An integrated model for Optimization of production-distribution inventory levels and routing Structure for a multi-period, multi-product, bi-echelon supply chain

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Abstract
In many multi-stage manufacturing supply chains, transportation related costs are a significant portion of final product costs. It is often crucial for successful decision making approaches in multi-stage manufacturing supply chains to explicitly account for non-linear transportation costs. In this paper, we have explored this problem by considering a Two-Stage Production-Transportation (TSPT). A two-stage supply chain that faces a deterministic stream of external demands for a single product is considered. A finite supply of raw materials, and finite production at stage one has been assumed. Items are manufactured at stage one and transported to stage two, where the storage capacity of the warehouses is limited. Packaging is completed at stage two (that is, value is added to each item, but no new items are created), and the finished goods inventories are stored which is used to meet the final demand of customers. During each period, the optimized production levels in stage one, as well as transportation levels between stage one and stage two and routing structure from the production plant to warehouses and then to customers, must be determined. We consider “different cost structures,” for both manufacturing and transportation. This TSPT model with capacity constraint at both stages is optimized using Genetic Algorithms (GA) and the results obtained are compared with the results of other optimization techniques of complete enumeration, LINDO, and C-plex.

Keywords
TSPT, Genetic Algorithms, complete enumeration, LINDO, C-plex

Introduction
To exploit economies of scale and order in large lots, the important issue in supply chain is to optimize the inventory level by considering various costs in
maintaining a high service level towards the customer. Since, the cost of capital tied up in inventory is more, the inventory decision in the supply chain should be coordinated without disturbing the service level. The coordination of inventory decision within an entity is viable, but not between the entities. So the integration of the entities to centralize the inventory control is needed.

Several factors influence inventory policy and practice for the firm includes distribution savings demand seasonality in production and purchasing economies with the desired level of customer service. Transportation is the spatial linkage for the physical flows of a supply chain. Today supply chain has to examine the effective use of supply chain with value added processes in distribution center with minimum inventory levels for delivering the product in time. The gateway of the supply chain is to know our customer and serve the needs by considering the aspects of speed and reliable service with the value propositions. The effective supply chain management needs to improve customer service, reduction of costs across the supply chain, optimal management of existing inventory with optimized manufacturing schedules. The impact of distribution inventory will enhance the customer value in the form of lower costs across the supply chain.

This project presents the Genetic Algorithm Distribution Inventory Model (GADIM) approach for optimizing the inventory levels of supply chain entities with the consideration of two-echelon plant to warehouse and warehouse to customers. This model is especially suitable for inventory control and cost reduction by unlocking the hidden profits through the genetic algorithm optimization. The generalized GADIM evaluate the transportation link between the two entities and finding the best route for transporting the product from plant to warehouse and then to customer. The demand of the customer is known in advance, so that the production rate and the inventory levels can be adjusted in the plant and warehouse. Since the logistics plays an important role in the supply chain that the two important sectors transportation and distribution inventory has taken into consideration for redesigning the allocation and routing through optimization. This approach is based on genetic algorithm. It searches the population of solutions of an optimization problem towards the improvement by simulating the natural search and selection process associated with natural genetics.
The model comprises the genetic algorithm optimization where the inventory level at the plant and warehouse are optimized according to the production capacity of the same. The distribution cost plays an important role in optimizing the inventory level, so that the total cost for the entire supply chain is minimized. The GADIM approach assure in minimizing the total cost of the supply chain by optimizing the inventory levels in accordance with production capacity of plant and warehouse.

**Literature Survey**

The literature review in aspect of distribution inventory management of supply chain, states clearly that no optimal inventory policy has been developed for a serial Supply Chain in view of the complexity of the problem. Hence the objective of the study is to optimize the inventory level for a two-echelon single product serial supply chain using Genetic Algorithm. So as to minimize the total supply chain cost comprising the distribution and production related cost.

Andi Cakravastia et al, developed a analytical model for the supplier selection process in designing a supply chain network. The constraints on the capacity of each potential supplier are considered in process. The assumed objective of the supply chain is to minimize the level of customer dissatisfaction, which is evaluated by two performance criteria.(1) Price (2) delivery lead time. The overall model operates at two levels of decision making : the operational level and the chain level. An optimal solution in terms of the models for the two levels can be obtained by using a mixed integer programming technique.

Beatriz Abdul-Jalbar et al, addresses a multi-echelon inventory system with one-warehouse and \( N \)-retailers. The demand at each retailer is assumed to be known and satisfied by the warehouse. Shortages are not allowed and lead times are negligible. Costs at each facility consist of a fixed charge per order and a holding cost.

The goal is to determine single-cycle policies which minimize the average cost per unit time, that is, the sum of the average holding and setup costs per unit time at the retailers and at the warehouse. They propose an \( O(N \log N) \) heuristic procedure to compute efficient single-cycle policies. This heuristic is compared with other approaches proposed by Schwarz, Graves and Schwarz and Muckstadt and Roundy.
They carry out a computational study to test the effectiveness of the heuristic and to compare the performance of the different procedures. From the computational results, it is shown that the new heuristic provides, on average, better single-cycle policies than those given by Muckstadt and Roundy.

A. Gunasekaran, C. Patel, Ronald E. McGaughey have studied that supply chain management has been a major component of competitive strategy to enhance organizational productivity and profitability. In recent years, organizational performance measurement and metrics have received much attention from researchers and practitioners. Performance measurement and metrics have an important role to play in setting objectives, evaluating performance, and determining future courses of actions. Performance measurement and metrics pertaining to SCM have not received adequate attention from researchers or practitioners. We developed a framework to promote a better understanding of the importance of SCM performance measurement and metrics.

Masao Yokoyama discusses a new model and its solution procedure for the commodity distribution system consisting of distribution centers and consumer points are discussed. Demand is assumed to be a random variable that obeys a known, stationery probability distribution. An integrated optimization model is built where both the order-up-to-R policy, which is one of the typical inventory policies of periodic review models, and the transportation problem are considered simultaneously. The assignment of consumer points to distribution centers is not fixed. The problem is to determine the target inventory and the transportation quantity in order to minimize the expectation of the sum of inventory related cost and transportation cost. Simulation and linear programming are used to calculate the expected cost, and a random local search method is developed in order to determine the optimum target inventory. He uses a genetic algorithm to test and compare with the proposed random local search method. The model and effectiveness of the proposed solution procedure are clarified by computational experiments.

Siddhartha S. Shyam proposed a model which significantly extends traditional facility locational models by introducing several logistical cost components such as holding, ordering and transportation costs in a multi
commodity, multi location framework. This paper provides an integrated model, and seeks to minimize total physical distribution cost by simultaneously determining optimal locations, flows, shipment composition and shipment cycle times.

A decision support system for supplier selection is developed by S. H.Ghodsypour et al. In this paper an integration of analytical hierarchy process and linear programming is proposed to consider both tangible and intangible factors in choosing the best suppliers and placing the optimum order quantities among them such that the value of purchasing becomes maximum. This model can be applied for supplier selection with or without capacity constraints.

Masood A. Badri proposes the use of AHP and multi-objective goal programming methodology as aid in making location-allocation decisions. The methodology presented can help facility planning authorities to formulate viable location strategies in the volatile and complex global decision environment.

A brief review of literature for modeling of multistage supply chain can be found in the paper by Benita M. Beamon. The above mentioned works leave scope for development of an optimization technique based on a meta heuristic approach. Thus we select the Genetic Algorithm to address the problem of “total cost optimization in a bi echelon multi-product and multi period supply chain environment”. We thereby also aim to study the other optimization techniques so as to compare and provide with the best optimization technique.

Research Gap Analysis

Aim

The aim of the paper is to optimize the total cost of a two stage multi product multi period supply chain environment using Genetic Algorithm Distribution Inventory model by taking into consideration all the constraints and assumptions. It also aims to provide a comparative study among the existing optimization techniques like LINDO, C-plex, Complete Enumeration and GADIM so as to provide the vendor with the best optimization technique depending on time and data constraints.

Objectives
The main objective of the present work is to demonstrate the importance of GADIM with relevance to supply chain management and logistics. There has been a growing interest in GA with respect to SCM and logistics. The main reason being that supply chain and logistics management is a collaborative function and a number of players are involved in the decision making process. So the approach used should support group tasking and GA is particularly suitable in that aspect. Moreover, in logistics decision making a number of non quantifiable criteria come into picture and GA handles this very effectively.

Another objective of this work is to incorporate customer in the logistics decision making process. Costs are often used as the main factor in logistics, where as enough attention is not paid to the various quantitative and qualitative customer service elements. The objective here is to develop a customer centric approach.

The present work is limited to evaluation of alternative links and nodes in a supply chain. This project aims to optimize the total cost of a multi product two stage multi period supply chain using GA. This work shall improve the productivity and minimize the losses of the two stage SCM thereby leading to optimized route based on the customer demand.

Scope

Supply chain is the emerging area with lot of problems in the production-distribution echelon and the problem has been less addressed and solved. Many Researchers have pursued their research in solving the production-distribution inventory management for a single echelon system (only supplier / manufacturer / distributor / retailer). But, (However) the Multi echelon distribution inventory management has received less attention from solution approaches. Some of the Researchers have tried this problem with the linear programming and Traditional Optimization technique. This problem hasn’t approached with the Non Traditional Optimization Technique. Thus we aim to approach this problem by using the meta- heuristic approach Genetic Algorithm. This method is capable of handling large data and particularly in this case where we shall deal with multi-product multi-period and two stage namely production, warehousing and
customer. Thereby on obtaining the result we shall compare it with the existing optimization techniques of complete enumeration, LINDO and C-plex.

**Problem Structure**

In many multi-stage manufacturing supply chains, transportation related costs are a significant portion of final product costs. It is often crucial for successful decision making approaches in multi-stage manufacturing supply chains to explicitly account for non-linear transportation costs. A two-stage supply chain that faces a deterministic stream of external demands for a multi product is considered. A finite supply of raw materials, and finite production at stage one has been assumed. Items are manufactured at stage one and transported to stage two, where the storage capacity of the warehouses is limited. Packaging is completed at stage two (that is, value is added to each item, but no new items are created), and the finished goods inventories are stored which is used to meet the final demand of customers. During each period, the optimized production levels in stage one, as well as transportation levels between stage one and stage two and routing structure from the production plant to warehouses and then to customers, must be determined.

**Problem Definition**

The investigation of Supply Chain Distribution Inventory Management has done using a model GADIM with a focus on inventory level at plant and warehouse, the production rate and production cost at the plant, and the transportation costs between the entities at all cross-levels. The three entities plant, warehouse and customers are considered in this supply chain model. The model is generalized by consideration of ‘Npl’ number of plants, ‘Nw’ number of warehouses and ‘Nc’ number of customers. The model GADIM suits for any kind of industry, which follows the three-entity supply chain model with finite number of plants, warehouses and customers. The Total demand of the customer is satisfied by the stock in the warehouse which inturn procure the product stock from the plant entity. The time factor and cost level in transporting the product between the three entities varies according to the logistics flow structure from the plant to the warehouse and then to customer of the supply chain model. So the delay factor may occur in accordance with the optimal transportation route. Since, there is a limited inventory level in the plant and warehouse it has a possibility of maintaining the Backlog
Inventory to satisfy the demand of the customers in the forthcoming period of this GADIM supply chain. The purpose of GADIM of supply chain is to provide a nearer optimal production rate for the plant, inventory level for the plant and warehouse by minimizing the total logistics costs of the entire supply chain. In designing the model, three “subsystems” need to be analyzed: (1) the production rate at each plant for all periods (2) the inventory at each plant and warehouse for all periods. (3) route between entities where the product is transferred from plant to warehouse and then to customer for all periods. A synthesis of these subsystems is used to minimize the total costs of supply chain.

**Methodology**

The methodology of Genetic Algorithm begins with defining the objective function. In our problem the objective function is to minimize the total cost. To obtain the objective function the constraints namely production cost, transportation cost, storage cost and customer demand are considered. The first stage involves initialization. The initial chromosomes are randomly generated. Next, these chromosomes are evaluated and after evaluation the best chromosome are selected. Then the selected chromosomes undergo crossover and mutation operations. After the mutation process the entire cycle is iterated till the best result is obtained. The results obtained are compared with the results of other optimization techniques of complete enumeration, LINDO, C-plex and the comparative result is tabulated.

**Assumptions**

The model assumes that a single product with a stochastic demand.

- The Total demand for all periods should be less than the Total production capacity and warehouse capacity.
- The Total production rate for all period is equal to total demand of all period.
- Time taken for transporting the product between the entities is homogenous and not taken into consideration.

**Constraints**

- **Manufacturing plant**
  - Product cost of each plant
  - Inventory cost of each plant
  - Products (production) capacity of each plant
- Inventory capacity of each plant

- **Warehouse**
  - Total cost from each plant to each warehouse
  - Inventory cost of each warehouse
  - Inventory capacity of each warehouse

- **Suppliers**
  - Total cost from each warehouse to each customer
  - Demand by each customer for one period

Taking into account the above said constraints, the following outputs are obtained:

1. Total cost for all periods in order to satisfy the demand constraint.
2. Optimal production rate in each period for all plant
3. Optimal warehouse stock in each period for all warehouses.
4. Optimal Routing (from plant to warehouse to customers) for all periods.

**Notations**

The definitions of notations are

- \( N_p \): Number of period
- \( N_{pl} \): Number of plant
- \( N_w \): Number of warehouse
- \( N_c \): Number of customer
- \( P_{ci} \): Production cost at \( i^{th} \) plant \( i = 1 \ldots N_{pl} \)
- \( ICCP_i \): Inventory carrying cost at \( i^{th} \) plant \( i = 1 \ldots N_{pl} \)
- \( ICCW_j \): Inventory carrying cost at \( j^{th} \) warehouse \( j = 1 \ldots N_w \)
- \( CDEM_{pk} \): Demand at the \( K^{th} \) customer at the period ‘\( p \)’ \( p=1 \ldots N_p \)
- \( PCAP_i \): Production capacity at the \( i^{th} \) plant
- \( WCAP_j \): Warehouse capacity at the \( j^{th} \) warehouse
- \( BO \): Backordering cost
- \( ICAP_i \): Inventory capacity at the \( i^{th} \) plant
- \( TCPW_{pij} \): Unit Transportation cost from source ‘\( i \)’ to destination ‘\( j \)’ for period ‘\( p \)’.
- \( TCWC_{pjk} \): Unit Transportation cost from source ‘\( j \)’ to destination ‘\( k \)’ for a period ‘\( p \)’.
- \( Pr_{pi} \): Production rate at \( i^{th} \) plant for a period ‘\( p \)’.
- \( UTPW_{pij} \): Unit transferred from \( i^{th} \) plant to \( j^{th} \) warehouse for a period ‘\( p \)’.
- \( UTWC_{pjk} \): Unit transferred from \( j^{th} \) warehouse to \( k^{th} \) customer for a
period ‘p’.

\[ \text{WHS}_{pj} \] Warehouse stock at \( j^{\text{th}} \) warehouse for a period ‘p’.

\[ \text{USC}_{pk} \] Unsatisfied demand at \( k^{\text{th}} \) customer for a period ‘p’.

\[ \text{ICP}_p \] Total inventory cost for all plant for a period ‘p’.

\[ \text{ICW}_p \] Total inventory cost for all warehouse for a period ‘p’.

\[ \text{TPC}_p \] Total production cost for a period ‘p’.

\[ \text{PCP}_i \] Production cost at \( i^{\text{th}} \) plant

\[ \text{PWTC}_p \] Plant to warehouse total transportation cost for a period ‘p’.

\[ \text{WCTC}_p \] Warehouse to customer total transportation cost for a period ‘p’.

\[ \text{SC}_p \] Shortage cost for a period ‘p’.

\[ \text{IP} \] Inventory in plant.

\[ \text{IW} \] Inventory in Warehouse.

\[ \text{TC} \] Total cost comprising production, inventory (plant and warehouse), transportation (plant to warehouse to customer) and shortage costs for all period.

**Mathematical Model**

The objective function of GADIM is to minimize the total cost of the supply chain. The model is interpreted in linear programming form as given below.

\[
\text{Min } TC = \sum_{p=1}^{N_p} \left( \text{ICP}(p) + \text{ICW}(p) + \text{TPC}(p) + \text{PWTC}(p) + \text{WCTC}(p) + \text{SC}(p) \right)
\]

- (1)

**Subject to Constraints,**

\[
\text{ICP} = \sum_{i=1}^{N_p} \left( \text{IP}(i) \times \text{ICCP}(i) \right)
\]

\[
\text{ICW} = \sum_{j=1}^{N_w} \left( \text{IW}(j) \times \text{ICCW}(j) \right)
\]

\[
\text{TPC} = \sum_{i=1}^{N_p} \text{PCP}(i)
\]

\[
\text{PCP}(i) = \sum_{j=1}^{N_w} \left( \text{UTPW}(i)(j) \times \text{PC}(i) \right)
\]

\[
\text{PWTC} = \sum_{j=1}^{N_w} \sum_{i=1}^{N_p} \left( \text{UTPW}(i)(j) \times \text{TCPW}(i)(j) \right)
\]

\[
\text{WCTC} = \sum_{k=1}^{N_c} \sum_{j=1}^{N_w} \left( \text{UTWC}(j)(k) \times \text{TCWC}(j)(k) \right)
\]
\[ SC = \sum_{k=1}^{Nc} (USC(k) \times BO) \]

The production rate should be less than or equal to the production capacity of each plant

\[ Pr(i) \leq PCAP(i) \]

The Total Production rate is equal to total customer demand

\[
\sum_{p=1}^{Np} \sum_{i=1}^{Npl} Pr(i)(p) = \sum_{p=1}^{Np} \sum_{k=1}^{Nc} CDEM(k)(p)
\]

The sum of units transferred from each plant to all warehouse should be less than or equal to production rate of the particular plant

\[ \sum_{j=1}^{Nw} UTPW(i)(j) \leq Pr(i) \]

The sum of units received from all plants to the warehouse should be less than or equal to the warehouse capacity

\[ \sum_{i=1}^{Npl} UTPW(i)(j) \leq WCAP(j) \]

Warehouse stock is equal to the sum of units received from all plant.

\[ WHS(j) = \sum_{i=1}^{Npl} UTPW(i)(j) \]

The warehouse stock should not exceed the warehouse capacity

\[ WHS(j) \leq WCAP(j) \]

The sum of units transferred from warehouse to all customers should be less than or equal to the warehouse stock

\[ \sum_{k=1}^{Nc} UTWC(j)(k) \leq WHS(j) \]
The sum of units received to customer from all warehouse should be less than or equal to customer demand

\[ \sum_{j=1}^{Nw} UTWC(j)(k) \leq CDEM(k) \]

Unsatisfied customer demand for the current period

\[ USC(k) = CDEM(k) - \sum_{j=1}^{Nw} UTWC(j)(k) \]

Next period demand is

\[ CDEM(k) = USC(k-1) + CDEM(k) \]

Methodology of GADIM

Our approach is based on Genetic Algorithm. GA’s are inspired by natural evolution. GA’s search populations of solutions of an optimization problem towards improvement by simulating the natural search and selection process associated with natural genetics. GA’s encodes an optimization problem into a set of genetic characteristics. Each solution is encoded in a string (called chromosome) of binary digits or integers. The method uses three natural operators selection, crossover and mutation to search populations of solutions towards improvement. GA’s starts with a random population of solutions to explore the solution space of a problem. Then GA’s attempt to improve the solutions through a number of iterations called generations. The performance of each solution to the problem is evaluated by fitness function, which corresponds to the objective function of the optimization problem. Each individual contributes to the next generation in proportion to its fitness. The selection of chromosomes for reproduction is biased so that the fitter one tends to reproduce more often than the less fit ones. Following parental selection, crossover and mutation are applied. Crossover combines materials from parents to produce their children. It also provides pressure for improvement or exploration. On the other hand, mutation makes small local changes of feasible solutions to provide diversity of population for wide exploration of feasible solutions. GA is a global search methodology whose convergence although possibly slow, has been proven (Holland, 1975). The mutation operator is usually defined to
ensure that the generated solutions will not get trapped in some local minimum. Moreover, the final solution does not depend on the initial solutions. Hence, in most badly the initial population is randomly generated. The chance of reaching a very good solution, if not optimal one, is very high and increases, of course, with the number of generations.

*The procedure of the classical GA is as follows*

1. Initialize a set of feasible solutions (i.e., initialize a population of chromosomes) randomly.
2. Evaluate each chromosome in the population by a fitness function.
3. Select chromosomes for reproduction.
4. Apply crossover and mutation on the selected chromosomes to produce new chromosomes.
5. Evaluate the new chromosomes.
6. If the stopping condition is reached, return the best solution; if not, go to 3.

For the single product GADIM, the chromosomes considered includes the production rate of the plant for all period, the inventory level at the warehouse for all period, total demand of customer for all period with the amount of product transferred from plant to warehouse and then to customer for all period. The chromosome is randomly generated by satisfying the feasibility condition for the process of Genetic Algorithm.

The example of the randomly created chromosome for the two periods, with the assumption of 2 plants, 3 warehouses and 3 customers in GADIM supply chain is considered for illustration.

**PERIOD 1**

**PERIOD 2:**

![Diagram](image-url)
The 10 Population size chromosomes are considered. The pop size randomly generated chromosomes are evaluated using the specified objective function (Eqn 1) after evaluating all the chromosomes. Select the better parent chromosome through the selection process. The Roulette wheel selection approach has been used in the selection process, where the fitness function is calculated as the inverse of the objective function i.e., 1/(1+TC). The Probability occurrence of each chromosome in the selection process is calculated and in turn the cumulative probability. The generated random numbers in the range 0-1 is compared with the cumulative probability to select the better parent chromosome for the next crossover process. The crossover operator suggested by Vignaux and Michalewicz for the transportation problem is considered with slightly modification for the crossover process for the present problem. For the crossover operation the production rate of the plant, inventory level of the warehouse, unit product transferred from plant to warehouse and then to customer. The crossover operator is suggested as a matrix for all the above measure where the transformation of the matrix with the viable feasibility condition in each chromosome is considered.

**Steps in crossover**

Assume that two matrices X1 and X2 are selected as parents for the crossover operation. The crossover is performed in three steps.

Step 1. Create two temporary matrices ‘D’ and ‘R’ as follows. Matrix ‘D’ keeps rounded average values from both parents, and matrix R keeps track of whether any rounded is necessary.

\[
D = \left\lfloor \frac{(X1 + X2)}{2} \right\rfloor \\
R = (X1 + X2) \mod 2
\]

Step 2. Divide (Split) matrixes ‘R’ into two matrices R1 and R2. It has many possible ways to divide ‘R’ into R1 and R2 while satisfying the condition.
Step 3. Then we produce two offspring of Y1 and Y2 as follows.

\[ Y1 = D + R1 \]
\[ Y2 = D + R2 \]

The illustrative example of the crossover operation for the pair of chromosome as shown below. The chromosome for the crossover is divided into two sections, 1. plant to warehouse for all period 2. warehouse to customer for all period.

1. Plant to warehouse for all period:

<table>
<thead>
<tr>
<th>First chromosome (X1):</th>
<th>Second chromosome (X2):</th>
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<tbody>
<tr>
<td>1 0 0 7 0</td>
<td>0 0 5 0 3</td>
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<tr>
<td>0 4 0 0 0</td>
<td>0 4 0 0 0</td>
</tr>
<tr>
<td>2 1 4 0 5</td>
<td>0 0 5 7 0</td>
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<tr>
<td>0 0 6 0 0</td>
<td>3 1 0 0 2</td>
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The temporary matrices ‘D’ and ‘R’ are

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<th>Matrix D</th>
<th>Matrix R</th>
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<tr>
<td>0 0 2 3 1</td>
<td>1 0 1 1 1</td>
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<tr>
<td>0 4 0 0 0</td>
<td>0 0 0 0 0</td>
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<tr>
<td>1 0 4 3 2</td>
<td>0 1 1 1 1</td>
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<td>1 0 3 0 1</td>
<td>1 1 0 0 0</td>
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Then divide R into R1 and R2 as follows:

<table>
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<tr>
<th>Matrix R1</th>
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<td>0 0 1 0 1</td>
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Finally, two offspring Y1 and Y2 are:

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Similarly the chromosomes are crossover for the warehouse to customer.

The next process mutation is carried over for the crossover chromosome in using the mutation operator devised by Vignaux and Michalewicz. So the procedure of mutation operation is performed in three steps.

Step 1. Make a sub matrix from a parent matrix. Randomly select \( i = 1 \) to \( N_{pl} \) for all period \( i \) and \( j = 1 \) to \( N_{w} \) for all period \( j \) columns to create a \( (p \times q) \) sub matrix ‘Y’

Step 2. Reallocate a commodity for the sub matrix. The available amount of commodity and demands for the sub matrix are determined. We can use the initialization procedure to assign new values to the sub matrix such that all constraints are satisfied.

Step 3. Replace the appropriate elements of the parent matrix by new elements from the reallocated sub matrix Y.
Select the chromosome randomly for the mutation process.

The illustrative example for the mutation from plant to warehouse for all period is.

**Chromosome X**

```
0 0 5 0 3
0 4 0 0 0
0 0 5 7 0
3 1 0 0 2
```

Select two rows (2,4) and three columns (2,3,5) randomly. The corresponding sub matrix and the reallocated sub matrix are

**Sub matrix Y**

```
4 0 0
1 0 2
```

**After Reallocation**

```
2 0 2
3 0 0
```

Then the Offspring after mutation is

**Offspring**

```
0 0 5 0 3
0 2 0 0 2
0 0 5 7 0
3 3 0 0 0
```

Similarly the chromosomes are mutated for warehouse to customer for all period.

The process is repeated with the mutated chromosome as an initial chromosome for the specified number of generations.

**Other Methodologies**

**Complete Enumeration**
Enumerative schemes have been considered in many shapes and sizes. The idea is fairly straightforward. Within a finite search space, or a discredited infinite search space, the search algorithm starts looking at objective function values at every point in the space, one at a time. Although the simplicity of this type of algorithms is attractive and enumeration is a very human kind of search (when the number of possibilities is small), such schemes must ultimately be discounted in the robustness race for one simple reason—lack of efficiency. Many practical spaces are too large to search one at a time and still have a chance of using the information to some practical end. Even the highly touted enumerative scheme—dynamic programming breaks down on problems of moderate size.

It is a method which is very time taking and elaborative. As this method involves one to go through all the various permutations and combinations it becomes virtually impossible to handle very large data. It is a very tedious as it is a manual process and the results obtained are near optimal and not necessarily the best.

**LINDO**

LINDO (Linear, INteractive, and Discrete Optimizer) is a software package tool and it is a convenient, but powerful tool for solving linear, integer, and quadratic programming problems. These problems occur in areas of business, industry, research and government. Specific applications areas where LINDO has proven to be of great use would include product distribution, ingredient blending, production and personnel scheduling, inventory management... The list could easily occupy the rest of this help file.

The guiding design philosophy for LINDO has been that, if a user wants to do something simple, then there should not be a large setup cost to learn the necessary features of LINDO. At the other extreme, LINDO has been used to solve real industrial linear, quadratic, and integer programs of respectable size. For commercial applications, LINDO is frequently used to solve problems with tens of thousands of constraints and hundreds of thousands of variables.

There are three basic styles of using the LINDO software. For small to medium sized problems, LINDO is simple to use interactively from the keyboard. Entering a
The model is quite easy to do. It's also possible to use LINDO with files created elsewhere, containing scripts of commands and input data, and producing files for reporting purposes. Finally, custom-created subroutines may be linked directly with LINDO to form solution containing both your code and the LINDO optimization libraries.

The LINDO software is designed to be simple to learn and to use. This is particularly true for small problems.

**C-Plex**

C-PLEX Interactive Optimizer is a command-line interactive program, provided in executable, ready-to-use form. It packs all the power and speed of CPLEX into an easy-to-use, easy-to-learn format, featuring a simple user interface and an extensive help system. New users become proficient and productive immediately. Just read in a problem, issue the "optimize" command and review results. ILOG CPLEX is designed to help solve corporate business problems, using the most sophisticated analytical techniques. Major companies and software providers in supply-chain planning, network design, transportation logistics, utilities and a variety of other industries depend on CPLEX in mission-critical resource allocation applications. CPLEX is at the core of countless cutting-edge commercial products worldwide.

**Illustrative example for GADIM**

The supply chain with 2 plants, 3 warehouses, and 3 customers for the two periods is taken into consideration. The production capacities for the two plants are 40 and 50. The inventory levels at the three warehouses are 30, 40 and 50. Total demands for the three customers for first period are 10, 20 and 30. For the second period are 20, 30 and 40. The production cost for the plant 1 and plant 2 are Rs 2 per unit and Rs 3 per unit respectively. The inventory carrying cost for the plant 1 and plant 2 are Rs 2 per unit per period and Rs 4 per unit period respectively. The inventory carrying cost for the
warehouse 1, warehouse 2 and warehouse 3 are Rs 3 per unit per period, Rs 4 per unit per period, Rs 5 per unit per period respectively. Inventory capacity in plant 1 and plant 2 are 15 units and 20 units respectively. The backordering cost is Rs 5 per unit.

The Transportation cost from plant to warehouse is

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

The Transportation cost from warehouse to customer is

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

This example problem is optimized using Genetic Algorithm by random generation of production rate in the plant, inventory level in the warehouse, unit of products transferred from plant to warehouse and then to customer with known demand for all periods.

Initially 10 chromosomes are randomly generated for the initialization process. The chromosomes are evaluated for roulette wheel selection process. And the selected chromosomes will undergo crossover and mutation operations. Finally mutated chromosomes are considered as a randomly initialized chromosome for the next generation. The process is repeated for 50 numbers (iteration or times) of generations.

**Results**

*Results Obtained Through GA*
The optimal solution for the illustrative example through the optimization by GA given below. Among the 50 iterations, at the end of 46\textsuperscript{th} Iteration nearer optimal result is achieved.

1. Optimal Production Rate

<table>
<thead>
<tr>
<th>Production Rate</th>
<th>Plant 1</th>
<th>Plant 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>17</td>
<td>47</td>
</tr>
<tr>
<td>Period 2</td>
<td>38</td>
<td>48</td>
</tr>
</tbody>
</table>

2. Optimal Warehouse Stock

<table>
<thead>
<tr>
<th>Warehouse stock</th>
<th>Warehouse 1</th>
<th>Warehouse 2</th>
<th>Warehouse 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>21</td>
<td>15</td>
<td>26</td>
</tr>
<tr>
<td>Period 2</td>
<td>20</td>
<td>20</td>
<td>50</td>
</tr>
</tbody>
</table>

3. Optimal Units transferred from plant to warehouse

<table>
<thead>
<tr>
<th>UTPW for Period 1</th>
<th>Warehouse 1</th>
<th>Warehouse 2</th>
<th>Warehouse 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant 1</td>
<td>0</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>Plant 2</td>
<td>21</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>UTPW for Period 2</td>
<td>Warehouse 1</td>
<td>Warehouse 2</td>
<td>Warehouse 3</td>
</tr>
<tr>
<td>-------------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
</tr>
<tr>
<td>Plant 1</td>
<td>11</td>
<td>1</td>
<td>28</td>
</tr>
<tr>
<td>Plant 2</td>
<td>8</td>
<td>19</td>
<td>21</td>
</tr>
</tbody>
</table>

4. Optimal Units transferred from warehouse to customer

<table>
<thead>
<tr>
<th>UTWC for period 1</th>
<th>Customer 1</th>
<th>Customer 2</th>
<th>Customer 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warehouse 1</td>
<td>9</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>Warehouse 2</td>
<td>0</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>Warehouse 3</td>
<td>1</td>
<td>4</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>UTWC for Period 2</th>
<th>Customer 1</th>
<th>Customer 2</th>
<th>Customer 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warehouse 1</td>
<td>7</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>Warehouse 2</td>
<td>0</td>
<td>14</td>
<td>6</td>
</tr>
<tr>
<td>Warehouse 3</td>
<td>13</td>
<td>14</td>
<td>23</td>
</tr>
</tbody>
</table>

5. Total Production Cost
6. Optimal Transportation cost (plant to warehouse to customer)

<table>
<thead>
<tr>
<th>Optimal Transportation cost</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant to Warehouse</td>
<td>270</td>
<td>350</td>
<td>620</td>
</tr>
<tr>
<td>Warehouse to Customer</td>
<td>335</td>
<td>560</td>
<td>895</td>
</tr>
<tr>
<td>Total</td>
<td>605</td>
<td>910</td>
<td>1515</td>
</tr>
</tbody>
</table>

7. Optimal Total cost

<table>
<thead>
<tr>
<th>Periods</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost</td>
<td>776</td>
<td>1146</td>
<td>1922</td>
</tr>
</tbody>
</table>

The following table gives the results obtained through C-plex and LINDO.

<table>
<thead>
<tr>
<th>Periods</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost</td>
<td>764</td>
<td>1153</td>
<td>1917</td>
</tr>
</tbody>
</table>

Optimality Test
<table>
<thead>
<tr>
<th>Model/Matrix</th>
<th>Complete enumeration / LINDO (a)</th>
<th>Genetic Algorithm (b)</th>
<th>b-a/a*100</th>
</tr>
</thead>
<tbody>
<tr>
<td>2<em>2</em>2</td>
<td>optimal</td>
<td>Near optimal</td>
<td>&lt;10%</td>
</tr>
<tr>
<td>2<em>2</em>3</td>
<td>optimal</td>
<td>Near optimal</td>
<td>&lt;10%</td>
</tr>
<tr>
<td>2<em>3</em>3</td>
<td>optimal</td>
<td>Near optimal</td>
<td>&lt;10%</td>
</tr>
<tr>
<td>2<em>3</em>4</td>
<td>optimal</td>
<td>Near optimal</td>
<td>&lt;10%</td>
</tr>
<tr>
<td>3<em>3</em>3</td>
<td>Not applicable</td>
<td>Optimal</td>
<td></td>
</tr>
<tr>
<td>3<em>3</em>4</td>
<td>Not applicable</td>
<td>Optimal</td>
<td></td>
</tr>
</tbody>
</table>

**Time Complexity**

<table>
<thead>
<tr>
<th>Model/Matrix</th>
<th>Complete enumeration / C-Plex (a)</th>
<th>Genetic Algorithm (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2<em>2</em>2</td>
<td>Min</td>
<td>less</td>
</tr>
<tr>
<td>2<em>2</em>3</td>
<td>Min</td>
<td>less</td>
</tr>
<tr>
<td>2<em>3</em>3</td>
<td>Average</td>
<td>less</td>
</tr>
<tr>
<td>2<em>3</em>4</td>
<td>High</td>
<td>less</td>
</tr>
<tr>
<td>3<em>3</em>3</td>
<td>High</td>
<td>less</td>
</tr>
<tr>
<td>3<em>3</em>4</td>
<td>Very high</td>
<td>less</td>
</tr>
</tbody>
</table>

The results obtained predict that Genetic algorithm is best suited for the real world application since it has nearest optimal solution and also less time consuming when compared to other existing tools. Moreover the tools like C-plex, LINDO cannot handle more than 150 constraints whereas Genetic Algorithm can handle very complex constraints.

Thus the optimization technique that has been evolved is more effective and the generalized tool developed provides flexible end user capabilities.

**Conclusions and Future Research Directions**
This Paper has presented a near optimal type inventory-logistics cost minimizing model for a production/distribution network with multiple plants supplying to a multiple warehouses, which in turn distributes to a large number of customers. The model was a synthesis of three components (1) the production rate at each plant for all periods (2) the inventory at each plant and warehouse for all periods. (3) route between entities where the product is transferred from plant to warehouse and then to customer for all periods. The decisions in the model were made through a comprehensive distribution-based cost framework that includes the inventory, transportation, and transit components of the supply chain.

The repeated trial runs on the test problem using GADIM show that the results obtained are encouraging. Early investigation shows that some modifications on the genetic search scheme such as using a newly modified crossover operator will further enhance its efficiency. One should see that the solution of Distribution Inventory is important for the supply chain industry. Thus our model demonstrates good ability in solving the Distribution Inventory problem can be applied to larger class of complex problem.

However, the model suffers from a number of limitations. Future research can address some of these issues:

• The model is limited to two-echelon. Extensions of the model need to be undertaken beyond two echelons.

• The model assumes a single product and stochastic demand for modeling simplicity. Better procedures need to be introduced to tackle the multi-product and probabilistic demand case.

• Time taken for transporting the product between entities has not taken into consideration.

References


Gunasekaran et al. (2004), A framework for supply chain performance measurement Int. J. Production Economics Volume 87, pp 333–347


