenboer & Marshman, 2016). This project was designed to give teachers a positive mathematical experience while deepening their content knowledge. In Section 4, I will discuss the limitations and the strengths of this project, recommendations for alternative approaches, scholarship, project development and evaluation, leadership and change, reflections on the importance of the work, and implications, applications, and directions for future research and social change.

Project Strengths and Limitations

A strength of this project is that the course draws heavily on the findings from the study to provide experiences for teachers that involve lots of collaboration with colleagues through meaningful and engaging tasks and activities. The activities and math tasks are designed for active learning. Participants will be generating many of the mathematical ideas on their own with colleagues versus listening to the facilitator lecture them. The tasks are also designed to have low entry points so all participants can access the ideas without difficulty. However, the tasks also have a high ceiling, meaning the mathematics is also complex enough for participants to go much deeper with the work if they are ready to think about more complex ideas.

A limitation to the project is that there is still a fair amount of work involved for participants, requiring a significant time commitment on the part of participants, which might turn some people off. One way to manage this limitation is to have some flexibility and understanding with participants who need additional time to turn assignments in. The course is not graded, so there is a flexibility for facilitators who lead the course. Another possibility is to eliminate the course textbook, *5 Practices for Orchestrating Productive Mathematical Discussions* (Smith & Stein, 2018) if the feedback from participants is that the workload is too much. As an alternative to reduce the amount of reading, I could use a short article based on the main ideas from the textbook (Smith, Hughes, Engle, & Stein, 2009).

Recommendations for Alternative Approaches

One need for an alternative approach is to make the whole course online. This is part of the contingency plan in case the campus is closed due to a quarantine; however, it is also a viable option to make the course accessible to a larger audience. Offering the course online means that teachers from around the world can participate depending on time zones and when the course is scheduled. Making the transition to an online format would only require converting the course materials to Google documents, slides, and forms and connecting with participants via Zoom.

Another alternative approach would be to break the 3 days up into six evening sessions and run the course during the school year from 5:30–8:30 p.m. This follows the structure and schedule of the other evening classes on campus and makes it accessible for teachers who work until 3:45 p.m. each day. With this model, the course would run much like it would in the summer, but the work would be spread out. Each full day in the summer is divided into a morning section and an afternoon section. Each section could be one evening class, allowing all the course materials to get covered.

Scholarship, Project Development and Evaluation, and Leadership and Change

This project study was a labor of love for me and one that helped me grow as an academic scholar. I have always been a practitioner at heart and have approached my work from a pragmatic point of view. Decisions I made about course development, structures, and revisions, were often based on my instincts and intuition as a practitioner. Though this seemed to work fine for me in the past, engaging in this project study brought the limitations to this approach to the surface.

I learned the value of reading current and relevant literature as it pertains to my work as a provider of professional learning. This process helped me develop the skills to read and apply ideas from empirical studies to my work at the college and has helped me develop an analytical lens as I encounter new ideas, or I am faced with any decisions that affect the professional development program at the college. Instead of only going with my instinct, I now consult literature and consider multiple sources of data in my decision-making.

Another important change that occurred in me is a greater appreciation and understanding of the use of data to inform my work. Again, prior to this experience, I would read evaluations and exit surveys and reflect on the information, but most changes to the courses were based on my own reflections on what I thought worked or did not work. Conducting in-depth interviews with participants and analyzing their responses for this study gave me a greater appreciation of the power of data collection and analysis as a mechanism for understanding my work more deeply. I learned more about what teachers want and need for professional learning as a result studying the data than I would have if I just read course evaluations.

The project development was also a different experience for me. In the past, courses were constructed based on learning outcomes I wanted for the participants. I focused primarily on what I would be teaching and less on how I would teach it. In the case of this project development, most of my focus was on how participants would engage and interact with the course material. The course design derived from the data and I found that when I need to make a decision, I would consider my findings before deciding on which direction to take. As a result, I believe the course structure is more coherent and will provide a better experience for participants than previous courses I designed.

My skills as a leader of national professional learning program and director of a graduate program have been strengthened as a result of this experience. I have been working in academia for 8 years without a terminal degree. Until now, I have approached my role as a leader as a pragmatic practitioner and not a scholar. However, I believe it is necessary to have both skillsets in my role and the latter was certainly developed and strengthened as a result of this project study.

Part of leadership is addressing change and this project gave me the opportunity to approach in various ways. I had to consider changes I needed to make in myself as I transitioned from thinking like a practitioner to thinking like a scholar. This transition was supported by my colleagues in the academic world along with my chair and second committee member for this project study. I also needed to consider making significant changes to my course design and structure based on my qualitative data analysis. These changes enhanced my skillset as a leader of professional and graduate education for math teachers and coaches.

The other aspect of leadership and change this project study addressed was affecting change in others, particularly those who might be resistant to such changes. The goals of the course I designed are to change teachers' perceptions of mathematics and to deepen their mathematical content knowledge. To do this effectively, I needed to design a course structure that blended the needs and wants of the teachers with my own goals for the course. Analyzing the responses of the participants in the study and using the findings to design the course allowed me to consider my audience and how best to meet their needs and move them forward in their professional learning.

Reflection on Importance of the Work

Mathematics education is an important field and one to which I have dedicated my entire career. It is critically important that all students have access to high quality and enriching experiences in mathematics and this requires that teachers have the knowledge and skills needed to teach the subject well (National Council of Teachers of Mathematics, 2014). Many elementary teachers do not have a deep conceptual understanding of mathematics to teach it well. Additionally, some of these teachers also have negative perceptions of mathematics that affect how they approach the subject with their students. This project was designed to address this challenge and make a significant change for teachers in mathematics. My hope is that as teachers take this course, they will discover that regardless of their age or experience, they can all learn mathematics on a conceptual level and develop a rich appreciation for the beauty of the subject. I want teachers to have transformative experiences as learners and then I want them to apply what they learned to change their practice in their own work contexts. By changing teachers' perceptions of mathematics, I hope to change students' experiences with mathematics.

From this whole process I learned how important it is to have a scholarly approach to the work I do as a leader of educators. My knowledge of professional learning in mathematics has become greatly enhanced because of all the literature I have read regarding this work. When I attend conferences and attend sessions, I see the work through the lens of a scholar, and I connect what I experience with what I read in the literature. This is a significant change in my habit of thinking, and it has strengthened me in this field.

Implications, Applications, and Directions for Future Research and Social Change

The immediate implication of this project is that it should affect some level of change in classroom teachers and encourage them to continue their professional learning with mathematics. I see the course I developed as a steppingstone to much greater opportunities for teachers to engage in this work. My hope is that I can scale this work up with my staff so we can run this course throughout the year in various formats from evening classes to online structures. It is quite possible for us to reach thousands of educators with this work and this will have a huge impact on social change.

For students that are successful with mathematics, certain career pathways may be more accessible to them. Conversely, for students who struggle with mathematics, access to more complex mathematics courses may be unavailable to them and they will have fewer opportunities with career pathways tied to mathematics. Teachers can affect how students perceive mathematics and perceive their own abilities with it. By strengthening teachers' knowledge, skills, and understanding of mathematics, my course can affect numerous students in a positive way.

Methodological Application

Another implication is that my thinking has been drastically altered as a result of this work. All future courses and professional learning models I develop will be approached by me as scholar and practitioner. Everything that I design will be backed up by research and will have practical applications so teachers can take what they learn and apply it in their own contexts. Having a scholarly approach to course development is a significant shift for me and my program and one that I think will greatly enhance everyone's experiences with it.

I anticipate working with my staff who also design courses for the college professional development program to help them ground their work in research. I will share my findings with my team, but we will also meet a few times a year to discuss current literature related to mathematics and professional learning. The field is ever evolving, and I would like my program to be on the forefront with this work. Our approach to course design and professional learning will be adapted as a result of the work on this project study.

Recommendations

This project study was small in nature and by no means can the findings be generalized to the greater population. Future research is certainly needed with a much larger population to get a better sense of the needs of teachers across the country and around the world. I am interested in considering the needs of classroom teachers that teach in high-needs districts as this was not a focus of my project study and I see that as a limitation to my findings.

Conclusion

Mathematics does not need to be scary or something to do for the sake of getting answers. It is a rich and beautiful subject that has not been taught well for years. Consider how educators approach reading instruction. A best practice in reading instruction is to focus on developing students' accuracy, fluency, and comprehension (Fountas & Pinnell, 2017). If teachers ignored comprehension, then students would learn that reading is just saying words really fast without needing to understand what they were saying. Yet, for years that is exactly how teachers approached math education (National Council of Teachers of Mathematics, 2014). The focus has been on accuracy and fluency with little or no regard for conceptual understanding. For example, a student may have memorized their times tables and the standard algorithm for multiplication but does not understand the concept of multiplication or the distributive property. That student may be able to calculate the correct answer, but with little understanding of why the algorithm works or what to do if they needed to use an alternative method. Therefore, it should come to no surprise that a large portion of the U.S. population has an unfavorable view of mathematics because they were never given the opportunity to understand it.

In this study, I aimed to raise teachers' awareness of the importance of comprehension in mathematics. Math must make sense for teachers so they can then ensure that it makes sense for their students. The course I developed is designed to give teachers the rich conceptual understand they need to begin teaching math in ways that help students develop that same deep understanding. By drawing upon my findings of how professional learning can support teachers and by considering the current literature on professional learning in mathematics, I can ensure that I will create experiences for teachers that will elevate the importance of meaning in mathematics. From there, my hope is that the experience sparks a change for teachers and puts them on a path to continue to learn and grow in this field.

References

Ahuja, R. (2019). Theory of professional competence in teaching of mathematics: Developing and explication through cross-cultural examination of teaching practices in India and the united states. *Journal of Mathematics Education at Teachers College*, *10*(1), 31-48. Retrieved from https://jmetc.columbia.edu/

Anderson, R. (2019). Networked professional development: An ecological perspective on mathematics teacher learning. *Conference Papers – Psychology of Mathematics & Education of North America*, 525-529. Retrieved from https://www.pmena.org/

Andrews, A., & Brown, J. (2015). The effects of math anxiety. *Education*, *135*(3), 362-370. Retrieved from https://www.projectinnovation.com/education.html

Baloglu, M., & Kocak, R. (2006). A multivariate investigation of the differences in mathematics anxiety. *Personality and Individual Differences, 40*(7), 1325-1335. doi:10.1016/j.paid.2005.10.009t

Barlow, A., Lischka, A., Willingham, J., Hartland, K., & Stephens, D. (2018). The relationship of implicit theories to elementary teachers' patterns of engagement in a mathematics-focused professional development setting. *Mid-Western Educational Researcher*, *30*(3), 93-122. Retrieved from https://www.mwera.org/

Barnes, Y., & Solomon, Y. (2014). Empowering teachers as learners: Continuing professional learning programmes as sites for critical development in pedagogical practice. *Workplace Learning in Teacher Education, 10,* 137-150. doi:10.1007/978-94-007-7826-9

Barrett, D. (2013). Preservice elementary teachers' attitudes improve and math anxiety decreases with focus upon manipulative use. *National Teacher Education Journal, 6*(2), 5-10. Retrieved from https://ntejournal.com/

Bartell, T., Webel, C., Bowen, B., & Dyson, N. (2013). Prospective teacher learning: Recognizing evidence of conceptual understanding. *Journal of Mathematics Teacher Education, 16*(1), 57-79. doi:10.1007/s10857-012-9205-4

Bazeley, P. (2009). Analyzing qualitative data: More than 'identifying themes. *Malaysian Journal of Qualitative Research, 2*, 6-22. Retrieved from https://www.qramalaysia.org/journals-vol3

Bell, C., Wilson, S., Higgins, T., & McCoach, D. (2010). Measuring the effects of professional development on teacher knowledge: The case of developing mathematical ideas. *Journal for Research in Mathematics Education, 41*(5), 479-512. Retrieved from https://www.nctm.org/publications/journal-for-research-in-mathematics-education/

Biccard, P. (2019). The professional development of primary school mathematics teachers through a design-based research methodology. *Pythagoras, 40*(1), 1-10. doi:10.4102/pythagoras.v40i1.515

Blank, R., & de las Alas, N. (2010). Effects of teacher professional development on gains in student achievement: How meta-analysis provides scientific evidence useful to education leaders. *Society for Research on Educational Effectiveness*, 1-24. Retrieved from https://www.sree.org/

Bonghanoy, G. B., Sagpang, A. P., Alejan, R. A., & Rellon, L. R. (2019). Transformative professional development for mathematics teachers. *Journal on Mathematics*

Education, *10*(2), 289-302. Retrieved from http://usnsj.com/index.php/JME

Brendefur, J., Thiede, K., Strother, S., Jesse, D., & Sutton, J. (2016). The effects of professional development on elementary students' mathematics achievement. *Journal of Curriculum and Teaching, 5*(2), 95-108. Retrieved from http://www.sciedupress.com/journal/index.php/jct

Brown, A. E. (2016). *Policy to practice: The perspectives of teachers and administrators on the implementation of common core utilizing action research to design a professional development model* (Doctoral dissertation). Retrieved from https://escholarship.org/uc/item/3w04r4zm

Bruner, J. S. (1966). *Toward a theory of instruction*. Retrieved from https://www.hup.harvard.edu/catalog.php?isbn=9780674897014

Campbell, P., & Malkus, N. (2011). The impact of elementary mathematics coaches on student achievement. *Elementary School Journal*, *111*(3), 430-454. Retrieved from https://www.jstor.org/journal/elemschoj

Carney, M., Brendefur, J., Thiede, K., Hughes, G., & Sutton, J. (2016). Statewide mathematics professional development: Teacher knowledge, self-efficacy, and beliefs. *Educational Policy, 30*(4), 539-572. Retrieved from https://journals.sagepub.com/home/epx

Cave, A., & Brown, C. (2010). When learning is at stake: Exploration of the role of teacher training and professional development schools on elementary students' math achievement. *National Forum of Teacher Education Journal, 20*(3), 1-21. Retrieved from http://www.nationalforum.com/Electronic%20Journal%20Volumes/Cave,%20Agnes%20When%20Learning%20is%20at%20Stake%20NFTEJ%20V20%20N3%202010.pdf

Center for Research in Mathematics and Science Education. (2010). *Breaking the cycle: An international comparison of U.S. mathematics teacher preparation*. Retrieved from http://www.educ.msu.edu/content/sites/usteds/documents/Breaking-the-Cycle.pdf

Chapman, O. (2015). Understanding and supporting mathematics teachers' knowledge for teaching. *Journal of Mathematics Teacher Education, 18*(2), 101-103. Retrieved from https://www.springer.com/journal/10857

Chin, E. T., Lin, Y. C., & Tuan, H. L. (2016). Analyzing changes in four teachers' knowledge and practice of inquiry-based mathematics teaching. *Asia-Pacific Education Researcher, 25*(5-6), 845-862. doi:10.1007/s40299-016-0304-3

Chitanana, L. (2012). A constructivist approach to the design and delivery of an online professional development course: A case of the iEARN online course. *International Journal of Instructions, 5*(1), 23-48. Retrieved from http://www.e-iji.net/

Cole, J., & Gardner, K. (1979). Topic work with first-year secondary pupils. In E. Lunzer & K. Gardner (Eds.), *The effective use of reading* (pp. 167-192). doi:10.1080/10862969409547853

Connelly, L. (2013). Grounded theory. *MEDSURG Nursing*, *22*(2), 124-127. Retrieved from http://www.medsurgnursing.net/cgi-bin/WebObjects/MSNJournal.woa

Cosby, M., Horton, A., & Berzina-Pitcher, I. (2017). Math is all around us: Exploring the teaching, learning, and professional development of three urban mathematics teachers. *Journal of Computers in Mathematics & Science Teaching, 36*(3), 287-305. Retrieved from https://www.aace.org/pubs/jcmst/

Creswell, J. (2009). *Research design: Qualitative, quantitative, and mixed methods approaches*. Retrieved from https://doaj.org/article/93549d21a31d4560bd57db4ac5c30b23

Cueto, S., Leon, J., Sorto, M. A., & Miranda, A. (2017). Teachers' pedagogical content knowledge and mathematics achievement of students in Peru. *Educational Studies in Mathematics*, *94*(3), 329-345. doi:10.1007/s10649-016-9735-2

Danish, J., Saleh, A., Andrade, A., & Bryan, B. (2017). Observing complex systems thinking in the zone of proximal development. *Instructional Science, 45*(1), 5-24. doi:10.1007/s11251-016-9391-z

Davidson, A. (2019). Ingredients for planning student-centered learning in mathematics. *Australian Primary Mathematics Classroom, 24*(3), 8-14. Retrieved from https://www.aamt.edu.au/Journals/Journals-Index/Australian-Primary-Mathematics-Classroom/

Day, N. K. (2013). A synthesis of action research on coaching. *International Journal of Educational Leadership: Theory and Practice*. doi:10.1080/13603124.2013.863386

Doerr, H. M., Goldsmith, L. T., & Lewis, C. C. (2010). *Mathematics professional development.* Retrieved from http://nctm.org/uploadedFiles/Research_and_Advocacy/research_brief_and_clips/Research_brief_15-Goldsmith(1).pdf

Drust, J. (2013). *Elementary school teachers' perceptions of the math coach approach to professional development* (Doctoral dissertation). Retrieved from https://about.proquest.com/products-services/dissertations/

Finlayson, M. (2014). Addressing math anxiety in the classroom. *Improving Schools, 17*(1), 99-115. Retrieved from https://journals.sagepub.com/home/imp

Fountas, I., & Pinnell, G. S. (2017). *Guided reading: Responsive teaching across the grades.* Portsmouth, NH. Heinemann. Retrieved from https://www.heinemann.com/products/e08684.aspx

Garcia, N., Shaughnessy, M., Xueying, J., Pfaffm, E., Mortimer, J., Cirino, N., . . . Robinson, D. (2018). Changing teaching practice: Examining professional development impact on mathematics discussion leading practice. *Conference Papers – Psychology of Mathematics & Education of North America,* 366-369. Retrieved from https://www.pmena.org/

Garet, M. S., Heppen, J. B., Walters, K., Parkinson, J., Smith, T. M., Song, M., . . . Wel, T. E. (2016). Focusing on mathematical knowledge: The impact of content intensive teacher professional development. Retrieved from https://ies.ed.gov/ncee/pubs/

Gee, D., & Whaley, J. (2016). Learning together: Practice-centered professional development to enhance mathematics instruction. *Mathematics Teacher Education and Development, 18*(1), 87-99. Retrieved from https://mted.merga.net.au/index.php/mted

Geist, E. (2015). Math anxiety and the “math gap”: How attitudes toward mathematics disadvantages students as early as preschool. *Education, 135*(3), 328-335. Retrieved from https://www.projectinnovation.com/education.html

Gersten, R., Taylor, M. J., Keys, T. D., Rolfhus, E., & Newman-Gonchar, R. (2014). Summary of research on the effectiveness of math professional development approaches. *Regional Educational Laboratory Southeast*, 1-16. Retrieved from https://ies.ed.gov/ncee/edlabs/projects/project.asp?ProjectID=391

Gibbons, L. K., & Cobb, P. (2017). Focusing on teacher learning opportunities to identify potentially productive coaching activities. *Journal of Teacher Education, 68*(4) 411 -425. doi:10.1177/0022487117702579

Goos, M., Bennison, A., & Proffitt-White, R. (2018). Sustaining and scaling up research-informed professional development for mathematics teachers. *Mathematics Teacher Education and Development, 20*(2), 133-150. Retrieved from https://mted.merga.net.au/index.php/mted

Gresham, G. (2018). Preservice to inservice: Does mathematics anxiety change with teaching experience? *Journal of Teacher Education, 69*(1), 90-107. doi:10.1177/0022487117702580

Grootenboer, P., & Marshman, M. (2016). The affective domain, mathematics, and mathematics education. In P. Grootenboer & M. Marshman, *Mathematics, Affect and Learning* (pp. 13-33). Singapore: Springer. Retrieved from https://www.springer.com/gp/book/9789812876782

Hancock, D. R., & Algozzine, R. (2006). *Doing case study research: A practical guide for beginning researchers.* New York: Teachers College. Retrieved from http://psychoarshad.com/file/file-download/99/

Hanushek, E., Peterson, P., & Woessmann, L. (2010). U*.S. math performance in global perspective: How well does each state do at producing high-achieving students?* (PEPG Report 10-19). Retrieved from http://hanushek.stanford.edu/publications/us-math-performance-global-perspective-how-well-does-each-state-do-producing-high

Harris, K., Sithole, A., & Kibirige, J. (2017). A needs assessment for the adoption of the Next Generation Science Standards (NGSS) in K-12 education in the United States. *Journal of Education and Training Studies, 5*(9), 54-62. Retrieved from http://home.redfame.com/jets/

Hill, H. (2007). Mathematical knowledge of middle school teachers: Implications for the no child left behind policy initiative. *Educational Evaluation and Policy Analysis, 29*(2), 95-114. Retrieved from https://journals.sagepub.com/home/epa

Hill, H., Beisiegel, M., & Jacob, R. (2013). Professional development research: Consensus, crossroads, and challenges. *Educational Researcher*, *42*(9), 476-487. Retrieved from https://journals.sagepub.com/home/edr

Hill, H., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406. Retrieved from https://journals.sagepub.com/home/aer

Hill, H., Rowan, B., & Ball, D. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407. Retrieved from https://journals.sagepub.com/home/jte

Hill, K. K., Bicer, A., & Capraro, R. M. (2017). Effect of teachers' professional development from MathforwardTM on students' math achievement. *International Journal of Research in Education and Science, 3*(1), 67-74. Retrieved from https://www.ijres.net/index.php/ijres

Holmqvist, M. (2017). Models for collaborative professional development for teachers in mathematics. *International Journal for Lesson and Learning Studies, 6*(3), 190-201. doi:10.1108/IJLLS-12-2016-0051

Isiksal, M., Curran, J., Koc, Y., & Askun, C. (2009). Mathematics anxiety and mathematical self-concept: Considerations in preparing elementary-school teachers. *Social Behavior and Personality, 37*(5), 631-644. https://www.sbp-journal.com/index.php/sbp

Jacobs, J., Seago, N., & Koellner, K. (2017). Preparing facilitators to use and adapt mathematics professional development materials productively. *International Journal of STEM Education*, 1, 1-14. doi:10.1186/s40594-017-0089-9

Jameson, M. M., & Fusco, B. R. (2014). Math anxiety, math self-concept, and math self-efficacy in adult learners compared to traditional undergraduate students. *Adult Education Quarterly, 64*(4), 306-322. doi:10.1177/0741713614541461

Jansen, A., & Bartell, T. (2013). Caring mathematics instructions: Middle school students' and teachers' perspectives. *Middle Grades Research Journal, 8*(1), 33-49. Retrieved from https://www.infoagepub.com/middle-grades-research-journal.html

Jayanthi, M., Gersten, R., Taylor, M. J., Smolkowski, K., & Dimino, J. (2017). Impact of the Developing Mathematical Ideas Professional Development Program on grade 4 students' and teachers' understanding of fractions. *Regional Educational Laboratory Southeast*, 3-47. Retrieved from https://ies.ed.gov/ncee/edlabs/regions/southeast/publications.asp

Jordan, A. (2014). Houston, we have a problem: Math and science education lags in the United States, but CASIS is working to reverse the trend. *IEEE Pulse, 4*(4)*, 46-51*. doi:10.1109/MPUL.2014.2321491

Karunakaran, M. (2020). Opportunities to decrease elementary prospective teachers' mathematics anxiety. *Mathematics Enthusiast, 17*(2/3), 469-492. Retrieved from https://scholarworks.umt.edu/tme/

Khaliqi, D. (2016). How common is the common core? A global analysis of math teaching and learning. *School Science and Mathematics, 116*(4), 199-212. doi:10.1111/ssm.12170

Knaus, M. (2017). Supporting early mathematics learning in early childhood settings. *Australasian Journal of Early Childhood, 42*(3), 4-13. doi:10.23965/AJEC.42.3.01

Kutaka, T., Smith, W., Albano, A., Edwards, C., Ren, L., Beattie, H., . . . , Lewis, W., Heaton, R., & Stroup, W. (2017). Connecting teacher professional development and student mathematics achievement: A 4-year study of an elementary mathematics specialist program. *Journal of Teacher Education*, *68*(2), 140-148.

doi:10.1177/0022487116687551

Kutaka, T., Ren, L., Smith, W., Bettie, H., Edwards, C., Green, J., . . . , Chernyavskiy, P., Stroup, W., Heaton, R., & Lewis, W. (2018). Examining change in K-3 teachers' mathematical knowledge, attitudes, and beliefs: The case of primarily math. *Journal of Mathematics Teacher Education*, *21*(2), 147-177. doi:10.1007/s10857-016-9355

Lachner, A., & Nückles, M. (2016). Tell me why! Content knowledge predicts process-orientation of math researchers' and math teachers' explanations. *Instructional Science, 44*(3), 221-242. doi:10.1007/s11251-015-9365-6

Le Cornu, R. (2016). Professional experience: Learning from the past to build the future. *Asia-Pacific Journal of Teacher Education, 44*(1), 80-101. doi:10.1080/1359866X.2015.1102200

Lesseig, K. (2016). Fostering teacher learning of conjecturing, generalizing, and justifying through mathematics studio. *Mathematics Teacher Education and Development*, *18*(1), 100-118. Retrieved from https://mted.merga.net.au/index.php/mted

Levi-Keren, M., & Patkin, D. (2016). Mathematics teachers' professional development program – needs and expectations. *International Journal for Mathematics Teaching & Learning*, 1-33. Retrieved from https://www.cimt.org.uk/ijmtl/index.php/IJMTL

Lindvall, J. (2017). Two large-scale professional development programs for mathematics teachers and their impact on student achievement. *International Journal of*

Science & Mathematics Education, *15*(7), 1281-1301. doi:10.1007/s10763-016-9750-x

Lodico, M., Spaulding, D., & Voegtle, K. (2010). *Methods in Educational Research: From Theory to Practice* (2nd ed.). San Francisco, CA: Jossey Bass. Retrieved from https://www.wiley.com/en-us/Methods+in+Educational+Research%3A+From+Theory+to+Practice%2C+2nd+Edition-p-9780470436806

Looney, L., Perry, D., & Steck, A. (2017). Turning negatives into positives: The role of an instructional math course on preservice teachers' math beliefs. *Education, 138*(1), 27-40. Retrieved from https://www.projectinnovation.com/education.html

Luebeck, J., Roscoe, M., Cobbs, G., & Scott, L. (2017). Re-envisioning professional learning in mathematics: Teachers' performance, perceptions, and practices in blended professional development. *Journal of Technology and Teacher Education*, *25*(3), 273-300. Retrieved from https://www.aace.org/pubs/jtate/

Ma, L. (2010). Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. New York, Routledge. Retrieved from https://www.academia.edu/12089767/Knowing_and_Teaching_Elementary_Mathematics

Maass, K., Swan, M., & Aldorf, A. (2017). Mathematics teachers' beliefs about inquiry-based learning after a professional development course - An international study. *Journal of Education and Training Studies*, 5(9), 1-17. Retrieved from http://redfame.com/journal/index.php/jets

Malik, S. A. (2017). Revisiting and re-representing scaffolding: The two gradient model. *Cogent Education, 4*(1). doi:10.1080/2331186X.2017.1331533

Malinsky, M., Ross, A., Pannells, T., & McJunkin, M. (2006). Math anxiety in preservice elementary school teachers. *Education, 127*(2), 274-279. Retrieved from https://www.projectinnovation.com/education.html

Martin, C., Polly, D., Mraz, M., & Algozzine, R. (2018). Teacher perspectives on literacy and mathematics professional development. Issues in Teacher Education, 27(1), 94-105. Retrieved from https://www.itejournal.org/

Martin, C., Polly, D., Mraz, M., & Algozzine, R. (2019). Examining focus, duration, and classroom impact of literacy and mathematics professional development. *Teacher Development, 23*(1), 1-17. doi:10.1080/13664530.2018.1530695

Massachusetts Department of Elementary and Secondary Education. (2017). Next Generation MCAS Achievement Results. Retrieved from http://profiles.doe.mass.edu/statereport/nextgenmcas.aspx

Mcdermott, B., & Tchoshanov, M. (2014). *Draw yourself learning and teaching mathematics: A collaborative analysis.* Proceedings of the 41st Annual Meeting of the Research Council on Mathematics Learning. San Antonio, TX. Retrieved from https://www.rcml-math.org/assets/Proceedings/rcml%20proceedings%202014.pdf

McNeill, J., Butt, G., & Armstrong, A. (2016). Developing collaborative approaches to enhance the professional development of primary mathematics teachers.

Education, *3*(13), 426-441.doi:10.1080/03004279.2014.973896

Merriam, S. B. (2009). *Qualitative research: A Guide to design and implementation.* San Francisco: Jossey-Bass. Retrieved from https://www.wiley.com/en-us/Qualitative+Research%3A+A+Guide+to+Design+and+Implementation%2C+4th+Edition-p-9781119003618

Merriam, S. B., & Tisdell, E. J. (2015). Qualitative research: A guide to design and implementation (4th ed). San Francisco, CA: John Wiley & Sons. Retrieved from https://www.wiley.com/en-us/Qualitative+Research%3A+A+Guide+to+Design+and+Implementation%2C+4th+Edition-p-9781119003618

Meryem, Ç. (2017). Examination of the relationship between the preschool teachers' attitudes towards mathematics and the mathematical development in 6-year-old preschool children. *Journal of Education and Learning*, *6*(4), *49-56*. Retrieved from http://www.ccsenet.org/journal/index.php/jel

Mhakure, D. (2019). School-based mathematics teacher professional learning: A theoretical position on the lesson study approach. *South African Journal of Education*, 39, 1-8. Retrieved from http://www.sajournalofeducation.co.za/index.php/saje

Miles, M. B., Huberman, A. M., & Saldaña, J. (2013). *Qualitative data anaysis: A methods sourcebook.* (3rd ed). Los Angeles, CA: Sage Publications, Inc. Retrieved from https://us.sagepub.com/en-us/nam/qualitative-data-analysis/book246128

Mntunjani, L., Stanley, A., & Sibawu, W. (2018). Foundation phase teachers' use of manipulatives to teach number concepts: A critical analysis. South African *Journal of Childhood Education 8*(1). Retrieved from https://sajce.co.za/index.php/sajce

Mohan, P., & Chand, D. (2019). Mathematics teachers' professional learning: Impact of story sharing and reflective dialogue. *Waikato Journal of Education*, 24(2), 73-81. Retrieved from https://wje.org.nz/index.php/WJE

Mutodi, P. (2014). Exploring mathematics anxiety: Mathematics students' experiences. *Mediterranean Journal of Social Sciences, 5*(1), 283-294. Retrieved from https://www.mcser.org/journal/index.php/mjss/index

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: Author. Retrieved from https://www.nctm.org/Store/Products/Principles-to-Actions--Ensuring-Mathematical-Success-for-All/

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for mathematics*. Washington, DC: Author. Retrieved from http://www.corestandards.org/Math/

National Professional Development Center on Inclusion. (2008). What do we mean by professional development in the early childhood education field? Chapel Hill: The University of North Carolina, FPG Child Development Institute. Retrieved from https://npdci.fpg.unc.edu/sites/npdci.fpg.unc.edu/files/resources/NPDCI_ProfessionalDevelopmentInEC_03-04-08_0.pdf

Novak, E., & Tassell, J. L. (2017). Studying preservice teacher math anxiety and mathematics performance in geometry, word, and non-word problem solving. *Learning and Individual Differences*, *54*, 20-29. doi:10.1016/j.lindif.2017.01.005

Olson, A., & Stoehr, K. (2019). From numbers to narratives: Preservice teachers' experiences with mathematics anxiety and mathematics teaching anxiety. *School Science and Mathematics, 119(2)*, 72-82. doi:10.1111/ssm.12320

Pain, K. (2015). *The effect of key features of high quality professional development on student achievement in reading and mathematics* (Dissertation). University of St. Francis, ProQuest Dissertations Publishing, 3687901. Retrieved from https://about.proquest.com/products-services/dissertations/

Parker, P. (2014). An examination of the impact of pedagogical preparation, teaching experience, and future career plan on mathematics graduate teaching assistants' efficacy. Dissertation, Georgia State University. Retrieved from http://scholarworks.gsu.edu/msit_diss/133

Patton, M. Q. (2015). *Qualitative research & evaluation methods integrating theory and practice* (4th ed). Thousand Oaks, CA: Sage Publications Inc. Retrieved from https://us.sagepub.com/en-us/nam/qualitative-research-evaluation-methods/book232962

Perdomo-Diaz, J., Felmer, P., Randolph, V., & González, G. (2017). Problem solving as a professional development strategy for teachers: A case study with fractions. EURASIA *Journal of Mathematics, Science, & Technology Education*, *13*(3), 987-999. doi:10.12973/eurasia.2017.00653a

Perry, R., & Lewis, C. (2010). Building demand for research through lesson study. In M. K. Stein & C. Coburn (Eds.), *Research and Practice in Education: Building Alliances, Bridging the Divide.* Lanham, Md.: Rowman & Littlefield Publishing Group. Retrieved from https://rowman.com/ISBN/9781442203648/Research-and-Practice-in-Education-Building-Alliances-Bridging-the-Divide

Peterson, P. E., & Ackerman, M. (2015). States raise proficiency standards in math and reading: Commitments to common core may be driving the proficiency bar upward. *Education Next, 3,* 16. Retrieved from https://www.educationnext.org/

Piaget, J. (1957). *Construction of Reality in the Child.* London: Routledge & Kegan Paul. Retrieved from https://www.routledge.com/The-Construction-Of-Reality-In-The-Child-1st-Edition/Piaget-Jean/p/book/9781315009650

Polly, D. (2017). Providing school-based learning in elementary school mathematics: The case of a professional development school partnership. *Teacher Development*, 21(5), 668-686. doi:10.1080/13664530.2017.1308427

Polly, D., Neale, H., & Pugalee, D. (2014). How does ongoing task-focused mathematics professional development influence elementary school teachers' knowledge, beliefs, and enacted pedagogies? *Early Childhood Education Journal 42*(1), 1-10. doi:10.1007/s10643-013-0585-6

Polly, D., Wang, C., Martin, C., Lambert, R., Pugalee, D., & Middleton, C. (2018). The influence of mathematics professional development, school-level, and teacher-level variables on primary students' mathematics achievement. *Early Childhood Education Journal, 46*(1), 31-45. doi:10.1007/s10643-017-0837-y

Postholm, M. B., & Wæge, K. (2016). Teachers' learning in school-based development. *Educational Research, 58*(1), 24-38. doi:10.1080/00131881.2015.1117350

Prast, E. J., Van de Weijer-Bergsma, E., Kroesbergen, E. H., & Van Luit, J. E. H. (2018). Differentiated instruction in primary mathematics: Effects of teacher professional development on student achievement. *Learning and Instruction, 54,* 22-34. doi:10.1016/j.learninstruc.2018.01.009

Reid, M. (2013). *Mathematics content knowledge, mathematics teacher efficacy, and pedagogy: An examination of the three constructs in mathematics preservice elementary education* (doctoral thesis). University of Calgary, Calgary, Canada. doi:10.11575/PRISM/26840

Reid, M., & Reid, S. (2017). Learning to be a math teacher: What knowledge is essential? *International Electronic Journal of Elementary Education, 9*(4), 851-872. Retrieved from https://www.iejee.com/index.php/IEJEE

Rieckhoff, B., & Larsen, C. (2012). The impact of a professional development network on leadership development and school improvement goals. *School-University Partnerships, 5*(1), 57-73. Retrieved from https://napds.org/journal/

Ruef, J., Willingham, J., & Sweeny, S. (2019). Re-envisioning "good at math:" A case study of positive transformation. *International Journal of Gender, Science, and Technology, 11*(3), 383. Retrieved from http://genderandset.open.ac.uk/index.php/genderandset/article/view/624

Rupam, S., & Gujarati, J. (2013). Moving toward positive mathematics beliefs and developing socio-mathematical authority: Urban preservice teachers in mathematics methods courses. *Journal of Urban Learning, Teaching, and Research, 9*, 100-111. Retrieved from https://aera-ultr.wixsite.com/ultr/journal-of-ultr

Sahin, S. M., & Özpinar, I. (2020). Reflections of mathematical and mathematics educational values on pedagogical formation students' teaching preferences. *Asian Journal of Education and Training, 6(1)*, 80-92. Retrieved from https://eric.ed.gov/?id=EJ1246042

Samford, W. (2013). *Exploring sustained change in teachers' beliefs after professional development.* Electronic Dissertation. Retrieved from https://etd.ohiolink.edu/

Santagata, R., Kersting, N., Givvin, K., & Stigler, J. (2011). Problem implementation as a lever for change: An experimental study of the effects of a professional development program on students' mathematics learning. *Journal of Research on Education Effectiveness, 4(1)*, 1-24. doi:10.1080/19345747.2010.498562

Sawyer, A. (2017). Factors influencing elementary mathematics teachers' beliefs in reform-based teaching. *The Mathematics Educator*, *26*(2), 26-53. Retrieved from https://openjournals.libs.uga.edu/tme

Schubert, E. (2019). Stopping the cycle of math anxiety: Recommendations for teachers and schools. *Delta-K, 55*(2), 15-24. Retrieved from https://eric.ed.gov/?id=EJ1165807

Shenton, A. (2003). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information*, 22, 63-75. doi:10.3233/EFI-2004-22201

Shriki, A., & Patkin, D. (2016). Elementary school mathematics teachers' perception of their professional needs. *Teacher Development, 20*(3), 329-347. doi:10.1080/13664530.2016.1155476

Simpson, A., & Linder, S. M. (2014). An examination of mathematics professional development opportunities in early childhood settings. *Early Childhood Educational Journal, 42*(5), 335-342. doi:10.1007/s10643-013-0612-7

Sloan, T. (2010). A quantitative and qualitative study of math anxiety among preservice teaches. *Educational Forum, 74*(3), 242-256. doi:10.1080/00131725.2010.483909

Smith, M., Hughes, E., Engle, R., & Stein, M. K. (2009). Orchestrating discussions. *Mathematics Teaching in the Middle School, 14*(9), 548-556. Retrieved from https://www.nctm.org/

Smith, M., & Stein, M. K. (2018). *5 practices for orchestrating productive mathematics* (2nd ed.). Reston, VA: National Council of Teachers of Mathematics. Retrieved from https://www.nctm.org/Store/Products/5-Practices-for-Orchestrating-Productive-Mathematics-Discussions,-2nd-edition-(eBook)/

Stake, R. (1995). *The art of case study research.* Thousand Oaks, CA: Sage Publications. Retrieved from https://us.sagepub.com/en-us/nam/the-art-of-case-study-research/book4954

Stephens, M., Landeros, K., Perkins, R., & Tang, J. (2016). *Highlights from TIMSS and TIMSS Advanced 2015: Mathematics and science achievement of U.S. students in grades 4 and 8 and in advanced courses at the end of high school in an international context* (p. 55). National Center for Education Statistics. Retrieved from https://nces.ed.gov/pubs2017/2017002.pdf

Stoehr, K. (2017). Building the wall brick by brick: One prospective teacher's experiences with mathematics anxiety. *Journal of Mathematics Teacher Education, 20*(2), 119-139. doi:10.1007/s10857-015-9322-y

Tabach, M., & Schwarz, B. (2018). Professional development of mathematics teachers toward the facilitation of small-group collaboration. *Educational Studies in Mathematics*, 3, 273-298. doi:10.1007/s10649-017-9796-x

Tasdan, B., & Koyunkaya, M. (2017). Examination of pre-service mathematics teachers' knowledge of teaching function concept. *Acta Didactica Napocensia, 4*(3), 1-18. doi:10.1007/s10649-017-9796-x

Taton, J. (2015). Much more than it's cooked-up to be: Reflections on doing math and teachers' professional learning. *Perspectives on Urban Learning 12*(1), 48-61. Retrieved from https://files.eric.ed.gov/fulltext/EJ1056673.pdf

Tchoshanov, M., Cruz, M., Huereca, K., Shakirova, K., Shakirova, L., & Ibragimova, E. (2017). Examination of lower secondary mathematics teacher' content knowledge and its connection to students' performance. *International Journal of Science & Mathematics Education, 15*(4), 683-702. doi:10.1007/s-015-9703-9

Thornton, J. S., Crim, C. L., & Hawkins, J. (2009). The impact of an ongoing professional development program on prekindergarten teachers' mathematics practices. *Journal of Early Childhood Teacher Education, 30,* 150–161. doi:10.1080/10901020902885745

Timmons-Brown, S., & Warner, C. (2016). Using a conference workshop setting to engage mathematics teachers in culturally relevant pedagogy. *Journal of Urban*

Mathematics Education, *9*(2), 19 - 47. Retrieved from http://ed-osprey.gsu.edu/ojs/index.html

Toraman, C., & Demir, E. (2016). The effect of constructivism on attitudes towards lessons: A meta-analysis study. *Eurasian Journal of Educational Research,* 62, 115-142. doi:10.14689/ejer.2016.62.8

Trent, J. (2012). Teacher professional development through a school-university partnership. What role does teacher identity play? *Australian Journal of Teacher Education, 37*(7), 1-106. Retrieved from https://ro.ecu.edu.au/ajte/

Ulrich, C., Tillema, E., Hackenberg, A., & Norton, A. (2014). Constructivist model building: Empirical examples from mathematics education. *Constructivist Foundations, 9*(3), 328-339. Retrieved from https://constructivist.info/

Vahedi, S., & Farrokhi, F. (2011). A confirmatory factor analysis of the structure of abbreviated math anxiety scale. *Iranian Journal of Psychiatry, 6*(2), 47-53. Retrieved from https://ijps.tums.ac.ir/index.php/ijps

Viadero, D. (2010). Future U.S. teachers rank in middle on math tests. *Education Week, 29*(29), 8. Retrieved from https://www.edweek.org/ew/articles/2010/04/16/29math.h29.html

Vistro-Yu, C. P. (2013). Cross-national studies on the teaching and learning of mathematics: Where do we go from here? *ZDM Mathematics Education, 45*(1), 145-151. Retrieved from https://www.springer.com/journal/11858

Vygotsky, L. S. (1978). Mind in society: The development of higher psychological processes. Cambridge, MA: Harvard University Press. Retrieved from https://www.hup.harvard.edu/catalog.php?isbn=9780674576292

Walker III, W. S. (2018). Teacher views of useful features of mathematics professional development and how the views influenced instruction. *Conference Papers – Psychology of Mathematics & Education of North America*, 426-429. Retrieved from https://www.pmena.org/proceedings/

Wenig, H. (2016). 6 x 7 = yikes! Coaching teachers past their math anxiety. *Exchange, 227*, 63-65. Retrieved from https://www.childcareexchange.com/article/6-x-7--yikes!-coaching-teachers-past-their-math-anxiety/5022763/

Yin, R. (2014). *Case study research: Design and methods* (4th ed.). Los Angeles: Sage Publications. Retrieved from https://www.researchgate.net/publication/308385754_Robert_K_Yin_2014_Case_Study_Research_Design_and_Methods_5th_ed_Thousand_Oaks_CA_Sage_282_pages

Zager, T. (2017). *Becoming the math teacher you wish you had*. Portland, ME: Stenhouse Publishers. Retrieved from https://www.stenhouse.com/content/becoming-math-teacher-you-wish-youd-had

Appendix A: The Project

Beyond Answer-Getting: Exploring Mathematical Practices and Developing Math Reasoning with All Students

Purpose

The purpose of this project is to have a 3-day course that changes teachers' perceptions of mathematics, builds their confidence in their ability to teach it well, and encourages them to want to pursue more professional learning experiences in mathematics. The course is designed to be very accessible for teachers and yet challenge them with engaging activities that build their mathematical content knowledge and relieve anxiety about teaching mathematics. In other words, teachers will be pushed to grow but in ways that do not intimidate them because they will have successful experiences throughout the course.

Goals

- Deepen teachers' mathematical and pedagogical content knowledge.
- Relieve anxiety about teaching mathematics.
- Provide teachers with positive learning experiences in mathematics.
- Boost teachers' confidence in their ability to teach mathematics well.

Learning Outcomes

As a result of this course teachers will:

- Understand how K to 8 teachers can support students with the following Standards for Mathematical Practice:
    - Math Practice 2: Look for and Express Regularity in Repeated Reasoning

  - Math Practice 4: Modeling with Mathematics
  - Math Practice 7: Look for and Make Use of Structure
  - Math Practice 8: Look for and Express Regularity in Repeated Reasoning (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010)
- How the Standards for Mathematics Practice can be leveraged in their classrooms.
- Have specific models and protocols for facilitating learning experiences that draw upon students' ideas and contributions.
- Know how to implement Smith and Stein's (2018) The *5 Practices for Orchestrating Productive Mathematical Discussions*.
- Be able to analyze and understand students' strategies and highlight connections between the strategies.

Target Audience

This course is designed for K to 5 teachers with a wide range of mathematical experiences, and elementary principals, special education teachers, and paraprofessionals who support work in mathematics classrooms. While the course is designed relieve anxiety for teachers who may be less comfortable with mathematics, it is designed to help confident teachers grow and develop through challenges and new ideas in the course as well.

Course Text

The 5 Practices for Orchestrating Productive Mathematics Discussions (Smith, & Stein, 2018).

Materials

- Projector, screen, and speakers for instructor's computer
- Snap cubes
- Base-5 blocks
- Base-10 blocks
- Graph paper
- Rulers
- Markers
- Chart paper
- Easel
- Index cards

Copies (the following documents are all available after the reference section below)

- Math Practice 7 and The Matrix
- Pre-Course Assignment
- Propeller Design Template
- Alphabet Counting Sheet
- Exploring Addition and Subtraction Strategies Sheet
- 3-Act Task Recording Sheet
- Base-5 Task Sheet
- Henrietta's Roost Case
- The Cycling Shop Task

- Terminating and Repeating Decimals Task
- Multiplying by 10 Task Sheet
- Exploring Structure Through Algebraic Reasoning Sheet
- Post-Course Reflection

Pre-Course Preparation

One month before the start of the course, participants will be sent a pre-course assignment (see below) that will be used to gauge their conceptual understanding of some elementary math concepts as well as their pedagogical knowledge as it relates to those concepts. The pre-course assignment will involve teachers analyzing student thinking through problems with whole numbers and fractions. The analysis requires teachers to have a broader understanding of the operations and a sense of how students make sense of the mathematics. If teachers have a more traditional approach to math instruction, they may have limited responses to the prompts.

Instructors will use participants’ responses to inform the way they facilitate the course. If the math content and pedagogical knowledge seem weaker, more emphasis should be placed on developing this knowledge for participants. If the responses indicate that teachers have a deeper conceptual understanding of the mathematics and their pedagogical knowledge is strong, the emphasis on the course will be on teaching practices. In both cases, the math content, pedagogy, and teaching practices will be parts of the course. Instructors will alter their pacing for each component based on participants’ needs.

Additionally, participants will be asked to read the first two chapters of the course textbook, The *5 Practices for Orchestrating Productive Mathematical Discussions* (Smith & Stein, 2018). This book, hereafter shortened to *5 Practices,* focuses on shifting the focus onto students' ideas as the main driver of the math learning rather than the teacher's direct instruction. Each of the practices discussed help teachers to plan for and facilitate meaningful discussions of mathematics with their students.

Day 1: Framing the Shifts in Math Teaching

Schedule

8:30 – 8:40 a.m.: Introductions

8:40 – 9:10 a.m.: Money Roll: Mathematical Modeling with 3-Act Tasks

9:10 – 9:30 a.m.: Looking Back

9:30 – 9:50 a.m.: Break

9:50 – 11:30 a.m.: Moving From Answer-Getting to Sense-Making

11:30 – 1:30 p.m.: Lunch and Reading

1:30 – 2:30 p.m.: Levels of Inquiry

2:30 – 3:00 p.m.: Discussion of Readings

3:00 – 3:20 p.m.: Break

3:20 – 3:40 p.m.: Open-Middle Tasks

3:40 – 4:20 p.m.: Task Analyses and Math Makeovers

4:20 – 4:30 p.m.: Wrap Up and Homework Assigned

Detailed Agenda Day 1

Introductions (10 Minutes, Slide 1)

Introduce yourself and share one interesting personal fact about yourself. Then ask each participant to do the same. Review basic logistics (bathrooms, water fountains, lunch procedure, etc.). Then begin the session.

Money Roll: Mathematical Modeling with 3-Act Tasks (30 Minutes, Slides 2-16)

The day will begin with a mathematical modeling task I created called *The Money Roll*. This will serve as the launch for the whole course.

- Show a 30-second video where I demonstrate rolling out a large roll of one-dollar bills that are all taped together. Participants' task is to figure out how many bills are in the whole role. (Slide 2)
- Show the measurement of a bill and the measurement of the whole roll using snap cubes as the unit of measure. (Slide 3)
- Make sure participants see that a bill is slightly larger than 8 cubes which should create some ambiguity in the task. Although there is only one correct answer, the hope is that with the ambiguity of the bill being slightly larger than 8 cubes, participants will come up with numerous different answers which will result in a robust debate among the group.
- Show the length of the whole roll measured in cubes. (Slide 4)
- Show the side by side view of the bill and the roll and tell participants they will work on solving the problem. (Slide 5)
- Allow participants about 10 minutes to work on the problem with their small groups.
- Then call groups back to debate their answers. Typical suggested answers are between 19 – 23 bills. Allow for some debate and encourage participants to defend their answers.
- Play the reveal video showing the actual answer and allow some discussion about the range of answers. (Slide 6)

After the reveal of the answer and some more debate about the differences in participants' thinking, show the equation of the math they just did, "179 ÷ 8 = __" (Slide

7) and ask if they would have been as engaged if the class began with this task versus *The Money Roll*. Then show Slide 8 with a word problem version of the Money Roll task. This is to set up a comparison between a task geared toward answer-getting and one geared toward problem solving and math reasoning. Show the slides that review why tasks like *The Money Roll* are more engaging than basic word problems and computation exercises.

- The Peak/End Rule (Slide 9): People judge their experience based on how they felt at the most intense moments and at the beginning and end. Show that peaks are positive experiences and pits are negative experiences (Heath & Heath, 2017).
- Football example (Slide 10): Show how football has lots of pit moments but the peak moment of game day is so high that lots of players have fond memories of the sport.
- Math class example (Slide 11). Show the peak/end rule in math class with the flat line with dips and no peaks. Ask how we can create peak moments in math class.
- 3 Part Recipe to Build Peak Moments (Slides 12 & 13). To build peak moments we need to boost sensory appeal, raise the stakes, and break the script. Have participants consider how the Money Roll task did all three things.
- What kinds of work are students doing? (Slide 14): Show the grid that highlights the four kinds of ways students engage in mathematics (Meyer,

2014). Ask participants to estimate what percentage of time is spent on each area.

The purpose in starting the course off this way is to get right to the point of moving beyond answer-getting in mathematics. It also leverages one of the key findings from the study that teachers like learning experiences when they can collaborate with colleagues to makes sense of complex mathematical ideas. End this section with the slide showing the course goals (Slide 15).

Looking Back (20 minutes, Slide 16)

After the launch task, participants will be asked to reflect for 5 minutes on their own experiences in mathematics when they were in elementary school.

- What do they remember from their experiences?
- What are some positive memories they have?
- What are some negative memories?

After some quiet reflection participants will pair off and share their experiences for 5 minutes. They will then return and collect and categorize the positive and negative experiences to better understand what works and what does not work when teaching math. Spend 1 minutes on this part.

Break (20 minutes)

Allow participants time to use the restroom, get coffee and water, and stretch their legs.

Moving From Answer-Getting to Sense-Making (100 Minutes, Slides 17-47)

The next part of the course will be devoted to why we are looking to move away from answer-getting mathematics to develop students' conceptual understanding. This section of the day has a blend of small interactive activities coupled with some presentation from the facilitator all aimed at framing the work. Topics that will get covered during this section include: Answer-getting versus problem solving, routine and adaptive expertise, the illusion of understanding in mathematics, the conceptual understanding pentagon, the concrete, representational, and abstract continuum, and shifting from the gradual release method of teaching mathematics. A key activity in this section will be when participants must operate in a base-5 number system. This activity pulls together all the above topics into one power experience that should clarify for participants why they need to shift their instructional practices.

Understanding the Shift (20 Minutes)

- Lost in Boston (Slides 17-19): Explain that memorizing procedures in mathematics is like memorizing directions to navigate Boston. In both cases, you may get where you are going, but you will feel lost the whole time.
- Use Slides 20-27 to compare answer-getting and problem solving and relate answer-getting to routine expertise and the illusion of understanding. This process involves the following steps:
    - Slide 20 is the title slide for this section.
    - Slide 21 explains routine expertise (Russell, 2010)

  - Slide 22: Illusion of Understanding (Flynn, 2017). Explain that this is the appearance that students understand a concept when they only have rote knowledge of it.
  - Slide 23: Take participants through the shapes task by asking them to memorize three “facts”. This is to help them experience the illusion of understanding.
  - Quiz participants using Slide 24. They should be 100% successful because the task was set up to support routine expertise.
  - Give participants an applied problem next (Slide 25) and ask them why they can’t answer it. They should say it’s because they have no understanding of the task. They just memorized information.
  - Share the answer and then give participants a minute to discover that the underlying structure is that they are adding up the number of sides each shape has. Show Slide 26 and ask them to apply their new understanding to the earlier work they memorized.
  - Give participants a new applied problem (Slide 27). They should be 100% successful, but this time it is because they understand the meaning of the work.
- Show Slide 28 with the *Shepherd Problem* and then show the video (Kaplinsky, 2013) of eighth graders trying to solve it as another example of routine expertise and the illusion of understanding. Ask for participant

reactions after the video. You can embed the video on Slide 29 or show it from YouTube.

- Slide 30: Explain the Conceptual Understanding Pentagon (Van de Walle, Karp, Lovin, & Bay-Williams, 2018)
- Share the common confusions third graders make when they memorize math facts without understanding the operation (Slide 31) and reference the conceptual understanding pentagon (Slide 32).
- Show Slide 33 as one more example of the illusion of understanding with Jared. Jared is a second grader whose mom had him memorize the standard algorithm for addition, but Jared really could only solve problems by counting tally marks. Display Slide 33 showing the three second graders' work using the standard algorithm, which includes Jared's work. Have them predict which of the three students is struggling in math. Most likely they will say Sharonda because she put the plus sign in the tens place. Reveal the students' second methods which show Jared using tally marks to solve the problem. Have participants comment on Jared's second method and let them know that Jared's mom had taught him how to use the standard algorithm even though he did not understand place value. This sets up the next activity. Show Slide 34 and say we can embrace the illusion of understand and give Jared an A for his work. However, we know that is not the right thing to do for him and his mathematical learning.

Calculating in Base-5 Intro (20 Minutes)

- Slides 35–41 will help you set this work up. Tell participants they will experience what it is like to be Jared (Slide 35).
- Slide 36: Give them a math problem to solve mentally. They will solve it in base 10 so the answer they give will be wrong. Do not tell them it is base 5. Break down the addition and tell participants they need to memorize the new math facts. They should see how the algorithm works but will not know why it works because the math will likely not make sense at this point. Give participants another problem and follow the same steps.
- When participants are thoroughly confused and frustrated, explain that this is what it is like to be Jared. He has memorized a procedure to get answers but does not know why it works and really struggles with the mathematics. His answer-getting method masks his confusion.
- Reveal that they are working in base 5 and have participants help the facilitator construct the first 25 numbers in the system using a base-5 representation (Slide 37).
- Give participants 30 minutes to work on the base-5 tasks (see **Base 5 Task Sheet** below). They should use base-5 blocks and visual representations to help them. Monitor the work and ask guiding questions as participants get stuck. Do not rescue participants by telling them what to do. Allow them to struggle productively during this task.

- After the task, have a 10-minute discussion about the experience of working with base 5 using concrete and visual representations (Slides 38 & 39). Participants should notice:
    - They constructed the understanding themselves.
    - The facilitator did not do any direct teaching.
    - The visual tools were necessary for understanding and the algorithm was not helpful until they understood how the math worked.
    - The facilitator supported productive struggle by asking questions rather than telling people how to do the work.
- Slide 40: Review adaptive expertise (Russell, 2010).
- Slide 41: Ask participants to connect the conceptual understanding pentagon to adaptive expertise.

Defining the Shift (20 Minutes. Slides 42-46)

- Connect the base-5 work to the concrete/representational/abstract continuum. Have participants comment on the importance of all three ways of thinking (Slides 42 & 43).
    - Make the comparison between being a math teacher and being either a lifeguard or swim instructor. Lifeguards rescue and eliminate struggle while swim instructors encourage some struggle so children learn to swim on their own (Slide 44).

- Discuss how we need to flip the gradual release method of *I Do, We Do, You Do* (Slides 45 & 46).

Lunch and Reading Break (2 Hours, Slide 47)

Participants will then break for lunch and will have assigned readings related to the work they just completed. The first will be Chapters 3 and 4 from the course textbook. The second is an article from the National Council of Teachers of Mathematics Teaching Middle School Mathematics Journal called *Never Say Anything a Kid Can Say* (Reinhart, 2000). Both readings have a focus on the power of using students' ideas in math class and how we should shift from the teachers being the deliverer of knowledge to a facilitator of learning experiences.

Levels of Inquiry (60 minutes, Slides 48-60)

The second half of the day will begin with an interactive task exploring the four levels of inquiry: Confirmation, Structured Inquiry, Guided Inquiry, and Open Inquiry. For this work, participants will use a propeller made of paper to go through four explorations. The purpose of this task is to give participants an opportunity to experience all four levels of inquiry as learners so they can see how their engagement and investment in the work build as they move up in the levels. The facilitator will then make comparisons between this experience and mathematics instruction. The interactive tasks are as follow:

- Slide 48: **Introduction.** Show participants how to make the propeller (see materials below). Then explain to them that they will confirm that propellers cause objects to spin when they fall. Have them do the experience and the reflect on the experience (Slide 49).

- Show the Levels of Inquiry (Slide 50) and explain they just did level one.
- Slide 51: **Confirmation.** Explain that the first level of inquiry is confirmation. Participants will be given the question they need to explore, the method they will use to the explore it, and the solution to the question before they begin their work. The participants' job is to confirm a principle that the facilitator already stated, in this case, that propellers cause an object to spin as it is dropped.
- **Structured Inquiry**. The exploration will continue with a structure inquiry experience. Show Slide 52 and explain structured inquiry. Then show Slide 53 with their challenge. In this case, participants will be given the question, "How does changing the position of the propellers affect an object when it is dropped?" They will also be given the method to explore this question. In this case, they need to drop the propellers and observe them as they fall. Then they will alternate the positions of the propellers and drop them again. They will not know the results so it should surprise them that the direction of the rotation changes when the positions are flipped. Once again, the participants will debrief how the experience felt for them as learners and what this kind of inquiry looks like in math class (Slide 54).

- **Guided Inquiry.** Show Slide 55 and explain guided inquiry. Then show Slide 56 and explain their new challenge. The next phase of this work is a guided inquiry where participants are given the question, "How does adding weight to their object affect how it falls?" This time, they can design their own methods for exploring this question and the results are unknown. I anticipate that participants will use paperclips that are provided to add weight to various locations on their propellers. Some propellers will work, and participants will notice that the rotation speed increases. Some propellers will not work when the weight is added too close to the top or on the actual propellers. When the propellers do not work because the weight is added to the wrong part of it, participants should note the issue and then work to find a position that results in a rotation. At the end participants will debrief again about what the experience was like for them as learners and what this kind of inquiry looks like in a math class (Slide 57).
- **Open Inquiry.** Show Slide 58 and explain open inquiry. Show Slide 59 and explain their new challenge. The final level of inquiry is open inquiry where participants generate the question, the method, and they do not know the results ahead of time. Facilitators should anticipate a wide range of possible questions to explore and a high level of engagement from the group. Participants will follow the same pattern from earlier and end with

a debrief on what this was like for them as learners and what this would look like in a math class (Slide 60).

Conclude, with a discussion with participants on their insights after this experience. The purpose of this exploration is to help participants experience their level of interest and engagement rise as the participants are given more autonomy and agency in their work. My hope is that participants will recognize that the same thing can happen in math class when students are given options and are less restricted by the one right way to approach a task. I also want participants to see how actual process is much more interesting than the product. In other words, the actual work toward solving a problem is where the interesting stuff happens. The results, while important, are much less interesting. Yet, the focus in math class for so long has been on the latter.

Discussion of Readings (30 Minutes, Slides 61 & 62)

Next, participants will have small group discussions on the readings for 20 minutes. Participants will be asked to discuss the points that resonated with their thinking, the points that challenged their thinking, and the questions these readings raised. They will also need to consider how these ideas might play out in their own classrooms. When participants return to the whole group, the facilitator will synthesize the discussion by asking the group the benefits and challenges of having students do more talking in math class with the teacher acting as a facilitator. The goal is to be sure participants can express both the positive and negative aspects of this shift. By allowing participants to share their concerns, the facilitator can be sure to address these concerns throughout the rest of the course. The whole group discussion should take 10 minutes.

Break (20 minutes)

Allow participants time to use the restroom, get coffee and water, and stretch their legs.

Open-Middle Tasks (20 Minutes, Slides 63-66)

Start with the title slide (Slide 63) and ask if anyone knows what open-middle problems are. Show Slide 64 and explain what they are if nobody knows. The facilitator will follow this experience with another math task for participants and connect it with the previous work on the levels of inquiry. This time the facilitator will use an open-middle task from Kaplinsky (2019) that could be categorized as a guided inquiry from the previous work. Show the task on Slide 65. This task asks participants to use the digits 1 to 9 exactly one time each to create an addition expression using three 3-digit numbers. This task is much more complicated than it looks and will take participants some time to solve. Small groups discuss their strategies and methods and participants will compare this work to a traditional addition problem involving three 3-digit addends. After some time, the full group will debrief the task and identify the most useful strategies for solving this type of problem (Slide 66).

Task Analyses and Math Makeovers (40 Minutes, Slides 67-70)

Show title slide (Slide 67). Following this work, participants will pivot toward looking at a task-analysis guide on Slide 68 to consider what makes a rich task that is geared toward problem solving compared to typical answer-getting exercises. Participants will practice using the guides to analyze a very prescribed fraction lesson (Slide 69). They will work to redesign the task to increase both the level of challenge and the level of engagement for students (Slide 70). The goal of this work is to empower teachers with the skills needed to adapt math tasks and lessons to increase engagement for students.

Homework (Slide 71)

Participants will be assigned Chapters 4 and 6 from the course text for homework. This is a lot of reading, so do not assign any written work. However, participants will need to be prepared to discuss the chapters from the lens of their own classrooms. A big part of this work is the implementation of these ideas so that participants are always thinking about what these ideas will look like in place in their classrooms.

Day 2: Explore Mathematical Practices and Developing Mathematical Reasoning Part I.

Schedule

8:30 – 8:40 a.m.: Combatting the Curse of Knowledge

8:40 – 9:10 a.m.: Alphabet Counting

9:10 – 10:35 a.m.: Mathematical Practice 2 and Exploring Students' Strategies

10:35 -10:55 a.m.: Break

10:55 – 11:25 a.m.: Discussion of Readings

11:25 – 11:30 a.m.: Review Lunch Assignments

11:30 – 1:30 p.m.: Lunch and Reading

1:30 – 2:00 p.m.: What is Mathematical Modeling

2:00 – 2:35 p.m.: Story Problems and Tasks to Support Mathematical Modeling

2:35 – 2:55 p.m.: Break

2:55 – 4:25 p.m.: Mathematical Modeling Through 3-Act Math Tasks

4:25 – 4:30 p.m.: Assign Homework

Detailed Agenda

Start with Day 2 title slide (Slide 72). The next two days of the course will focus more heavily on the Standards for Mathematical Practice (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and deepening teachers' mathematical content knowledge. The teaching practices will be analyzed by having participants reflect on their roles and the facilitator's role within each major math experience. The facilitator will lead the participants through the math much in the same way that participants should be leading their students. The facilitator will use the *5 Practices* method from Smith and Stein (2018) and will have the participants lead the work while the facilitator asks guiding questions and helps synthesize the groups ideas.

The focus on Day 2 will be on the two Standards for Mathematical Practice that deal with contextualizing and decontextualizing in mathematics. The first in Math Practice 2: Reason Quantitatively and Abstractly. This one is all about students making sense of the mathematics by contextualizing abstract concepts to make sense of the numbers and symbols within a particular mathematical idea. Math Practice 2 also involves students decontextualizing a situation to understand the relationship between the symbols and their concrete referents. In other words, students think contextually and abstractly to make sense of mathematical concepts. Math Practice 4 will be covered in the afternoon. This one also deals with contextualizing and decontextualizing in mathematics but for the purposes of problem solving.

Combatting the Curse of Knowledge (10 Minutes, Slides 73 & 74)

- Introduce the participants to the concept of the curse of knowledge by sharing the definition and asking them to consider why this applies to the math teaching world (Heath & Heath, 2017).
- Introduce the Alphabet Counting activity.

Alphabet Counting (30 Minutes, Slides 75 & 76)

Pass out the **Alphabet Counting Sheet** (see below). For this task, the number system has been replaced with letters and participants need to count and compare objects, skip count, count backwards, and use a counting strategy to add. Because this system is unfamiliar to the participants, they will need to work hard to make sense of the system. Some participants may create visual references. Other participants may use their fingers. In both cases, all participants will be moving from abstract to concrete referents and will get a feel for the complexity of this work.

Participants will debrief the experience from both the mathematical lens and from the facilitation lens. The facilitator should ask participants to first discuss how the task was set up in a way for them to engage in productive struggle and for them to discover the key ideas on their own. Then participants should consider the facilitator's role in bringing out the key ideas from the activity. The participants should notice that all the ideas were generated from the group and that the facilitator just asked questions to guide the learning.

Mathematical Practice 2: Reasoning Abstractly and Quantitatively (85 Minutes, Slides 77-87)

- Introduce Mathematical Practice 2 by sharing the definition. Allow participants to discuss which parts make sense and which seem confusing. Work to clarify any misunderstandings (Slide 77).
- Give two mental math problems.
    - Start with 34 + 28 and then record solutions (Slide 78).
    - Then have participants work on 53 – 17 and record their solutions (Slide 79).
    - Walk through the steps participants will need to take for their small group work (Slides 80-85).
- Next, pass out the **Analyzing Student Strategies Sheet** (see below). Participants will work in small groups analyzing students' methods for solving addition and subtraction problems. For each method, participants must write a generalizable verbal description of the strategy, create a base-10 representation of the strategy, use a number to model the strategy, and to create a story context where each element of the strategy is accounted for. Creating visual representation, verbal descriptions, and contextualized situations to reflect the mathematics within a given strategy is difficult work but it will help participants gain a richer understanding of the alternative methods for solving addition and subtraction problems.

- After participants have worked through each strategy with their small groups, call them back to debrief. Ask them what new insights they gained, what was challenging, and what key mathematical ideas underlie each strategy. In the end, participants will have a clearer understanding that all generalizable strategies are basically algorithms and depending on the scenario, they might be more efficient than the standard algorithm that has been used for so long in our country (Slides 86 & 87).

Discussion of Readings (30 Minutes, Slides 88 & 89)

The morning will end with another discussion of the readings. This time participants will be asked to identify when they saw elements of the first two practices in the work the facilitator did with them in the morning. Participants will then look at a math lesson they will teach in the school year to see how they might begin designing the lesson with the *5 Practices* (Smith & Stein, 2018) in mind. Have them spend 20 minutes in small groups and then call them back and share the whole group discussion prompt. The whole group discussion should last 10 minutes.

Review Lunch Assignments (5 minutes, Slide 90)

The assigned lunch reading will be Chapter 7 of the 5 Practices textbook (Smith & Stein, 2018) and the article *Modeling with Mathematics Through Three-Act Tasks* (Fletcher, 2016).

Lunch and Reading (2 Hours)

Mathematical Practice 4: Modeling with Mathematics

The afternoon session will focus on Math Practice 4: Modeling with Mathematics. This practice involves students abstracting a contextualized situation to better understand the mathematical components and the relationship between the quantities involved. Students then use this information to solve a problem. Math modeling involves approximation and considering the variability that exists within our world. Thus, when people abstract a situation to create a mathematical model, they may discover that their model has inaccuracies once we apply our results back to the context. Mathematical models often need to be revised as part of the problem-solving process.

In addition to working on math modeling tasks, participants will analyze classroom video from kindergarten and second grade that demonstrate how elementary students begin to mathematize the world around them. They will also analyze different story problems to identify those that have elements of mathematical modeling and those that do not. This understanding is critical because I want teachers to leave with applicable skills that they can use in their classroom to support this work.

What is Mathematical Modeling? (30 Minutes, Slides 91-117)

Begin with Slide 91 and explain what the 8 Standards for Mathematical Practice are. Then show Slide 92 which gives the definition of mathematical modeling. Ask participants what that means (Slide 93). Show the abbreviated definition on Slide 94 (Illustrative Mathematics, 2014).

To launch this work, lead participants through a story involving mathematical modeling in a lighthouse:

- Show Slide 95 with the lighthouse photo and then Slide 96 from inside the lighthouse looking up and ask participants what they notice and wonder. You want somebody to wonder how many steps are in the whole lighthouse. Once that question surfaces, ask what information they would need to calculate how many steps there are.
- Show the Slide 97 with the cross-section of the lighthouse and have them calculate the total number of staircases. Then show the slide that has the number of steps for a particular staircase (Slide 98).
- Show the cross-section again on Slide 99 and ask participants to write an equation and answer how many steps are in the whole lighthouse. Then reveal the answer which will show they are all incorrect. Display Slide 100 with the equation on it.
- Reveal the answer of how many steps there are on Slide 101.
- Ask them to look at the cross-section again and consider what might be wrong. Then have them revise their model again (Slide 102). Continue this process for all the variations on Slides 103-110.
- Throughout the process, participants will create mathematical models only to find that they are incorrect. They will revise their models based on new information and continue this process until the end of the story when the final critical piece of information is revealed (Slides 111-114). This task is designed to give all participants a true understanding of what mathematical modeling is. This understanding is important because typically there are a lot of misconceptions among teachers about this practice standard that should be cleared up.

- Discuss the concrete to abstract continuum and how it relates to mathematical modeling (Slides 115-117).

Story Problems and Tasks to Support Mathematical Modeling (35 Minutes, Slides 118-133)

Next, participants will explore various math tasks and story problems geared toward mathematical modeling. Present each task on the slides and give participants 5 minutes to work on the task with a partner and discuss how the task support mathematical modeling. They should notice that unlike traditional story problems and math tasks that present all the numbers participants need to solve the problem, these tasks require participants to generate additional numbers and patterns in order to solve the problems. Go through the tasks in the following order:

- Marble Jar (Slides 118-120)
- Quick Images (Slides 121-124)
    - Do the task, then have participants discuss how it supports mathematical modeling in the primary grades.
- Six Crayons in All (Slides 125-127)
    - Do the math and then have participants discuss how it supports mathematical modeling in the primary grades.
- Henrietta's Roost Case (Slides 128-132) – Have teachers do the task and then read the case study found in the materials below.
- Cycling Shop Task (Flynn, 2016; Slide 133)

Break (20 Minutes)

Mathematical Modeling Through 3-Act Math Tasks (90 Minutes, Slides 134-142)

The culmination of this work on mathematical modeling will be with three-act math tasks (Fletcher, 2016). These tasks are designed with three acts, much the same way some books and plays are designed.

- **Act 1** involves hooking the audience, getting them curious about the situation, and identifying the problem. This is typically done by showing a short video or compelling image that has a mathematical element to it. Participants share observations and questions they have after watching the video. Typically, one of the questions is the actual question the task is designed to answer.
- **Act 2** involves participants figuring out want information they need (or do not need) to answer the selected question. It is also where participants work to use the information that they have to solve the problem.
- **Act 3** begins with people discussing their solutions. This discussion often involves some debate when there is more than one potential solution. Finally, there is the actual reveal of the solution which is typically in the form of a short video or another compelling image with the answer easily seen.

Launch (20 Minutes)

- Start with the title slide (Slide 134) and explain what 3-act math tasks are and how they engage students (Meyer, 2011). Launch the work on three-act math tasks with a task I made called *Burger Up*. The task begins with a short video (Monster burger Act 1; Flynn, 2019) of a promotion for a food challenge from a Chicago-based restaurant. The video shows a massive cheeseburger being served and held up to the screen (Slide 135).

- Ask participants to talk in their groups about what they notice and wonder about the video. Ideally, someone will wonder how much the burger weighs. Once that question is on the table, move onto Act 2 (Slide 136).
- Ask the participants what information they need to answer the question of how much the burger weighs. The participants can brainstorm lots of possible ways they can figure out the weight of the burger using math. The information included in the task is all the ingredients that make up the burger and their weights in ounces. The challenge with this task is that the weights of the ingredients are all before cooking. This is what makes *Burger Up* a great math modeling task. Participants must consider that some of the weight will be lost during cooking. This ambiguity will result in a range of answers from participants.
- For Act 3, participants will defend their answers and provide sound mathematical arguments to back up their reasoning. After their responses are shared, the facilitator will show the Act 3 video (Monster burger Act 3; Flynn, 2019) which reveals the actual weight of the burger (8 pounds) (Slide 137). Participants will look back at their mathematical models and have discussions about what would need to be revised in them. Finally, the facilitator will debrief the task by first asking participants to talk about the

mathematics and then by having them reflect on their roles as learning and the facilitator's role in bringing their learning forward (Slides 138-141).

Selecting and Facilitating 3-Act Tasks (70 Minutes)

- Show the assignment on Slide 142. Participants will work in groups to look for existing three-act math tasks online. Numerous math educators create and share them online for free so there is no shortage of great three-act math tasks to draw from. For example, Graham Fletcher has one that helps students explore multiplication and division strategies using his task "Array-bow of Colors" (Fletcher, 2015). In this task, Act 1 opens with a jar being filled with Skittles. Act 2 gives students the amount of skittles in one package and the total number of packages used to fill the jar. Act 3 reveals the total number of Skittles by making an array of piles Skittles with 10 Skittles in each pile.
- The groups will select one task and use the 5-Practices planning protocol to design a learning experience with their task.
- Bring the group back together and show Slide 142 outlining the jigsaw discussion. Then jigsaw the groups and each member will facilitate their task to their new small group. This will give each member an opportunity to practice facilitating a task using The *5 Practices* (Smith & Stein, 2018).
- They will then debrief the experience.

Assign Homework (5 Minutes, Slide 143)

The homework for the evening will be to finish the *5 Practices* Smith & Stein, 2018) book.

Day 3: Explore Mathematical Practices and Developing Mathematical Reasoning Part II.

Schedule:

8:30 – 8:35 a.m.: Defining Mathematical Practice 8: Looking for and Expressing Regularity in Repeated Reasoning

8:35 – 9:05 a.m.: Visual Patterns

9:05 – 9:50 a.m.: Terminating and Repeating Decimals

9:50 – 10:10 a.m.: Break

10:10 – 11:00 a.m.: The Path of the Billiard Ball

11:00 - 11:25 a.m.: Discussion of Reading

11:25 – 11:30 a.m.: Work Assigned

11:30 – 1:30 p.m.: Lunch and Reading

1:30 – 2:00 p.m.: Defining Mathematical Practice 7: Looking for and Making Use of Structure and Comparing the Standard to the Movie The Matrix

2:00 – 2:40 p.m.: Multiplying by 10

2:40 – 3:00 p.m.: Using Structure to Support Fluency

3:00 – 3:20 p.m.: Break

3:20 – 3:40 p.m.: Routines to Support Mathematical Practices 7 and 8

3:40 – 4:20 p.m.: Designing Routines and Applying Course Ideas

4:20 – 4:30 p.m.: Final Reflections

4:30 p.m.: Course Evaluations

Detailed Agenda

Start with the title slide (Slide 144).The math practices for this day are Math Practice 7: Look For and Make Use of Structure and Math Practice 8: Look For and Express Regularity in Repeated Reasoning (citation). The two practices are related and often do not happen apart from one other. When one notices regularity in mathematics, there is always some underlying structure that is causing that regularity. For example, a participant might notice that when he or she adds one to a factor in a multiplication expression, the product increases by the other factor. That participant may test that idea through repeated reasoning and then express the regularity in the form of a generalized claim. All work is an example of Math Practice 8. If the student then explores why that pattern exists, they are working on Math Practice 7 because they are looking for the underlying structure. Once they understand the structure, they can then use that in problem solving settings. This last day will focus on the relationship between both practices.

Defining Mathematical Practice 8: Looking For and Expressing Regularity in Repeated Reasoning (5 Minutes)

- Review the definition of Math Practice 8: Look For and Express Regularity in Repeated Reasoning. Ask participants to share their understanding of this standard. Then work to clarify any misconceptions.

Visual Patterns (30 Minutes, Slides 145 & 146)

The class will begin by focusing on looking for regularity. This is challenging work because it requires participants to explore lots of instances within a given scenario to see regularity and make general claims about it. For an easy open, the facilitator will use visual patterns as they are easily accessible to all learners but can be very complicated structurally. The steps are as follow:

- Show Slide 145 with the first visual pattern.
- Ask participants what they notice and wonder about the images. The goal is to get them to look at various aspects of the visual pattern. People typically want to jump right to discovering the rule for the pattern, but I want to slow this work down to allow participants time to notice lots of different forms of regularity. It is likely that new ideas will be generated from this work because participants will take the time to analyze the visual structure rather than just jump to an abstract formula.
- Show Slide 146. Participants will then work to describe the pattern using verbal descriptions rather than a using a formula. Encourage participants to slow down and move from their tendency to jump to an "answer" because the actual interesting work is in the process of discovery through slow exploration. Slowing down also forces them to look at the mathematics through concrete and visual lenses which strengthens their conceptual understanding of the ideas.
- The final work for participants is to create a generalizable rule for the pattern expressed both verbally and numerically.
- Debrief the activity by focusing on the nature of the task and the intentional instructional decisions that were in place that allowed the participants to dig into the work and focus more on the process than the end result. Participants will then

discuss their roles in the work and the role of the facilitator in orchestrating a productive mathematical discussion. This discussion will help participants see the *5 Practices* (Smith & Stein, 2018) from both the lens of the learner and the lens of the facilitator.

Terminating and Repeating Decimals (45 Minutes, Slides 147-149)

- Show Slide 147 and ask participants to discuss the questions.
- Pass out the **Terminating and Repeating Decimals Task Sheet** (see below). This next math task involves participants exploring the results of division of unit fractions. They will work in small groups to explore what happens with different denominators by working out the division by hand. Participants will notice that some unit fractions produce terminating decimal quotients while others produce repeating decimal quotients. As they do this work, they will begin to make conjectures as to why this is happening.
- When participants return, discuss the math (Slide 148 and use Slide 149 so all the unit fractions can be sorted. Debrief the activity with participants in the same way the visual patterns task was debriefed, by focusing on the nature of the task and the intentional instructional decisions that were in place that allowed the participants to dig into the work and focus more on the process than the end result.
- Participants will then discuss their roles in the work and the role of the facilitator in orchestrating a productive mathematical discussion. This discussion will help

them see the *5 Practices* (Smith & Stein, 2018) from both the lens of the learner and the lens of the facilitator.

Break (20 Minutes)

The Path of the Billiard Ball (60 Minutes, Slides 150-154)

The final task for participants to explore for Math Practice 8 is called *The Path of the Billiard Ball* (Jacobs, 1970). The scenario is that there is a fictitious billiard table where they ball always starts in the lower left corner and always travels at a 45-degree angle. The ball never stops moving until it hits a corner. When this action is performed on a grid while tracing the path of the ball, numerous interesting patterns emerge. Slides 150 to 154 will set the task up for participants.

- Give each participant graph paper, rulers, and colored pencils to explore billiard tables of different dimensions. *The Path of the Billiard Ball* is an open-ended task meaning there is not one result, conclusion, or conjecture.
- Participants are tasked with noticing and expressing regularity in their repeated reasoning. They could take notes on the different patterns they notice and express them in the form of a generalized statement.
- The participants will conclude with a discussion of the mathematics that was uncovered and then they will debrief the work as it pertains to the *5 Practices* (Smith & Stein, 2018). Connections will be made to the levels of inquiry from Day 1 because this is an example of an open inquiry experience where participants generated their own questions and testing it with different methods they design.

- Participants will be asked to rate this experience in terms of engagement as well as the opportunity to develop mathematical insight.

Discussion of the Reading (25 Minutes, Slide 155)

Participants will talk in small groups about the ways in which elements of the *5 Practices* already exist in their classrooms and the changes teachers must make in order to integrate them fully into their work.

Work Assigned (5 Minutes, Slide 156)

For their lunch reading, participants will read Deborah Schifter's (2018) chapter called Early Algebra as Analysis of Structure: A Focus on Operations from the book *Teaching and Learning Algebraic Thinking* with 5 to 12-Year-Olds edited by Karen Kieran. This chapter pulls together the work from the morning and prepares them for the afternoon's work. The chapter also gives participants a K to 5 view of this work in the classroom.

They will also read a short essay written by the researcher comparing the movie The Matrix to Math Practice 7. The essay is found below.

Lunch and Reading (2 Hours)

The afternoon portion of the class will focus on Math Practice 7 from the Common Core State Standards. This standard in titled: Look For and Make Use of Structure. To help participants make sense of this standard, the facilitator will compare this practice standard with the movie *The Matrix* because both have to do with power one gets once he or she understands the underlying structure of the system they are in. This analogy will carry forward for the rest of the class as participants work through the mathematics and think of their own shifts as math teachers.

Defining Mathematical Practice 7: Looking for and Making Use of Structure and Comparing the Standard to the Movie *The Matrix* (30 Minutes)

Start with Slide 157 and ask participants about what it means to look for and make use of structure in mathematics. Show Slides 158 to 161 reviewing the definition of Mathematical Practice 7. Then Have participants get together in small groups and talk about the reading comparing the movie The Matrix to mathematics education and ask them to describe their reactions to it and to think of experiences in their own life that relate to the comparison.

Connecting the Matrix to Our Work

Start by showing Slide 162 and ask the participants to discuss their reactions to the reading about The Matrix.

Share Slide 163 showing Jared's work from Day One. Walk through the kinds of mathematical experiences Jared had to get unplugged from the Matrix and ask participants to comment on how those experiences supported his overall learning in mathematics (Slides 164-173). Then tell participants they are going to do some work to unplug themselves even further from the Matrix.

Multiplying By 10 (30 Minutes, Slides 174-177)

Show Slide 174 and 175 to review the task. Pass out the **Multiplying by 10 Task Sheet** (see below). Participants will explore the rules for multiplying by 10, 100, 1,000, 10,000 and so on. Most people have learned the "just add zeros" rule. However, this is an incorrect statement and it does not get at the mathematics underlying the rule. Adding zero results in no change to the number so we are not actually adding zeros when we multiply by a number that is a multiple of 10. To uncover the underlying structure involved, participants will be asked to represent the result of multiplying 23 by 1, 10, 100, 1,000, and 10,000 using base-10 blocks. At first this will appear to be an easy task for participants, but soon their limitations regarding representing large values with base-10 blocks will push their thinking.

- Assign participants to small groups and share the instructions for the task.
- Make sure everyone has access to a set of base-10 blocks.
- Participants start with a unit cube to represent one, then move to a rod of ten cubes to represent ten, and then they create a flat made up of ten 10-rods to represent 100.
- When participants have ten flats, the result is a much larger cube that now represents 1,000. The base-ten blocks only have premade blocks to represents, ones, tens, hundreds, and thousands.
- When participants have to represent 10,000, they will likely struggle as they have not considered the pattern of cube, rod, flat, cube, rod, flat, etc. before. They will try to make a new shape using ten of the 1,000 blocks. After trial and error someone will notice that you can stack them to make a giant rod of 10,000. From there they should see that they can then take ten of those 10,000 rods to make a giant flat of 100,000. Ten of those big flats can be placed together to make a cube

that is worth 1,000,000. This pattern repeats indefinitely, and participants should be amazed when they see this.

- As with the earlier work in the course, a big part of this task is giving participants experiences where they make sense of these ideas for themselves. There is magical moment of discovery that the facilitator is trying to achieve, and it is a transformative experience for participants when they “get it”. I want their experiences as learners to leave such a lasting impression that they want to go back to their classrooms and create similar experiences for their students to discover key mathematical ideas. This shift from direct teaching to facilitating engaging learning experiences is possible and quite easy once participants realize they need to step back a bit and allow their students time to explore the mathematics for themselves.
- Use Slides 176 and 177 for the debrief. The facilitator will follow the same debriefing format that was used for the other tasks. First the mathematics will be unpacked. This discussion will support the development of participants’ mathematical content knowledge. Then participants will consider what it takes to facilitate a task like this. The goal is for participants to recognize that the facilitator needed to have a rich understanding of this mathematics in order to guide the discovery among the participants. If the facilitator did not deeply understand the mathematics, the facilitation would not have worked. This is a key point that participants need to know for their own practice. They must understand

the mathematics deeply for themselves before they can facilitate this way with their students.

Using Structure to Support Fluency (20 Minutes, Slides 178-181)

The facilitator will then shift the focus of the work on Math Practice 7 to have participants consider how understanding structure can lead to mathematical reasoning which leads to procedural fluency. Fluency is such a big concern for teachers, and it is important to make sure it is addressed in the short course.

- Slide 178 asks participants to discuss their definition of fluency. Solicit lots of responses so you have a wide range of definitions.
- Show the Common Core definition of fluency on Slide 179.
- Show Slide 180 with the quote about fluency practice and its relationship to math anxiety (Boaler, 2015).
- Show Slide 181 and tell the group they are going to have an experience that should help them see the connection between high pressure and the inability to access readily available knowledge. Tell them they are going to do a count around the room task much like students do in elementary school when they learn to skip count. This time, the adults are going to count by 12s. This should make many of them feel pretty anxious.
- Ask the participants to talk about these feelings and why this task is causing these feelings.

- Then let participants off the hook by saying you will facilitate the task in a way that draws upon structure and understanding without stressing people out. Here's the plan:
    - Everyone should stand up. When someone wants to share a number, they can raise their hand. The facilitator will call on a person to share the number and will either write it on chart paper.
    - Pause periodically to ask the participants to discuss any patterns they see or any strategies they use (e.g., add 10 and then add 2)
    - Also, take time whenever you cross over a century for people to have time to mentally calculate the next number. Strategies like this reduce anxiety and help people focus on the structure of counting by twelves.
    - At the end, ask participants to discuss how the way you facilitated the task was different from how they originally thought the task was going to go and what the benefits were from your approach.
    - Finally, explain that when students understand structure, they can use that knowledge to become more flexible and fluent with their computation.

Break (20 Minutes)

Routines to Support Mathematical Practices 7 and 8 (20 Minutes, Slides 182-191)

The final work for the day and the course involves exploring instructional routines that come from Russell, Schifter, Kasman, Bastable, and Higgins (2017). Participants can use these in their own classrooms that develop their students' algebraic reasoning through explorations with regularity and structure. Display Slide 182 that references where this work derives and walk the participants through the routine.

- The routine begins with participants noticing some regularity in mathematics (Slide 183).
- They then express that regularity as a general claim (Slide 184).
- Next, they test their claim with lots of repeated reasoning to make sure their claim works (Slide 185).
- Then they have to create a representation-based argument to prove their claim works for a whole class of instances (e.g., all whole numbers; Slide 186).
- Slides 187 and 188 show examples of students' representation-based arguments.
- They then work to revise and/or extend their claim if necessary (Slide 189).
- Finally, they consider how they can apply their claim in problem solving situations (Slide 190).

Show Slide 191 and go over the directions for the small group work. Pass out the **Exploring Structure Through Algebraic Reasoning Sheet** (See below). Participants will first walk through this routine by following the work of a second-grade class. For each part of the routine, the participants will do the work for themselves as adult learners and then they will see how second graders did the same work. This experience gives participants the view of this work from both the learner perspective and the teacher perspective.

Once participants have gone through the routine with second grade, they will be given a new scenario to work on in small groups following the same progression of instructional routines.

Designing Routines and Applying Course Ideas (40 Minutes)

- Participants will be broken up by grade level bands so the work they do is closely tied with the mathematics their students do.
- When participants finish their work, they will then create a similar progression they can use with their students to support this work on developing algebraic reasoning.
- After the participants return to whole group, the facilitator will debrief the mathematics as well as the learning experience with them.
    - Participants will address questions they have regarding the implementation of this work with their own students.

Final Reflections (10 Minutes, Slide 192)

- This reflection is a conversation with participants and not related the formal post-course reflection participants need to complete as their final assignment.
- The facilitator will then pivot the discussion to focus on participants' own growth and development over the course of the last three days.
- Participants will be given a prompt called "I Used to Think and Now I Think". This sentence frame asks participants to reflect on how their thinking has shifted by choosing a way they used to think about mathematics or math teaching and what their current thinking is based on the course experiences.

- The facilitator will end the course by having participants share their responses to this prompt.
- Participants will be given a post-course assignment (see below) to help gauge what each of them got out of the course. This will be due one week from the date it is assigned. Participants will also be given a short course evaluation (see below) to give feedback on what worked and what needs to be improved with the course.

Appendix A: Project References

Boaler, J. (2015). Fluency without fear. *Youcubed*. Retrieved from https://youcubed.org/wp-content/uploads/2017/09/Fluency-Withpout-Fear-1.28.15.pdf

Fletcher, G. (2015, September 30). *Array-bow of colors Act 1* [Video]. Retrieved from https://vimeo.com/140928901

Fletcher, G. (2016). *Modeling with mathematics through three-act tasks. Teaching Children Mathematics Blog.* Reston, VA: National Council of Teacher of Mathematics. Retrieved from https://www.nctm.org/Publications/Teaching-Children-Mathematics/Blog/Modeling-with-Mathematics-through-Three-Act-Tasks/

Flynn, M. (2016). The cycling shop. *Teaching Children Mathematics, 23*(1), 10-13. doi:10.5951/teacchilmath.23.1.0010

Flynn, M. (2017) *Beyond answers: Exploring mathematical practices with young children*. Portsmouth, NH. Stenhouse Publishers. Retrieved from https://www.stenhouse.com/content/beyond-answers

Flynn, M. (2019, January 1). *Monster burger Act 1* [Video]. Retrieved from https://vimeo.com/308977948

Flynn, M. (2019, January 1). *Monster burger Act 3* [Video]. Retrieved from https://vimeo.com/308978881

Heath, C., & Heath, D. (2007). *Made to stick: Why some ideas survive while others die*. New York, NY. Penguin Random House LLC. Retrieved from https://heathbrothers.com/made-to-stick

Heath, C., & Heath, D. (2017). *The power of moments: Why certain experiences have extraordinary impact*. New York, NY. Penguin Random House LLC. Retrieved from https://heathbrothers.com/the-power-of-moments

Jacobs, H. (1970). The path of the billiard ball. In *Mathematics: A Human Endeavor Edition 3*. New York, NY. Freeman, W. H. & Company.

Kaplinsky, R. (2019). *Open middle math: Problems that unlock student thinking.* Portsmouth, NH: Stenhouse Publishers. Retrieved from https://www.stenhouse.com/content/open-middle-math

Kaplinsky, R. (2013). *How old is the shepherd* [Video]? YouTube. Available https://www.youtube.com/watch?v=kibaFBgaPx4

Meyer, D. (2011). The three acts of a mathematical story. *dy/dan – less helpful*. Retrieved from https://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/

Meyer, D. (2014). Real work v. real world. *dy/dan – less helpful.* Retrieved from https://blog.mrmeyer.com/2014/developing-the-question-real-work-v-real-world/

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for mathematics*. Washington, DC: Author. Retrieved from http://www.corestandards.org/Math/

Reinhart, S. (2000). Never say anything a kid can say. *Mathematics Teaching in the Middle School, 5*(8), 478-483. doi:10.5951/mathteachmiddscho.24.1.0019

Russell, S. J. (2010). Learning whole number operations in elementary school classrooms. In D. Lambdin, & F. K. Lester Jr (Eds.), *Teaching and Learning Mathematics: Translating Research for Elementary School Teachers* (pp. 1-8). Reston, VA: National Council of Teachers of Mathematics

Russell, S. J., Schifter, D., Kasman, R., Bastable, V., & Higgins, T. (2017). *But why does it work?* Portsmouth, NH: Heinemann.

Schifter, D., Bastable, V., & Russell, S. J. (2017a) *Developing mathematical ideas: Building a system of tens*. Reston, VA: National Council of Teachers of Mathematics.

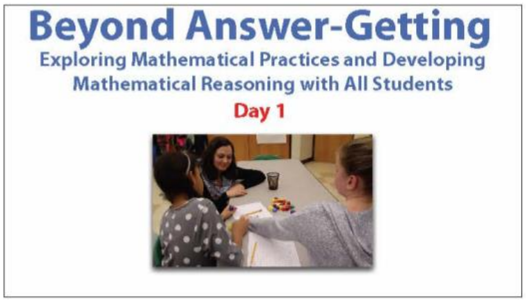
Schifter, D., Bastable, V., & Russell, S. J. (2017b). *Developing mathematical ideas: Patterns, Functions, and Change casebook.* Reston, VA: National Council of Teachers of Mathematics.

Schifter, D. (2018). Early algebra as analysis of structure: A focus on operations. In C. Kieran (Ed.), *Teaching and learning algebraic thinking with 5-12-year olds: The global evolution of an emerging field of research and practice* (pp. 309-327). ICME-13 Monographs. Cham, Switzerland: Springer International Publishing. https://doi.org/10.1007/978-3-319-68351-5

Smith, M., & Stein, M. K. (2018). *5 practices for orchestrating productive mathematics* (2nd ed.). Reston, VA: National Council of Teachers of Mathematics. Retrieved from https://www.nctm.org/Store/Products/5-Practices-for-Orchestrating-Productive-Mathematics-Discussions,-2nd-edition-(eBook)/

Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2018). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 3-5* (Vol. II, 3rd ed.). New York, NY, Pearson. Retrieved from https://www.pearson.com/us/higher-education/program/Van-de-Walle-Teaching-Student-Centered-Mathematics-Developmentally-Appropriate-Instruction-for-Grades-3-5-Volume-II-with-Enhanced-Pearson-e-Text-Access-Card-Package-3rd-Edition/PGM291139.html

Appendix A: Project Presentation Slides



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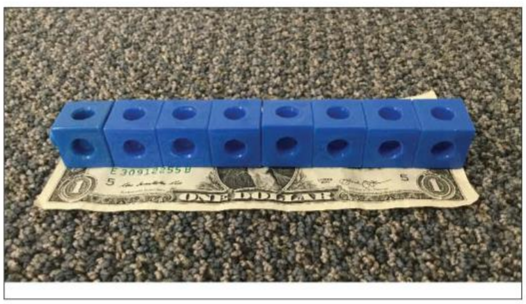

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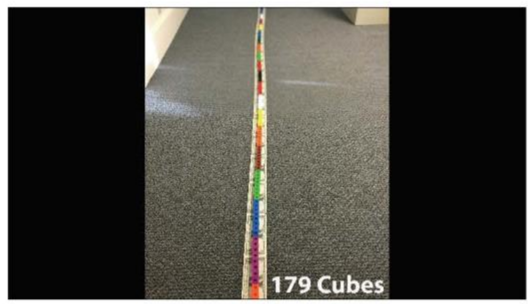

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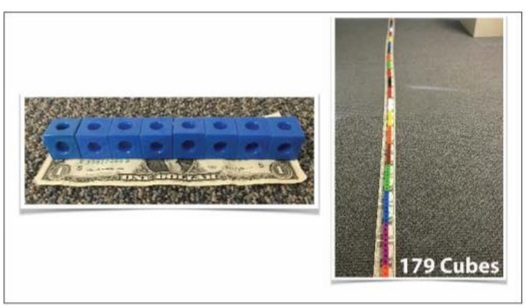

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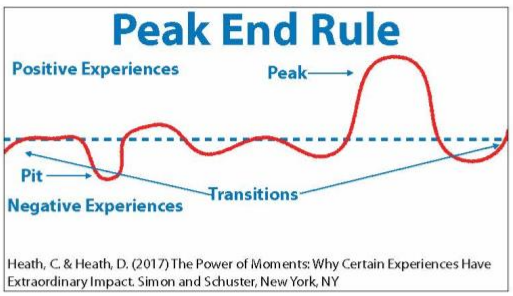
6

179 ÷ 8 =

7

A roll of dollar bills was taped together and was 179 cubes long. If one dollar is 8 cubes long, how many dollars are in the roll?

8



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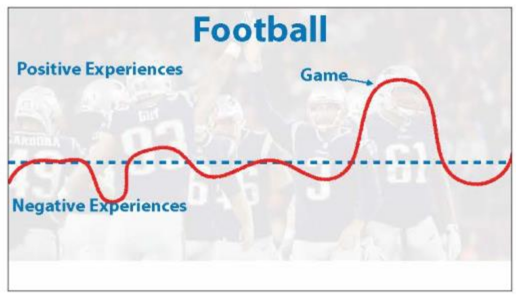

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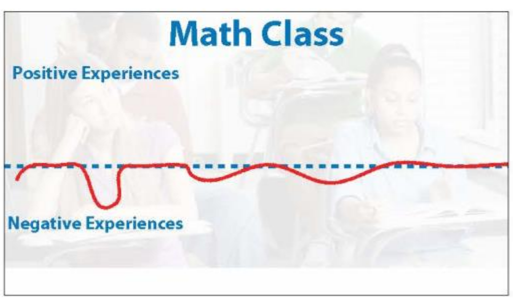


11

3-Part Recipe to Build Peaks

Boost Sensory Appeal

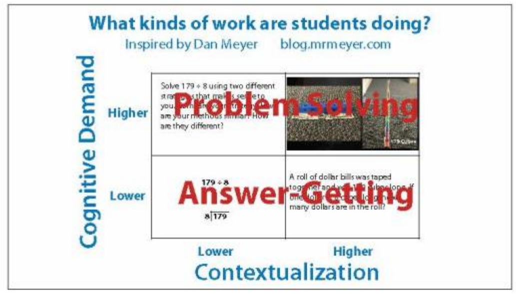
Raise the Stakes

Break the Script

Heath, C. & Heath, D. (2017) The Power of Moments: Why Certain Experiences Have Extraordinary Impact. Simon and Schuster, New York, NY

12

13



14

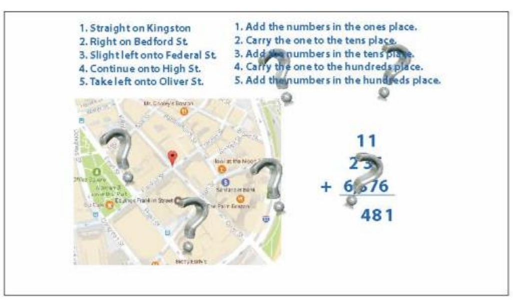
Course Goals

- Framing the challenge with shifting our teaching practices
- Unpack and experience the Standards for Mathematical Practice as learners
- Consider how this work will play out in our different settings

15

What was your experience like in math class as en elementary student?

16



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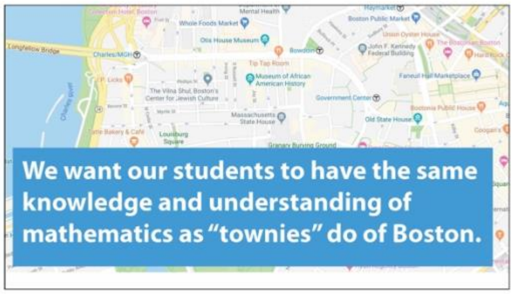


19

Answer-Getting

vs.

Problem Solving

20

Routine Expertise

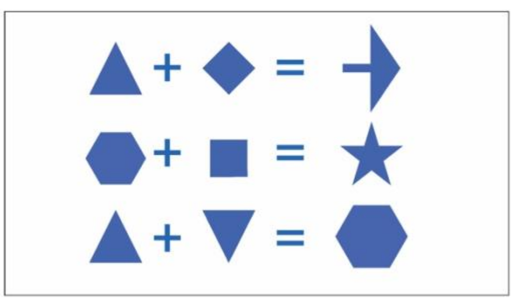
- Knowledge memorized by rote.
- Knowledge that can be used effectively with familiar tasks but not with new ones, even those that differ only slightly from the original one.
- This knowledge is learned and stored in isolation

Russell, S.J. (2010). Learning whole number operations in elementary school classrooms. In Teaching and Learning Mathematics: Translating Research for Elementary School Teachers, edited by Diana V. Lambdin and Frank K. Lester Jr., pp. 1–8. Reston, VA: NCTM

21

Illusion of Understanding

22



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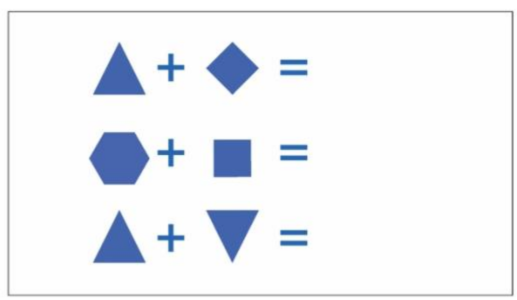

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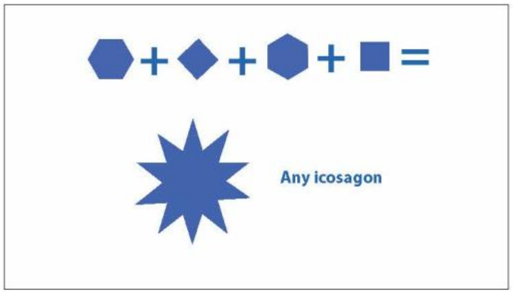

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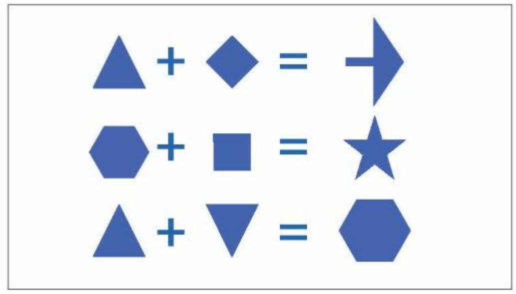

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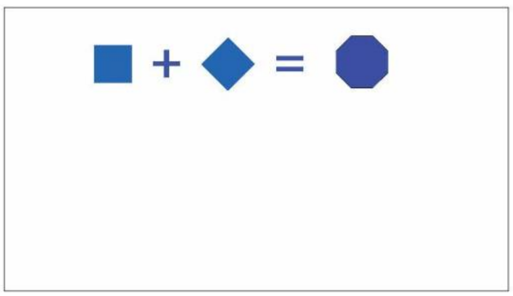

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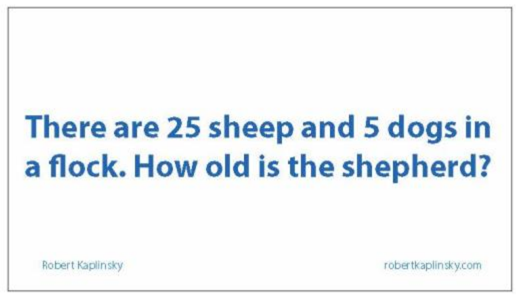

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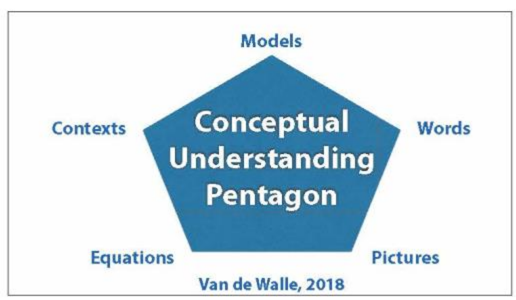

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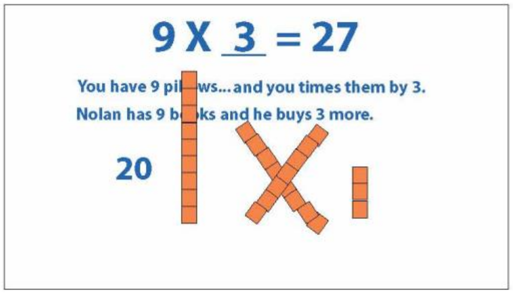

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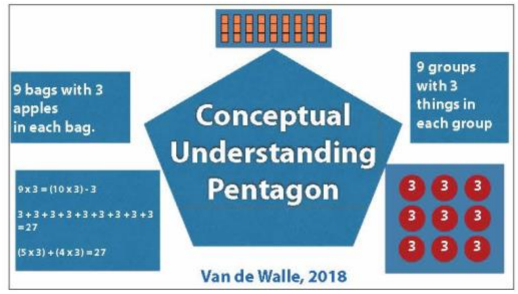

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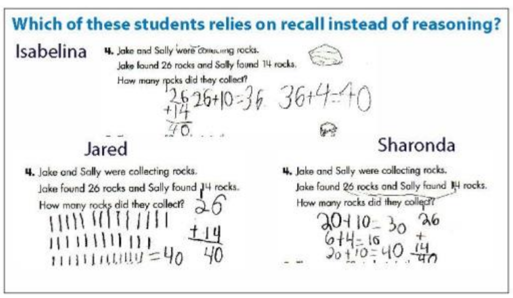

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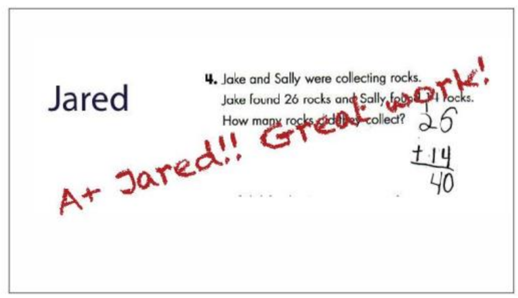

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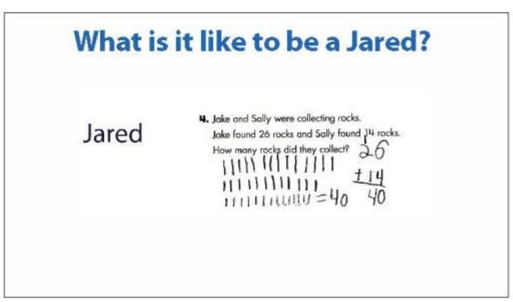

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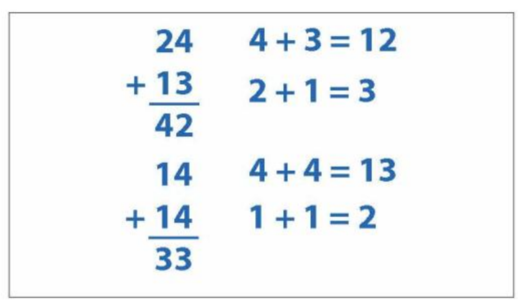

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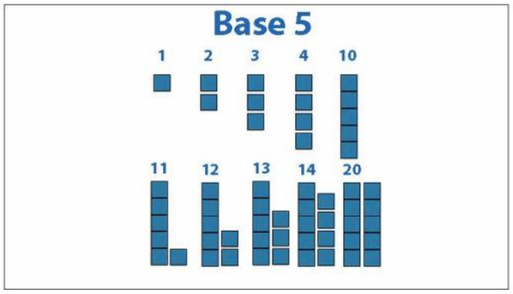

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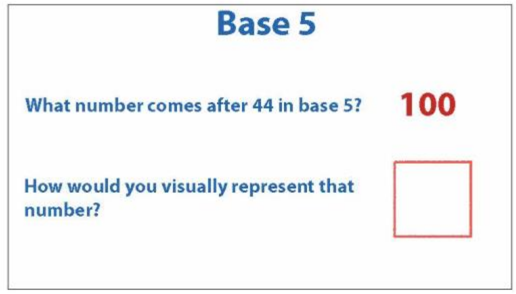

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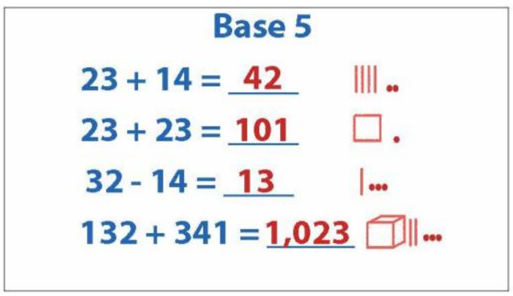

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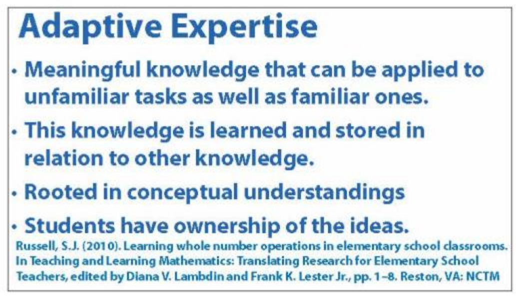

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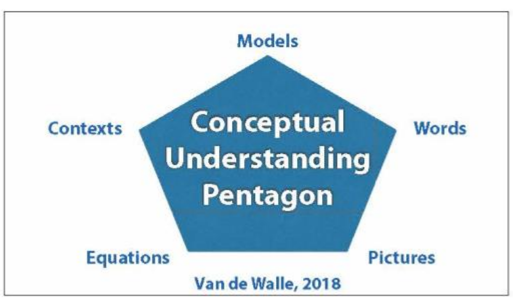

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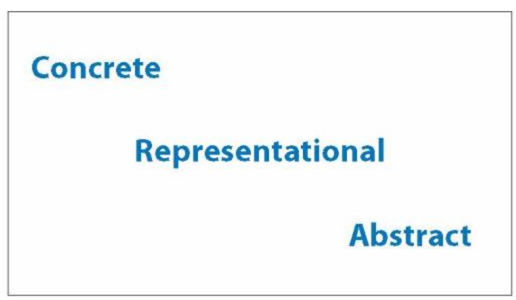

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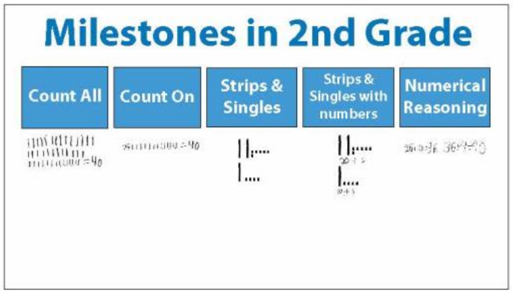

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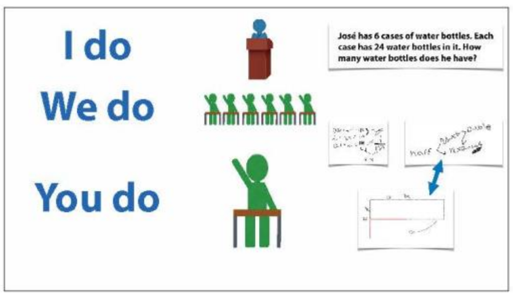

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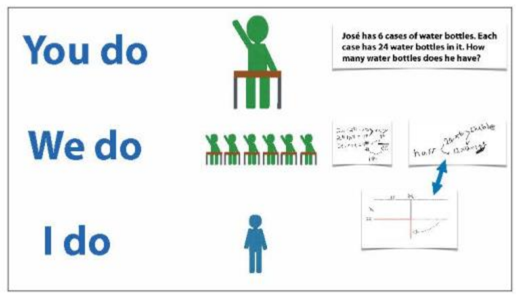

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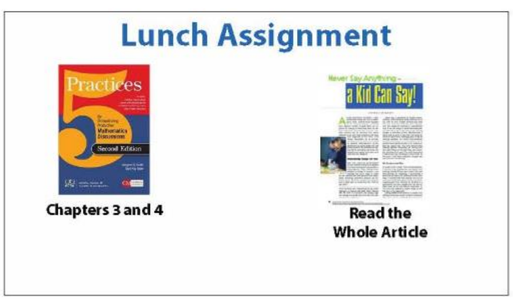

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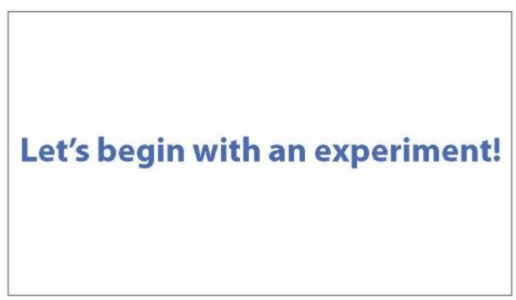

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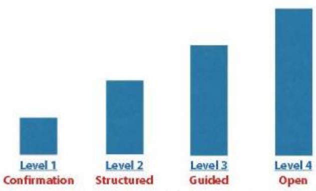
What was your experience like as a learner?

What would this experience look like in math class?

49

Levels of Inquiry



Banchi, H., Bell, R. (2008). The Many Levels of Inquiry. Science and Children, 46 (2), 26-29

50

Level 1
Confirmation

Students confirm a principle through a prescribed activity in which the results are known in advance.

Given Question → Given Method → Given Results

Banchi, H., Bell, R. (2008). The Many Levels of Inquiry. Science and Children, 46 (2), 26-29

51

Level 2
Structured Inquiry

Students investigate a teacher-presented question through a prescribed procedure.

Given Question → Given Method → Results Unknown

Banchi, H., Bell, R. (2008). The Many Levels of Inquiry. Science and Children, 46 (2), 26-29

52

Level 2

Question: How do the positions of the propellers affect how the object falls?

Method: Drop your object and observe. Then fold the propellers in the opposite direction and drop it again.

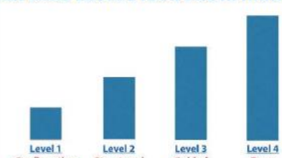
Discuss what you noticed and why you think that happened?

Banchi, H., Bell, R. (2008). The Many Levels of Inquiry. Science and Children, 46 (2), 26-29

53

What was your experience like at Level 2?

What does Level 2 look like in math?



Banchi, H., Bell, R. (2008). The Many Levels of Inquiry. Science and Children, 46 (2), 26-29

54

Level 3
Guided Inquiry

Students investigate a teacher-presented question using student designed/selected procedures.

Given Question → Student Designed or Selected Procedures → Results Unknown

Banchi, H., Bell, R. (2008). The Many Levels of Inquiry. Science and Children, 46 (2), 26-29

55

Level 3

Question: How does adding weight to your object affect how it spins?

Design a method to explore this question using objects at your table.

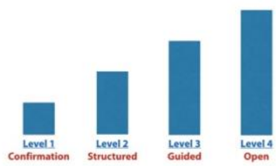
Discuss what you noticed and why you think that happened?

Banchi, H., Bell, R. (2008). The Many Levels of Inquiry. Science and Children, 46 (2), 26-29

56

What was your experience like at level 3?

What does Level 3 look like in math?



Banchi, H., Bell, R. (2008). The Many Levels of Inquiry. Science and Children, 46 (2), 26-29

57

Level 4
Open Inquiry

Students investigate topic-related questions that are student formulated through student designed/selected procedures.

Student Formulated Questions → Student Designed or Selected Methods → Results Unknown

Banchi, H., Bell, R. (2008). The Many Levels of Inquiry. Science and Children, 46 (2), 26-29

58

Level 4

What other questions could you investigate?

Design a method to explore your question.

Discuss what you noticed and why you think that happened?

Banchi, H., Bell, R. (2008). The Many Levels of Inquiry. Science and Children, 46 (2), 26-29

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Level 4

What would a Level 4 experience look like in math?

Banchi, H., Bell, R. (2008). The Many Levels of Inquiry. Science and Children, 46 (2), 26-29

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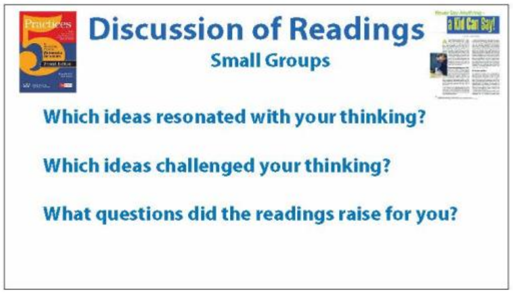

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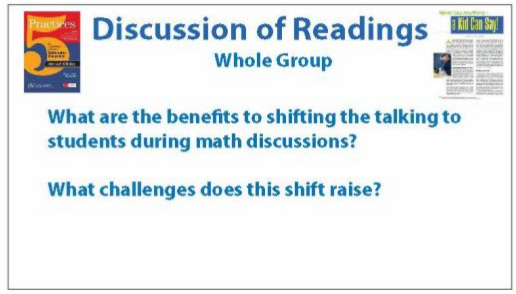

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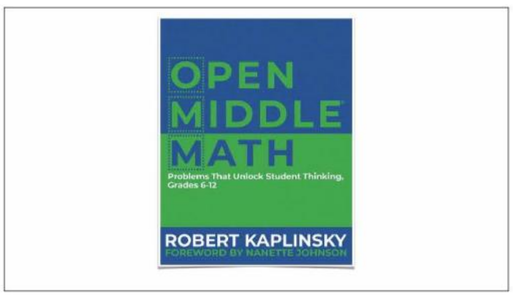

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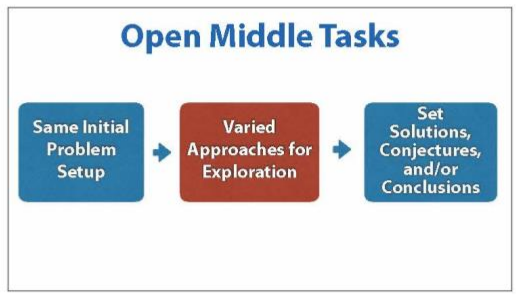

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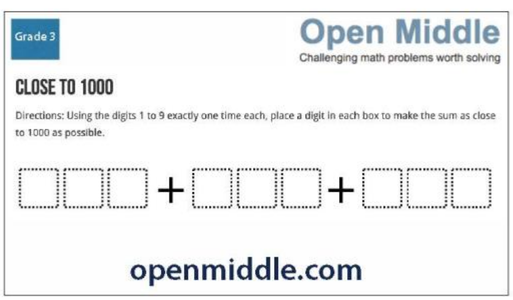

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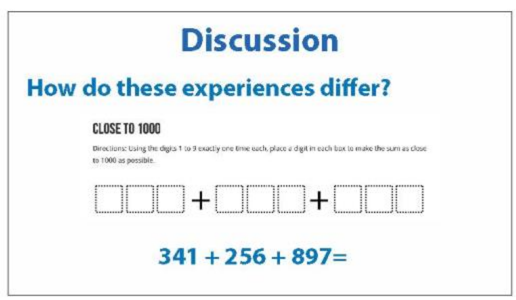


66

Making Over Math Tasks

67

Task Analysis

- Does the task evoke curiosity?
- Does it have multiple entry points?
- Does it involve high cognitive demand?
- Does it allow for multiple solution pathways?
- Does it engage a wide range of learners?
- Does it allow for extensions?
- Does it feel satisfying?

68

Fraction Word Problems
Multiplying With Whole Numbers

When you multiply a fraction with a whole number, first you must write the multiplication equation.

Example: Tammy drank $\frac{2}{3}$ gallon of lemonade. Susie drank 3 times more. How much did Susie drink?

1. Write multiplication equation. $\frac{2}{3} \times 3$
2. Write the whole number as a fraction by putting 1 as the denominator.
   $3 = \frac{3}{1}$ ← numerator ← denominator
3. Multiply the numerator with the numerator, multiply the denominator with the denominator.
   $\frac{2}{3} \times \frac{3}{1} = \frac{2\times3}{3\times1} = \frac{6}{3}$

Solve the word problems by multiplying fractions.

Rose ate $\frac{1}{8}$ of the soup in the pot. Kristi ate 4 times more that Rose did. How much soup did Kristi have?

1. Write multiplication equation.
2. Write the whole number as a fraction by putting 1 as the denominator.

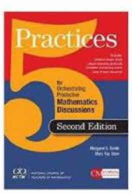
69

Math Tasks Need a Makeover

- In your small groups, take the task in your packet and analyze it using our Task Analysis Guide and the Depth of Knowledge form.
- Then work with your group to redesign the task to increase the level of engagement and depth of knowledge level of the task.

70

Homeworks



- Read Chapters 5 and 6
- As you do, please consider what these practices will look like in your classroom
- What are you already doing that supports this work?
- What changes will you need to make to accommodate this approach?

71

Beyond Answer-Getting

Exploring Mathematical Practices and Developing Mathematical Reasoning with All Students

Day 2

72

The Curse of Knowledge

The curse of knowledge is a cognitive bias that occurs when an individual, communicating with other individuals, unknowingly assumes that **the** others have **the** background to understand.

Heath & Heath (2007)

73

Combatting The Curse of Knowledge

We need opportunities that put us in the learner's shoes so we experience productive struggle.

We also need to challenge our assumptions on the effectiveness of past practices.

74

Alphabet Counting

75

Reflection

What was challenging about counting with the alphabet?

How does your experience connect to the experience of young children learning to count?

What supports helped you in this activity? What are the implications for your work with students?

76

MP2
Reason Abstractly and Quantitatively

- **Make sense of quantities and their relationships in problem situations**
- **Decontextualize — to abstract a given situation and represent it symbolically**
- **Contextualize — to probe into the referents for the symbols involved**
- **Create a coherent representation of the problem at hand**
- **Consider the units involved**
- **Attend to the meaning of quantities, not just how to compute them**
- **Know and flexibly use different properties of operations**

77

34 + 28

78

53 - 17

79

Exploring Strategies

Section 1 illustrates four different strategies for calculating 34 + 28. Section 2 illustrates five strategies for calculating 53 – 17.

Directions

- Examine each method carefully, first trying some similar problems using the same strategy.
- Then write a generalized verbal description of the strategy.
- Demonstrate the method with manipulatives, diagrams, and a story context. In particular, try number lines and place value representations.

80

Verbal Description

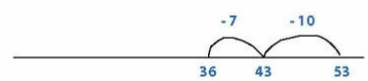
53 - 17 = __
53 -10 = 43
43 - 7 = 36

- Keep the first number whole.
- Decompose the subtrahend (second number) by place value
- Subtract the largest place value of the subtrahend from the minuend (first number) and record the difference.
- Subtract the next largest place value from the difference and record the new difference.
- Continue step 4 until you have subtracted all the place values.
- The final difference is the answer to the whole problem.

81

Number Line

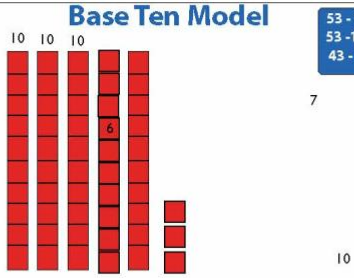
53 - 17 = __
53 -10 = 43
43 - 7 = 36



82

Base Ten Model

53 - 17 = __
53 -10 = 43
43 - 7 = 36



83

Story Context

53 - 17 = __
53 -10 = 43
43 - 7 = 36

Pencils come in boxes of 10. Sean had 53 pencils and wanted to share 17 with Collin. He gave him one full box of 10 pencils and had 43 total pencils leftover. Then he gave him 7 loose pencils. He now has 36 pencils left.

84

Exploring Strategies

Section 1 illustrates four different strategies for calculating 34 + 28. Section 2 illustrates five strategies for calculating 53 – 17.

Directions

- Examine each method carefully, first trying some similar problems using the same strategy.
- Then write a generalized verbal description of the strategy.
- Demonstrate the method with manipulatives, diagrams, and a story context. In particular, try number lines and place value representations.

85

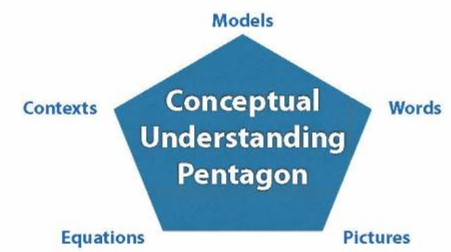
Discussion

What new understanding did you develop?

What was challenging?

What mathematical ideas that underlie these strategies.

86



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Discussion of Readings
Small Groups

What elements of the 5 Practices were evident in the work we did today?

Look at the math lesson you brought. How will you incorporate the 5 Practices into that work?

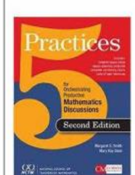
88

Discussion of Readings
Whole Group

Which one of the four practices discussed so far feels the most complicated to implement? Why?

89

Lunch Reading



Read Chapter 7

Read the NCTM article Modeling with Mathematics Through Three-Act Tasks

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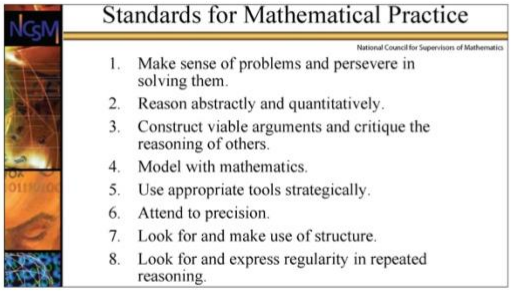

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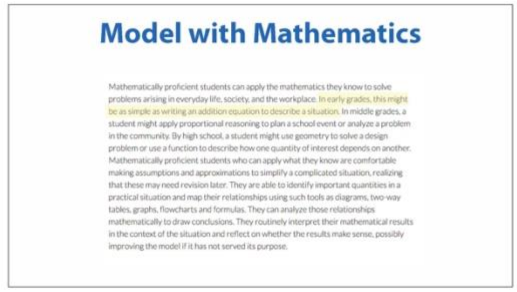

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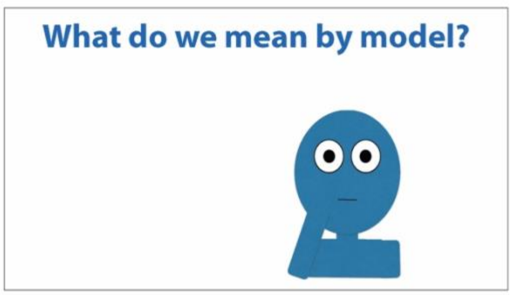

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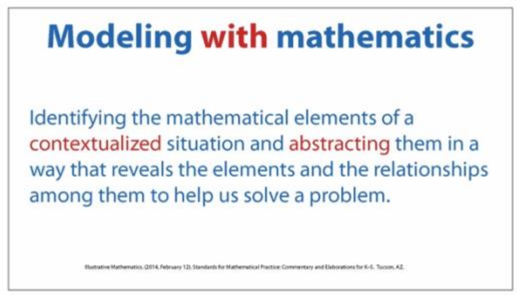

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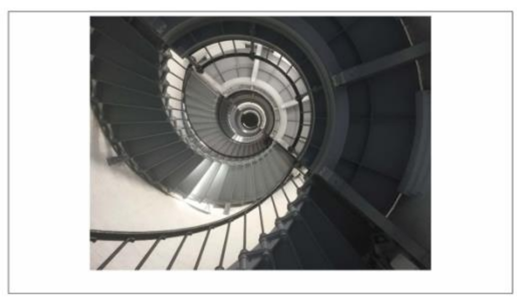

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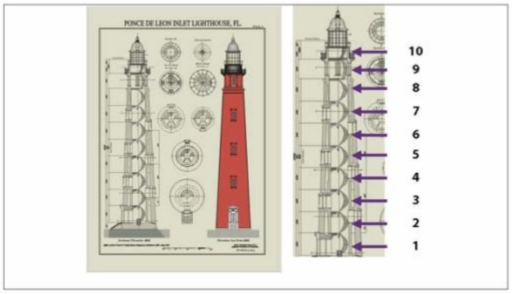

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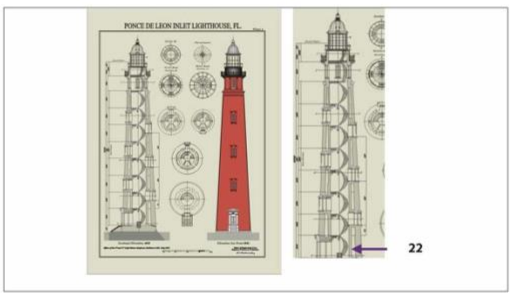

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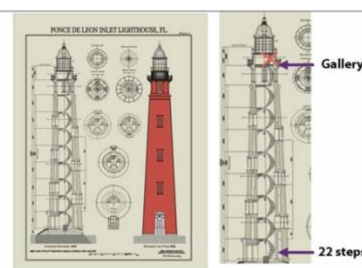
22 x 10 = 220

100

101



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~~22 x 10 = 220~~

22 x 9 = 198

103

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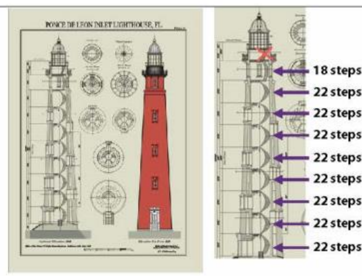

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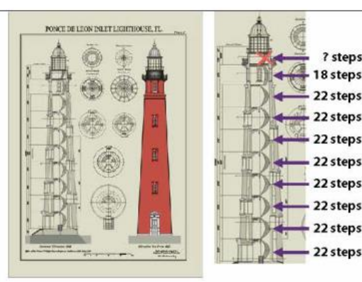
~~22 x 10 = 220~~

~~22 x 9 = 198~~

(22 x 8) + 18 = 194

106



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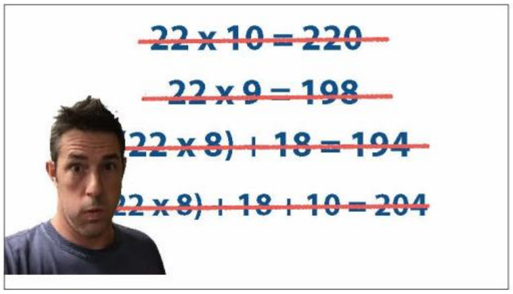

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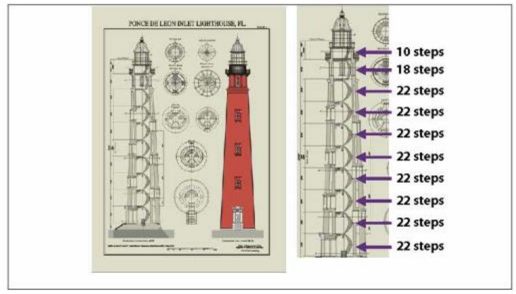

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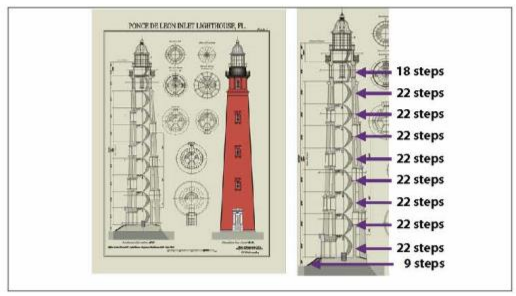

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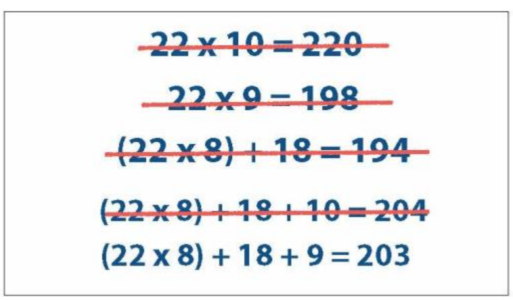

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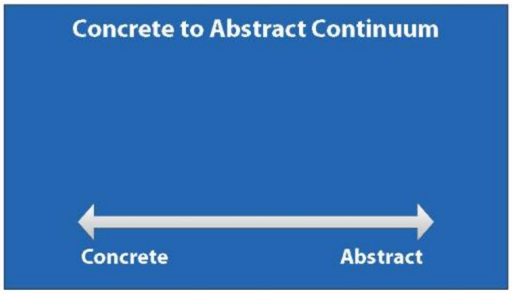

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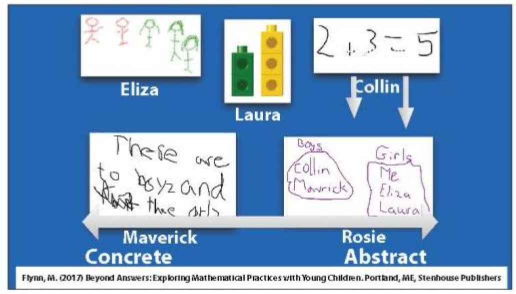

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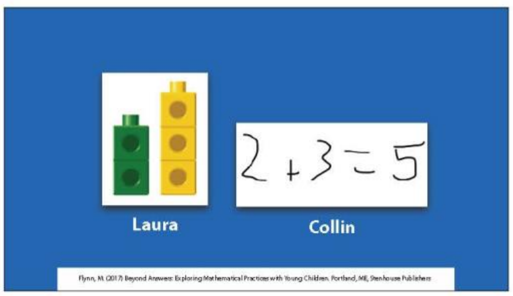

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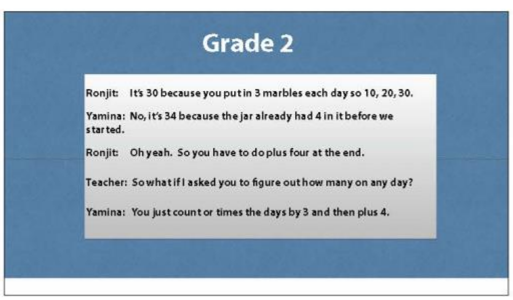

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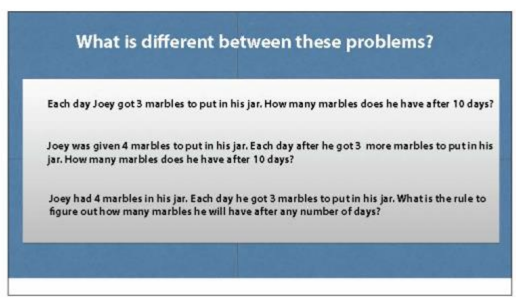

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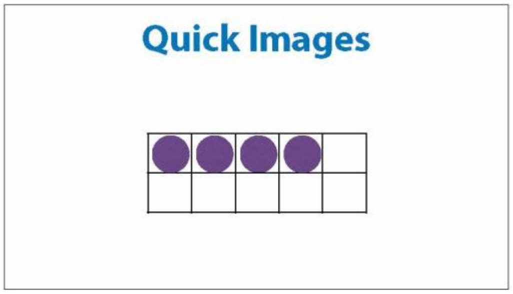

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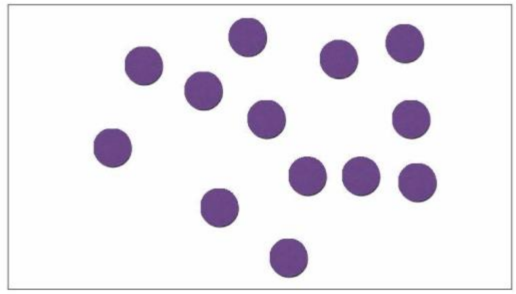

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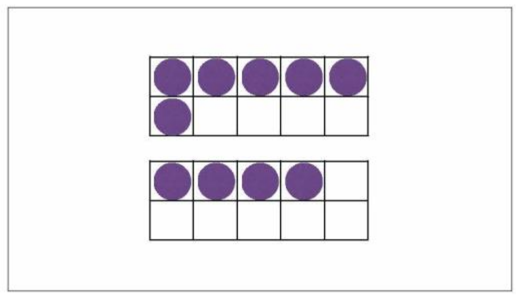

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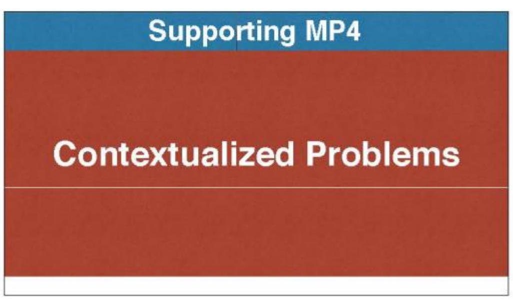

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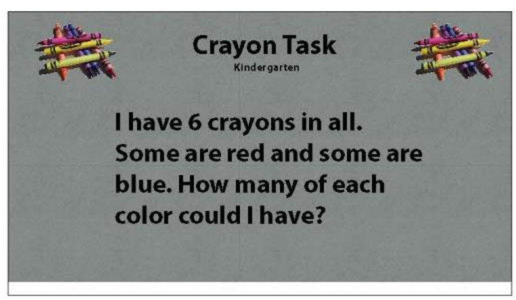

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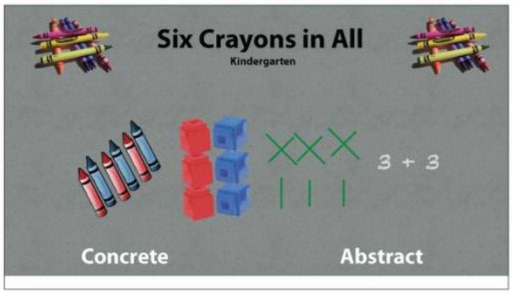

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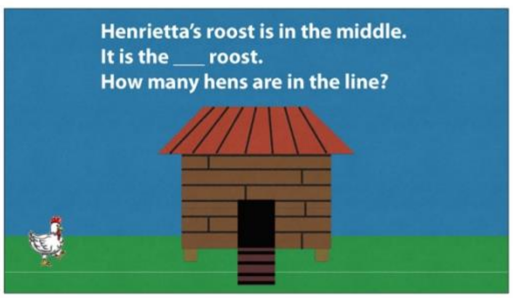

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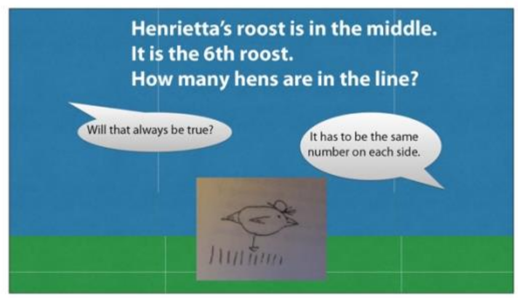

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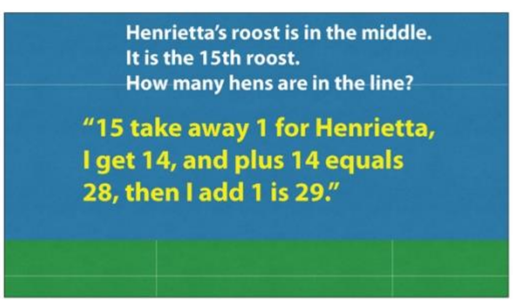

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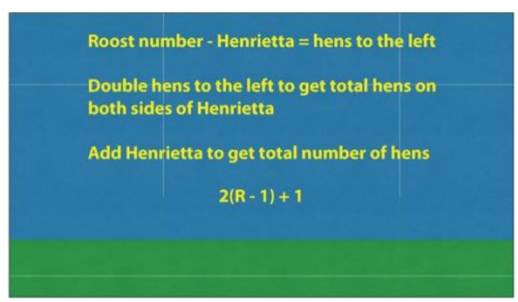

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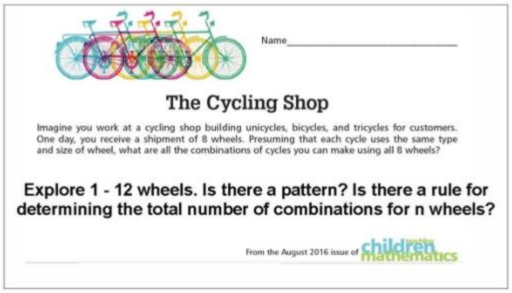

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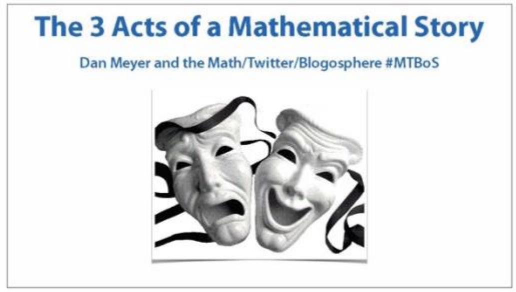

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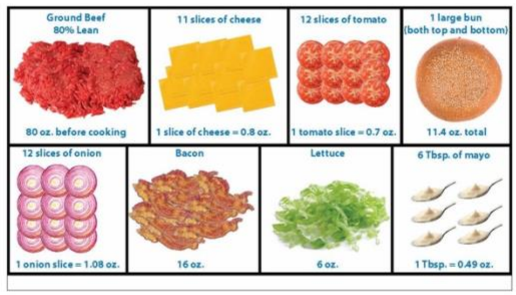

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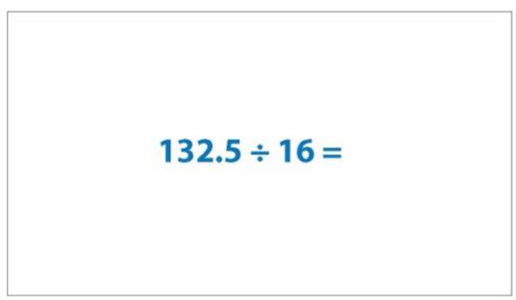

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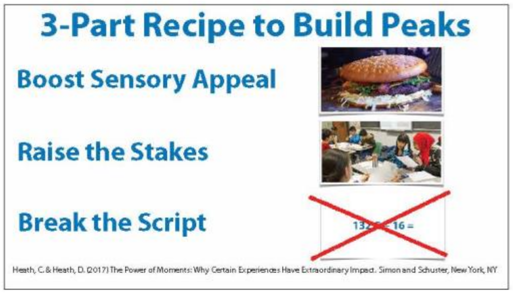

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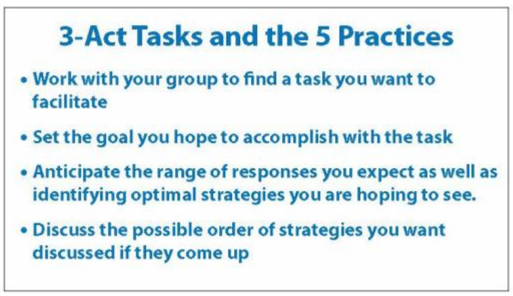

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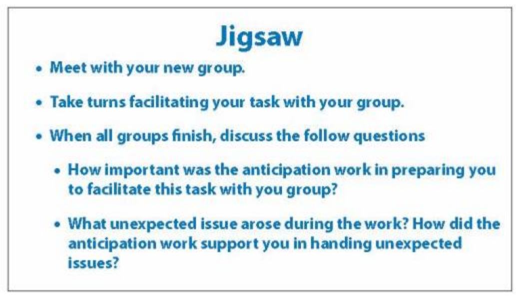

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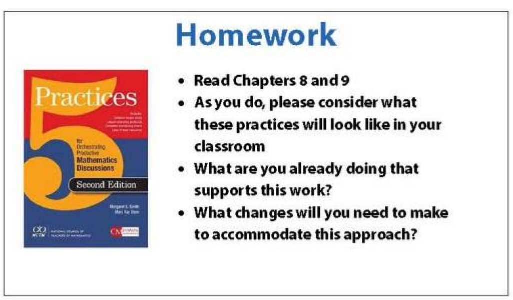

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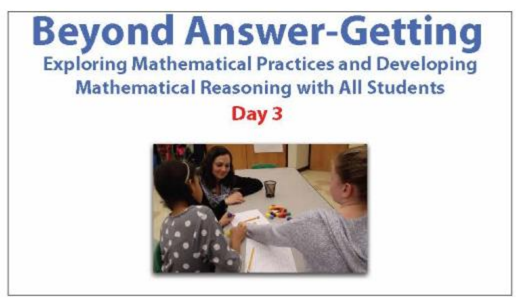

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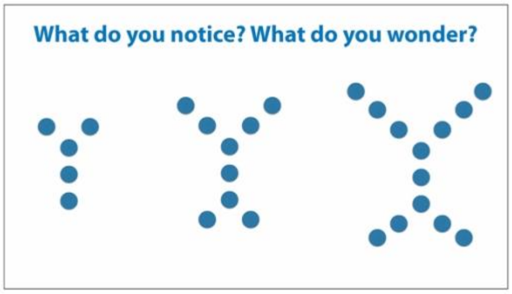

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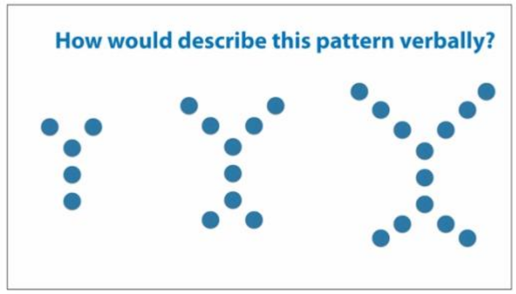

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- **What pairs of numbers, when divided, produce terminating decimals? Why?**
- **What fractions, in lowest terms, produce repeating decimals? Why?**

147

Discussion

- **What divisors of unit fractions produce terminating decimals?**
- **What divisors of unit fractions produce repeating decimals?**

148

Terminating	Repeating
$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{8}$	$\frac{1}{3}$ $\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{9}$

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The Path of the Billiard Ball

Adapted from The Path of the Billiard Ball by Harold Jacobs, 1994

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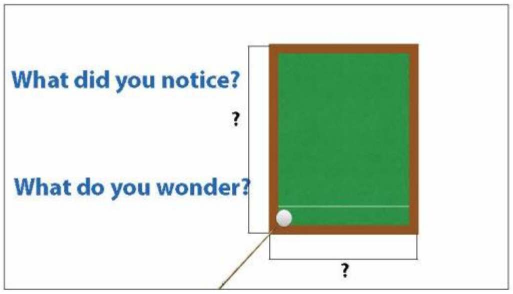

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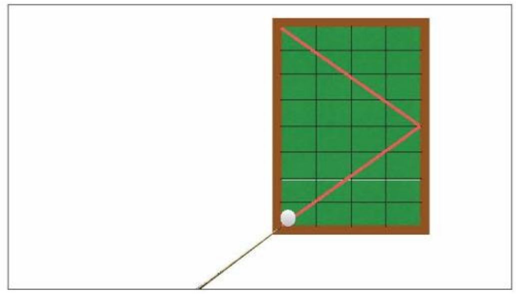
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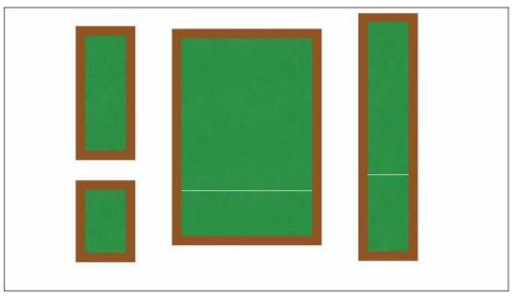
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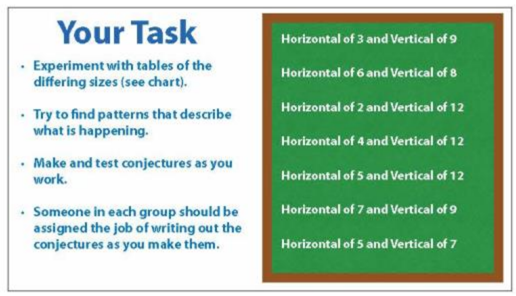

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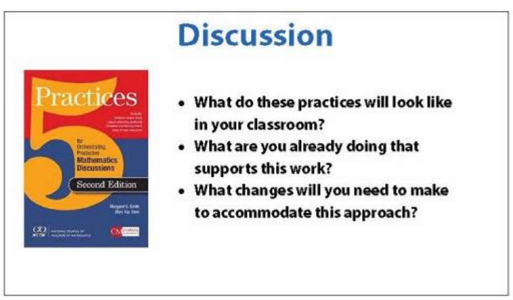

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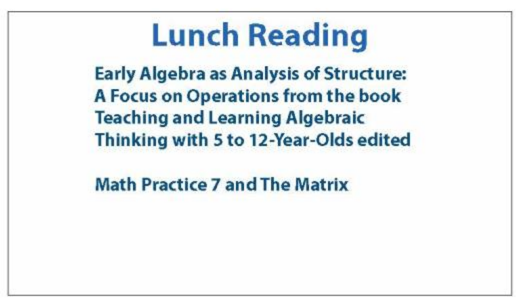

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What does it mean to look for and make use of structure?

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Mathematical Practice 7
Look For and Make Use of Structure

- When younger students recognize that adding 1 results in the next counting number, they are identifying the basic structure of whole numbers.

4 + 1 = 5

+1

1 2 3 4 5

Illustrative Mathematics. (2016). Elementary Elaborations of the Standards for Mathematical Practice

158

Mathematical Practice 7
Look For and Make Use of Structure

- When older elementary students calculate 16 x 9, they might apply the structure of place value and the distributive property to find the product: 16 x 9 = (10 + 6) x 9 = (10 x 9) + (6 x 9)

	10	6
9	10 x 9	6 x 9

Illustrative Mathematics. (2016). Elementary Elaborations of the Standards for Mathematical Practice

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Mathematical Practice 7
Look For and Make Use of Structure

- In many cases, they have identified and described these structures through repeated reasoning (MP8)

Illustrative Mathematics. (2016). Elementary Elaborations of the Standards for Mathematical Practice

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Mathematical Practice 7
Look For and Make Use of Structure

Mathematical Practice 8
Look For and Express Regularity in Repeated Reasoning

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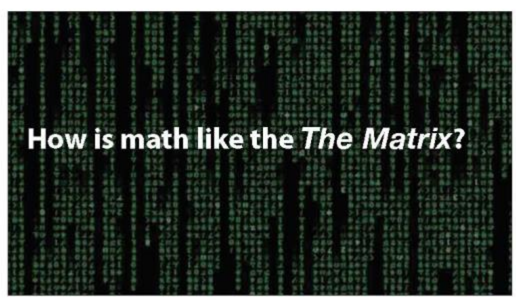

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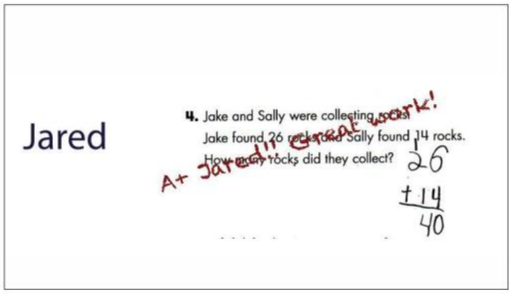

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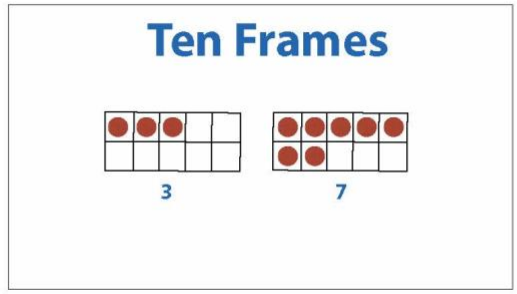

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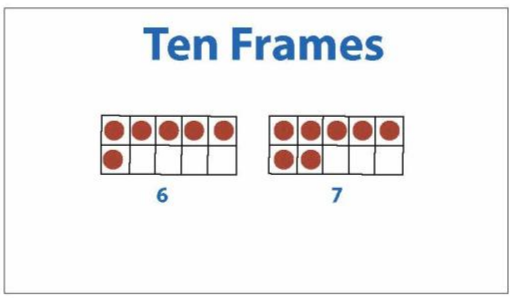

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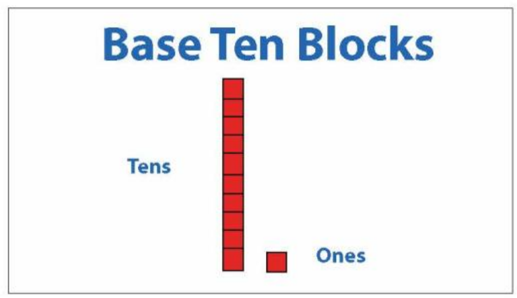

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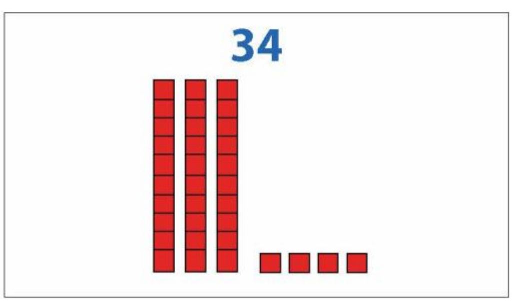

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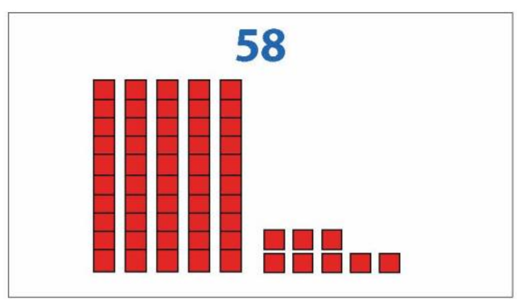

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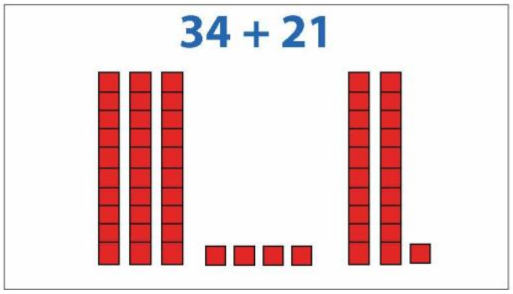

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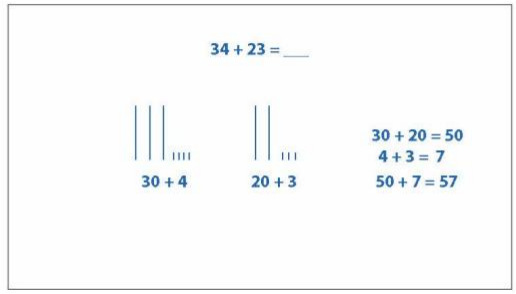

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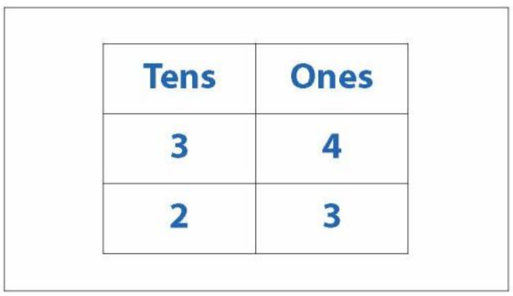

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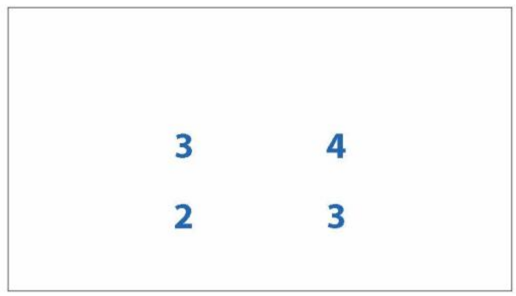

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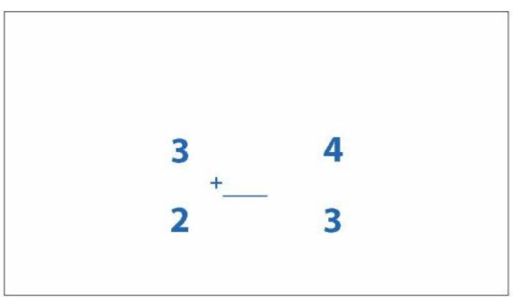

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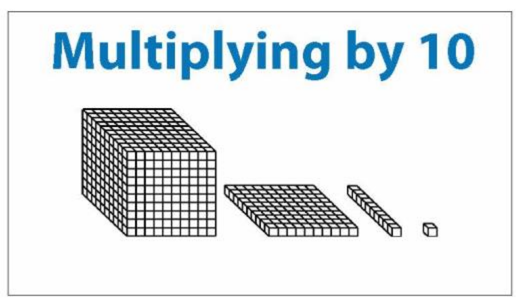

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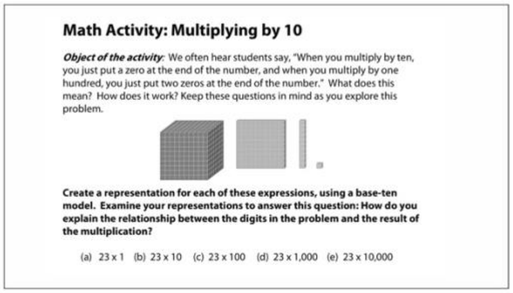

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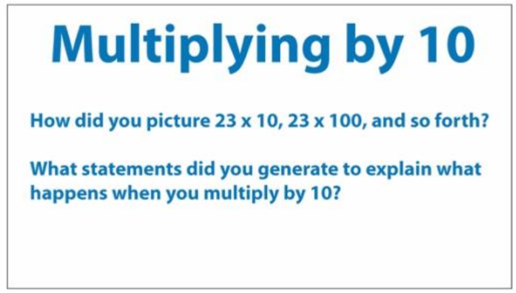

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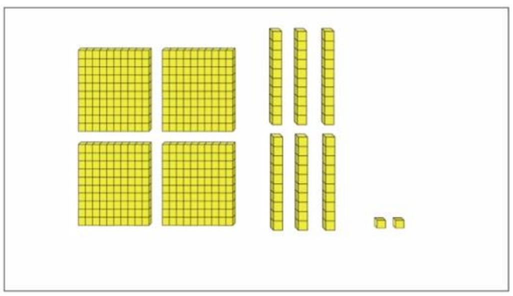

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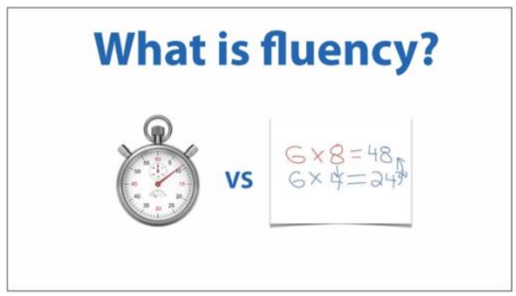

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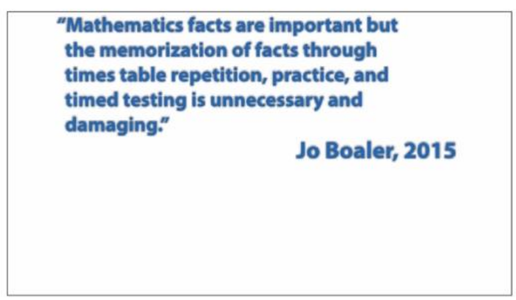

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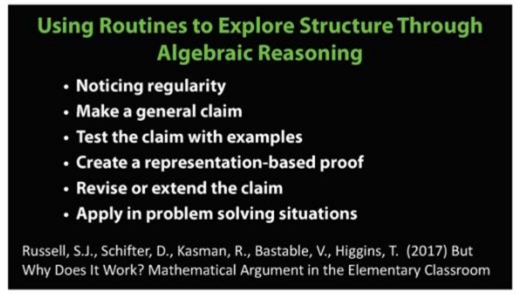

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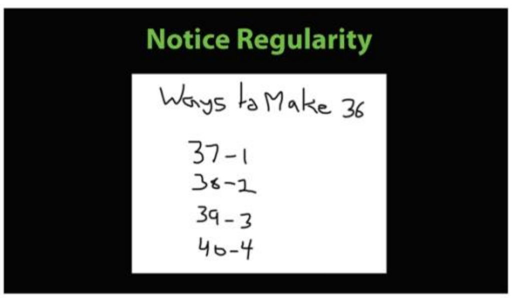

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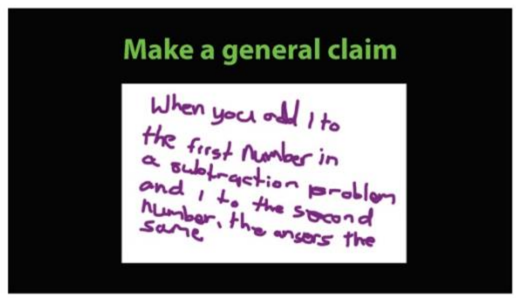

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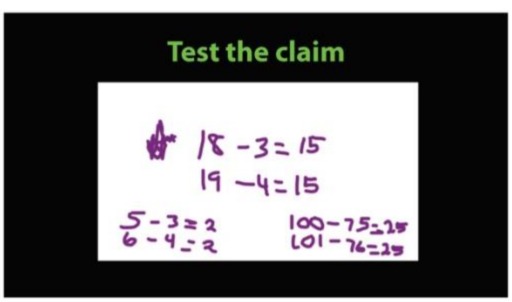

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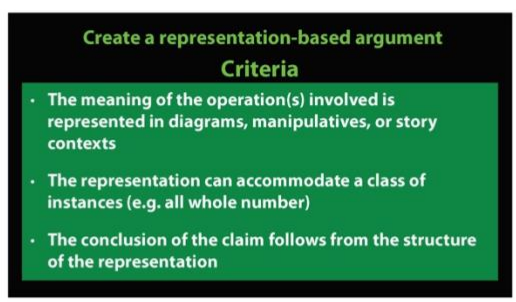

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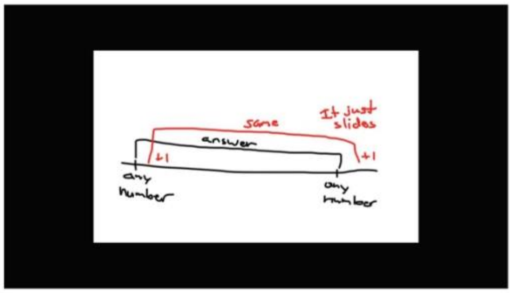

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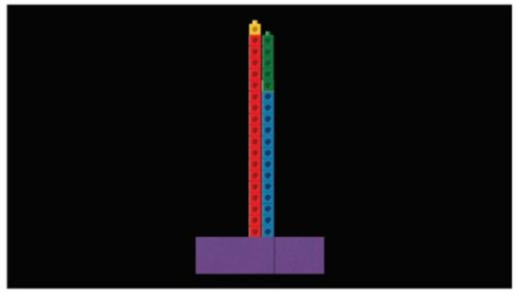

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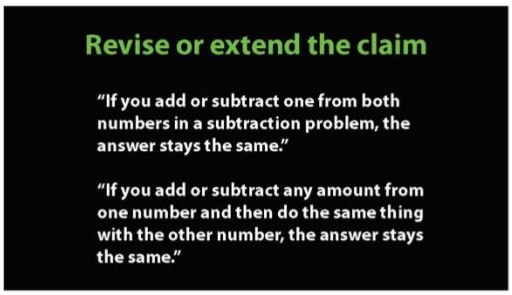

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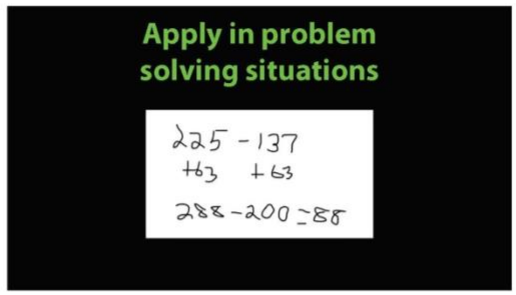

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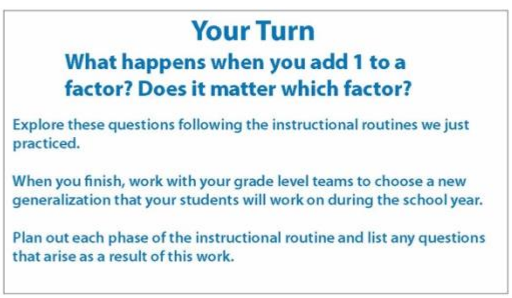

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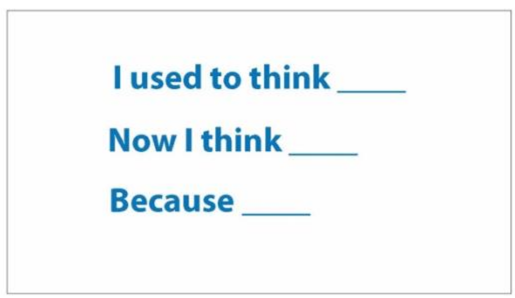

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Appendix A: Project Assignments and Course Evaluation

Math Practice 7 and The Matrix

The Matrix is a science fiction film about a character named Neo played by Keanu Reeves. Neo is a computer programmer who's just living his life like the rest of us. One day Neo gets a call from a man named Morpheus, played by Lawrence Fishburne, who tells Neo that the world he lives in is not real. Machines have taken over the world and they are using humans as a power source. Neo and most of humanity are actually plugged into a computer simulation called The Matrix that makes them all think they're living their day-to-day lives when in fact they're prisoners to the machines.

Morpheus offers to unplug Neo so he can join his crew and work to free other humans. Once outside the Matrix, Neo learns about the underlying structure of the Matrix and the dangers of going back in to free more humans. He learns about the agents, like Agent Smith, whose sole purpose is to protect the Matrix and kill anyone trying to free humans.

Neo needed training before he could reenter the Matrix. He needed to learn combat skills so he could protect himself, but more importantly, his training was designed to help him see the Matrix for what a really was, a computerized system that was governed by rules. He needed to free his mind of these rules and once he learned how he would be able to bend and break them, allowing him to do things like stop bullets, run up walls, and even fly.

So, what does any of this have to do with math? Well most of us grew up in the *Matrix of Math*. This too was a system governed by rules. There was one way to add, one way to subtract, one way to multiply, and one way to divide. We weren't encouraged to stray from these algorithms. In fact, we were conditioned to use them so much that for many adults today, that's the only way they can do these operations.

The *Matrix of Math* is rule-driven mathematics without concern for understanding. It's about speed and accuracy at all costs, even if that means we hinder student learning. The developer of this course once tutored third graders to help them get ready for the state test. Their teachers we deep in the *Matrix of Math* and it was easily evident. For example, one of the sessions was launched with what the developer thought was an easy mental math exercise. The class was asked to solve 200 - 198. However, shortly after posting this problem some of the kids started crying. When asked what was wrong, one student expressed how others were feeling. He said, "I can do it if you let me use my notebook and pencil, but I can't keep track of all that borrowing in my head." That's what being in the Matrix looks like. They only had one way to think about subtraction. They weren't even looking at the numbers.

The group was told not to worry about subtraction. They were asked, "What can you tell me about those numbers?"

- One kid suggested, "They're big."
- "Yes, what else can you tell me?"
- Another kid said, "They're close to each other."
- "Absolutely! How close are they?"

- After a slight pause few kids exclaimed, “Two…Two!!”

For just a moment, they were unplugged from the Matrix. They could see that there was more to subtraction than just the algorithm. They needed much more experience to be fully unplugged, but there was hope.

Now compare the experience of the third graders with that of Keisha, a second grader in one of the developer’s colleague’s classes. The developer was observing in her class and the teacher posed 85 – 37 as a mental math task. Keisha, very quickly had her hand in the air and was excited to share her thinking. She said, “I changed the problem to make it easier. I knew you needed to add 3 to 37 to make 40 and I knew if I did the same thing to 85 it would give me a new problem with the same answer. So ,I added 3 to 85 and got 88. Now it’s 88 – 40 which equals 48.”

This is the mathematical equivalent of stopping bullets. Keisha was unplugged from the Matrix. She didn’t see operations as commands to follow memorized procedures. She understood how the structure of subtraction and how it behaved. She could then use that understanding to think flexibly and make the math easier. Keisha was not an anomaly either. She and her classmates had been exploring the structure of addition and subtraction with their teacher all year long. The understanding they developed throughout the year actually unplugged them all from the Matrix. All students deserve opportunities to be unplugged.

Pre-Course Assignment

Beyond Answer-Getting: Exploring Mathematical Practices and Developing Mathematical Reasoning with All Students

Dear Participant,

Please respond in writing to the attached four problems and email your work to your course instructors by Friday before the start of class. I want you to capture your present thinking regarding these ideas prior to your formal class experience. Please use full sentences to explain your thinking in this document. I expect this assignment will take approximately 1 to 2 hours to complete. **Also please add your full name to the header (above).** Thank you.

I want to capture your present thinking regarding these ideas prior to your formal class experience, I am asking for you to work on these questions alone and without reference to mathematics websites or resources. Please use full sentences to explain your thinking in this document. **Please include your full name and grade level on your response and in the file name when you save it (e.g., Beyond Answer-Getting Pre-Course Assignment Smith)**

Sincerely,
(Signature of Course Instructor)

Pre-Course Assignment Problem 1

Students Janae, Tom, Bert, and Betsy demonstrated four different approaches to the same problem, 68 + 24 = ?

Janae's approach: 68 + 24 = ? 60 + 20 = 80 8 + 4 = 12 80 + 12 = **92**	**Tom's approach** 68 + 24 = ? 24 – 2 = 22 Take the -2 and add it to the 68 (68 + 2 = 70.) Add the 22 that was the answer from 24-2. 22 + 70 = **92**
Bert's thinking: 68 + 24 = ? Take the 24 and break it into smaller pieces. 24 = 5 + 5 + 5 + 5 + 4 Add the pieces onto the 68, one at a time, by counting up. 68 + 5 = 73 73 + 5 = 78 78 + 5 = 83 83 + 5 = 88 88 + 4 = **92**	**Betsy's thinking:** 68 +24 8 + 4 = 12 Carry the 1 and add it to the 6 + 2. 1 + 6 + 2 = 9 So the answer is **92.**

For each student, answer the following:

a. How do you, yourself, understand why this procedure produces a correct answer. You may use diagrams, story contexts, number lines or manipulatives to illustrate.

b. Will this procedure work on other addition problems? How do you know?

c. Would you want to have this student share his or her procedure with the whole class? Why or why not? (Janae; Tom; Bert; Betsy)

Pre-Course Assignment Problem 2

Consider the responses of these students who were asked to determine 27 x 4 in two different ways:

Jen's approaches	**Joel's approaches:**
First way: 2 x 27 = 54 2 + 2 = 4 4 x 27 = 108 54 + 54 = 108 I found out that 2 x 27 = 54 and then I added 54 + 54. Second way: 20 x 4 = 80 7 x 4 = 28 I did 80 + 28, 20 x 4, and 7 x 4 and then added them together.	First way: Well, 2 x 27 = 54, so 54 + 54 = **108.** Second way: First I added up all of the 20s and got 80 and then I added up all the sevens and got 28 and then I added 80 to 28 and got **108**.

For each student (Jen, then Joel) answer the following:

a. How do you, yourself, understand why this procedure produces a correct answer. You may use diagrams, story contexts, number lines or manipulatives to illustrate.

b. Will this procedure work on other multiplication problems? How do you know?

c. Would you want to have this student share his or her procedure with the whole class? Why or why not? (Jen's first and second ways; Joel's first and second ways)

Pre-Course Assignment Problem 3

Imagine you gave the following problem to your third-grade students,

I have 24 balloons to give out to my friends in bunches of 4. How many of my friends will get a bunch of balloons?

Among your students' work were the following five responses:

Josh 4 + 4 = 8, 8 + 8 = 16, 16 + 8 = 24
Marilyn 24 – 4 = 20; 20 – 4 = 16; 16 – 4 = 12; 12 – 4 = 8; 8 – 4 = 4 6 friends
Stacey The answer is 6 because 6 x 4 = 24
Max xxxx xxxx xxxx xxxx xxxx xxxx 6

a. How do you, yourself, understand why each produces a correct answer. You may use diagrams, story contexts, number lines or manipulatives to illustrate.

b. Explain why you would or would not want each student share his or her procedure with the whole class.

c. Assuming you gave this assignment at the beginning of a unit on division and these responses are representative of your class as a whole, what would you do next?

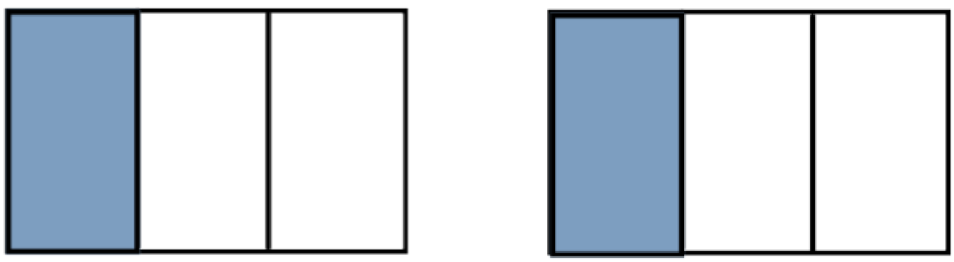
Pre-Course Assignment Problem 4

Here is a problem: There are 2 brownies to be shared among 3 children. What size portion will each child get? Show with a diagram.

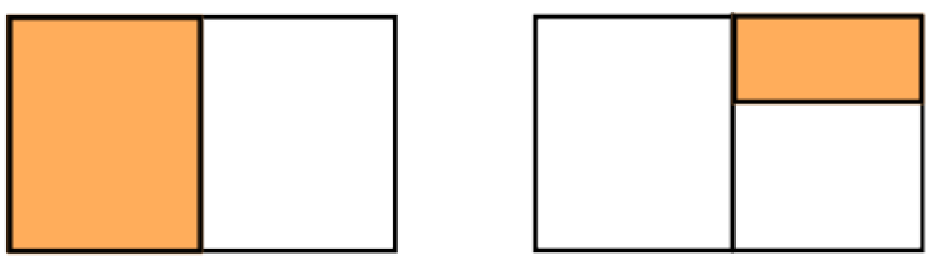
a. What is your answer to this question?

b. What number or numerical expression is indicated by the shaded portion in each students' diagrams?

Nia



Washawn



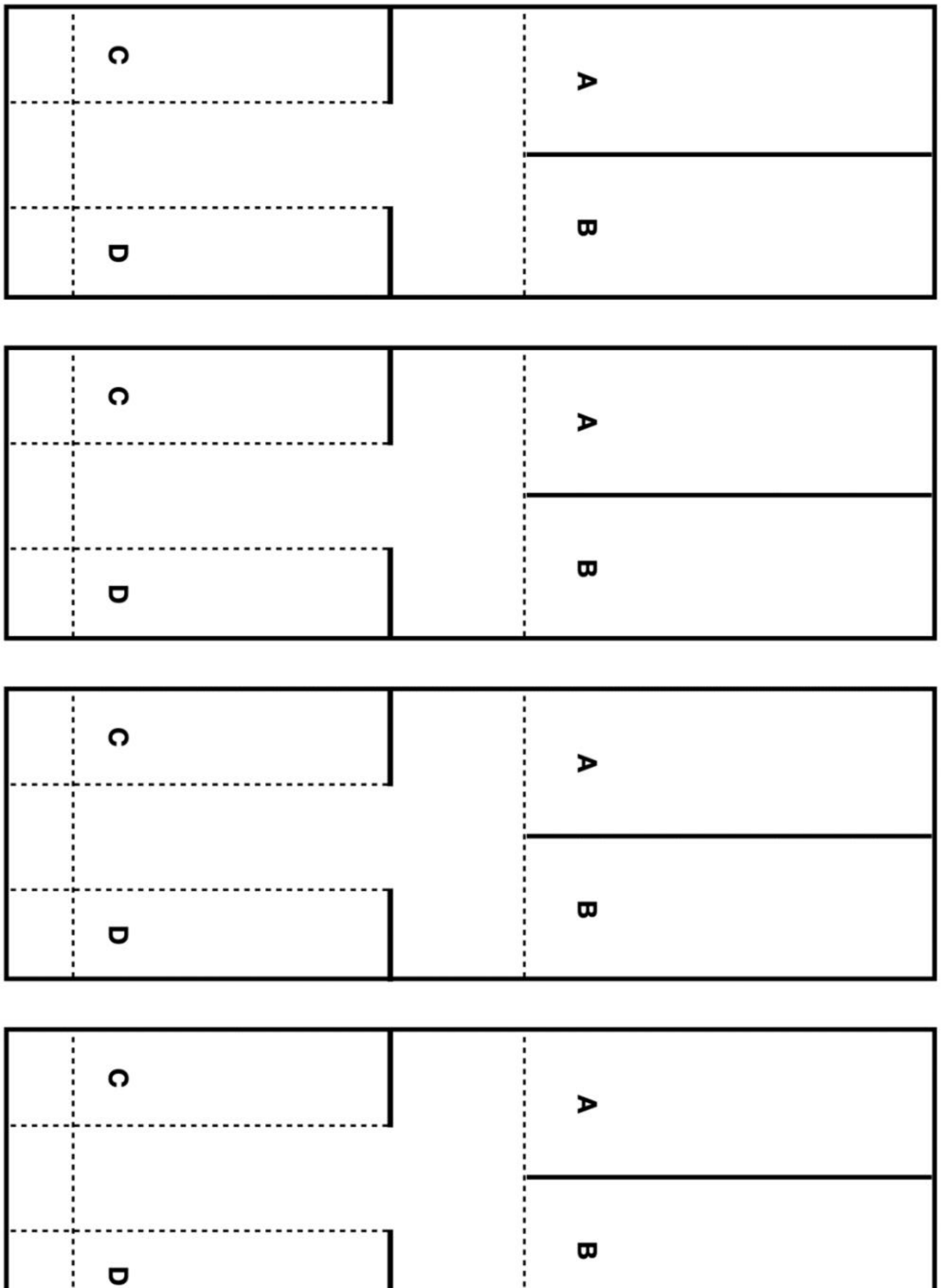
c. Which of these children has correctly shown the size of a portion?

____ Nia ____ Both

____ Washawn ____ Neither

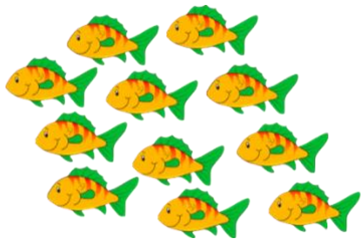
Explain why:

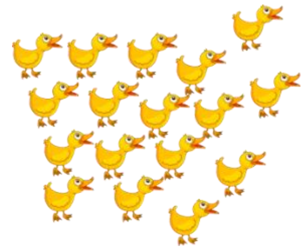
Propeller Design Template



Alphabet Counting Sheet

Use the alphabet as numbers to solve the following problems:





How many fish are there? ____ How many ducks are there? ____

Are there more fish or more ducks? __________

How many more? ________ (show how you figured it out)

Draw "e" circles and count them backwards.

Count by "c"______, ______, ______,

What is "d" + "e?" ______

How did you get your answer?

Exploring Addition and Subtraction Strategies Sheet

Analyzing Student Strategies Sheet

Section 1 illustrates four different strategies for calculating 34 + 28. Section 2 illustrates five strategies for calculating 53 – 17. Examine each method carefully, first trying some similar problems using the same strategy. Then write a verbal description of the strategy. Demonstrate the method with manipulatives, diagrams, or a story context. In particular, try number lines and place value representations. Finally, write out the mathematical ideas you see in each procedure.

Section 1 34 + 28

1(a) 30 + 20 = 50 4 + 8 = 12 50 + 12 = 62	1(b) ^{1}3 4 + 2 8 6 2
1(c) 34 + 20 = 54 54 + 8 = 62	1(d) 34 + 28 = 32 + 30 = 62

Section 2 53 – 17

2(a) 53 – 10 = 43 43 – 7 = 36	2(b) 53 – 17 = 56 – 20 = 36
2(c) ^{4}5 ^{1}3 – 1 7 3 6	2(d) 17 + 3 = 20 20 + 30 = 50 50 + 3 = 53 Answer is 3 + 30 + 3 = 36
2(e) 5 ^{1}3 – $_{2}$1 7	

3 6

From Schifter, D., Bastable, V., & Russell, S. J. (2017a). *Developing mathematical ideas: Building a system of tens*. Reston, VA: National Council of Teachers of Mathematics.

3-Act Task Recording Sheet

What do you notice?	What do you wonder?

What question are you trying to answer?

Write an estimate that's too low.	Write an estimate that's too high.	What is your actual estimate?

Show your estimate on a number line with your too low and too high.

Workspace
What is the answer to your question?

Base-5 Task Sheet

1. Starting at with the top left box, write one number in each box, starting at 1 and moving to the right as you add the next counting number in base 5. Draw a representation of each number as you go.

2. Make a number line that goes from 1 to 100 in base 5

3. Solve the following problems.

   a. 24 + 13 =

   b. 132 + 341 =

   c. 32 – 14 =

   d. 320 – 14 =

   e. What is your answer to this addition problem and why does that make sense?

   13 + 13 + 13 + 13 + 13 =

   f. 24 x 3 =

   g. 24 x 30 =

   h. 24 x 10 =

   i. 24 x 13 =

4. Make up some computation problems of your own to try.

From Schifter, D., Bastable, V., & Russell, S. J. (2017a). *Developing mathematical ideas: Building a system of tens.* Reston, VA: National Council of Teachers of Mathematics.

Henrietta's Roost Case

Ms. Nolan – Grade 1, May

TEACHER: Let's try another one. What if Henrietta is 8th and in the middle?

DIANE: 15 (This was done very quickly and without paper and pencil.)

TEACHER: How did you get that?

DIANE: I took away Henrietta from the 8 and I had 7, and so I put 7 on the other side.

TEACHER: Try 15.

DIANE: 29

TEACHER: How did you get this one so fast?

DIANE: 15 take away 1 for Henrietta, I get 14, and so 14 plus 14 equals 28, then I add 1 is 29.

TEACHER: You seem to have a strategy that works for solving this problem. Will it always work?

DIANE: Yes, I think so.

TEACHER: What would it take to convince you that it would always work?

DIANE: Do it a really lot of times and if they all come out right!

Diane spoke this last statement with conviction. I was so excited by her thinking and her ability to articulate her thought process. In my mind I could see two equations taken together, something like $x - 1 = y$ and $y + y + 1 = n$, where x is Henrietta's position in line, y is the number of hens on either side of Henrietta, and n is the total number of hens.

I realize that Diane did not have the vocabulary or symbols to write these equations, yet somehow she had so clearly defined the steps she used each time. Is this the beginning of algebraic thinking?

From Schifter, D., Bastable, V., & Russell, S. J. (2017b). *Developing mathematical ideas: Patterns, functions, and change casebook.* Reston, VA: National Council of Teachers of Mathematics.

Terminating and Repeating Decimals Task Sheet

1. Examine what happens when you divide by 3. Work out the following:

   1 ÷ 3   2 ÷ 3   3 ÷ 3   4 ÷ 3   5 ÷ 3   6 ÷ 3   7 ÷ 3   etc.

   What patterns do you notice? How do you explain them?

2. Examine what happens when you divide by 5. Work out the following:

   1 ÷ 5   2 ÷ 5   3 ÷ 5   4 ÷ 5   5 ÷ 5   6 ÷ 5   7 ÷ 5   etc.

   What patterns do you notice? How do you explain them?

3. Why does dividing by 5 produce such different results than dividing by 3?

4. Examine what happens when you divide by 4. Work out the following:

   1 ÷ 4   2 ÷ 4   3 ÷ 4   4 ÷ 4   5 ÷ 4   6 ÷ 4   7 ÷ 4   etc.

   What patterns do you notice? How do you explain them?

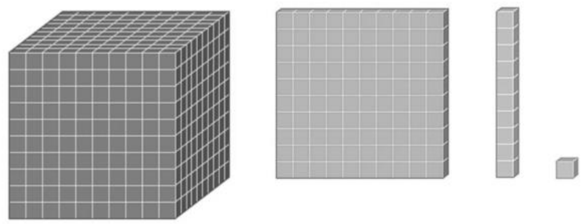
5. Consider the other digits. Which produce results like dividing by 5? Which produce results like dividing by 3? How do you explain that?

6. *Summarizing:* Consider $\frac{1}{n}$ for various values of n, such as n = 2, 3, 4, … For what values of n does the decimal expression for $\frac{1}{n}$ end? For what values of n does the decimal expression for $\frac{1}{n}$ not end? How can you explain why this happens?

From: Schifter, D., Bastable, V., & Russell, S. J. (2017a). *Developing mathematical ideas: Building a system of tens.* Reston, VA: National Council of Teachers of Mathematics.

Multiplying By 10 Task Sheet

Object of the activity: We often hear students say, "When you multiply by ten, you just put a zero at the end of the number, and when you multiply by one hundred, you just put two zeros at the end of the number." What does this mean? How does it work? Keep these questions in mind as you explore this problem.



Create a representation for each of these expressions, using a base-ten model. Examine your representations to answer this question: How do you explain the relationship between the digits in the problem and the result of the multiplication?

(a) 23 x 1 (b) 23 x 10 (c) 23 x 100 (d) 23 x 1,000 (e) 23 x 10,000

From Schifter, D., Bastable, V., & Russell, S. J. (2017a). *Developing mathematical ideas: Building a system of tens*. Reston, VA: National Council of Teachers of Mathematics.

Exploring Structure Through Algebraic Reasoning Sheet

1. **Noticing regularity** (MP8)

"Each time I make the second number bigger, my answer gets smaller."

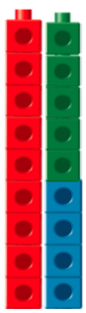
2. **Make a general claim** (MP8)

"If you make the number you are subtracting bigger by one, the answer will get smaller by one."

3. **Test the claim with examples** (MP8)

7 – 3 = 4	19 – 11 = 8	52 – 24 = 28
7 – 4 = 3	19 – 12 = 7	52 – 25 = 27

4. **Create a representation-based argument** (MP3 and MP7)



5. **Revise or extend the claim** (MP8)

"If you increase the number you're subtracting by any amount, the answer will get smaller by that same amount."

6. **Apply in problem solving situations (MP7)**

I can solve 53 – 37 by changing it to make it easier. I'll add 3 to 37 to make it 53 – 40 which I can solve in my head and get 13. Then to solve 53 – 37, I just need to add 3 to the 13 to get 16.

From Russell, S. J., Schifter, D., Kasman, R., Bastable, V., & Higgins, T. (2017). *But why does it work?* Portsmouth, NH: Heinemann.

Course Evaluation

Name (Optional) ___________________

Please rate the following aspect of the course:

	Excellent	Good	Satisfactory	Fair	Poor
The Facilitator					
Course Content					
Course Structure					
Class Assignments					
Homework					

What elements of the course was most useful to you?

What elements of the course need improvement?

Please comment on the instructor's facilitation of the course.

How would you rate your overall experience in the course?

Excellent	Good	Satisfactory	Fair	Poor

Any additional comments?

Post-Course Assignment

Beyond Answer-Getting: Exploring Mathematical Practices and Developing Mathematical Reasoning with All Students

Review your written responses to the pre-course assignment and reflect on what you have learned during your time with us. Use track changes and revise your responses where necessary. When appropriate, reference specific elements from the course to help me see the connections you are making between the course content and your prior knowledge.

Then write a 2-page reflective essay on all your learning from this course. Be sure to reference what you learned about mathematical content as well as mathematics teaching.

Email your response to me by (insert date). Thank you

Sincerely,

(Course Instructor)

Appendix B: Interview Questions

Pre-Course Interview:

1. When you consider mathematics, what feeling and connotations does the subject provoke in you? Why? (RQ1 & RQ3)
2. What was your experience in mathematics as an elementary and middle school student and how do you think these experiences contributed to how you perceive mathematics today? (RQ1 & RQ3)
3. When you consider teaching mathematics to students, what feeling and connotations does this provoke in you? Why? (RQ1 & RQ3)
4. What was your experience in mathematics during your teacher preparation program and how do you think these experiences contribute to how you perceive teaching mathematics? (RQ1 & RQ3)
5. How would you describe your mathematical content knowledge? (RQ1)
6. What constitutes effective mathematics instruction? (RQ1 & RQ2)

Post-Course Interview

7. Reflecting on your experience during the college math course, what components stand out as having a positive effect on your learning and why? (RQ5)
8. Reflecting on your experience during the college math course, what components seemed to have little or no effect on your learning and why? (RQ5)
9. Did any component of the course have a negative effect on your learning? If so, please describe. (RQ5)

10. How have your perceptions of math and math instruction changed as a result of your experience during the college math course? (RQ5)
11. Thinking ahead to the coming school year, will you change anything about your approach to teaching mathematics and working with students? If so, what experiences from this course affected these decisions? If not, why? (RQ3 & RQ5)
12. When you consider mathematics, what feeling and connotations does the subject provoke in you? Why? (RQ4)
13. When you consider teaching mathematics to students, what feeling and connotations does this provoke in you? Why? (RQ3)